New physics simulations at colliders

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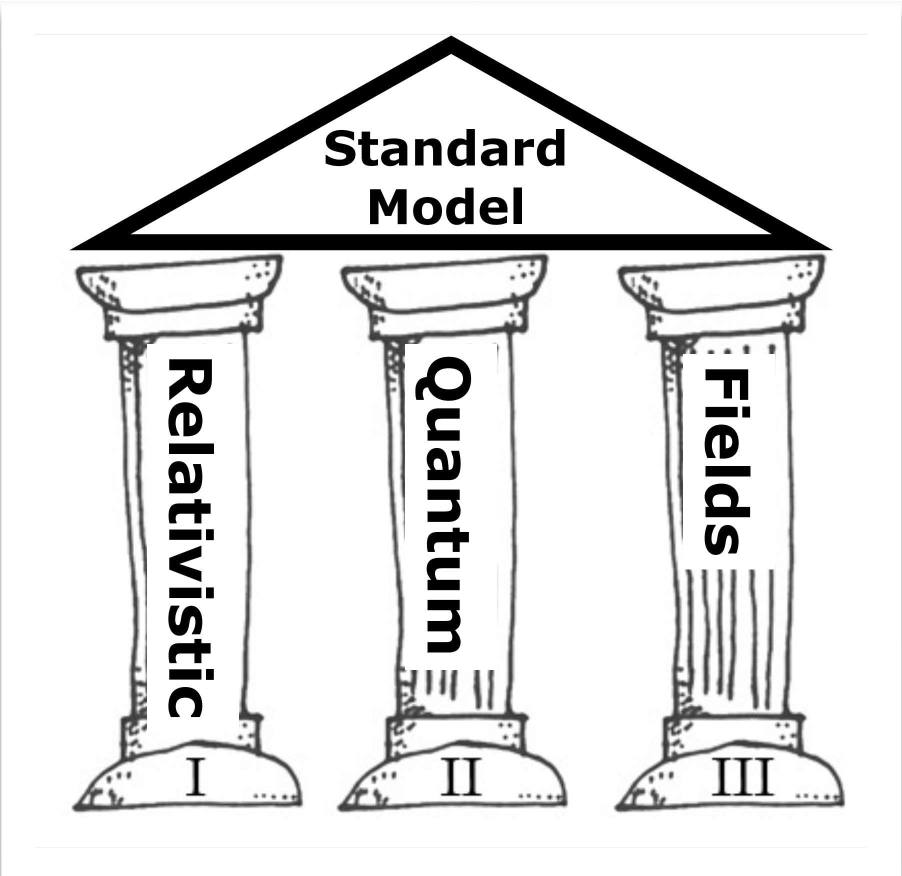
Beyond the Standard Model

NPAC lecture - March 01, 2021

Outline

1. The Standard Model of particle physics and Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
4. Parton showers, hadronisation & underlying event
5. Summary

The pillars of the Standard Model



- ✦ The Standard Model in a nutshell
  - ✤ Quantum field theory
    - ★ Quantum mechanics ⤳ $\hbar/S$ is not small
    - ★ Special relativity ⤳ v/c is not small
    - ★ Field theory ⤳ *particles embedded as field excitations*

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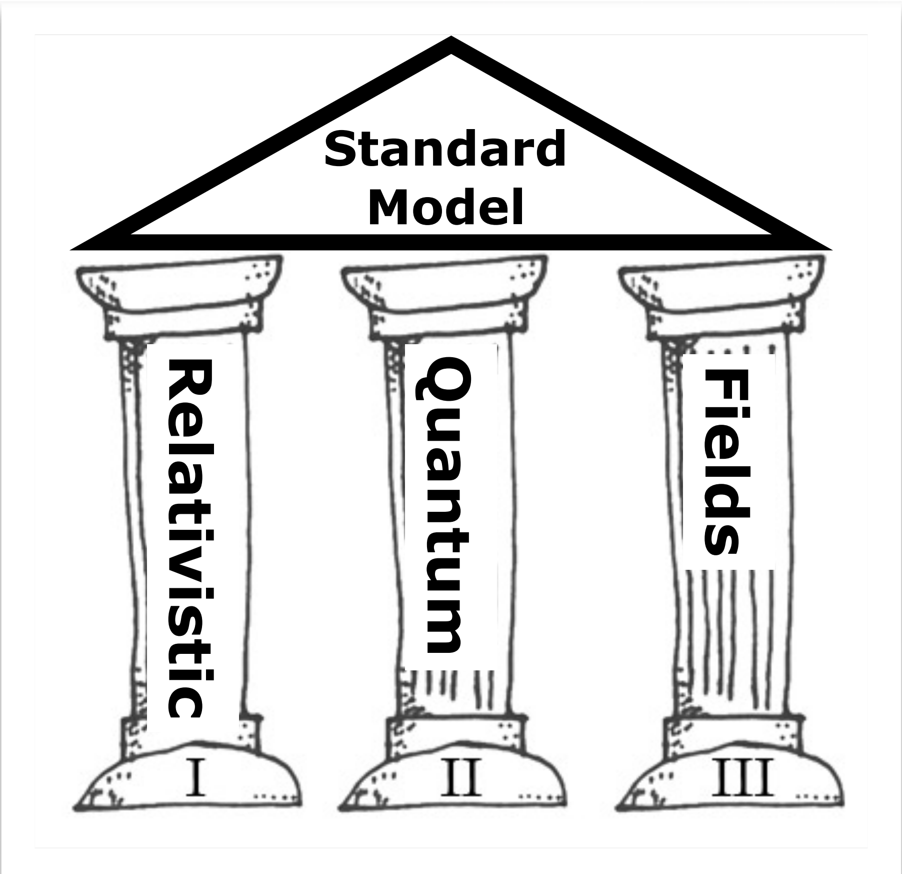

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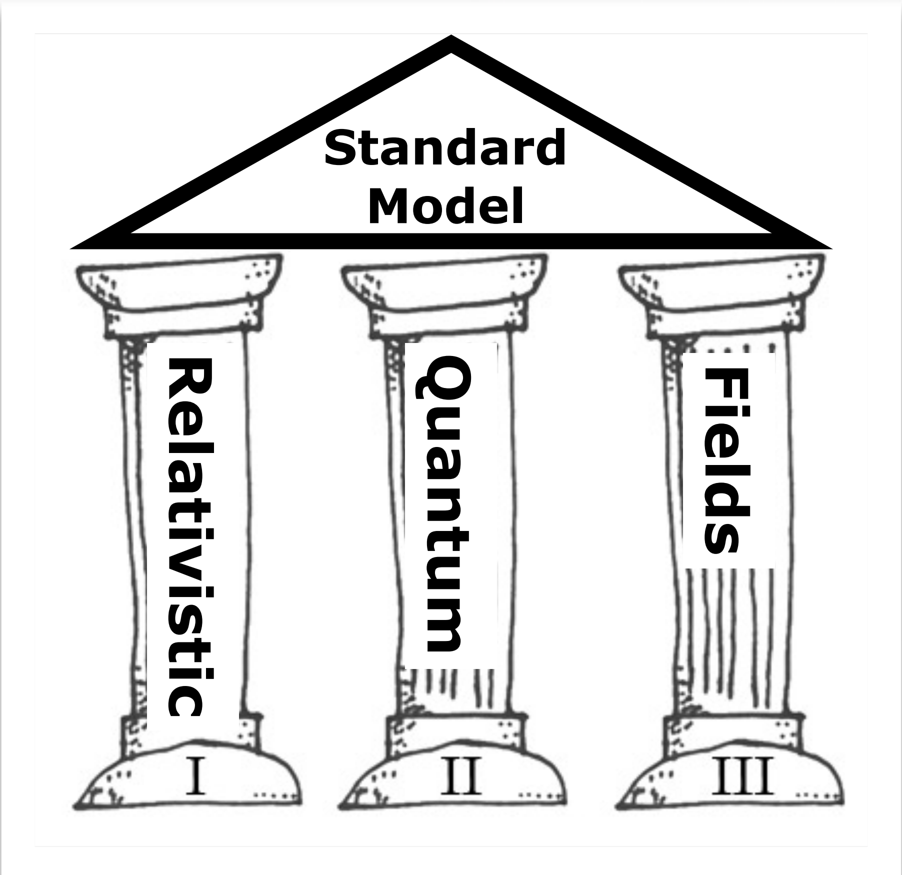

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- Fun facts
  - ★ 15 Nobel prizes, plenty of discoveries
  - ★ Technological challenges (experiments)
  - ★ Extremely precise measurements and predictions

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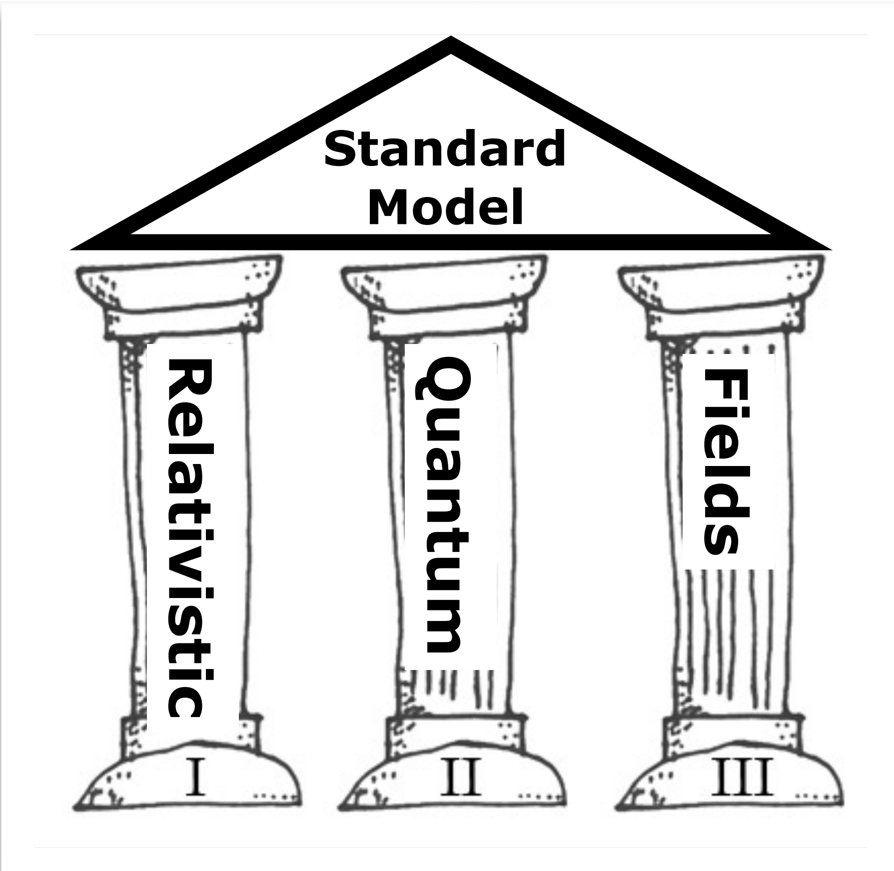

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✦ Particle physics is all about symmetries

✤ Energy-momentum conservation (translation invariance)

✤ Particule nature (Lorentz invariance)

✤ Charge - parity - time reversal (*CPT* invariance)

✤ Gauge invariance (fundamental interactions) ⇝ the intrinsic symmetry of the world

Three generations of fermions

✦ Elementary matter: we have <u>three</u> families of particles (+ antiparticles)

u	c	t
d	s	b
e	μ	τ
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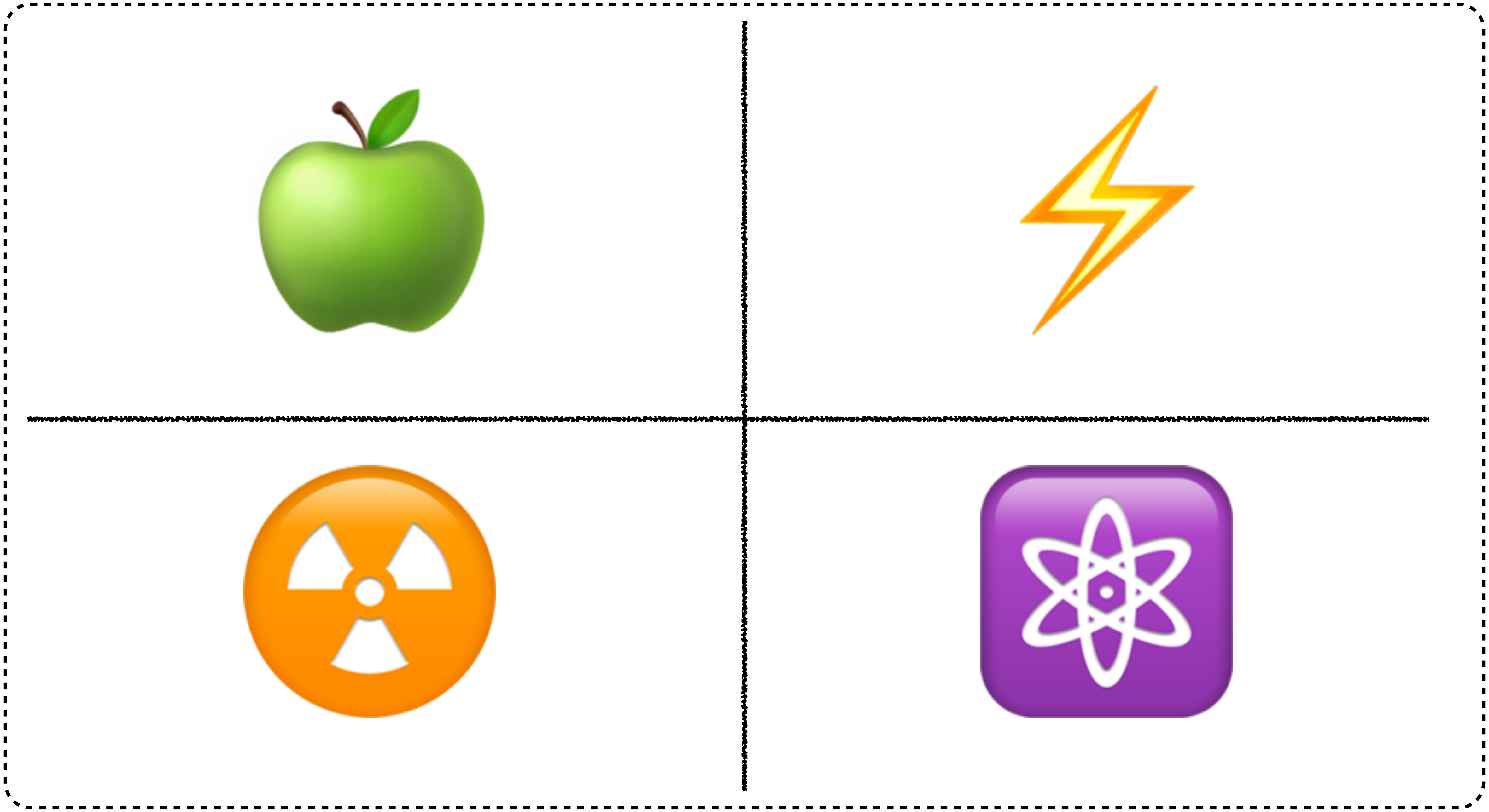
✤ First generation
- ★ Sufficient to describe all matter

✤ Second and third generations
- ★ More massive than the first generation
- ★ Identical quantum properties
- ★ Unstable (disappeared after the big bang)
- ★ Only accessible from highly-energetic collisions (LHC, cosmic rays)

The 3+1 fundamental interactions



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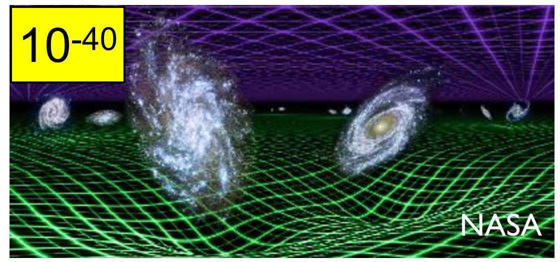

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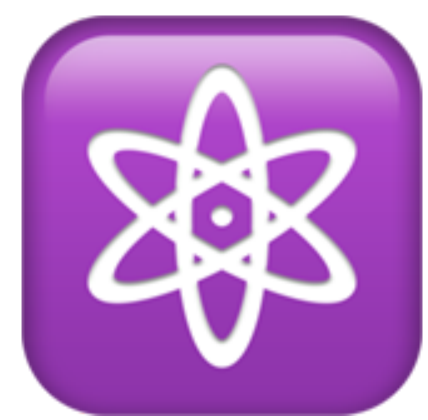
  - ↝ Gravitation
  - ↝ Stars, galaxies, …
- ✤ Extremely weak (10^{-40})
- ✤ Not included in the Standard Model



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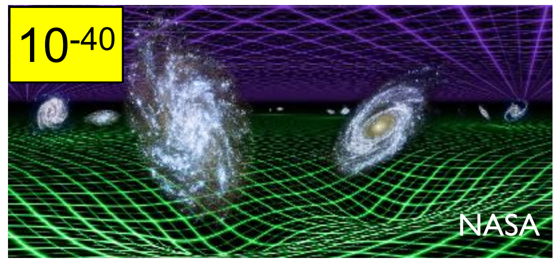

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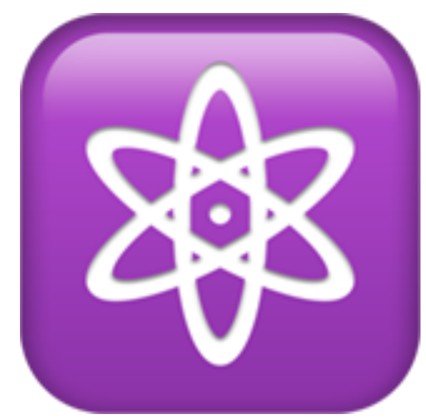

