

III. Strong dynamics 1506. 01961

III.1 Dimensional transmutation

- Another popular attempt to solve the hierarchy problem is to make the Higgs composite with a finite size R_H .
 - \Rightarrow The Higgs is now a bound state of a new strong force, with a confinement scale $m_* = 1/R_H \sim 1 \text{ TeV}$.
- The hierarchy problem is solved because
 - the Higgs behaves as elementary at low energy \rightarrow quadratic grows with E .
 - At $E \sim m_*$, we probe the internal structure of the Higgs and the finite size effects take over. The growing behavior with E is replaced by a peak followed by a steep fall.
- \Rightarrow no Higgs particle should be present in the theory for $E > m_*$.
- (As for QCD when we move from the quark to the baryon/pion picture)
- The composite sector must however not generate a new hierarchy problem when one sets m_* at the TeV scale, in particular as it emerges at a high scale $\Lambda_{UV} \gg 1 \text{ TeV}$.

⇒ The composite sector should not include operators with scaling dimension well below 4 (for instance, $\mu^2 \phi^\dagger \phi$ has $d=2 \rightarrow$ BAD) \equiv no unprotected scale.

i.e. no parameters with large positive dimensionality

⇒ one known example: QCD @ low-energy (no weak bosons + no heavy quark)

The only dimensional parameters are the quark masses \Rightarrow protected by the chiral symmetry \rightarrow act like if $d=4$.

Reminder: QCD with N_f massless quarks:

$$\mathcal{L}_0 = \sum_f \bar{q}_f i \not{D} q_f - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$

The Lagrangian is invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_L \times U(1)_R$
} large global symmetry

This symmetry is however broken spontaneously:

• $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ (the axial part is broken)

$N_f = 3 \rightarrow 8$ NG bosons (massless): $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$

But quarks are massive $\Rightarrow 8$ PNGB bosons that are much lighter than any other states -

→ $\langle \bar{L}_L L_R + \bar{R}_R L_L \rangle \propto \Lambda_{\text{QCD}}^3$

$$D U(1)_L \times U(1)_R = U(1)_V \times U(1)_A$$

\uparrow \uparrow
 baryon number net axial symmetry

per center
 (Adler-Bell-Jackiw anomaly)
 cannot be compensated even by a counterterm.

Chiral perturbation theory: $\Delta m < \Lambda_{QCD} \rightarrow$ mass-difference effects can be treated perturbatively.

In QCD, the confinement scale is given by

$$\log \frac{\Lambda_{QCD}}{m^*} = \frac{1}{18} \frac{16\pi^2}{g_s^2(\mu_Z)} \quad (RGE) \quad (\Lambda_{UV} = \mu_Z)$$

A scale m^* is generated although there is no dimensionful parameter in the Lagrangian \equiv dimensional transmutation.

In the composite case, we would have

$$\boxed{\log \frac{\Lambda_{UV}}{m^*} = \frac{8\pi^2}{b_1 g^2(\Lambda_{UV})}}$$

\rightarrow we can dynamically generate the confinement scale m^* from an asymptotically free theory in which $g^2(\Lambda_{UV})$ is small.

The theory we have contains thus 2 sectors:

- 1) The elementary sector with all SM particles except the Higgs
 - ↳ weakly coupled gauge theory
 - ↳ no Yukawa terms (as no Higgs)

2) The composite sector

↳ need to respect the SM gauge symmetries because the SM bosons are (phenomenologically) elementary

↳ Exact symmetry group $\mathcal{G} \supset SU(2) \times U(1)$

↳ difference with QCD (can be alleviated if needed)

gauged as usual

↳ EW communication between the 2 sectors

Analogy with QCD: 1) leptons + photons

2) Quarks ; $\mathcal{G} = SU(3)_L \times SU(3)_R \supset U(1)_{em}$.

