UNIVERSITE DE LYON

II. Supersymmetry

1401.6277

ISBN : 978-2-7298-6318-0

II.1. The Coleman-Mandula and Haag-Lopuszanski-Sohnius theorems

Supersymmetry automatically stems from Noether theorem and spin-statistics theorem.

We consider a theory with bosonic fields ϕ^a and fermionic field ψ^i.

The theory is invariant under a symmetry:

$$\phi^a \to \phi^a + \overbrace{(B^1_A)^a{}_b\,\phi^b}^{\delta_A \phi^a}$$

$$\psi^i \to \psi^i + \underbrace{(B^2_A)^i{}_j\,\psi^j}_{\delta_A \psi^i} \qquad (1)$$

$\Big\{$ B^1_A, B^2_A = symmetry generators acting on the bosonic/fermionic sectors respectively

The Lagrangian is invariant: $\delta_A \mathcal{L}$ is a total derivative:

$$\delta_A \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi^a}\,\delta_A \phi^a + \frac{\delta \mathcal{L}}{\partial \psi^i}\,\delta_A \psi^i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)}\,\delta_A(\partial_\mu \phi^a) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^i)}\,\delta_A(\partial_\mu \psi^i)$$

$$= \partial_\mu\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)}\,\delta_A \phi^a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^i)}\,\delta_A \psi^i\right]$$

(cf. Euler-Lagrange + integration by parts)

$$= \partial_\mu j^\mu_A$$

Noether theorem implies that the temporal $\underbrace{\text{component}}_{\hookrightarrow \text{ charge } B_A}$ of the current, integrated over space, is conserved:

$$B_A = -i \int d^3x \left[\underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi^a)}}_{\Pi_a} \delta_A \phi^a + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_0 \psi^i)}}_{\rho_i} \delta_A \psi^i \right]$$

$\equiv$ momentum densities conjugate to the fields

$$= -i \int d^3x \left[\Pi_a (B_A^1)^a{}_b \phi^b + \rho_i (B_A^2)^i{}_j \psi^j \right] \quad \text{cf. (1)}$$

The algebra of the B-charges can then be deduced:

$$[B_A, B_B] = -i \int d^3x \left[\Pi_a [B_A^1, B_B^1]^a{}_b \phi^b + \rho_i [B_A^2, B_B^2]^i{}_j \psi^j \right]$$

because
$$\left.\begin{aligned} [\phi^a(t,\vec{x}), \Pi_b(t,\vec{y})] &= i\delta^3(\vec{x}-\vec{y})\,\delta^a{}_b \\ \{\psi^i(t,\vec{x}), \rho_j(t,\vec{y})\} &= i\delta^3(\vec{x}-\vec{y})\,\delta^i{}_j \end{aligned}\right\} \text{spin-statistics}$$

(all other (anti) commutators vanish)

The symmetry algebra must close

$$\Rightarrow [B_A^1, B_B^1] = i f_{AB}{}^C B_C^1$$
$$[B_A^2, B_B^2] = i f_{AB}{}^C B_C^2$$

} the combination of two symmetry operations is by definition a symmetry

$\Rightarrow$ the $f_{AB}{}^C$ constant must be identical

so that $\boxed{[B_A, B_B] = i f_{AB}{}^C B_C}$

Moreover, the matrix product is associative $\Rightarrow$ Jacobi identities are satisfied

$\Rightarrow$ the B-charges fullfil a Lie algebra.

Coleman + Mandula $\rightarrow$ Phys. Rev. 159 (1967) 1251. proof that the corresponding symmetry algebra is always $\mathfrak{g} \equiv \underbrace{iso(1,3)}_{\text{Poincaré}} \times \underbrace{\mathfrak{g}_{int}}_{\text{compact Lie group}}$