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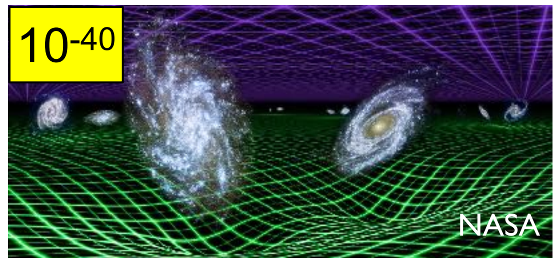

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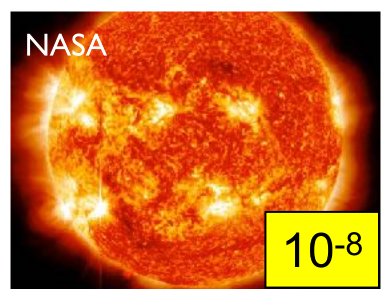

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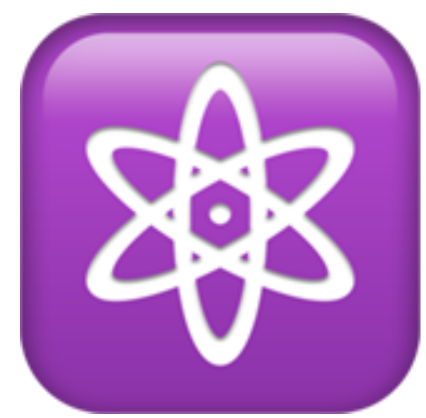

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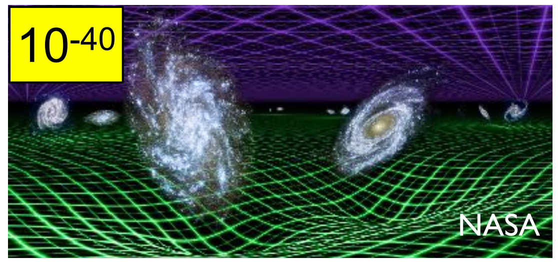

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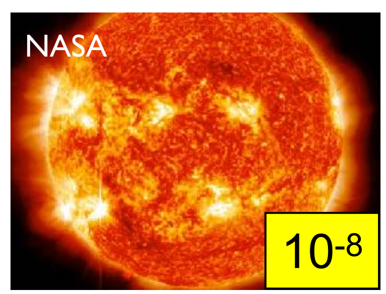

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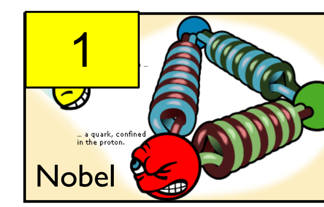

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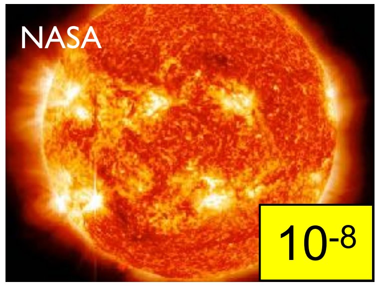

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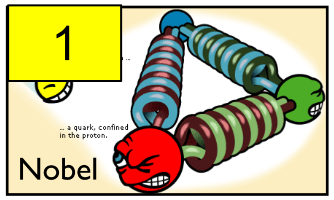

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And the Higgs boson?

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✦ The Standard Model is a greatly successful theory

- ✤ The fundamental building blocks of nature and 3 fundamental forces are unified
- ✤ Features:
  - ★ The quantum world brings infinities
  - ★ The fundamental interactions are linked to gauge symmetries
- ✤ Symmetries dictate requirements that must be fulfilled
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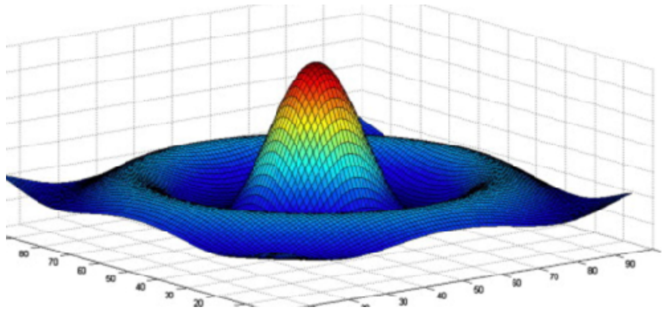
✦ The Brout-Englert-Higgs mechanism saves the day

✓ Prevents the theory from collapsing

✓ Cures the infinities (e.g. weak-boson scattering)

✓ Introduction of the particle masses in a symmetric way



[Englert & Brout (PRL`64); Higgs (PRL`64)]

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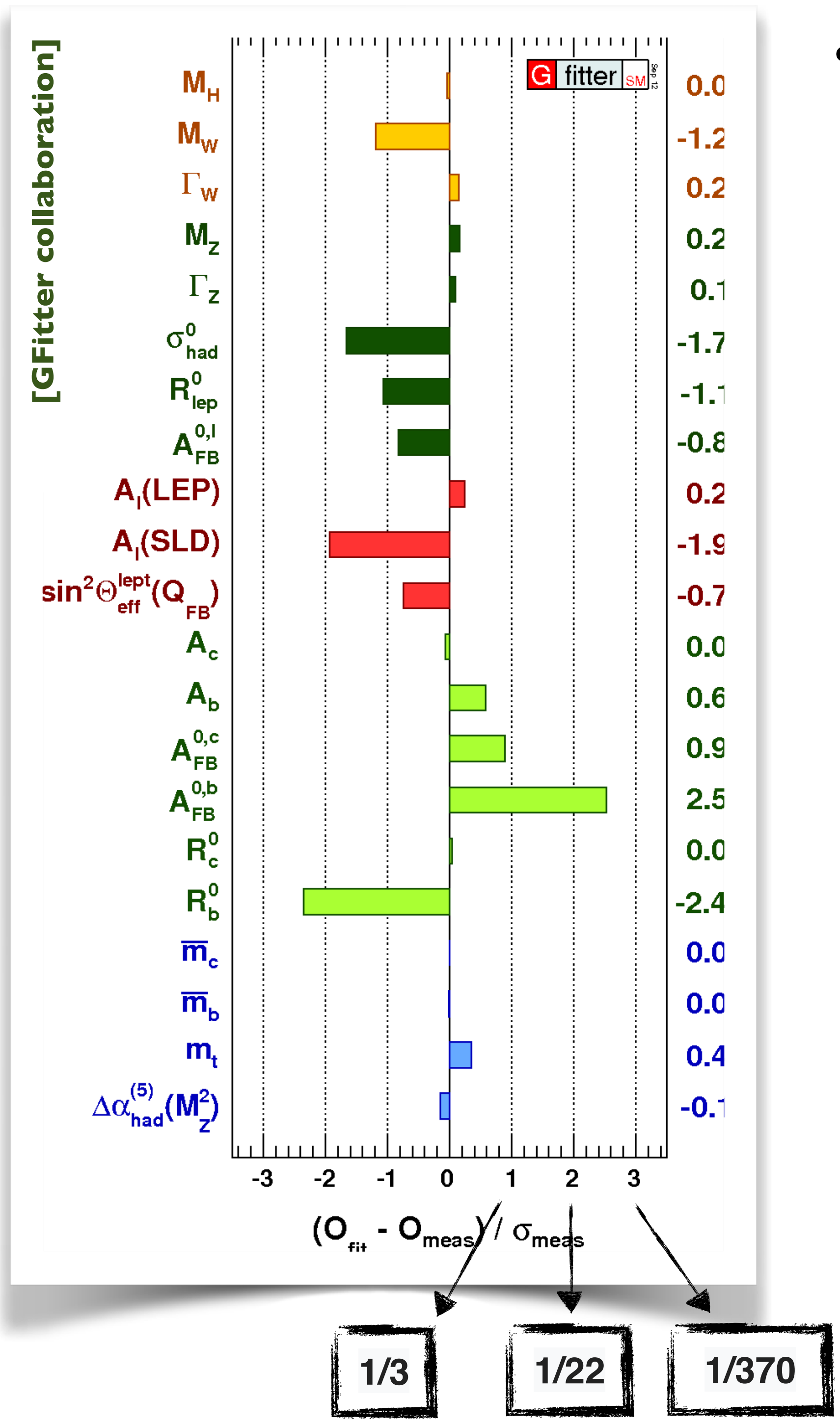

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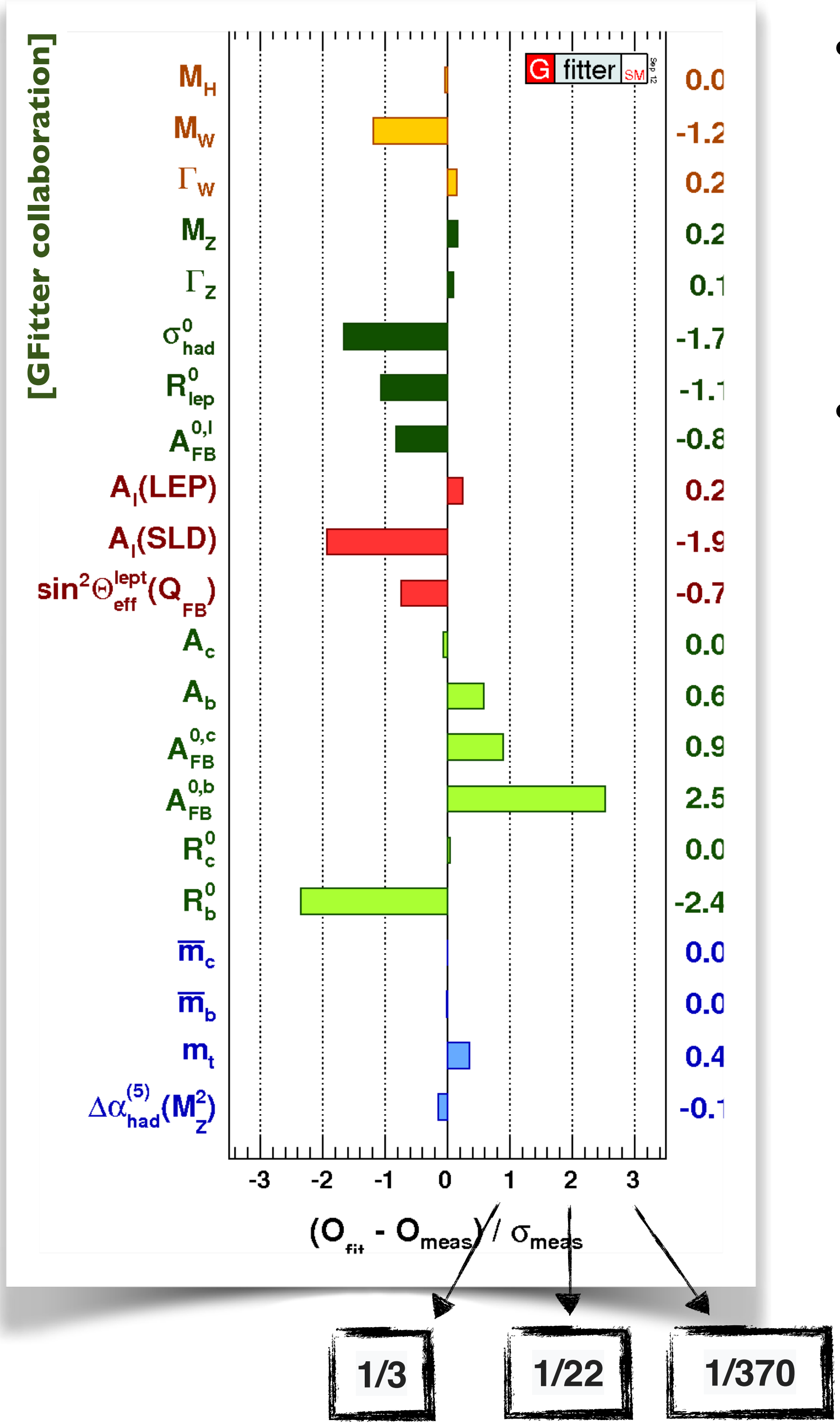

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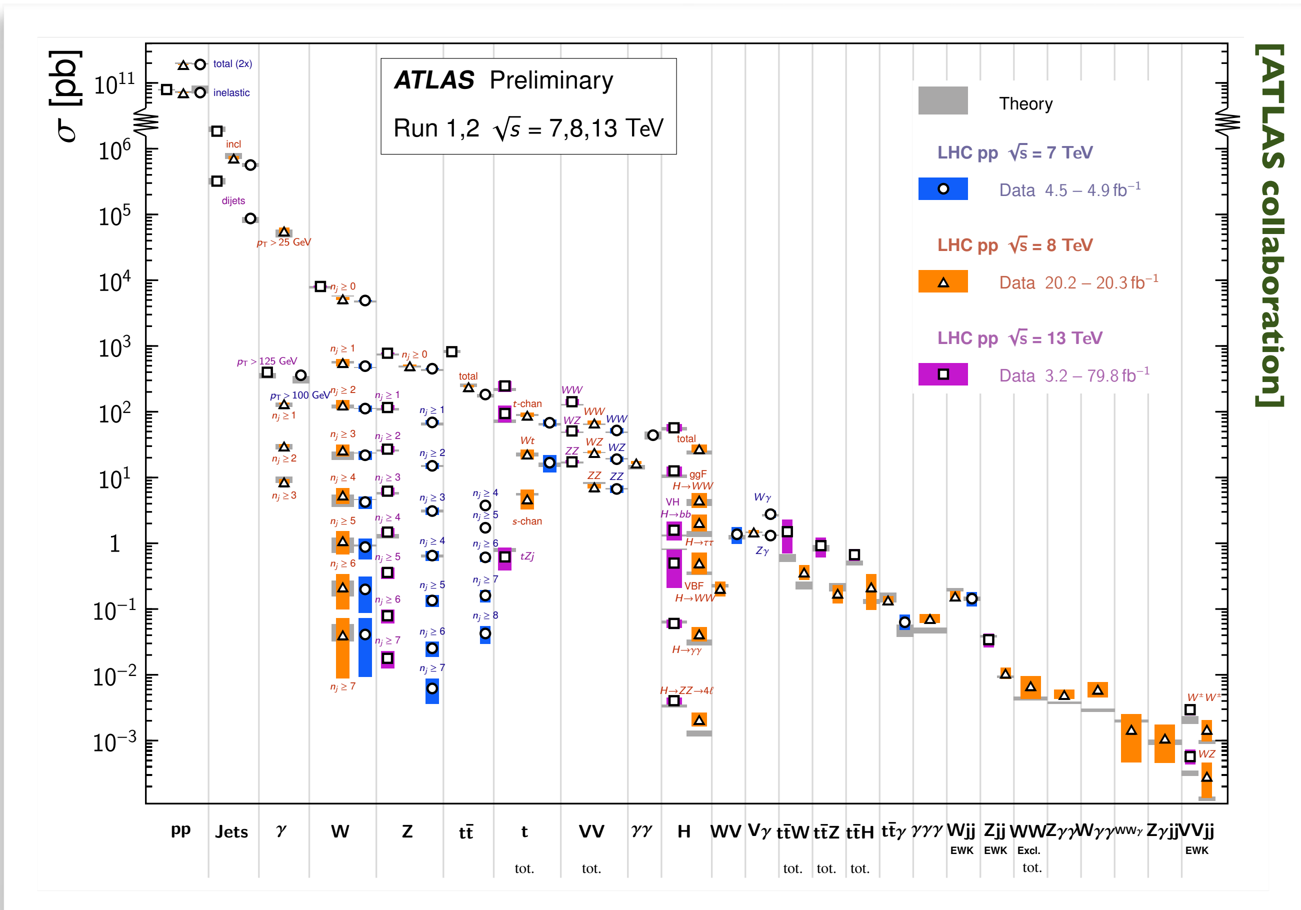
✤ Standard Model cross sections at the LHC



The Standard Model, beyond and simulations

✦ The Standard Model describes amazingly well the elementary world

✤ The LHC will hunt for deviations in precise parameter measurements

✤ Some of the Higgs sector will stay unknown for the next decades…

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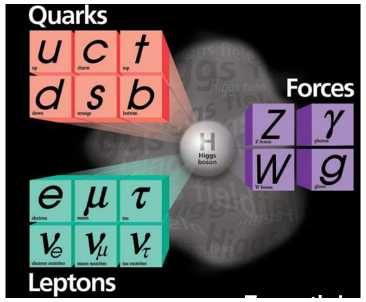

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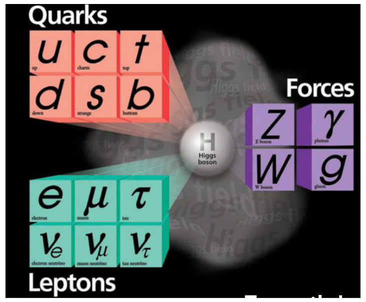


✦ The Standard Model is for sure only the tip of the iceberg

- ✤ Dark matter, neutrino masses, hierarchy problem, unification, *etc.*
- ✤ New physics is at the heart of the particle physics research programme
  - ★ Theoretically and experimentally
- ✤ Precision simulations tools are mandatory!

BSM simulations: where are we?