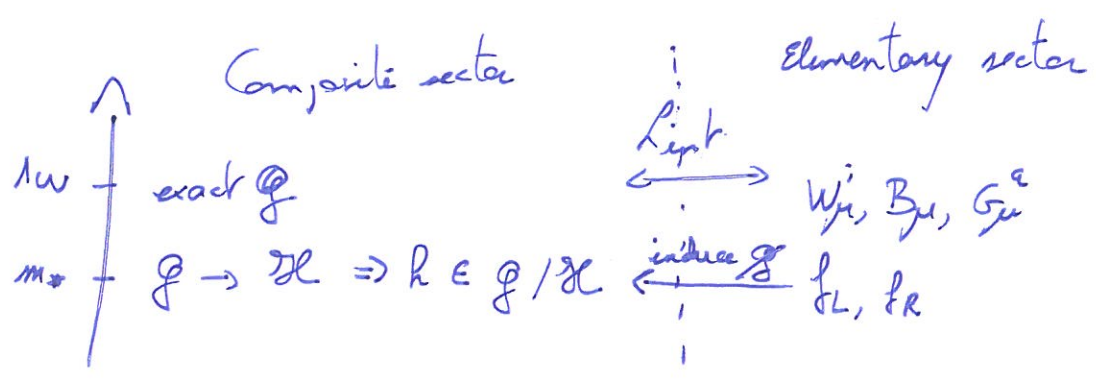
but here, \mathcal{G} is broken by the quark masses

→ not the case for composite BSM.

• At a scale m^* , $\mathcal{G} \rightarrow \mathcal{H} \Rightarrow$ massless NG bosons in the \mathcal{G}/\mathcal{H} coset.
 → the Higgs is one of them, acquires a mass + potential → breaks the EW symmetry

The symmetry group \mathcal{G} is explicitly broken by the elementary sector
 \Rightarrow the Higgs is in fact a PNG boson.

\Rightarrow This trick makes the model phenomenologically viable: the Higgs is not a bound state emerging from the composite sector $\rightarrow m_H$ can be in the TeV / multi-TeV range. \Rightarrow lighter PNG bosons + heavier resonances



The breaking is small \rightarrow weak perturbation (as in QCD where the weak interactions are a small perturbation of the strong dynamics). This also explains why h is light and acts as an elementary particle. \rightarrow tuning.

IV.2 Vacuum misalignment

- We have a composite sector symmetry \mathcal{G} , and a vacuum state invariant under $\mathcal{H} \subset \mathcal{G}$.
- The spontaneous breaking $\mathcal{G} \rightarrow \mathcal{H}$ induces the appearance of NG bosons in \mathcal{G}/\mathcal{H} .
- \mathcal{H} contains $SU(2)_L \times U(1)_Y \equiv G_{EW}$.
- \mathcal{G} has to be large enough so that at least one Higgs doublet is present in the coset.

• Notation: we consider a vacuum \vec{F} and a set of generators of $\mathcal{G} \equiv \{T_a, \hat{T}_a\}$ so that $T_a \vec{F} = 0$ and $\hat{T}_a \vec{F} \neq 0$

local transformations in the \hat{T}_a direction \uparrow
 $\equiv \mathcal{H}$ broken \uparrow
 \Downarrow contains the G_{EW} generators $\equiv \mathcal{G}/\mathcal{H}$

The NG bosons $\hat{\theta}_a$ are: $\vec{\phi}(x) = e^{i\hat{\theta}_a(x)\hat{T}_a} \vec{F}$

\rightarrow we have at least 4 of them (\equiv 1 Higgs doublet)

- They develop vacuum expectation values $\langle \hat{\theta} \rangle \neq 0$ that breaks G_{EW} .

example:



\mathcal{H}

$\mathcal{G} = SO(3)$
 $\mathcal{H} = SO(2)$

$\|\vec{F}\| = \text{scale of the } \mathcal{G} \rightarrow \mathcal{H} \text{ breaking}$
 $v = f \sin \theta$
 \uparrow
vev breaking $SO(2)$

$\langle \hat{\theta} \rangle$ is derived by minimization of the PN & B potential

• We need a small breaking:

$$\xi \equiv \frac{v^2}{f^2} = \sin^2(\theta) \ll 1.$$

The SM limit is retrieved for $f \rightarrow \infty$ ($\xi \rightarrow 0$): the composite scale is sent to ∞ . \equiv decoupling of the composite sector.