$\Rightarrow$ Poincaré $\equiv$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\nu\sigma} M^{\rho\mu} - \eta^{\mu\sigma} M^{\rho\nu} + \eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\rho} M^{\nu\sigma})$$
$$[M^{\mu\nu}, P^\rho] = -i(\eta^{\nu\rho} P^\mu - \eta^{\mu\rho} P^\nu)$$
$$[P^\mu, P^\nu] = 0$$

$$[T_a, T_b] = i f_{ab}{}^c T_c \Big\} \equiv \mathfrak{g}_{int}$$

$\mathfrak{g}_{int}$ and $ISO(1,3)$ factorize

and: $[P^\mu, T_a] = [M^{\mu\nu}, T_a] = 0$

This theorem can be generalized by introducing fermionic symmetry operators changing the spins of the particles:

$$\phi^a \to \phi^a + (F_I^1)^a{}_i \psi^i$$

$$\psi^i \to \psi^i + (F_I^2)^i{}_b \phi^b$$

Noether and spin-statistics yield a Lie superalgebra:

$$[B_A, B_B] = i f_{AB}{}^C B_C$$

$$\{F_I, F_J\} = Q_{IJ}{}^C B_C \quad + \text{super-Jacobi identities}$$

$$[B_A, F_I] = i R_{AI}{}^J F_J$$

Nucl. Phys. B 88 (1975) 257.

Haag-Lopuszanski-Sohnius generalizes the Coleman-Mandula theorem

The algebra $g = g_0 \oplus g_1$ (g_0: bosonic, g_1: fermionic)

$g_0 = iso(1,3) \times g_{int}$ (Coleman-Mandula)

g_1 = operators non-trivially charged under g_0.

Simplest option for g_1: $N=1$ Majorana spinor: $g_1 \equiv \{Q_\alpha\} \oplus \{\bar{Q}^{\dot\alpha}\}$

($\alpha = 1,2$; $\dot\alpha = 1,2$ $\Rightarrow$ 2 component fermions)

$\hookrightarrow$ charged under $iso(1,3)$

One can easily derive the Poincaré superalgebra:

Poincaré
$$\left\{\begin{array}{l} [M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\nu\sigma}M^{\rho\mu} - \eta^{\mu\sigma}M^{\rho\nu} + \eta^{\nu\rho}M^{\mu\sigma} - \eta^{\mu\rho}M^{\nu\sigma}) \\ [M^{\mu\nu}, P^{\rho}] = i(\eta^{\nu\rho}P^{\mu} - \eta^{\mu\rho}P^{\nu}) \\ [P^{\mu}, P^{\nu}] = 0 \end{array}\right.$$

g_{int} $\quad [T_a, T_b] = i f_{ab}{}^{c} T_c$

Col.-Mand. $[T_a, P^{\mu}] = [T_a, M^{\mu\nu}] = 0$

g_1
$$\left\{\begin{array}{l} [Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \qquad [\bar{Q}^{\dot\alpha}, M^{\mu\nu}] = (\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}\bar{Q}^{\dot\beta} \\ \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^{\mu}{}_{\alpha\dot\alpha}P_\mu \\ [Q_\alpha, P_\mu] = [\bar{Q}_{\dot\alpha}, P_\mu] = \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \\ [T_a, Q_\alpha] = [T_a, \bar{Q}_{\dot\alpha}] = 0 \qquad (Q, \bar{Q} \text{ are gauge singlets}) \end{array}\right.$$

II.2 Representations of the Poincaré superalgebra

II.2.1 Generalities

Reminder: the representations of the Poincaré algebra are characterized by two Casimir operators:

$$C_2 = P^{\mu}P_{\mu} \qquad \equiv \text{mass}$$

$$C_4 = W^{\mu}W_{\mu} \text{ with } W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^{\nu}M^{\rho\sigma} \qquad \equiv \text{spin}$$

$\tilde{W}_\mu = W_\mu - \frac{1}{4}\bar{Q}\bar{\sigma}_\mu Q$

C_4 does not commute with the $(Q, \bar{Q})$ changes $\Rightarrow$ a supermultiplet contains fields of different spins. C_4 has to be generalized $\to$ $C_4 = \tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu}$ with $\tilde{W}_{\mu\nu} = \tilde{W}_\mu P_\nu - \tilde{W}_\nu P_\mu$

Moreover, the fermion number operator $(-1)^N$ has a vanishing trace for a given multiplet:

$\hookrightarrow \begin{cases} +1 \text{ for bosons} \\ -1 \text{ for fermions} \end{cases}$

$$(-1)^N Q_\alpha = - Q_\alpha (-1)^N \quad (1) \quad (Q_\alpha \text{ is a fermion})$$

$$\rightarrow Tr\left[(-1)^N \{Q_\alpha, \bar{Q}_{\dot\alpha}\}\right] = Tr\left[(-1)^N Q_\alpha \bar{Q}_{\dot\alpha} + (-1)^N \bar{Q}_{\dot\alpha} Q_\alpha\right] \quad \text{cf. } (1)$$

$$= Tr\left[- Q_\alpha (-1)^N \bar{Q}_{\dot\alpha} + (-1)^N \bar{Q}_{\dot\alpha} Q_\alpha\right] = 0 \quad (\text{cyclicity})$$

$$= Tr\left[(-1)^N 2\sigma^\mu{}_{\alpha\dot\alpha} P_\mu\right]$$

$$\Rightarrow Tr\left[(-1)^N\right] = 0$$

$\Rightarrow$ one multiplet contains the same number of bosonic and fermionic dofs

II.2.2. Massless representation

We use the standard frame for the particle momentum: $p^\mu = (E, 0, 0, E)$

Under a Lorentz transformation: $p^\mu \rightarrow p'^\mu = \left(e^{-\frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}}\right)^\mu{}_\sigma \, p^\sigma$

We need to determine the little group $\rightarrow$ the transformations so that $p^\mu \rightarrow p'^\mu = p^\mu$.

$\Rightarrow$ rotations around the z-axis $\Rightarrow$ the generator is $M = M^{12}$.

In principle, translations must also be accounted for: $\begin{cases} T^1 = M^{10} - M^{13} \\ T^2 = M^{20} - M^{23} \end{cases}$

$\rightarrow$ the little group is $ISO(2)$ (rotations + translations in 2D)

We however set the eigenvalues to 0 (we do not want any continuous degree of freedom).

Without supersymmetry, C_2 and C_4 are the two Casimir $\Rightarrow$ a state is thus characterized by

$$|0, 0; p^\mu, \lambda\rangle$$

C_2 $\quad$ C_4 $\quad$ momentum $\quad$ eigenvalue of Π $\equiv$ helicity

Let us work out the little algebra in supersymmetry

$$[\Pi, Q_1] = \frac{1}{2} Q_1 \qquad [\Pi, \bar{Q}_{\dot{1}}] = -\frac{1}{2}\bar{Q}_{\dot{1}}$$

$$[\Pi, Q_2] = -\frac{1}{2} Q_2 \qquad [\Pi, \bar{Q}_{\dot{2}}] = -\frac{1}{2}\bar{Q}_{\dot{2}} \qquad \{Q_i, \bar{Q}_{\dot{j}}\} = 4E\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (1)$$

a vacuum state annihilated by Q_2

$$\Rightarrow 0 \overset{(1)}{=} \langle\Omega|\{Q_2, \bar{Q}_{\dot{2}}\}|\Omega\rangle = \langle\Omega| Q_2 \bar{Q}_{\dot{2}}|\Omega\rangle = \| \bar{Q}_{\dot{2}}|\Omega\rangle \|^2$$

$$\Rightarrow Q_2 = \bar{Q}_{\dot{2}} = 0$$ (unitarity) $\Rightarrow$ only 2 active supercharges Q_1 and $\bar{Q}_{\dot{1}}$

We define $a^\dagger = \frac{1}{2\sqrt{E}}\bar{Q}_{\dot{1}}$ and $a = \frac{1}{2\sqrt{E}} Q_1 \Rightarrow \begin{cases} \{a, a\} = \{a^\dagger, a^\dagger\} = 0 \\ \{a, a^\dagger\} = 1 \end{cases}$

$\Rightarrow$ standard annihilation and creation operators.

What do they do?

$$\circ \; M\,a\,|0,0;p^\mu,\lambda\rangle = \big(aM + \underbrace{[M,a]}_{=1/2\,a \text{ cf.}(1)}\big)\,|0,0;p^\mu,\lambda\rangle = (\lambda+\tfrac{1}{2})\,a\,|0,0;p^\mu,\lambda\rangle$$

$$\circ \; M a^\dagger|0,0;p^\mu,\lambda\rangle = (\lambda-\tfrac{1}{2})\,a^\dagger\,|0,0;p^\mu,\lambda\rangle$$

$\Rightarrow$ a and $a^\dagger$ changes the helicity by half a unit.