✦ New physics simulations - a challenge

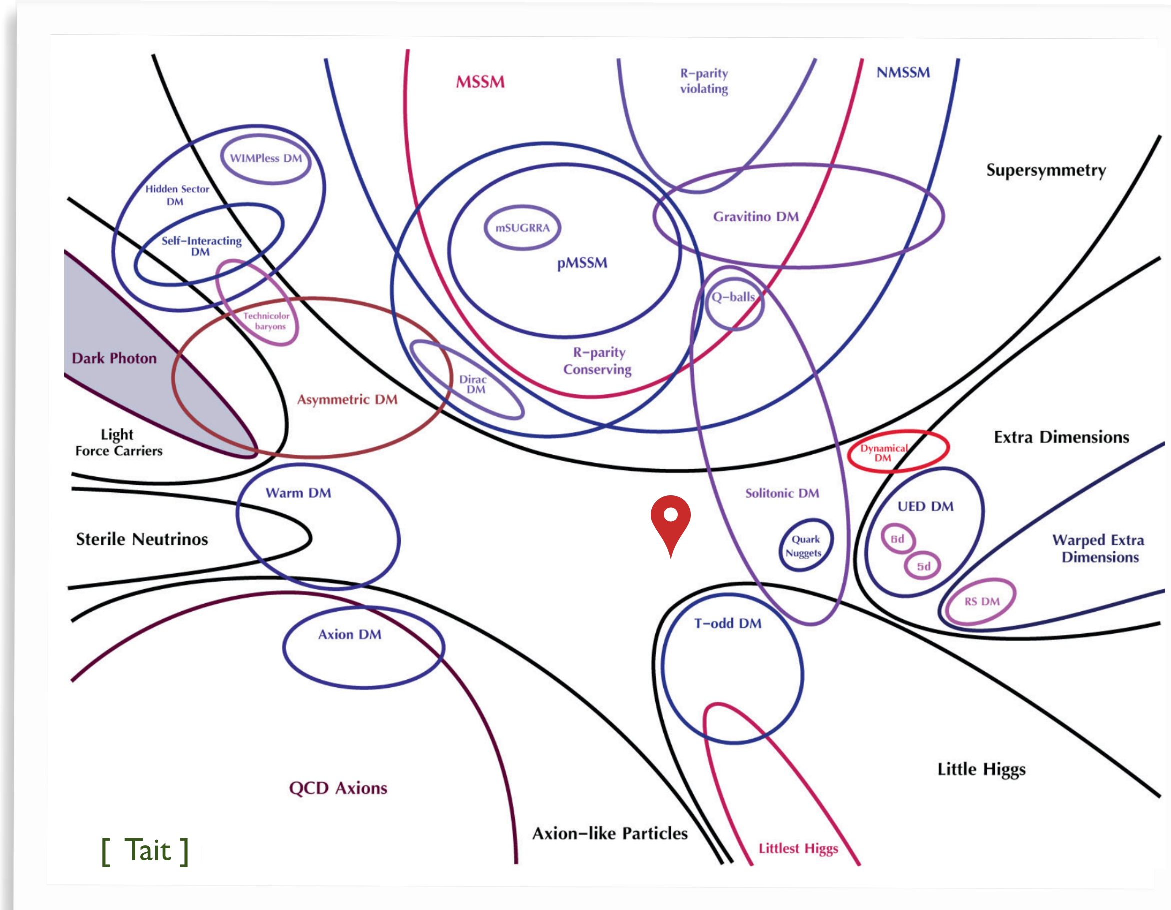
✤ No sign of new physics

✤ SM-like measurements

↝ no leading candidate theory

✤ Plethora of models to consider

↝ many implementations in tools



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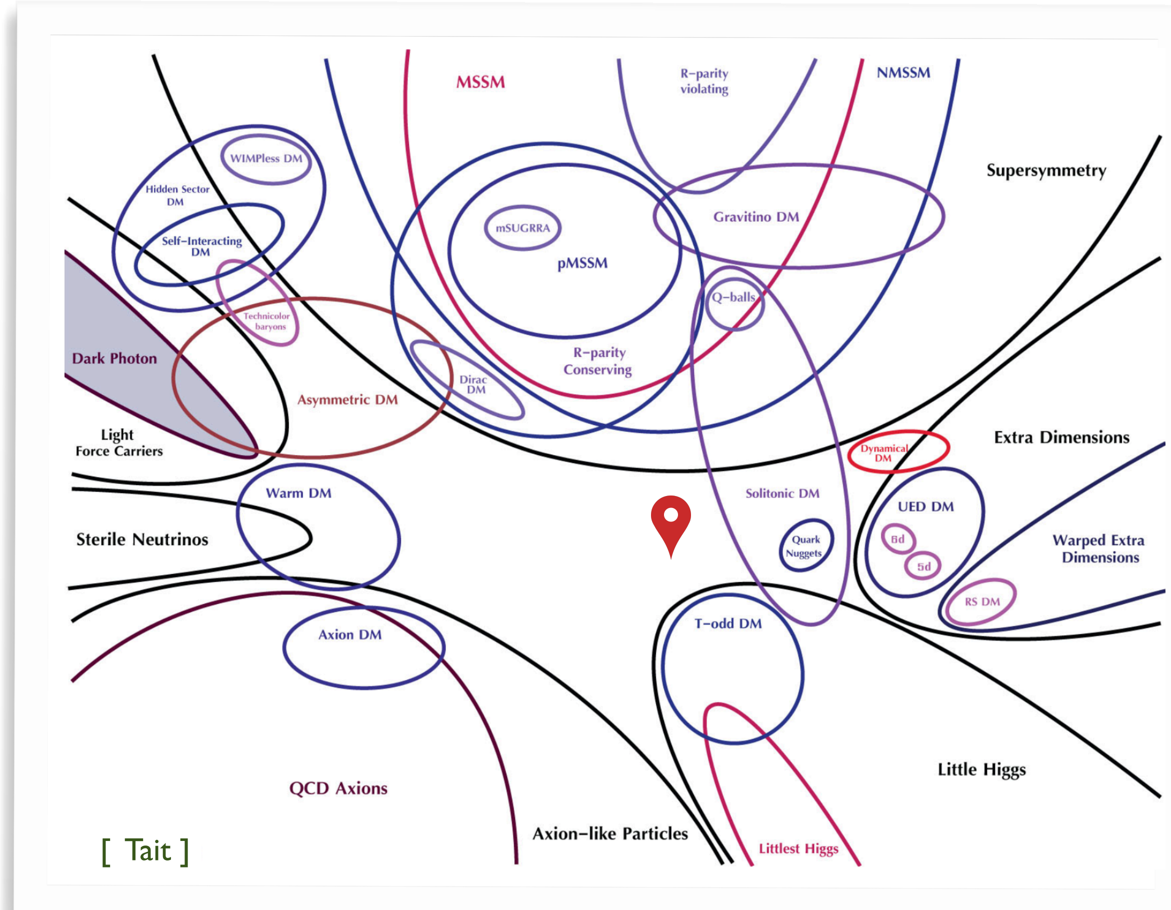
Despite of this, new physics is standard today



✦ New physics is standard

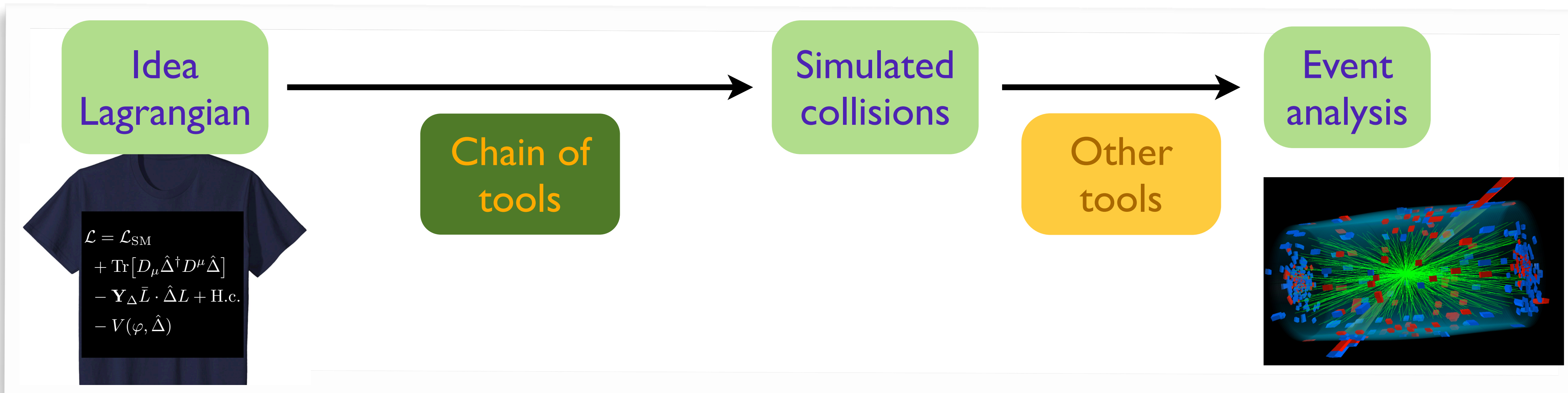
- ✤ 20-25 years of developments ↝ LO simulations are bread and butter
- ✤ Simulations at the NLO QCD accuracy easily achieved
  - ★ For any model/process (↝ MADGRAPH5_aMC@NLO)

From Lagrangians to events

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Streamlining the connection of a physics models to events

- ✤ Any new physics model can be implemented
- ✤ Easy to validate and maintain



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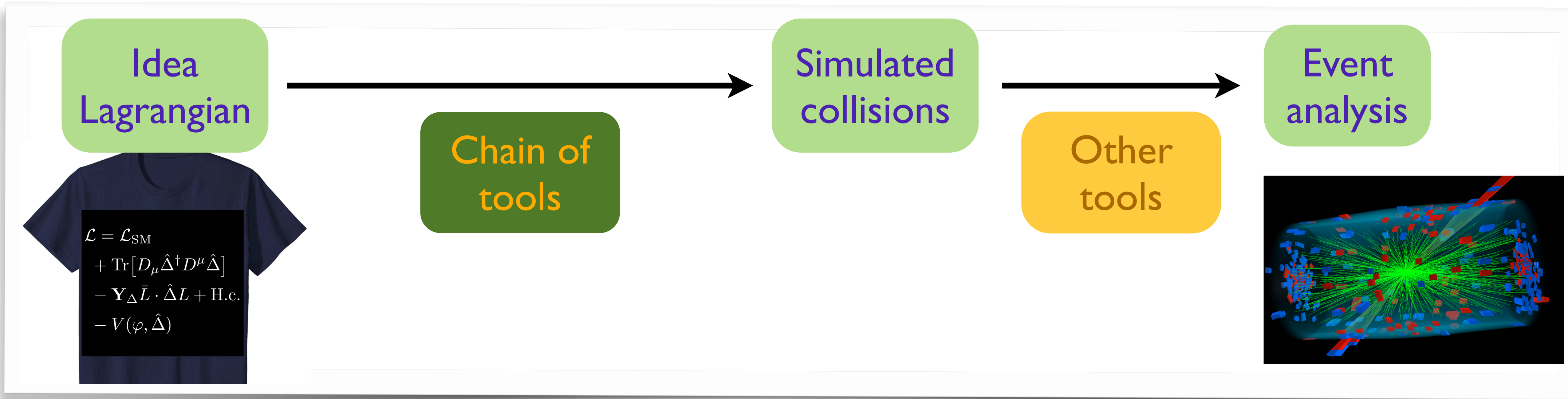
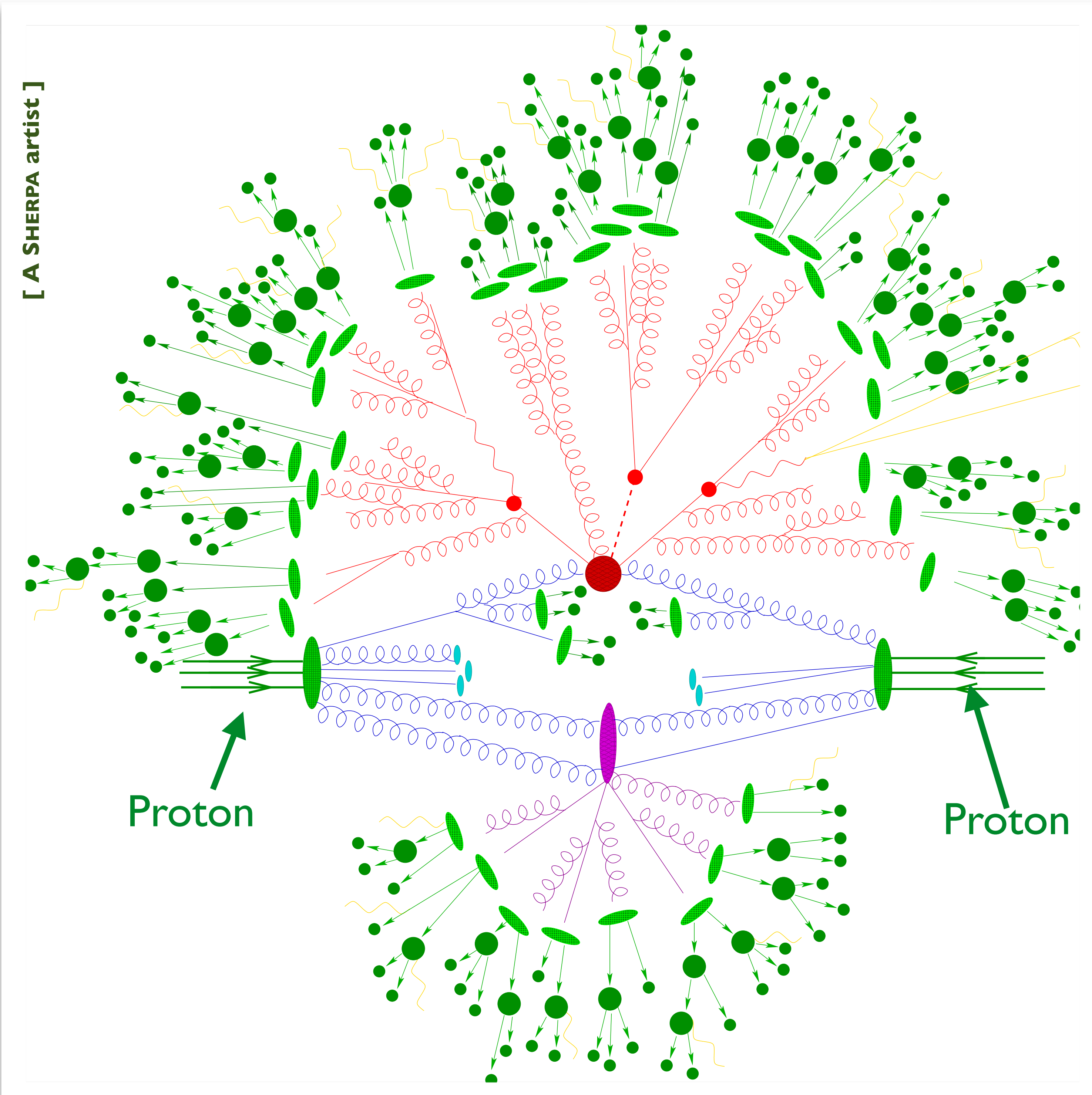


✦ Why a chain of several tools?