- $\xi \ll 1$ may sound unnatural (ξ should be α_1)
 - $\xi \sim 0.1 \rightarrow$ reasonable fine-tuning
 - Built-in mechanism (e.g. little-Higgs program where gauge couplings are enlarged \rightarrow hard to get $m_H \sim 125$ GeV)

III.5 A minimal example

- The simplest option: $G = SO(5)$ } $G/\mathcal{H} \rightarrow 4$ PNGB bosons
 $\mathcal{H} = SO(4)$
- Fundamental representation of $SO(5)$: 10 antisymmetric real 5×5 matrices
 $SO(4)$: 6 " " " 4×4 matrices
 ($\hookrightarrow SO(4) \sim SU(2)_L \times SU(2)_R$ and we identify $(\mathbf{3}_R \sim \mathbf{Y})$
 \rightarrow $\mathbf{6}_{2L}$ is included)
- We now consider the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{g_*^2}{8} (\vec{\phi} \cdot \vec{\phi} - f^2)^2$$

$\vec{\phi}$ is here an $S(5)$ fiveplet and the theory is invariant under global $SU(5)$ transformations:

$$\vec{\phi} \rightarrow e^{i\alpha_A T^A} \vec{\phi} \quad \text{with } A=1,2,\dots,5$$

Here, the $SO(5)$ generators in the fundamental are:

$$T^a = \begin{bmatrix} t_L^a & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} t_R^a & 0 \\ 0 & 0 \end{bmatrix} \quad \text{with } (t_L^a)_{ij} = \frac{1}{4} \text{Tr}[\bar{\sigma}_i^\dagger \sigma^a \bar{\sigma}_j]$$

$$(t_R^a)_{ij} = \frac{1}{4} \text{Tr}[\bar{\sigma}_i \sigma^a \bar{\sigma}_j^\dagger]$$

where $\bar{\sigma}_i = \{i\sigma_a, 1\}$

$$(T^{\hat{a}})_{ij} = -\frac{i}{\sqrt{2}} (\delta_i^{\hat{a}} \delta_j^{\hat{b}} - \delta_j^{\hat{a}} \delta_i^{\hat{b}})$$

6 $SO(4)$ generators
4 broken generators

$\vec{\phi}$ acquires a vev $\langle \vec{\phi} \rangle$: $S(5) \rightarrow SO(4)$

vacuum configuration: $\vec{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f \end{pmatrix}$

We will parameterize $\vec{\phi}$ around the vacuum:

renormalization to get canonical dim. terms

4 NG bosons

$$\hat{\phi}(x) = e^{i\frac{\vec{r}}{f} \cdot \frac{1}{\sqrt{2}} \hat{a}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f + \sigma(x) \end{pmatrix}$$

we in fact have: one "radial" coordinate $\sigma(x)$
four "angular" coordinates $\frac{\vec{r}}{f}(x)$

describes fluctuations (regenerate field) around the broken generators

4x4 vector

$$\Rightarrow \hat{\phi}(x) = \begin{pmatrix} \mathbb{1}_{4 \times 4} - (1 - \cos \frac{\pi}{f}) \frac{\vec{\pi} \vec{\pi}^T}{\pi^2} & \sin \frac{\pi}{f} \frac{\vec{\pi}}{\pi} \\ -\sin \frac{\pi}{f} \frac{\vec{\pi}}{\pi} & \cos \frac{\pi}{f} \end{pmatrix} \begin{pmatrix} \vec{0}_{4 \times 4} \\ f + \sigma(x) \end{pmatrix}$$

with $\pi^2 = \sqrt{\vec{\pi} \cdot \vec{\pi}}$

(= exp (i \frac{\sqrt{3}}{f} \frac{\vec{\pi} \cdot \vec{a}}{\pi} \hat{a}))

$$\Rightarrow \hat{\phi}(x) = (f + \sigma(x)) \begin{pmatrix} \sin \frac{\pi(x)}{f} \frac{\vec{\pi}(x)}{\pi} \\ \cos \frac{\pi(x)}{f} \end{pmatrix}$$

(actually a general expression for any SO(N) / SO(N-1) set)

We can now work out the Lagrangian.

