V. Other theories in a nutshell

V.1. GUTs

- Simplifying the gauge group structure
- Embedding quarks and leptons into a single representation
- Coupling unification

Method: We pick up $G \supset SU(3)_c \times SU(2)_L \times U(1)_Y$
- The SM must be reproduced at low energy
- G breaking should preserve QCD and EM
- Matter is chiral

$\rightarrow G = \begin{cases} SU(N) & N > 4 \\ SO(4N+2) & N \geq 2 \\ E_6 \end{cases}$ interesting cases

$\mathcal{L}_{GUT} = \mathcal{L}_{gauge} + \mathcal{L}_{Yuk} + \mathcal{L}_{breaking}$

$\mathcal{L}_{Yuk}$: scalar-fermion interactions $\neq$ fermion masses after sym breaking

$\mathcal{L}_{breaking}$: breaking mechanism $G \rightarrow G_{SM} \rightarrow SU(3)_c \times U(1)_{EM}$

Example: $SU(5)$: We include $SU(3)$ and $SU(2)$ in a single 5×5 matrix

$$\begin{pmatrix} SU(3) & LQ \\ LQ^\dagger & SU(2) \end{pmatrix}$$

LQ = 12 new gauge bosons = leptequarks

This matrix is traceless $\rightarrow$ hypercharge is quantized $\rightarrow$ electric charge is quantized

$$Y = \begin{pmatrix} 1/3 & & & & \\ & 1/3 & & 0 & \\ & & 1/3 & & \\ & 0 & & -1/2 & \\ & & & & -1/2 \end{pmatrix} \rightarrow Q = \begin{pmatrix} 1/3 & & & & \\ & 1/3 & & 0 & \\ & & 1/3 & & \\ & 0 & & 0 & \\ & & & & 1 \end{pmatrix}$$

$\downarrow$ $Y+T_3$

$\rightarrow L_L$ and D_R are combined.

→ $5 \equiv L_L + d_R$

10 ↝ the rest

↳ antisym $\begin{pmatrix} (U_{R\,3\times3}) & (Q_{L\,3\times2}) \\ -(Q_{L\,1\times3}) & (e_R) \end{pmatrix}$

2 representations for all matter fields

- Breaking: $SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}$

Simplest choice: $h_{24} \sim 24$; $h_5 \sim 5$

- Advantages: • gauge coupl. unif.
  • partial unif. of matter repr.
  • electric charge quant.
- Problems: proton decay, magnetic monopoles, ...

⇒ $SO(10)$, E_6: • full matter repr. unification
• N_R (ν physics)
• New $U(1)$'s after sym. breaking ⇒ Z'
• E_6 inspired by string theory
• no gauge coupl. unification without extra matter

V.2. Extra-dimensions

- Core idea: spacetime is not 4D
- The easiest example: $R^4 \times$ circle of radius R (the 5th dim is periodic)

↳ Massless 5D fields = towers of 4D fields. All 4D fields are massive

$$\phi(x^4, y) = \sum_n \phi_n(x^4) \exp\left[\frac{i n y}{R}\right]$$

↳ K-G eq: $\Box\phi = 0 \Leftrightarrow \left[\Box - \partial_y^2\right]\phi = 0 \to \left[\Box + \frac{n^2}{R^2}\right]\phi_n(x^4) = 0$

- No observation $\Rightarrow$ R is small $\rightarrow$ compact E.D.

- less naive: in the 1920s, gravity and EM were tried to be unified:

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & A_\mu = g_{\mu 4} \\ A_\mu = g_{4\mu} & \phi = g_{44} \end{pmatrix}$$ 5D metric includes ED in 4D

↳ Extension to all interactions: 11D $\rightarrow$ mirror fermions

- less naive II: Randall-Sundrum:
  - The SM lives on a 3-brane (4D spacetime)
  - Gravity lives in the bulk (5D spacetime)
  - large ED (TeV-scale)
  - KK parity $\rightarrow$ DM, MET signature, ...