- Phenomena at colliders occur at different scales ⤳ factorisation

Deciphering a proton-proton collision



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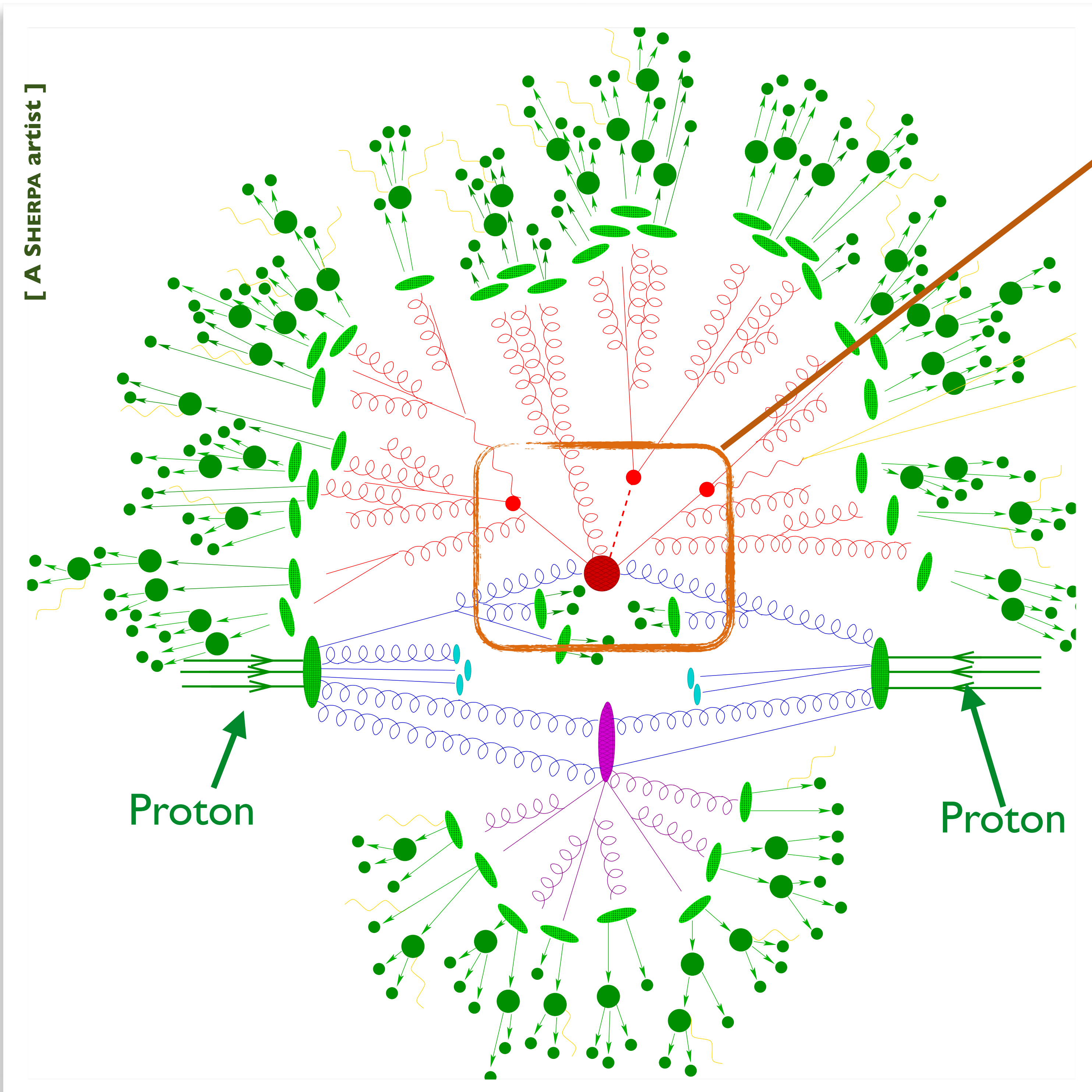

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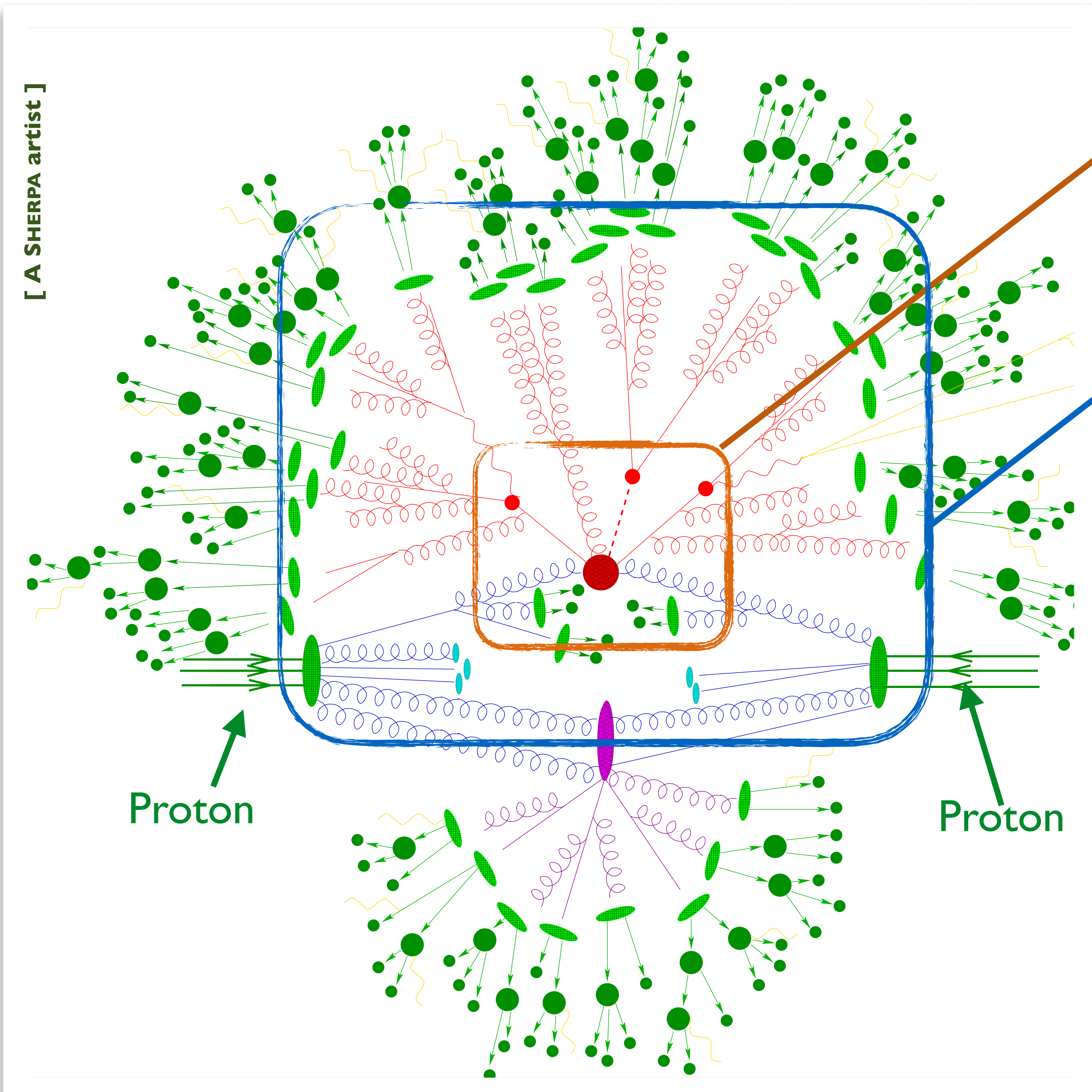

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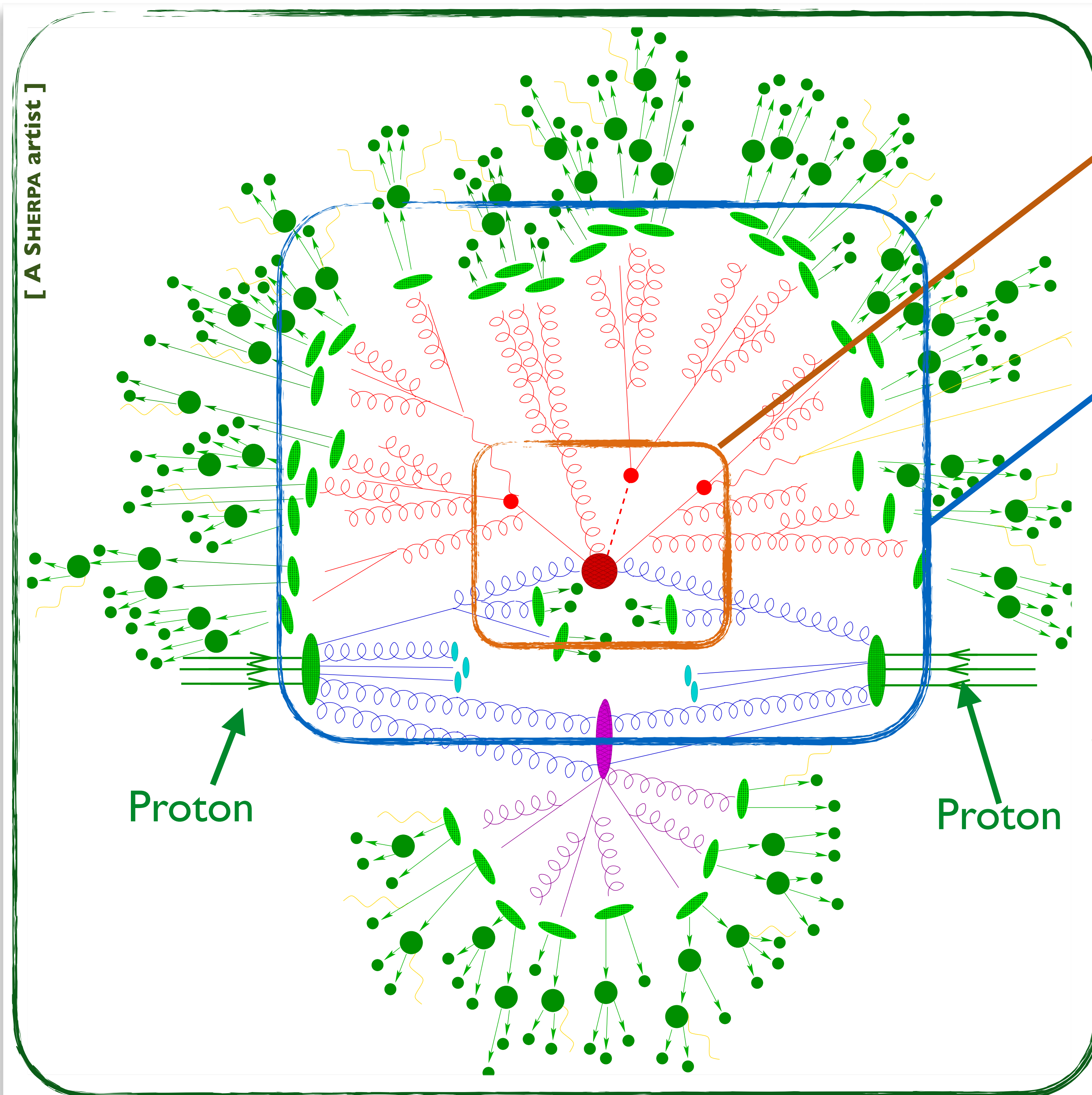

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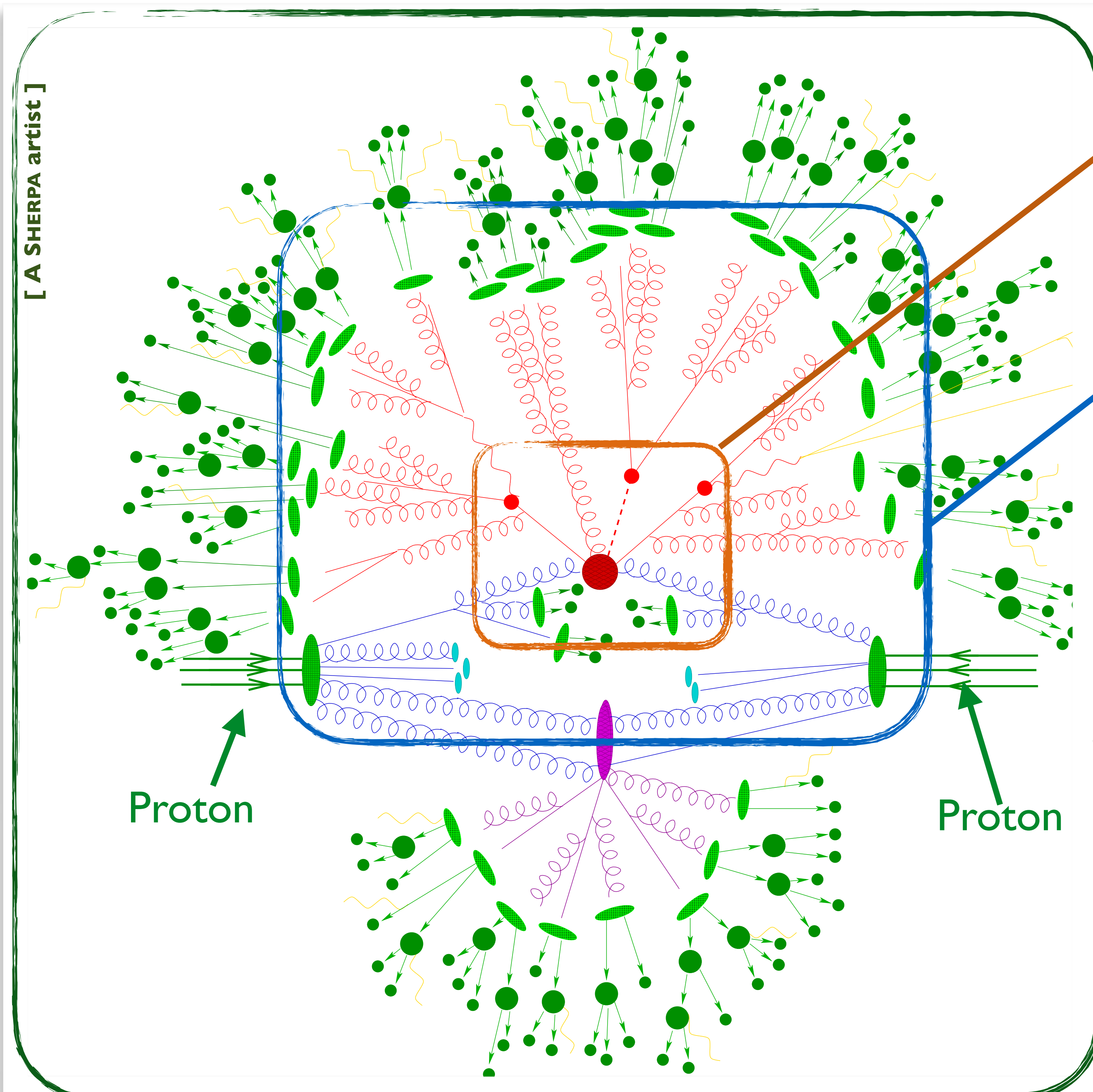

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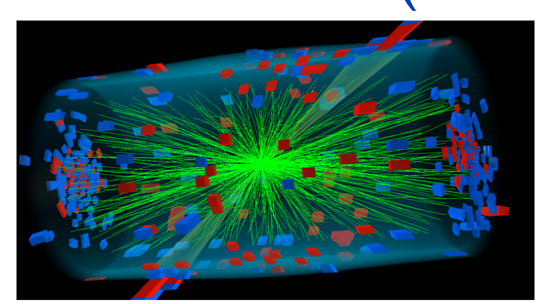
- ✦ Detector sim. (sub-MeV)

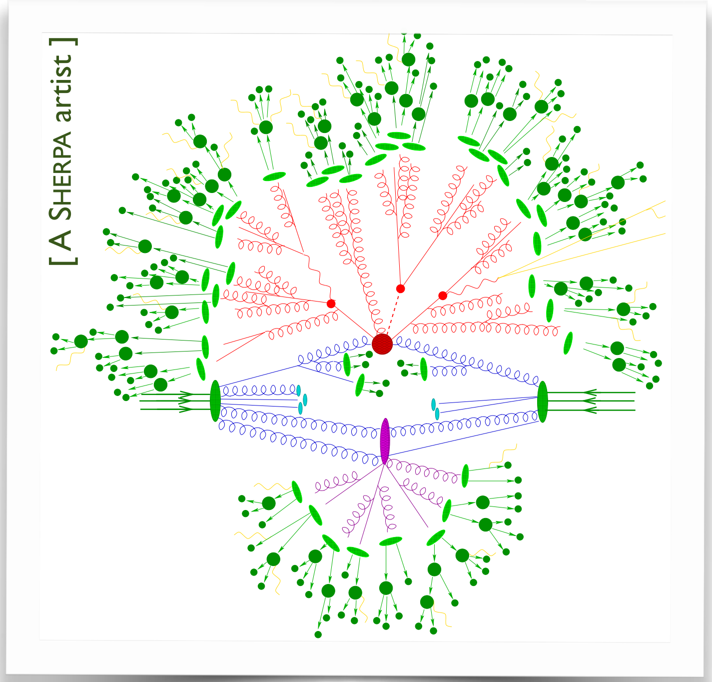


Monte Carlo simulations for proton collisions

✦ Multi-scale problem ↝ factorisation

- ✤ TeV scale: hard scattering (new physics?)
- ✤ Down to Λ_{QCD}: QCD environment
- ✤ Down to sub-MeV: interactions with a detector

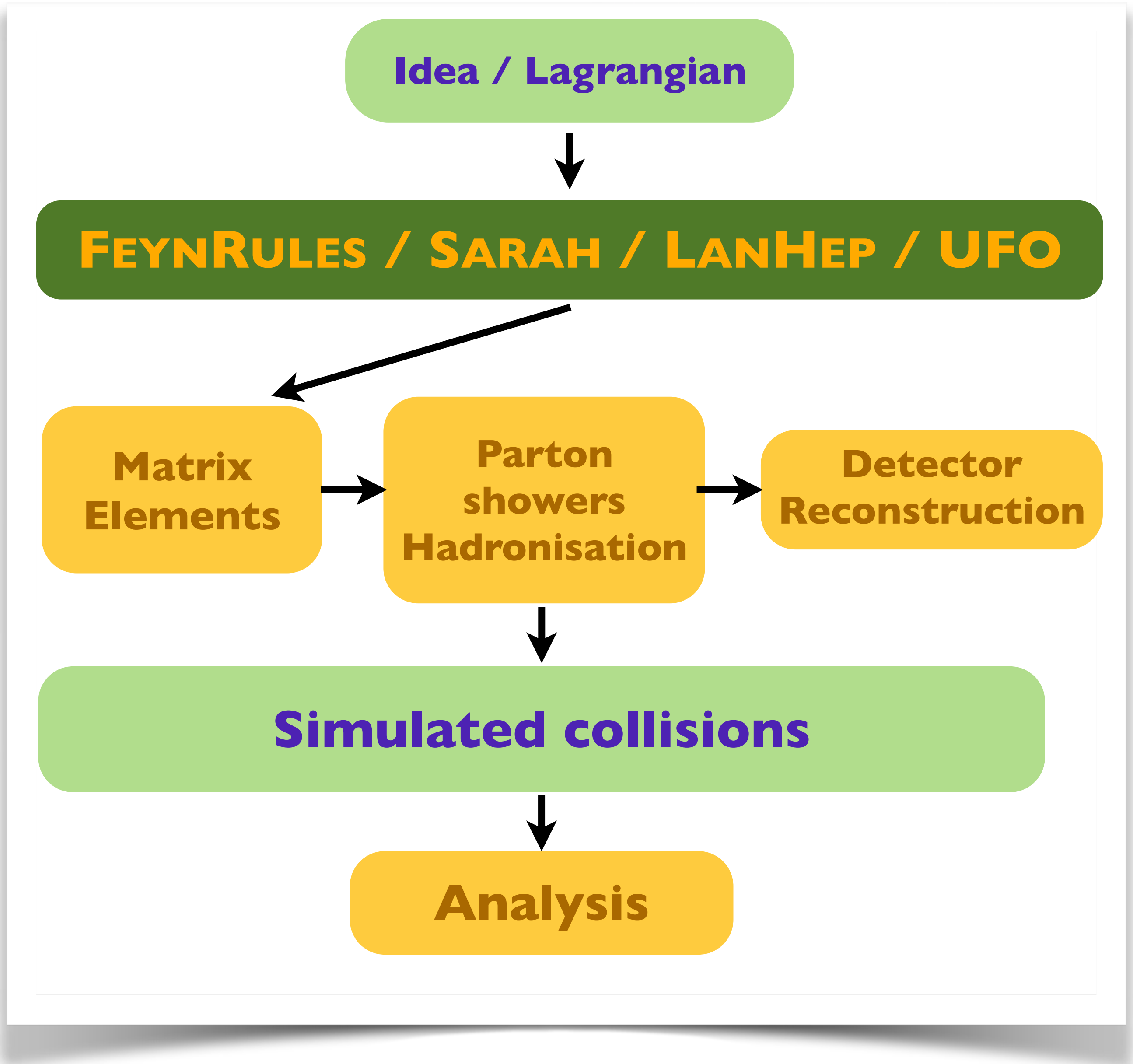
Tools and methods for each step



Making new physics a standard

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Tools connecting an idea to simulated collisions