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \vec{\phi})^T \cdot \partial^\mu \vec{\phi} - \frac{f^2}{8} (\vec{\phi} \cdot \vec{\phi} - f^2)^2$$

with $\partial_\mu \vec{\phi} = \partial_\mu \sigma \begin{pmatrix} \sin \frac{\pi}{f} \frac{\vec{\pi}}{\pi} \\ \cos \frac{\pi}{f} \end{pmatrix} + (f + \sigma) \begin{pmatrix} -\frac{\partial_\mu \pi}{f} \cos \frac{\pi}{f} \frac{\vec{\pi}}{\pi} + \sin \frac{\pi}{f} \frac{\partial_\mu \vec{\pi} \cdot \vec{\pi} - \pi \partial_\mu \pi}{\pi^2} \\ \frac{\partial_\mu \pi}{f} \sin \frac{\pi}{f} \end{pmatrix}$

$$\Rightarrow \mathcal{L}_\phi = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \left(1 + \frac{\sigma}{f}\right)^2 \left(\frac{f^2}{12} \sin^2 \frac{\pi}{f} \partial_\mu \vec{\pi}^\top \cdot \partial^\mu \vec{\pi} + \frac{f^2}{4\pi^2} \left(\frac{\pi^2}{f^2} - \sin^2 \frac{\pi}{f} \right) \partial_\mu \vec{\pi}^\top \partial^\mu \vec{\pi} \right) - \frac{g_*^2}{8} \sigma^4 - \frac{g_*^2}{2} f \sigma^3 - \frac{g_*^2}{2} f^2 \sigma^2$$

▷ We can Taylor-expand this Lagrangian around $\pi = 0$ (regular point)
 → many multi-NG-boson interactions ($\propto \frac{1}{f^n}$ as we always have $\frac{\pi}{f}$ in the Lagrangian) with up to 2 derivatives

▷ f plays the role of the "Higgs decay constant" (as for the pion decay constant in QCD)

▷ $\vec{\pi}$ describes the dynamics of 4 massless bosons (as 4 broken generators \hat{T}^a)

▷ $g_* \in [1, 4\pi] \Rightarrow$ separation of the f and m_* scales
 ▷ σ is massive: $m_\sigma = m_* = g_* f$ (see last term) (connected to the UV via ξ)

→ σ -resonance in analogy with the QCD resonances (but multi-TeV f data)

$m_* \rightsquigarrow \Lambda_{\text{QCD}}$ in QCD = confinement scale of the composite sector
 $g_* \rightsquigarrow$ low-energy coupling of the strong sector
 ↳ could be perturbative or even $\approx 4\pi$ (depends on the # colors)

We now need to identify the $SO(4)$ Higgs doublet. \rightarrow what are the Lagrangian symmetries?

$\Rightarrow SO(4)$ group under which $\vec{\pi}$ is a four-plet:

$$\vec{\pi} \rightarrow e^{i a \vec{t}^a} \vec{\pi} \quad \vec{\pi} \sim \begin{bmatrix} X_{4 \times 4} & 0 \\ 0 & 0 \end{bmatrix}$$

$$a \text{ in the } \vec{\phi} \text{ language: } \hat{\phi} \rightarrow e^{i a \vec{T}^a} \hat{\phi}$$

the 4×4 sub-block of the unbroken generators

\equiv linearly realized symmetry (linear and homogeneous on the field)