We can define the content of a supermultiplet by starting from a vacuum state of helicity λ ($|\Omega_\lambda\rangle$) corresponding to the state with the highest multiplicity $\Rightarrow$ $\cdot\; a|\Omega_\lambda\rangle = 0$ (we cannot raise the helicity)

$\cdot\; a^\dagger|\Omega_\lambda\rangle \equiv$ state with a helicity $\lambda-\frac{1}{2}$

$\cdot\; a^\dagger a^\dagger|\Omega_\lambda\rangle = 0$

$\rightarrow$ one supermultiplet contains 2 states:

Matter supermultiplet

Helicity	$\lvert\Omega_{1/2}\rangle$	$\lvert\Omega_{-1/2}\rangle$
1/2	1	
0	1	1
-1/2		1
	# of states	

Gauge supermultiplet

Helicity	$\lvert\Omega_1\rangle$	$\lvert\Omega_{-1}\rangle$
1	1	
1/2	1	
0		
-1/2		1
-1		1

We need to double the states because of CPT

$$\boxed{\begin{array}{l}\text{Matter : 1 complex scalar + 1 Weyl fermion}\\ \text{Gauge : 1 real vector + 1 Majorana spinor}\end{array}}$$

(Nb: gravitation multiplet : 1 spin-two field + 1 Rarita-Schwinger field)
Rk: off-shell Weyl = 4 dof ⇒ F terms added to matter supermultiplets (complex scalar)
. D-term (real scalar) added for the gauge supermultiplets for the same reason

II.2.3 Massive representations

In the standard frame, $p^\mu = (m, 0, 0, 0)$ → the little algebra is $so(3)$.
Without supersymmetry:

$C_2 = m^2$ and $C_4 = m^2 M^2$ ⇒ a state is labelled as $|m, j; p^\mu, j_3\rangle$

M^2 → eigenvalue : $j(j+1)$; j_3 → eigenvalue of M'^2.

With supersymmetry : $C_4 \to -2m^4 Y^2$ with $Y_i = M_i - \frac{1}{4m} \bar{Q} \bar{\sigma}_i Q$ $(M_i = M_{jk})$

Y = superspin (eigenvalue is $y(y+1)$)

A supersymmetric state is characterized by : $|m, y\rangle$. We determine the particle content with creation and annihilation operators:

$a^\dagger_{1,2} = \frac{1}{\sqrt{2m}} \bar{Q}_{1,2}$ $\quad a_{1,2} = \frac{1}{\sqrt{2m}} Q_{1,2}$ so that $\begin{cases}\{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0\\ \{a_i, a_j^\dagger\} = \delta_{ij}\end{cases}$

We start with the vacuum state $|\Omega\rangle = |m, j ; p^\mu, j_3\rangle$ and act on it with $a^\dagger_{1,2}$ (we assume it is annihilated by $a_{1,2}$).

We can show that $M_3\, a^\dagger_{1,2}|\Omega\rangle = (j_3 - \frac{1}{2})\, a^\dagger_{1,2}|\Omega\rangle$

$$M_3\, a_1^\dagger a_2^\dagger |\Omega\rangle = (j_3 - 1)\, a_1^\dagger a_2^\dagger |\Omega\rangle$$

Matter multiplet : we start from $|\Omega\rangle$ of spin 1/2 $\rightarrow$ 1 Majorana fermion
1 complex scalar

Gauge supermultiplet : $|\Omega\rangle$ of spin 1 $\rightarrow$ two spin-0
2 pairs of spin 1/2
1 spin-1
} doubling because of CPT.