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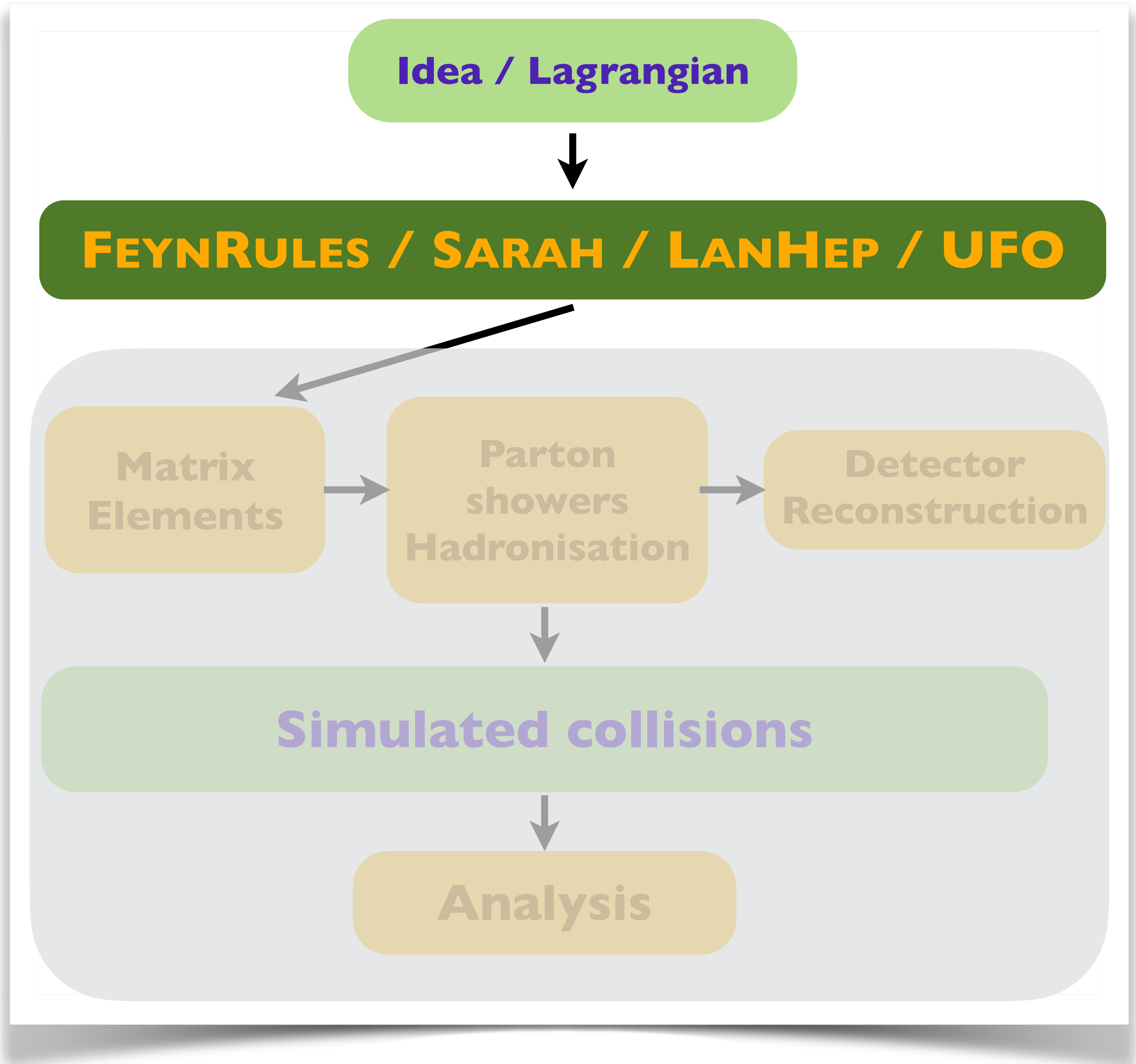

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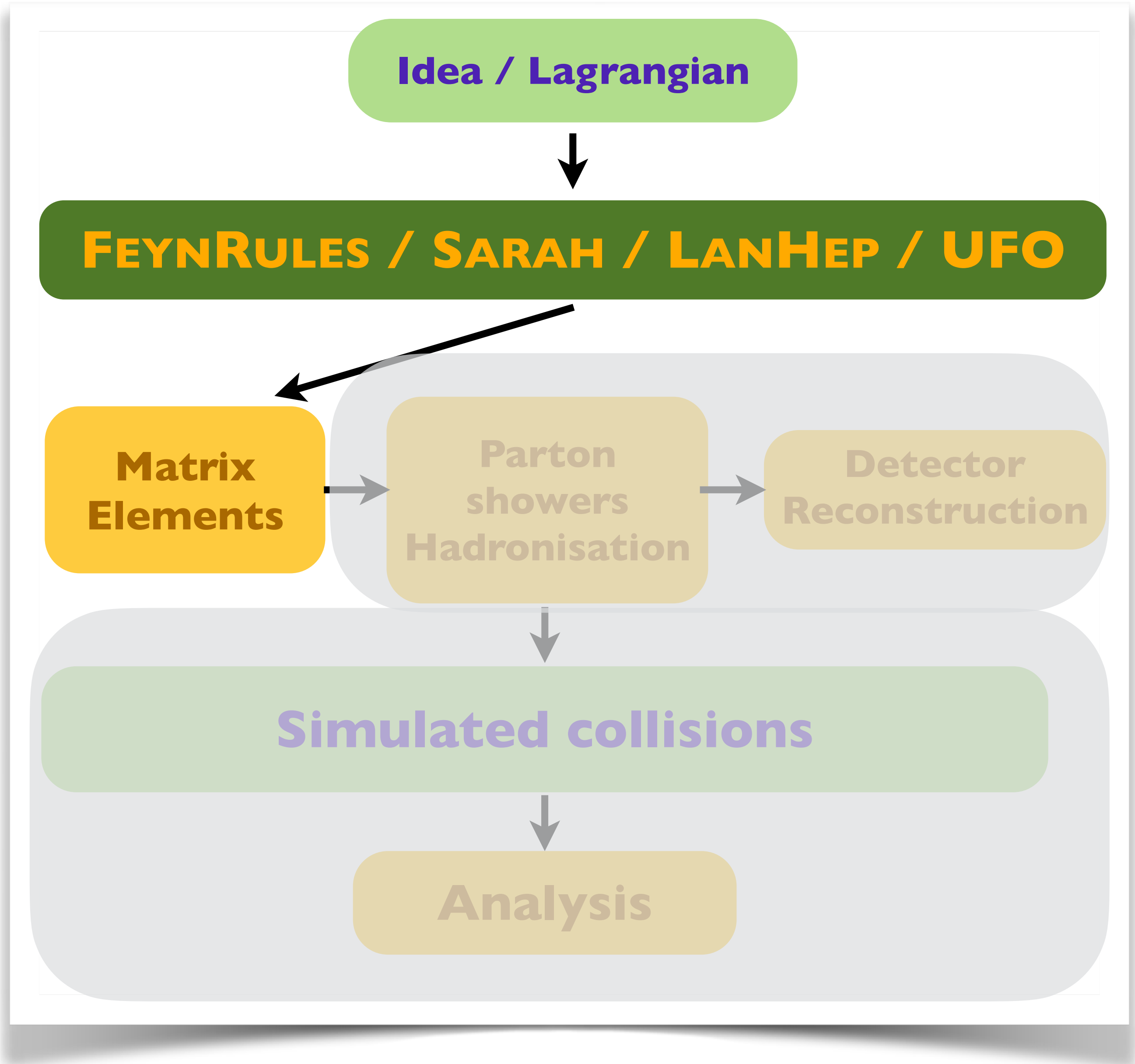

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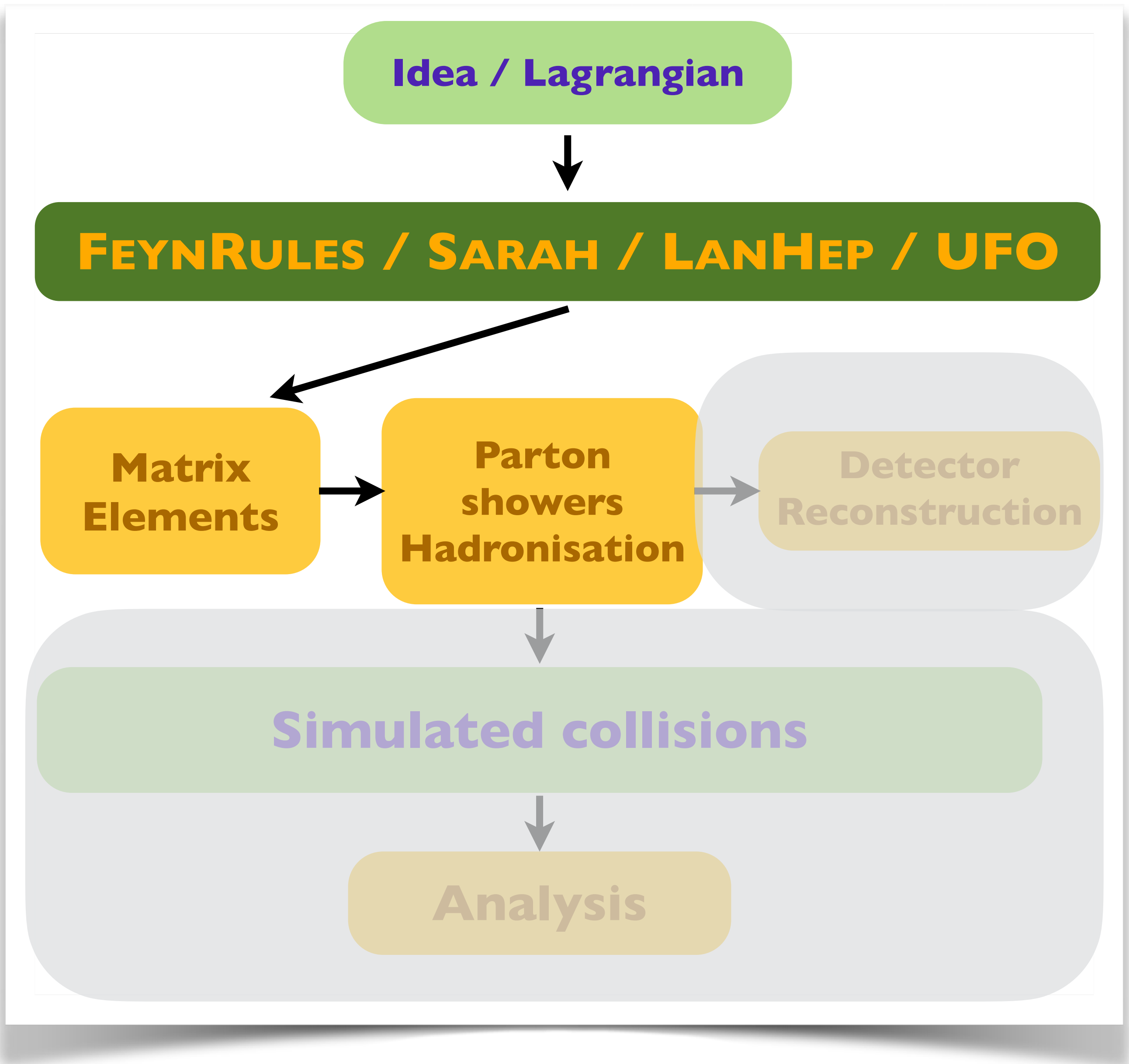

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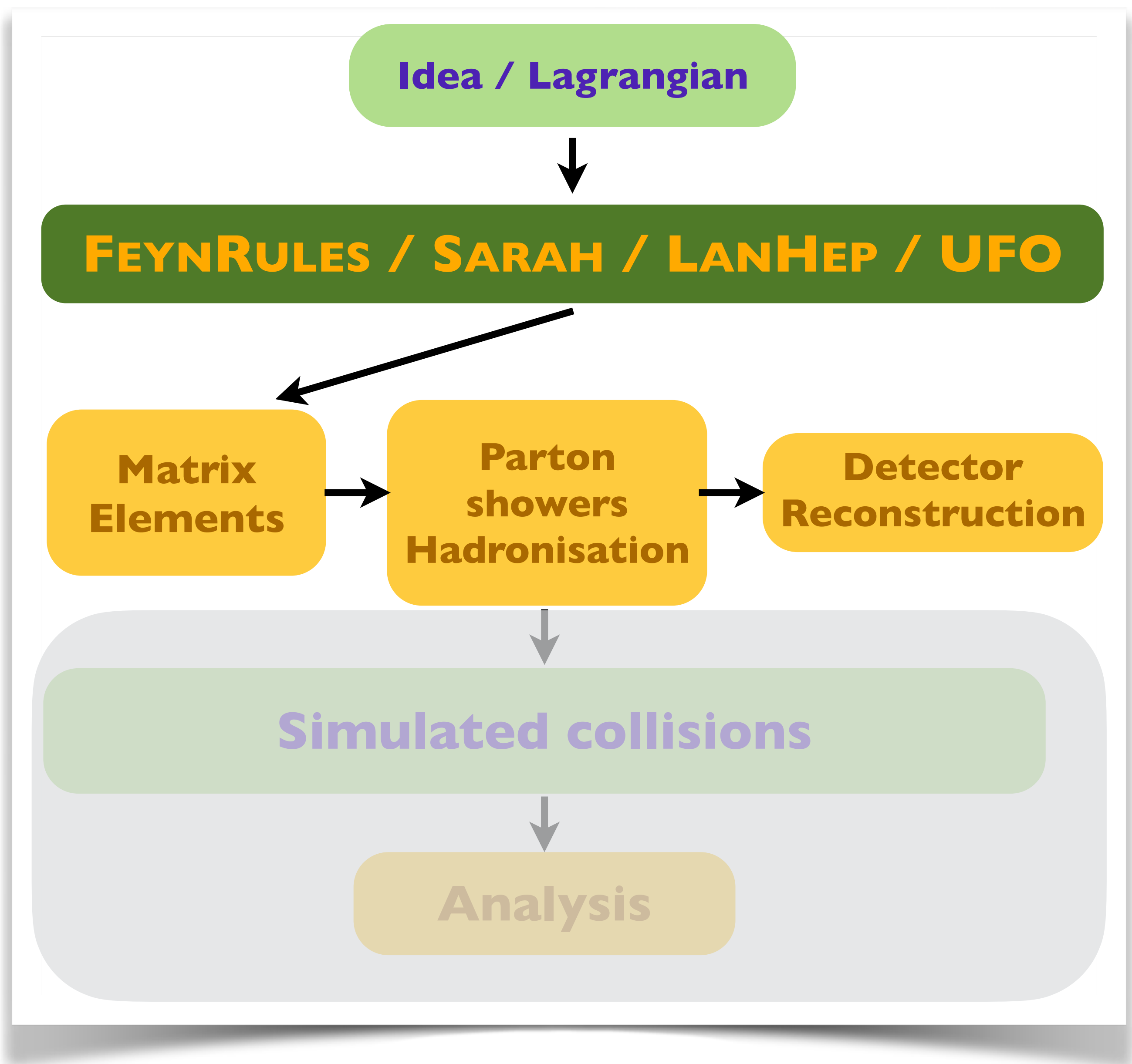

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- QCD environment
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- Detector simulation
  - ★ Simulation of the detector response
  - ★ Object reconstruction