Reminder: $SO(4) \simeq SU(2)_L \times SU(2)_R$

\Rightarrow The $SO(4)$ fourplet can be expressed as a $(1, 2)$ bidoublet (pseudo real matrix)

$$\Sigma = \frac{1}{\sqrt{2}} (i \vec{\sigma}_\alpha \vec{\pi}^\alpha + \pi^4) \quad (\alpha = 1, 2, 3)$$

$$= \begin{pmatrix} \frac{\pi^4 + i \pi^3}{\sqrt{2}} & \frac{\pi^2 + i \pi^1}{\sqrt{2}} \\ \frac{\pi^2 - i \pi^1}{\sqrt{2}} & \frac{\pi^4 - i \pi^3}{\sqrt{2}} \end{pmatrix} \equiv \begin{pmatrix} H^c, H \\ \sqrt{2} i \sigma_2 H^\dagger \end{pmatrix}$$

$$\Rightarrow H = \begin{pmatrix} H_u \\ H_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^2 + i\pi^1 \\ \pi^4 - i\pi^3 \end{pmatrix} \Rightarrow \frac{\vec{\pi}}{\pi} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i(H_u - H_u^+) \\ H_u + H_u^+ \\ i(H_d - H_d^+) \\ H_d + H_d^+ \end{pmatrix}$$

Under an $SU(2)_L \times SU(2)_R$ transformation: $\Sigma \rightarrow g_L \Sigma g_R^+$

$$\Rightarrow \delta_L \Sigma = i \delta_\alpha^L \frac{\sigma^a}{2} \Sigma \rightarrow SU(2)_L \text{ doublet}$$

$$\delta_R^3 \Sigma = -i \delta_\beta^R \Sigma \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \rightarrow U(1), \text{ quantum number is } 1/2 \text{ (} H_c \text{ is } -1/2)$$

$$4 \equiv (2, 2) \rightarrow 2_{1/2}$$

We can show that the Lagrangian is also invariant under

$$\vec{\pi} \rightarrow \vec{\pi} + \pi \cot \frac{\pi}{f} \vec{\alpha} + \left(\frac{f}{\pi} - \cot \frac{\pi}{f} \right) (\vec{\alpha}^T \cdot \vec{\pi}) \frac{\vec{\pi}}{\pi}$$

$$\approx \vec{\phi} \rightarrow \vec{\phi} + i \alpha_a \hat{T}^a \vec{\phi}$$

= symmetry in the broken generator direction, but non-linear and non-homogeneous ($\vec{0} \rightarrow f \vec{\alpha} \neq 0$)

→ conversely, any non-zero vec can be brought back to $\vec{0}$
→ no effect

The EW interactions are finally introduced by gauging the $SU(2)_L \times U(1)_Y$ subgroup of $SO(4)$.

$$D_\mu \vec{\Phi} \rightarrow D_\mu \vec{\Phi} = \left(\partial_\mu - ig W_{\mu i} T_L^i - ig' B_\mu T_R^3 \right) \vec{\Phi}$$

We add $\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^i W_i^{\mu\nu}$ to the lagrangian

In terms of the Higgs, \mathcal{L}_Φ become (ignoring the σ -pieces)

$$\mathcal{L}_\Phi = \dots + \frac{f^2}{2|H|^2} \sin^2 \frac{\sqrt{2} |H|}{f} D_\mu H^\dagger D^\mu H + \frac{f^2}{8|H|^4} \left(2 \frac{|H|^2}{f^2} - \sin^2 \frac{\sqrt{2} |H|}{f} \right) D_\mu |H|^2 D^\mu |H|^2$$

\Rightarrow Adopting the unitary gauge $H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix}$

$$\mathcal{L}_\Phi \approx \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{g^2}{4} \frac{v^2}{f^2} \sin^2 \frac{v+h}{f} (W_{\mu\nu}^+ W^{\mu\nu-} + \frac{1}{2c_w^2} Z_\mu Z^\mu) + \dots$$

$$\Rightarrow m_W = c_w m_Z = \frac{g g'}{2} \sin \frac{v}{f} \equiv \frac{1}{2} g v \quad v \approx 246 \text{ GeV}$$

$$\Rightarrow \xi = \frac{v^2}{f^2} = \sin^2 \frac{v}{f} \Rightarrow v \text{ is connected to the Higgs decay constant}$$