II.3 Supersymmetric Lagrangians

II.3.1 Matter sector (Wess-Zumino model)

kin. term for the scalar

· Matter supermultiplets : $\mathcal{L} = \partial_\mu \phi_i^\dagger \partial^\mu \phi^i + \frac{i}{2}\left(\psi_i \sigma^\mu \partial_\mu \bar{\psi}_i - \partial_\mu \psi_i \sigma^\mu \bar{\psi}_i\right)$
$(i = 1, 2, \ldots, n)$

$+ F_i^\dagger F^i$

auxiliary F-terms (vanish on-shell)
(Eq of motion : $F = 0$.)

kin terms for the Weyl fermion

The interaction are obtained by adding all terms allowed by supersymmetry

$$\begin{cases} \phi \to \phi + \delta_\varepsilon \phi = \phi + \sqrt{2}\, \varepsilon . \psi \\ \psi \to \psi + \delta_\varepsilon \psi = \psi - i\sqrt{2}\, \sigma^\mu \bar{\varepsilon}\, \partial_\mu \phi - \sqrt{2} F \varepsilon \\ F \to F + \delta_\varepsilon F = F - i\sqrt{2}\, \partial_\mu \psi \sigma^\mu \bar{\varepsilon} \end{cases}$$

(ε = fermion = parameter of the transformation)

Let $W(\phi)$ be a holomorphic function of the scalar fields ϕ^i

$$\mathcal{L}_{int} = -W_i F^i - W^{*i} F_i^\dagger - \frac{1}{2} W_{ij}\, \psi^i . \psi^j - \frac{1}{2} W^{*ij}\, \bar{\psi}_i . \bar{\psi}_j$$

$$W_i = \frac{\partial W}{\partial \phi^i} \; ; \; W_{ij} = \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \qquad (W^* = \text{complex conjugate})$$

The equations of motion for the F-terms are now: $F^i = W^{*i}$

$$\Rightarrow \mathcal{L}_F = F_i^\dagger F^i - W_i F^i - W^{*i} F_i^\dagger = -W_i W^{*i} = -F_i^\dagger F^i$$

$\Rightarrow$ this gives the F-term contribution to the scalar potential: $\boxed{V_F = W^{*i} W_i}$

II.3.2 Gauge sector

Let G be one factor of the theory gauge group.

$$\mathcal{L}_{gauge} = \underbrace{-\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}}_{\text{vector Lagr.}} + \underbrace{\frac{i}{2}\left(\lambda_a \sigma^\mu D_\mu \bar{\lambda}^a - D_\mu \lambda_a \sigma^\mu \bar{\lambda}^a\right)}_{\text{gaugino Lagr.}} + \underbrace{\frac{1}{2} D^a D_a}_{\substack{\text{auxiliary D term} \\ \text{(vanish on-shell)}}}$$

This lagrangian is SUSY-invariant:

$$A_\mu \to A_\mu + \delta_\varepsilon A_\mu = A_\mu + i(\varepsilon\sigma_\mu\bar\lambda - \lambda\sigma_\mu\bar\varepsilon)$$

$$\lambda \to \lambda + \delta_\varepsilon\lambda = \lambda + iD\varepsilon + \frac{1}{2}\sigma^\mu\bar\sigma^\nu\varepsilon F_{\mu\nu}$$

$$D \to D + \delta_\varepsilon D = D + \varepsilon\sigma^\mu\partial_\mu\bar\lambda + \partial_\mu\lambda\sigma^\mu\bar\varepsilon$$

This lagrangian is invariant under a gauge transformation too:

$$\begin{cases} A_\mu \to A_\mu + \partial_\mu\chi \\ \lambda \to \lambda \\ D \to D \end{cases}$$

We still need to make the matter sector lagrangian gauge invariant.
→ covariantization:

$$\begin{aligned}\mathcal{L}_{SUSY} = {} & D_\mu\phi_i^\dagger D_\mu\phi^i + \frac{i}{2}\left(\psi^i\sigma^\mu D_\mu\bar\psi_i - D_\mu\psi^i\sigma^\mu\bar\psi_i\right) + F_i^\dagger F^i \\ & - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \frac{i}{2}\left(\lambda_a\sigma^\mu D_\mu\bar\lambda^a - D_\mu\lambda_a\sigma^\mu\bar\lambda^a\right) + \frac{1}{2}D^a D_a \\ & \underbrace{+ i\sqrt{2}g\,\bar\lambda^a\cdot\bar\psi_i(T_a\phi)^i - i\sqrt{2}g\,\phi_i^\dagger(T_a\psi)^i\cdot\lambda^a - g D^a\phi_i^\dagger(T_a\phi)^i}_{\text{necessary for the SUSY invariance}}\end{aligned}$$

(g = coupling constant; T_a = rep. matrices of G)