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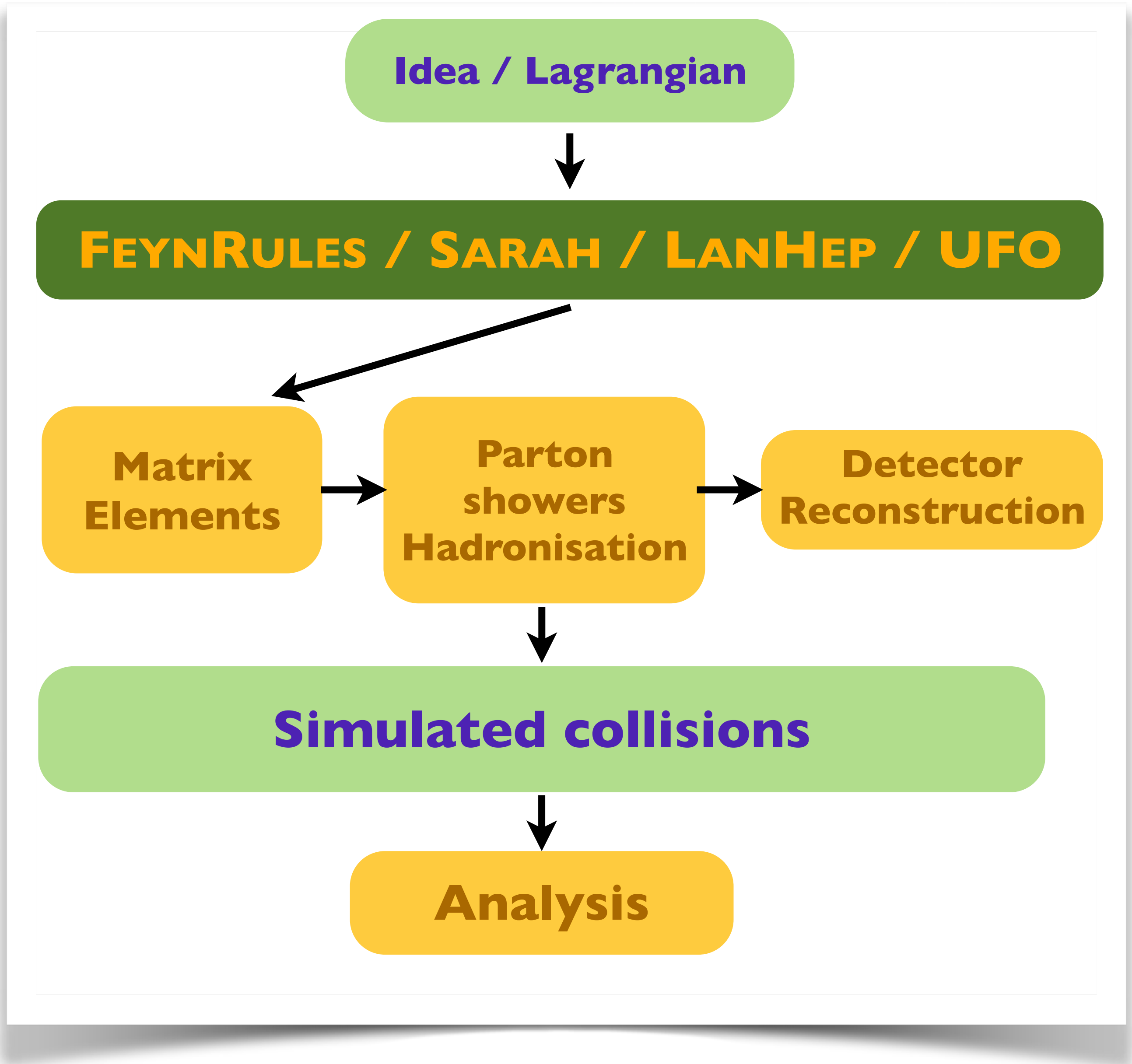

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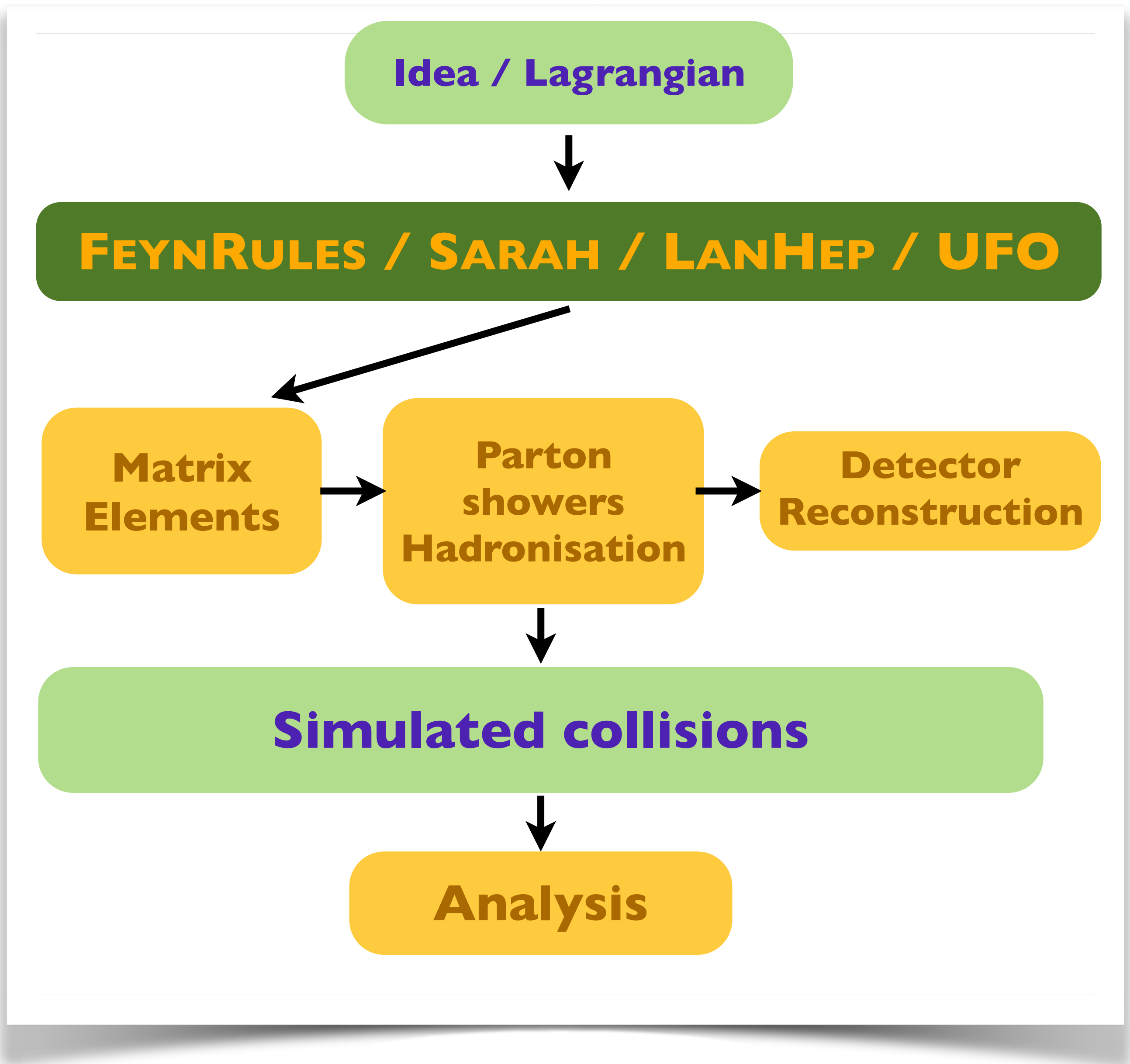
  - ★ Monte Carlo integration
  - ★ Events
- ✤ QCD environment
  - ★ Parton showering
  - ★ Hadronisation
  - ★ Underlying event
- ✤ Detector simulation
  - ★ Simulation of the detector response
  - ★ Object reconstruction
- ✤ Event analysis
  - ★ Signal/background analysis
  - ★ LHC recasting

Making new physics a standard

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Tools connecting an idea to simulated collisions



✤ Model building — Part 1

✤ Hard scattering
- ★ Feynman diagram / amplitude generation
- ★ Monte Carlo integration
- ★ Events

Part 2

✤ QCD environment
- ★ Parton showering
- ★ Hadronisation
- ★ Underlying event

Part 3

Outline

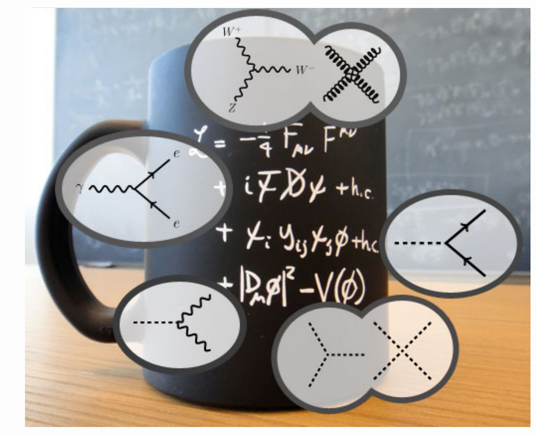
1. The Standard Model of particle physics and Monte Carlo simulations
2. **Implementing models into Monte Carlo event generators**
3. From models to hard-scattering events
4. Parton showers, hadronisation & underlying event
5. Summary

New physics simulations: the ‘how-to’

✦ Implementing new physics in Monte Carlo programs

✤ Definition: particles, parameters & vertices (≡ Lagrangian)

⇝ translated in some programming language



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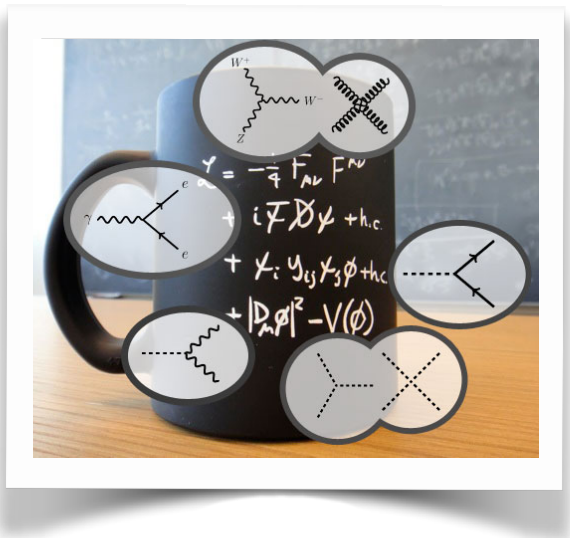

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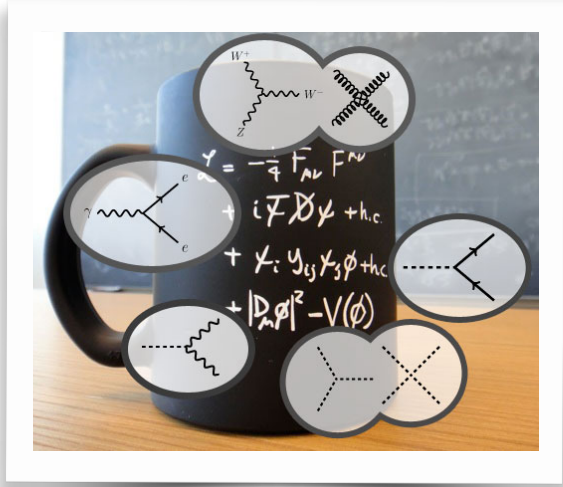

✦ Systematisation / automation

✤ Highly redundant (each tool, each model)

✤ No-brainer tasks (from Feynman rules to code)

From models to LO and NLO simulations

- The FEYNRULES platform (since 2009)
  - From Lagrangians to files in a programming language
    - Few limitations (spin, colour representation, EFT)
    - Renormalisation in the on-shell scheme

[Christensen & Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr & BF (CPC'14)]
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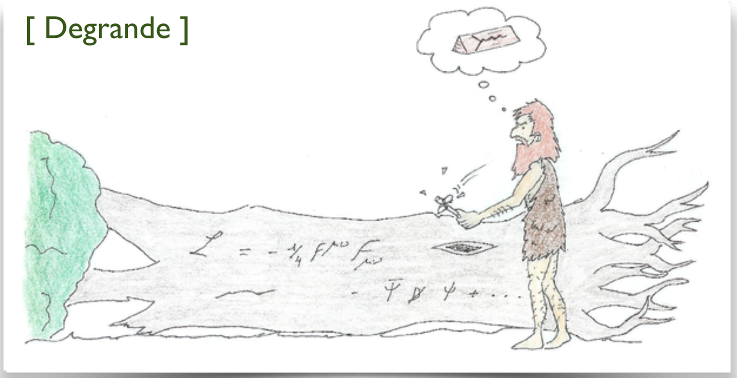
[Degrande]



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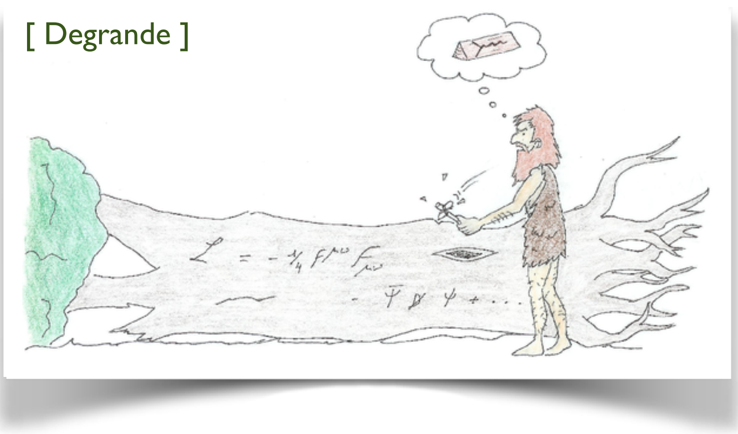

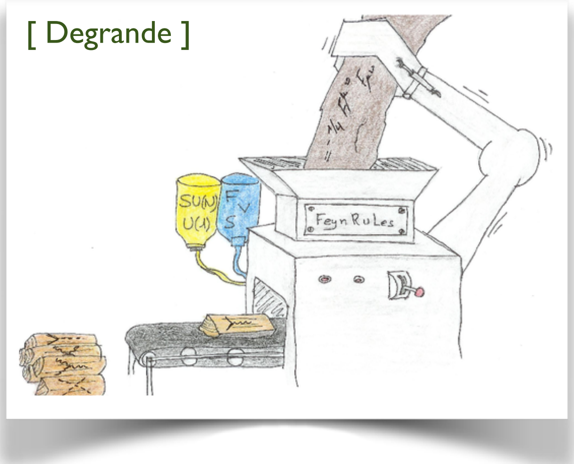

✦ Automation

✤ Working environment: MATHEMATICA

- ★ Flexibility, symbolic manipulations, design of new methods
- ★ Many built-in methods (superspace, spectrum, decays, NLO)

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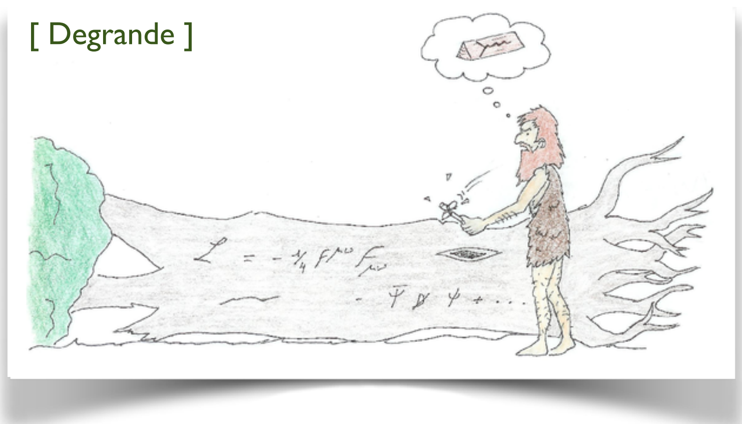

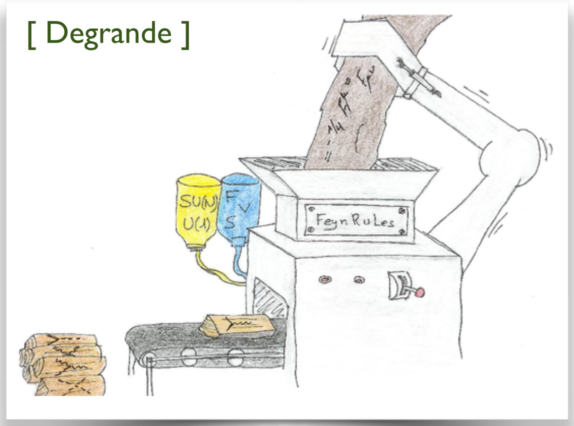

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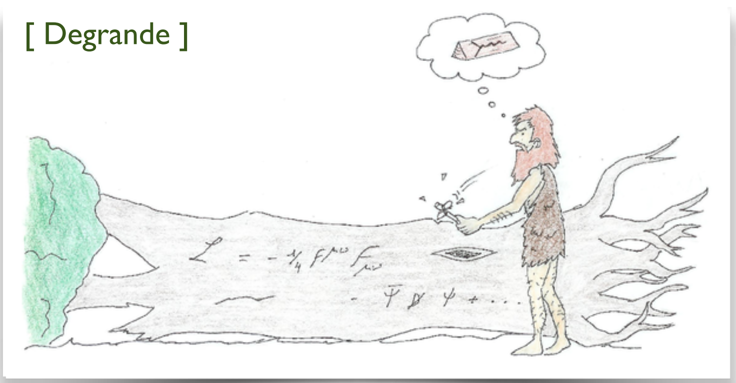

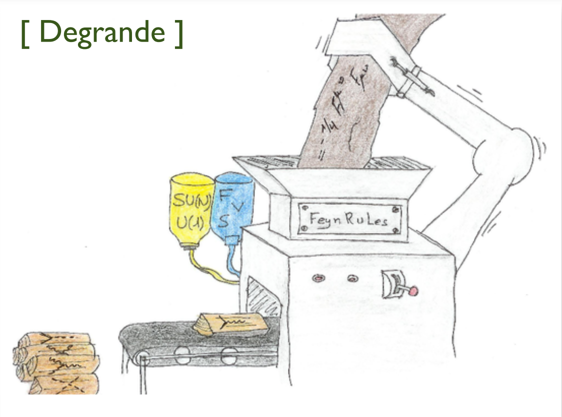

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✦ SARAH and LANHEP pursue a similar goal

- ✤ No NLO, different built-in methods, …

More about interfaces

- Each interface dedicated to a given tool is specific
  - Removal of vertices not compliant with the tool
    - Colour structures
    - Lorentz structures
  - Translation to a specific format and programming language

↝ not efficient
↝ a unique translation and the tools parse it

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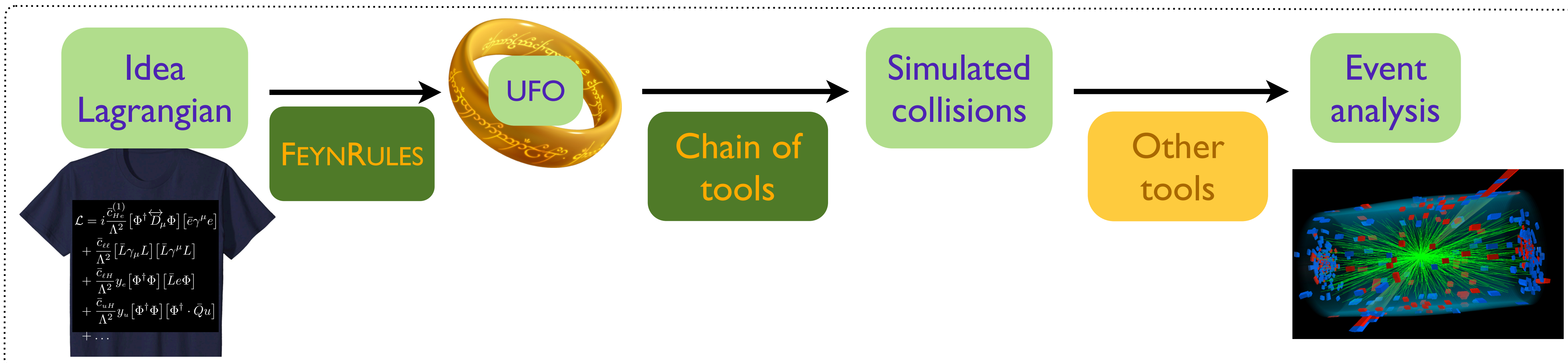
- ✦ One format to rule them all!