The lagrangian also contains VVh^2 interactions: *constrained experimentally*

$$\mathcal{L}_\Phi \approx \frac{g^2 v^2}{4} (W_{\mu\nu}^+ W^{\mu\nu-} + \frac{1}{2c_w^2} Z_\mu Z^\mu) \left(2\sqrt{1-\xi} \frac{h}{v} + (1-2\xi) \frac{h^2}{v^2} - \frac{4}{3} \xi \sqrt{1-\xi} \frac{h^3}{v^3} + \dots \right)$$

Another interesting point is that $P = 1$ at all orders in 1/f as the model embeds the custodial symmetry. ($SO(4) \rightarrow SO(3) \cong$ fix the W/Z mass ratio)

\Rightarrow all viable models must actually include the custodial symmetry, or ξ will be unacceptably small (fine-tuning).

III.4. Partial compositeness

III.4.0 Generalities

\equiv tool to introduce the fermion masses (the Higgs does not couple to fermions)

The SM fermions are elementary and coupled to the composite sector via linear op.

$$\mathcal{L}_{int} = \frac{\lambda_L^L}{\Lambda_{UV}^{d_L-5/2}} \bar{Q}_L \cdot O_F^L + \frac{\lambda_R^R}{\Lambda_{UV}^{d_R-5/2}} \bar{\psi}_R \cdot O_{f,R}^R + \frac{\lambda_D^R}{\Lambda_{UV}^{d_D-5/2}} \bar{d}_R \cdot O_{d,R}^R + \dots$$

\Rightarrow give rise of the mixing of elementary fields with composite ones. The λ parameters control the composite fractions, after evolution to m_{UV} :

$$\lambda_i(m_{UV}) = \lambda_i \left(\frac{m_{UV}}{\Lambda_{UV}} \right)^{d_i - 5/2}$$

large Yukawas can be generated for large scale separation, if $d > 5/2$

\Rightarrow 3rd generation $\Rightarrow d > 5/2$; large α for other generations (\rightarrow smaller masses)

reached for @LHC

- In details, let's assume 3 resonances Q, U and D that are generated at the scale $m_{*} \equiv p_{*}$.
- These resonances must have the same quantum numbers as the SM counterparts (but are in fact embedded in larger repr. like $SO(4)$ in the minimal model).
- The resonance mass are generated by the string dynamics (not the EW/SB) \Rightarrow Dirac mass terms \Rightarrow vector-like fermions
- Each operator related to the SM masses is expected to be capable to excite from the vacuum a single-particle state:

$$\begin{aligned} \langle 0 | O_{qF}^L | Q \rangle &\neq 0 \\ \langle 0 | O_{uF}^R | U \rangle &\neq 0 \\ \langle 0 | O_{dF}^R | D \rangle &\neq 0 \end{aligned} \quad \int \rightarrow \text{generates mixing, with the quarks}$$

III.4.2 The $SO(5) / SO(4)$ example

- At the high scale, operators are written as $SO(5)$ invariants
 - \rightarrow choice of the representation \rightarrow model building tasks
 - \rightarrow Only operators with the lowest dimension matter (otherwise, running effects tame them down),
- We need operators including the $2_{1/6}$, $1_{2/3}$ and $1_{-1/3}$ of G_{EW} \rightarrow they don't exist in $SO(5)$, \Rightarrow we need to extend the composite global symmetry

• The simplest option is $SO(5) \times U(1)_X \rightarrow SO(4) \times U(1)_X$ ($\times SU(3)_C$)
 The hypercharge now arises from both Tr^3 (of $SU(3)_C$) and $U(1)_X$:

$$Y = Tr^3 + X$$

(Higgs + gauge bosons are neutral \rightarrow pheno is unchanged)

• The simplest representation is the fundamental one (the 5). If $X = 2/3$:

$$5_{2/3} \rightarrow 4_{2/3} \oplus 1_{2/3} \rightarrow 2_{7/6} \oplus 2_{1/6} \oplus 1_{2/3}$$