This lagrangian is invariant under a SUSY-gauge transformation

$$\begin{cases} \delta\phi = \sqrt{2}\,\varepsilon\cdot\psi \\ \delta\psi = -i\sqrt{2}\,\sigma^\mu\bar{\varepsilon}\, D_\mu\phi - \sqrt{2}\,\varepsilon F \\ \delta F = -i\sqrt{2}\, D_\mu\psi\sigma^\mu\bar{\varepsilon} + 2ig\,\bar{\varepsilon}\cdot\bar{\lambda}\,\phi \end{cases} \qquad \begin{cases} \delta A_\mu = i(\varepsilon\sigma_\mu\bar{\lambda} - \lambda\sigma_\mu\bar{\varepsilon}) \\ \delta\lambda = iD\varepsilon + \frac{1}{2}\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu} \\ \delta D = \varepsilon\sigma^\mu D_\mu\bar{\lambda} + D_\mu\lambda\sigma^\mu\bar{\varepsilon} \end{cases}$$

The covariant derivative now appear in the transformation laws.

The eqs of motion for D are now: $D^a = g\,\phi_i^\dagger (T^a\phi)^i$

$\Rightarrow$ the scalar potential is $\boxed{V = \frac{1}{2} D_a D^a + F^i F_i^\dagger}$

II. 4 Supersymmetry breaking

For a given supermultiplet, the mass is given (P^2 is a Casimir)

$\Rightarrow$ a particle and its superpartner have the same mass

$\Rightarrow$ excluded experimentally (no selectron at 511 keV, ...)

$\Rightarrow$ SUSY is a broken symmetry

1) The breaking has to be soft to keep the hierarchy problem fixed

2) We have a nice tool: spontaneous symmetry breaking.

IV.4.1 The Goldstone theorem for SUSY

· Spontaneous SUSY breaking: $|\Omega\rangle$ is the vacuum state and $Q_\alpha|\Omega\rangle \neq 0$ (and $\bar{Q}_{\dot\alpha}|\Omega\rangle \neq 0$) $\Rightarrow$ the vacuum state is not supersymmetric.

· $$\langle\Omega|E|\Omega\rangle = \langle\Omega|P^0|\Omega\rangle \underset{(\text{as } \{Q,\bar{Q}\}\,\propto\, P)}{=} \frac{1}{4}\Big(\|Q_1|\Omega\rangle\| + \|Q_1^\dagger|\Omega\rangle\| + \|Q_2|\Omega\rangle\| + \|Q_2^\dagger|\Omega\rangle\|\Big) \neq 0$$

$\Rightarrow$ the energy is a sum of 4 norms $\Rightarrow$ it is positive

$$\Rightarrow \langle V\rangle = \langle F^i F_i^\dagger + \tfrac{1}{2} D^a D_a\rangle > 0$$

$\Rightarrow$ SUSY can be broken by F-terms or D-terms.

· At the minimum: $$\left\langle \frac{\partial V}{\partial \phi_j}\right\rangle = \left\langle F^i \underbrace{\frac{\partial F_i^\dagger}{\partial \phi_j}}_{=W_{ij}} + D^a \underbrace{\frac{\partial D_a}{\partial \phi_j}}_{=g(\phi^\dagger T_a)_j} \right\rangle = 0$$

↳ looks like a gauge transf.

$$\boxed{\begin{pmatrix} \langle W_{ij}\rangle & g\langle(\phi^\dagger T_a)_j\rangle \\ g\langle(\phi^\dagger T_a)_i\rangle & 0 \end{pmatrix}\begin{pmatrix}\langle F^i\rangle \\ \langle D_a\rangle\end{pmatrix} = 0} \Leftarrow \quad \delta_\omega W^* = W^{*i}\,\delta_\omega \phi_i^\dagger = F^i\big(i g\, \omega^a (\phi^\dagger T_a)_i\big) = 0$$

(ω: gauge transf. param.)

characterization of the vacuum state.