The Universal Feynman Output

✦ The UFO in a nutshell

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]
[Degrande, Duhr, BF, Hirschi, Mattelaer, Shao (*in prep.*)]

- ✤ UFO ≡ Universal FEYNRULES output ⇝ Universal Feynman Output
  - ★ Universal as not tied to any specific Monte Carlo program
- ✤ Set of PYTHON files to be linked to any code
- ✤ This module contains all the model information
  - ★ All colour/Lorentz structures
  - ★ NLO ingredients (optional: need for FEYNRULES)

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✦ The UFO is a standard

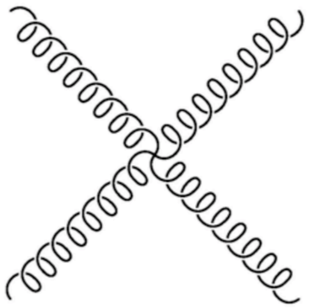
ALOHA | GOSAM | HERWIG ++ | MADANALYSIS 5 | SHERPA

MADGRAPH5_aMC@NLO | WHIZARD | LANHEP | SARAH

Interactions: the key strategy

✦ Decomposition in a spin x colour basis (coupling strengths ≡ coordinates)

✤ Example: the quartic gluon vertex



$$
\begin{aligned}
& ig_s^2\, f^{a_1a_2b} f^{ba_3a_4}\, \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}\right) \\
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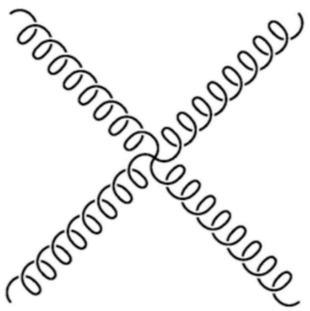

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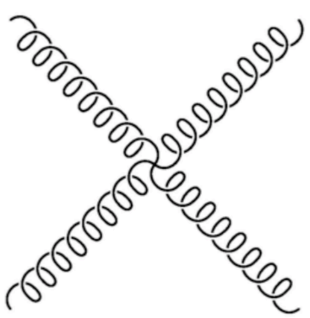
✤ UFO version



$$
\left(f^{a_1 a_2 b} f^{b a_3 a_4},\ f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3}\right)
\times \begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix}
\begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix}
$$

★ 3 elements for the colour basis
★ 3 elements for the spin (Lorentz structure) basis
★ 9 coordinates (6 are zero)

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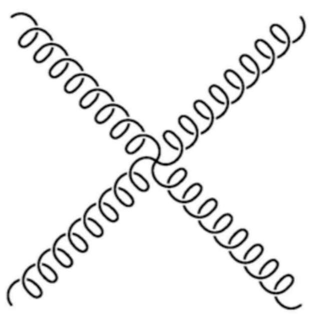

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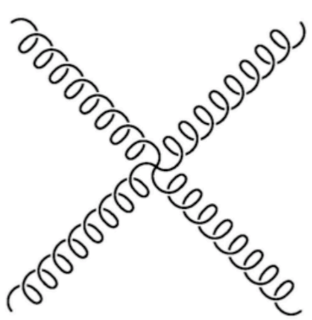

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$$

Recycling the duplicates in the implementation

[across vertices]

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- ★ 9 coordinates (6 are zero)

NLO cross sections

✦ Contributions to an NLO result in QCD

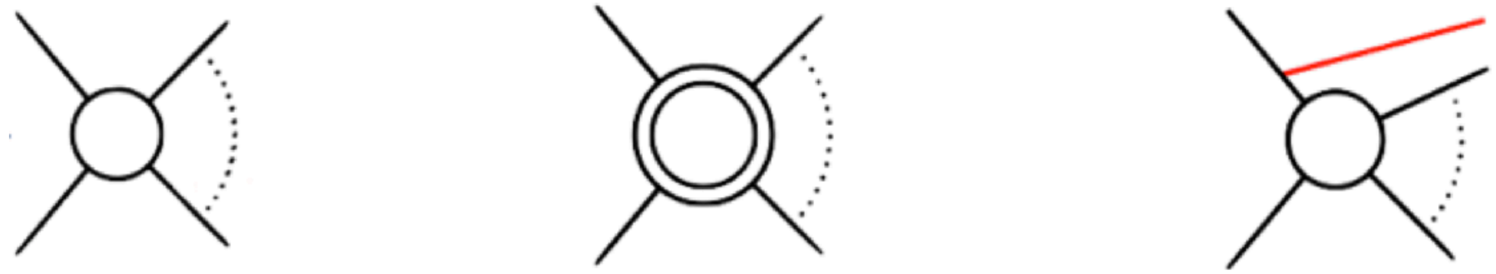
✤ Three ingredients: the Born, virtual loop and real emission contributions

$$\sigma_{NLO} = \int \mathrm{d}^4\Phi_n \mathcal{B} \quad + \quad \int \mathrm{d}^4\Phi_n \int_{\mathrm{loop}} \mathrm{d}^d\ell \; \mathcal{V} \quad + \quad \int \mathrm{d}^4\Phi_{n+1} \; \mathcal{R}$$

Born

Virtuals: one extra power of α_s and divergent

Reals: one extra power of α_s and divergent



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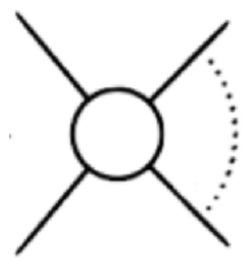

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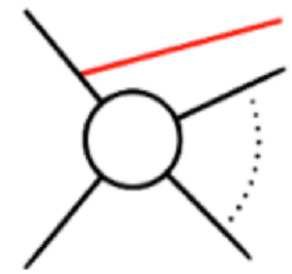


Extra information needed

Loop calculations

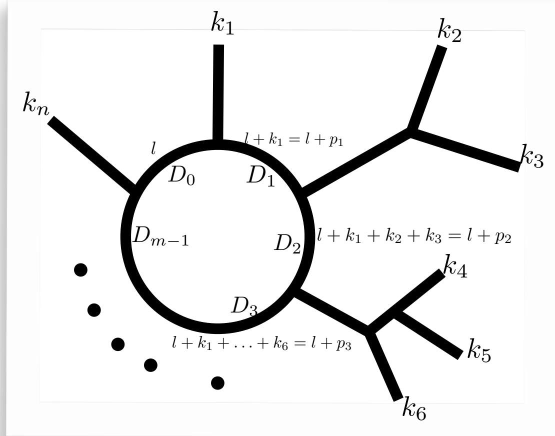
✦ Dimensional regularisation: calculations in $d = 4 - 2\varepsilon$

✤ Divergences explicit ($1/\varepsilon^2$, $1/\varepsilon$)

✤ Numerical methods work in 4 dimensions ⇝ R_1 / R_2 terms

$$\int d^d\ell \frac{N(\ell,\tilde{\ell})}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} \quad \text{with } \bar{\ell} = \ell + \tilde{\ell}$$

D-dim 4-dim (-2ε)-dim



[Ossala, Papadopoulos, Pittau (NPB'07; JHEP'08)]

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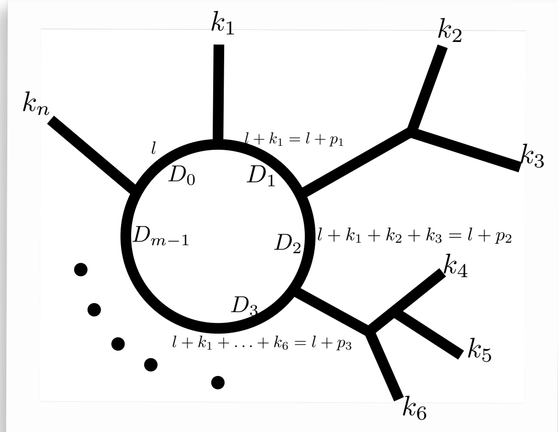

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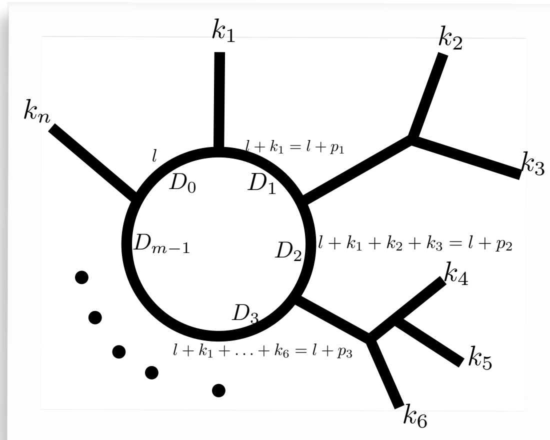

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✤ Renormalisable theory: finite number of R_2's

★ Seen as extra diagrams with special Feynman rules (⇝ *R_2 Feynman rules*)

★ Connected to the UV structure of the integrals (like the UV counterterms)

★ Can be derived from the bare Lagrangian ⇝ NLOCT

Loop calculations

- ✦ Dimensional regularisation: calculations in $d = 4 - 2\varepsilon$
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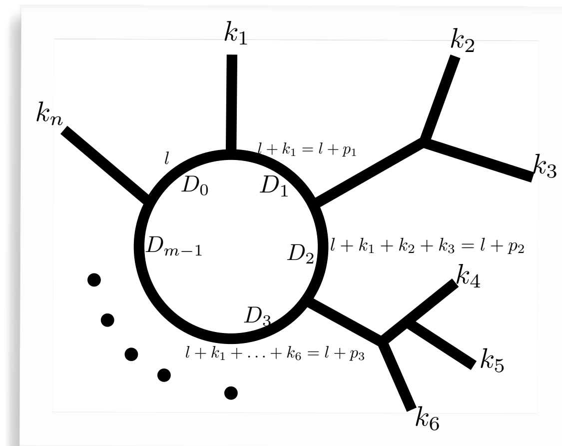

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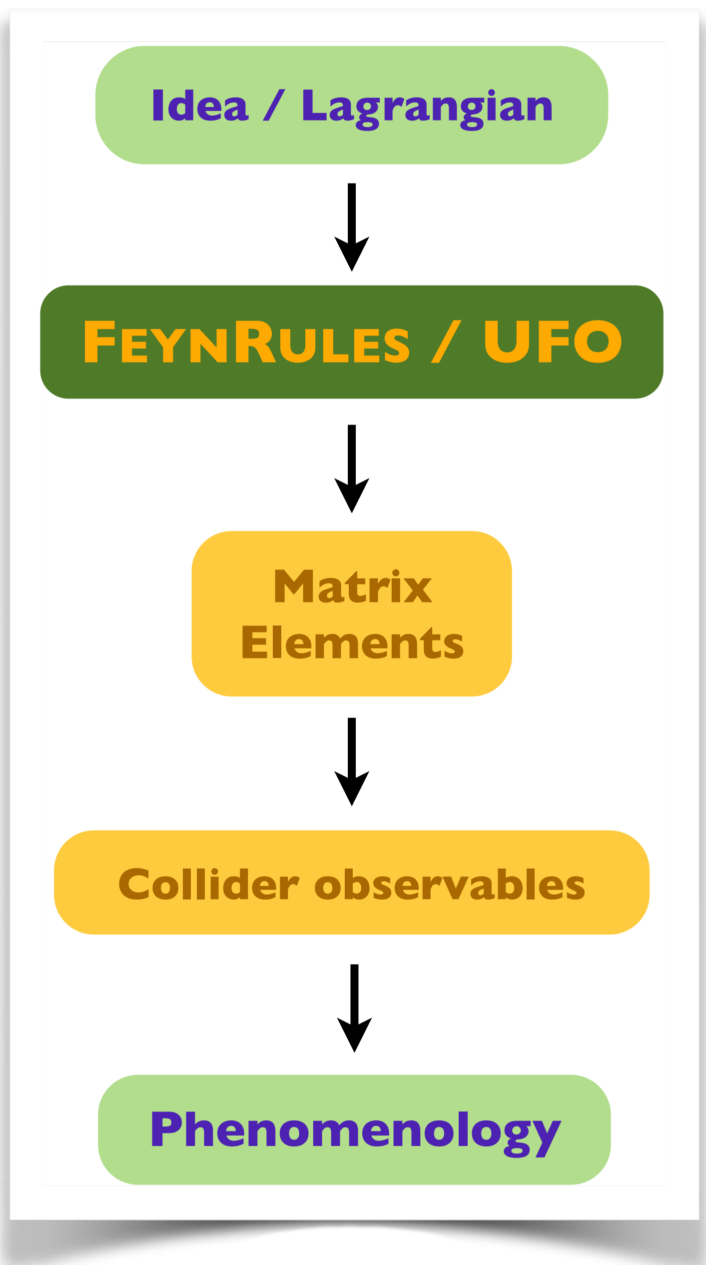
UFO @ NLO

Automated NLO simulations



✦ Model building: from Lagrangian to tools

✤ FEYNRULES $\oplus$ MOGRE $\oplus$ NLOCT $\rightsquigarrow$ UFO @ NLO

✤ General on-shell renormalisation scheme

[Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ; Degrande (CPC'15)]
[Degrande, Duhr, BF, Mattelaer & Reither (CPC'12)]
[Frixione, BF, Hirschi, Mawatari, Shao, Sunder & Zaro (JHEP`19)]

✦ Hard scattering

✤ Feynman diagram, matrix elements

✤ MG5aMC $\rightsquigarrow$ predictions at LO/NLO

[Alwall *et al.* (JHEP'14)]