(SO(4))

(GEM)

only the j th component couples to Tr^3

\hookrightarrow coupling to Q_L, U_R

$$\Rightarrow L_{int}^{(1)} = \tilde{\lambda}_0^R \bar{u}_R (O_{u,F}^R)_5 \subset \tilde{\lambda}_0^R \bar{U}_R^I (O_{u,F}^R)_I$$

with $I = 1, 2, \dots, 5$

$$= (\dots, u_R)^T$$

$\underbrace{\hspace{2cm}}_{4_{2/3}} \quad \underbrace{\hspace{2cm}}_{1_{2/3}}$

$$L_{int}^{(2)} = \tilde{\lambda}_0^L \bar{q}_L (O_{q,F}^L)_5 \subset \tilde{\lambda}_0^L \bar{Q}_L^I (O_{q,F}^L)_I$$

$$= (\underbrace{-ib_L, -b_L, -ib_L, b_L, \dots}_{4_{2/3}}, \dots)_{1_{2/3}}^T$$

• In general, one dresses the source with the Goldstone matrix
(see z formalism)

$$\left(\Theta_R^4, U_R^1 \right)^T = U[\vec{\pi}]^{-1} U_R \quad \left(U[\vec{\pi}] = e^{i \frac{\sqrt{2}}{f} \vec{\pi}_a(x) \hat{T}^a} \right)$$

$$\left(Q_L^4, Q_L^1 \right)^T = U[\vec{\pi}]^{-1} Q_L$$

(this allows to turn a \mathcal{J} index into an \mathcal{X} index)

$$\Rightarrow \bar{Q}_L^4 \cdot U_R^4 + \bar{Q}_L^1 U_R^1 = \bar{Q}_L \Theta_R = 0 \Rightarrow \text{only } \bar{Q}_L^4 \cdot U_R^4 \text{ or } \bar{Q}_L^1 \cdot U_R^1 \text{ should be used.}$$

$$\begin{aligned} \Rightarrow \mathcal{L}_{\text{Yuk}} &= -c \frac{\lambda_Q^L \lambda_U^R}{g_0^2} m_Q \bar{Q}_L^1 U_R^1 \\ &= -c \frac{\lambda_Q^L \lambda_U^R}{g_0^2} m_Q \frac{1}{2\sqrt{2}|H|} \sin \frac{2\sqrt{2}|H|}{f} \bar{Q}_L^1 H^c u_R + \text{h.c.} \end{aligned}$$

$$\Rightarrow m_Q = \frac{c \lambda_Q^L \lambda_U^R}{g_0^2} m_Q \frac{\sqrt{\xi(1-\xi)}}{\sqrt{2}} \quad \text{and } g_Q = \frac{1-2\xi}{\sqrt{1-\xi}} y_{\bar{Q}}$$

The bottom sector is built similarly, from $5_{-113} \rightarrow 2_{116} \oplus 2_{516} \oplus 1_{-113}$

$$Q = \frac{4}{\sqrt{2}} (-ib_L, b_L, ib_L, b_L, 0)^t$$

$$D = (a, a, a, b_R)^t$$

• other option: take the spinorial rep: $4_{116} \rightarrow (2, 1)_{116} \oplus (1, 2)_{116}$

$$\rightarrow 2_{116} \oplus 1_{213} \oplus 1_{-113}$$

1 family (of quarks)

$$Q_L = (b_L, b_L, a, a)^t$$

$$U_R = (a, a, b_R, a)^t$$

$$D_R = (a, a, 0, b_R)^t$$

III.5 Higgs potential

The generation of the Higgs potential (that generates EWSB) is quite complex. It comes from the explicit breaking of the Goldstone symmetry by the elementary sector

$\hat{H} = \text{Goldstone} \rightarrow \text{no potential}$

→ loop-induced

$$\rightarrow V = -\alpha f^2 \sin^2 \frac{H}{f} + B f^2 \sin^4 \frac{H}{f}$$

↑
↑
 EW gauge fermions only
 → fermions

→ generation of the Higgs mass.