Laboratoire d'Excellence LIO - *Institut des Origines de Lyon*

Contact - Bruno Guiderdoni, responsable scientifique et technique
9 avenue Charles André, 69230 Saint Genis Laval / bruno.guiderdoni@univ-lyon1.fr

- We can calculate the fermion mass matrix at the minimum:

$$M = \begin{pmatrix} \langle W_{ij} \rangle & g \langle (\phi^+ T_b)_j \rangle \\ g \langle (\phi^+ T_a)_i & 0 \end{pmatrix} \quad (\text{in the } (\psi^i, \sqrt{2}\, i \lambda^b) \text{ basis})$$

$\Rightarrow |\psi_G\rangle = \frac{\sqrt{2}}{2} \langle F_i^+ \rangle |\psi^i\rangle + \frac{i}{2} \langle D^a \rangle |\lambda_a\rangle$ is massless

It is the Goldstino $\Rightarrow$ SUSY version of the Goldstone theorem.

- The interaction of the goldstino can be derived from the conservation of the Noether current

IV.4.2. Constraints on SUSY-breaking

- SUSY-breaking via D-terms or F-terms is not satisfactory (yields phenomenologically not acceptable light states).

- We can derive a rule from the trace of all mass matrices:

$$\mathrm{STr}[M^2] = \sum_{l=0,1/2,1} (-)^{2l}(2l+1)\,\mathrm{Tr}[M_l^2] = 2g^2 \langle \phi^+ T_a \phi \rangle \mathrm{Tr}[T_a]$$

supertrace

M_0 = scalar mass matrix
$M_{1/2}$ = fermion " "
M_1 = vector " "

$\Rightarrow$ hard to accomodate after pluging in all SM masses ($\to$ very light states)

- Solution: SUSY-breaking via non-renormalizable interactions or radiatively

  ▷ SUSY is broken in a hidden sector that has no or reduced coupling to the visible sector

  ▷ Gravity-mediated supersymmetry breaking, gauge-mediated SUSY breaking, etc...

They all in general yield the following soft terms:

- gaugino mass terms
- scalar mass terms
- multiscalar interactions of the form of the superpotential

IV.5. The Minimal Supersymmetric Standard Model

IV.5.1 Particle content and Lagrangian

- Simplest SUSY model → direct supersymmetrization of the SM

  ▷ All SM fermions are included in matter supermultiplets

  ↳ sfermions: squarks, sleptons, sneutrinos

  ▷ All SM gauge bosons are included in gauge supermultiplets

  ↳ gauginos: bino, winos, gluino

SM superpartner $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \quad = (3, 2, \tfrac{1}{6})$$

$$U_R = u_R^c \quad \tilde{u}_R^{+} \quad = (3, 1, -\tfrac{2}{3})$$

$$D_R = d_R^c \quad \tilde{d}_R^{+} \quad = (3, 1, \tfrac{1}{3})$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix} \quad = (1, 2, -\tfrac{1}{2})$$

$$E_R = e_R^c \quad \tilde{e}_R^{+} \quad = (1, 1, 1)$$

$$V_B = (B_\mu, \tilde{B}) \ (1,1,0)$$

$$+ \quad V_W = (W_\mu^i, \tilde{W}^i) \ (1,3,0)$$

$$V_G = (G_\mu^a, \tilde{g}_\mu^a) \ (8,1,0)$$

Matter supermultiplets are left-handed → conjugate right-handed fields

The Higgs sector needs two doublets ⇒ mass terms for up-type and down type states in the superpotential + anomaly cancellation

$$H_U = \begin{pmatrix} H_U^+ \\ H_U^0 \end{pmatrix} \quad \begin{pmatrix} \tilde{H}_U^+ \\ \tilde{H}_U^0 \end{pmatrix} \quad (1, 2, \tfrac{1}{2})$$

$$H_D = \begin{pmatrix} H_D^0 \\ H_D^- \end{pmatrix} \quad \begin{pmatrix} \tilde{H}_D^0 \\ \tilde{H}_D^- \end{pmatrix} \quad (1, 2, -\tfrac{1}{2})$$

$$W = \tilde{u}_R^{+} y^u \tilde{Q}_L \cdot H_U - \tilde{d}_R^{+} y^d \tilde{Q}_L \cdot H_D - \tilde{e}_R^{+} y^e \tilde{L}_L \cdot H_D + \mu H_U \cdot H_D$$

(in flavor space)

We ignored B and L violating terms: $\frac{1}{2}\lambda\,\tilde{L}_L\tilde{L}_L\,\tilde{e}_R^{+} + \lambda'\,\tilde{L}_L\cdot\tilde{Q}_L\cdot\tilde{d}_R^{+} + \frac{1}{2}\lambda''\,\tilde{u}_R^{+}\tilde{d}_R^{+}\tilde{d}_R^{+}$
$- \kappa\,\tilde{L}_L\cdot\tilde{H}_U$ induce proton decay

→ R-parity conservation: $R = (-1)^{3B+L+2S}$

⇒ SUSY particles are pair-produced at colliders
⇒ the lightest SUSY particle is stable ⇒ dark matter candidate if neutral and non-colored

⇒ B and L are conserved

- After SUSY and EW-symmetry breaking, all particle with the same color representation, electric charge and spin mix:
  - $H_U^0, H_D^0 \to h, H, a + G^0$ eaten by the Z-boson
  - $H_U^+, H_D^- \to H^{\pm} + G^{\pm}$ eaten by the $W^{\pm}$-boson
  - $\tilde{H}_U^0, \tilde{H}_D^0, \tilde{W}_3, \tilde{B} \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$ (neutralinos)
  - $\tilde{H}_U^+, \tilde{H}_D^-, \tilde{W}_3^{\pm} \to \tilde{\chi}_1^{\pm}, \tilde{\chi}_2^{\pm}$ (charginos)
  - Sfermion mixing $(\tilde{f}_L, \tilde{f}_R) \to (\tilde{f}_1, \tilde{f}_2)$ (in principle, could be flavor violating → 6×6 mixings; in practice, only relevant if the mass of the corresponding SM particle is large:
    $\tilde{f}_R^{+}\, T\, \tilde{F}_L\cdot H \sim m_f A_f\, \tilde{f}_R^{+}\tilde{f}_L$ in concrete soft SUSY breaking models

LIO
UNIVERSITE DE LYON

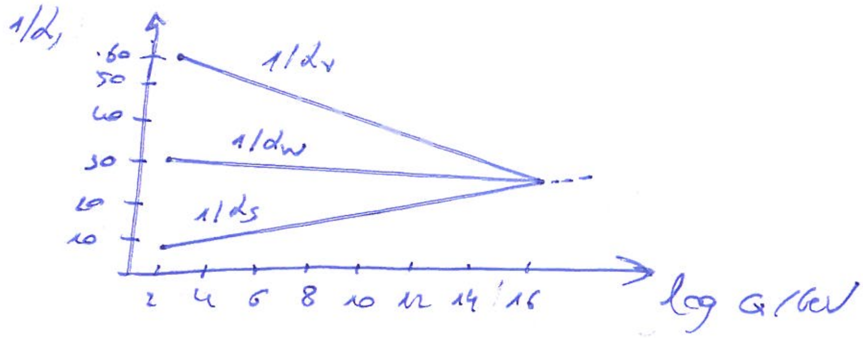
II.5.2. Phenomenological consequences

- The hierarchy problem is fixed:

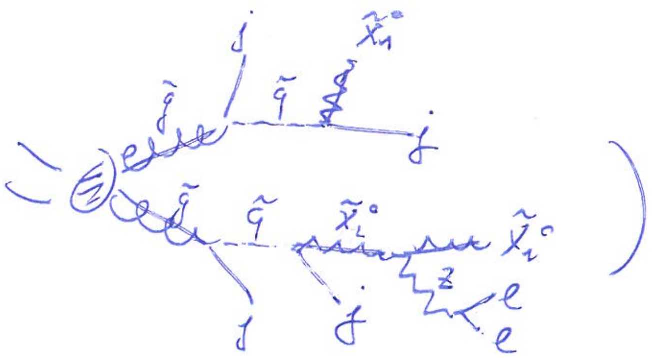
$\longrightarrow \delta m_R^2 \sim m_{\tilde g}^2 \log \frac{m_{\tilde g}^2}{\Lambda_{UV}^2}$

we expect TeV-scale SUSY

- The gauge coupling RGEs are modified $\to$ unification at about 10^{16} GeV.



- R-parity + colorless and neutral LSP $\longrightarrow$. Dark matter candidate
  . Missing energy at colliders
  ↳ typical SUSY signal



$\not E_T$ + jets + leptons