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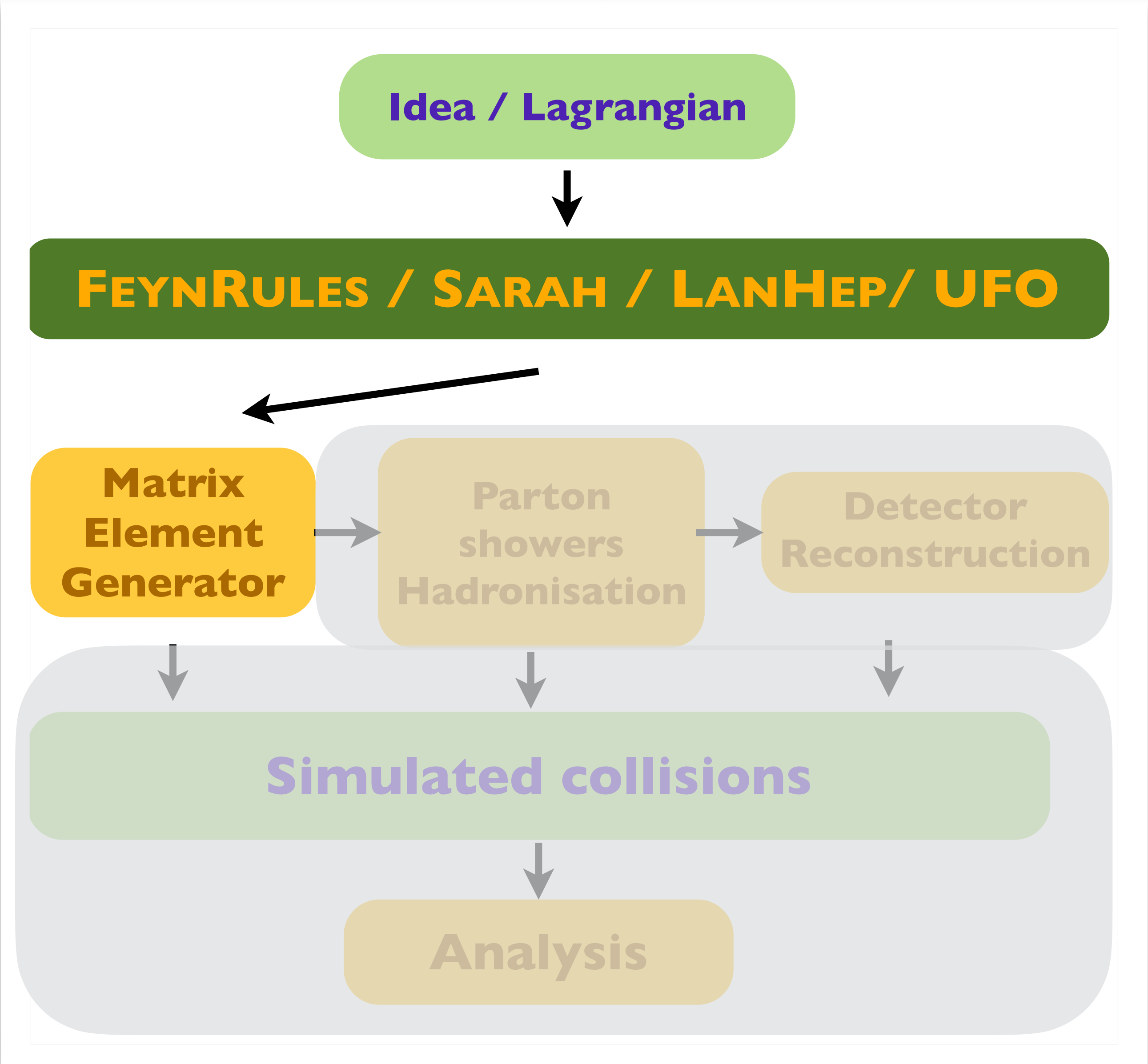
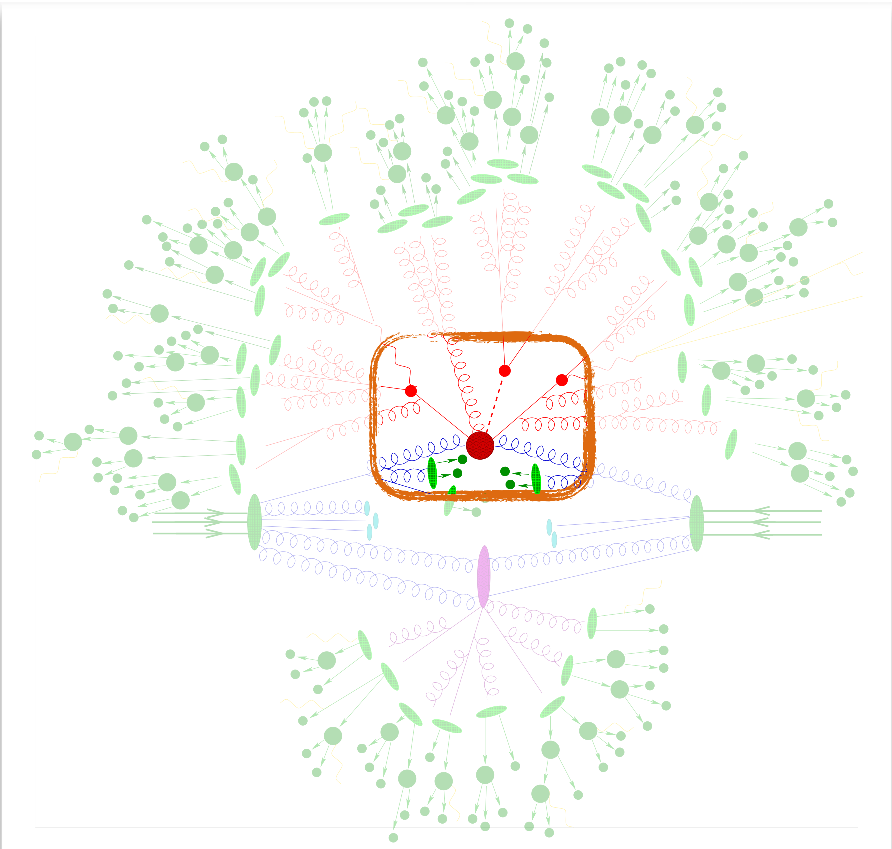
Back to the simulation chain

✦ Tools connecting an idea to simulated collisions



✤ Hard scattering process
- ★ Feynman diagram / amplitude generation
- ★ Monte Carlo integration
- ★ Event generation



QCD 101: predictions at the LHC

✦ Distribution of an observable ω: the QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{ab} \int \mathrm{d}x_a \, \mathrm{d}x_b \, \mathbf{f_{a/p_1}}(x_a; \mu_F) \, \mathbf{f_{b/p_2}}(x_b; \mu_F) \, \frac{\mathrm{d}\sigma_{\mathbf{ab}}}{\mathrm{d}\omega}(\ldots, \mu_F)$$

✤ Long distance physics: the parton densities

✤ Short distance physics: the differential parton cross section $\mathrm{d}\sigma_{ab}$

✤ Separation of both regimes ⇝ the factorisation scale μ_F

★ Choice of the scale ⇝ theoretical uncertainties

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✦ Short distance physics: the partonic cross section

✤ Order by order in perturbative QCD: $\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \alpha_s \, \mathrm{d}\sigma^{(1)} + \ldots$

★ More orders ⤳ more precision

★ Truncation of the series and α_s ⤳ theoretical uncertainties

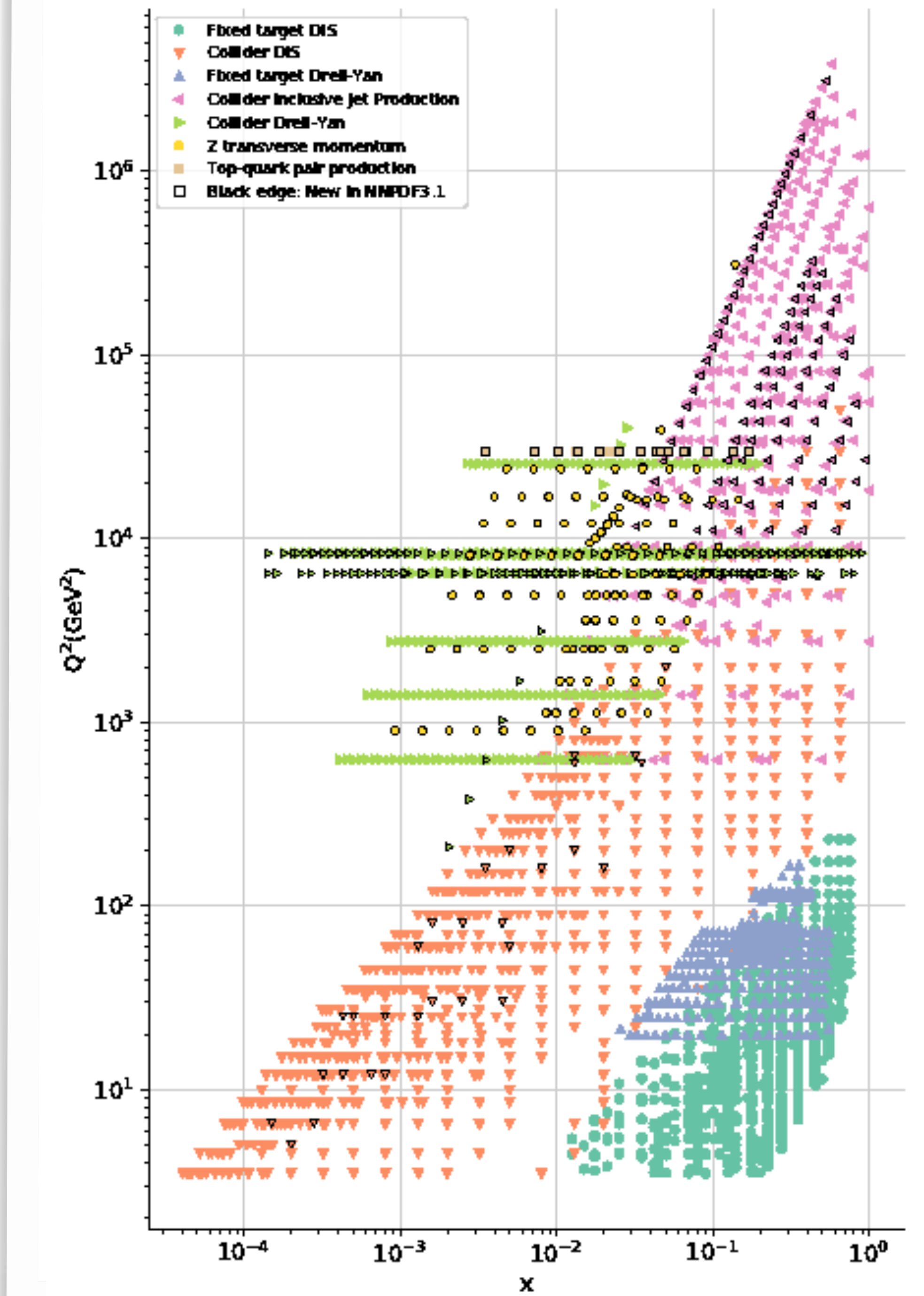
Feynman diagrams (from UFOs)

Parton densities

✦ Long distance physics: parton densities

✤ Relate hadrons to their content

[NNPDF 3.0 (EPJC'17)]



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✤ Depend on the momentum fraction *x* of the parton in the proton

✤ Depend on a scale Q

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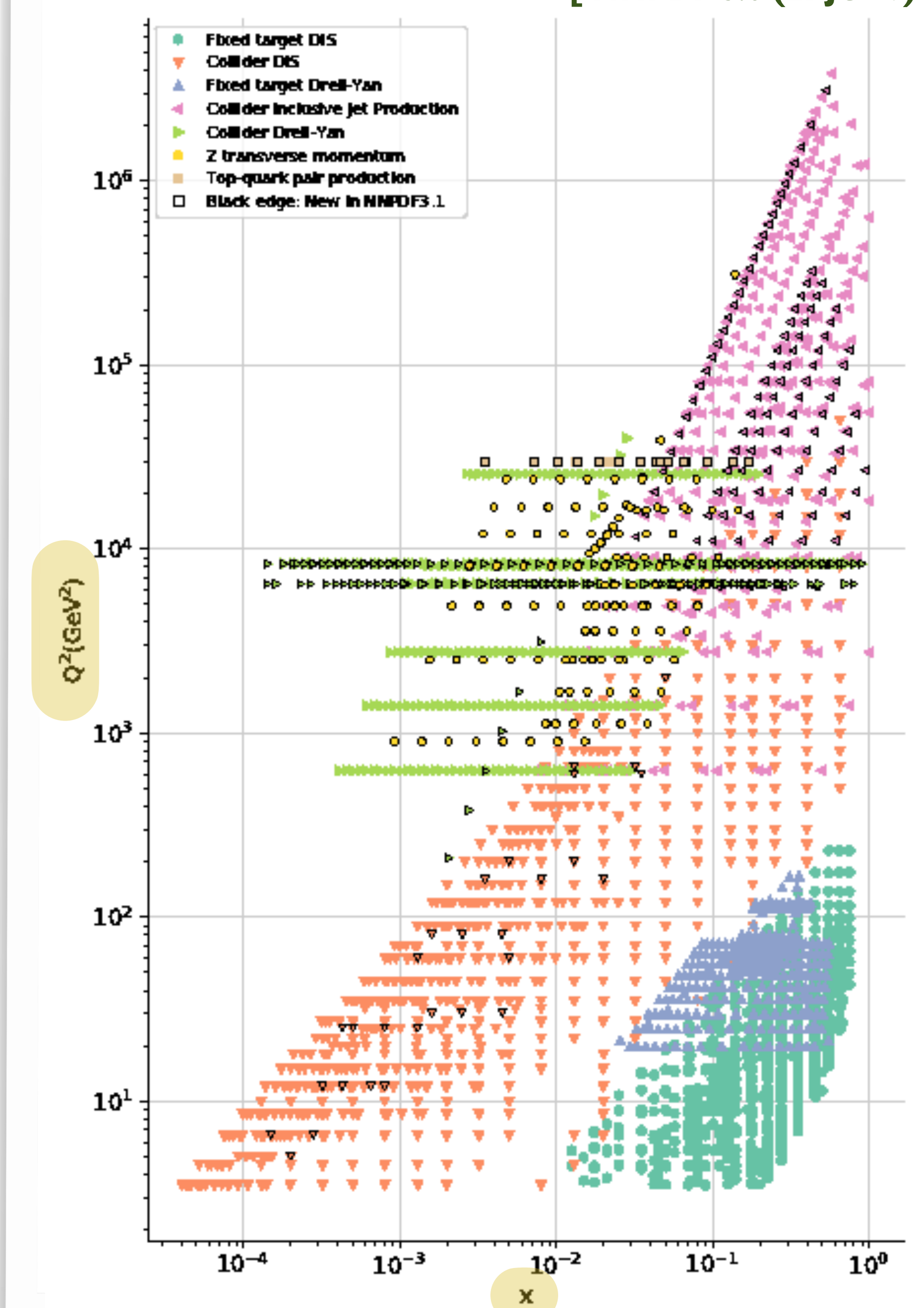

Parton densities

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✤ Depend on the momentum fraction *x* of the parton in the proton

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✤ Fitted from experimental data [in some kinematical regimes (x,Q)]

✤ Evolution driven by QCD (DGLAP/BFKL)

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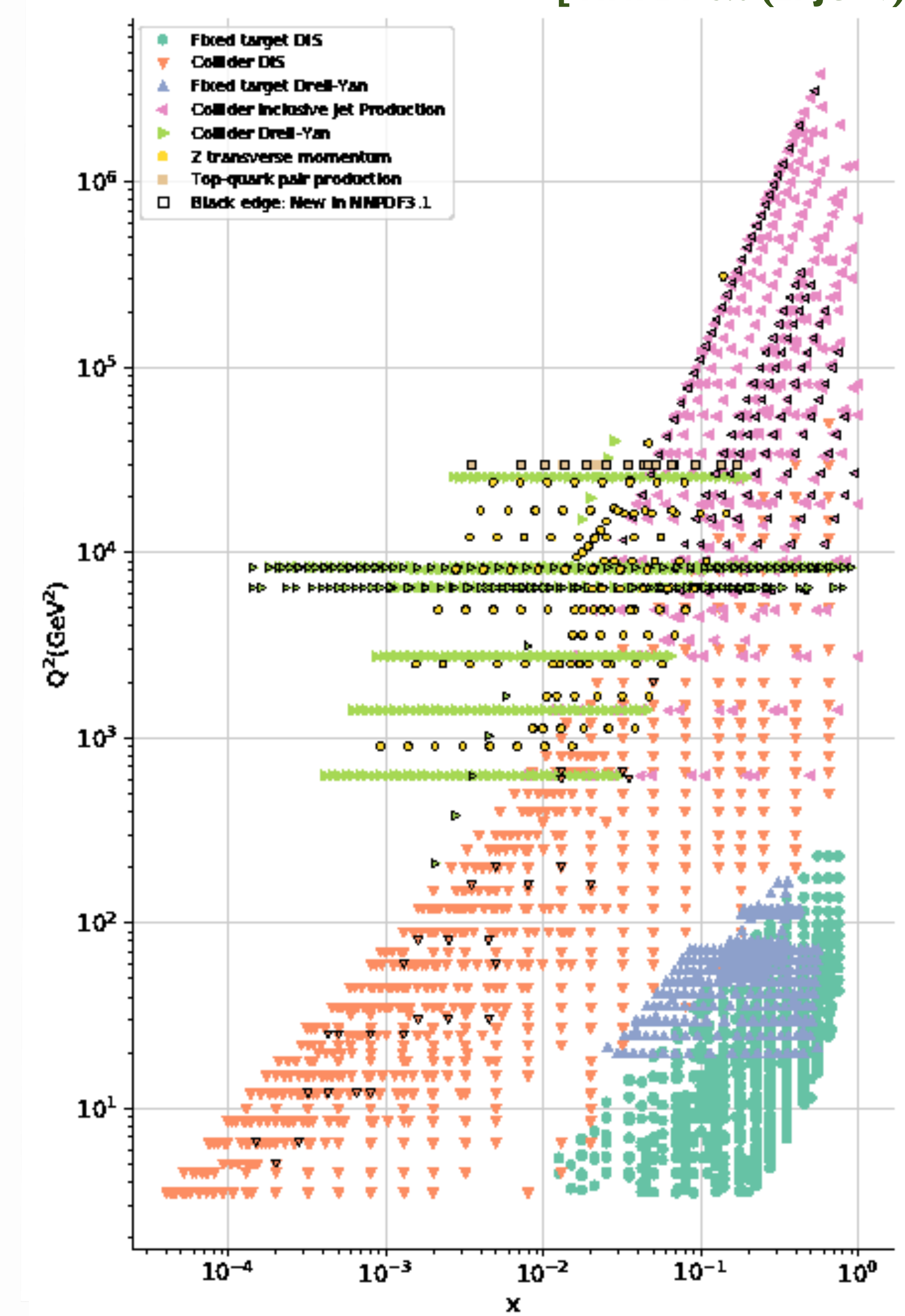
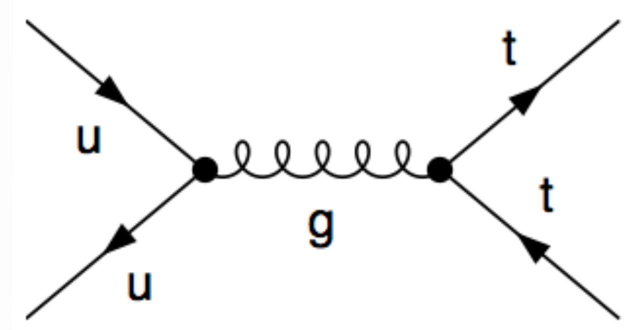


Feynman diagram calculations

- Direct squared matrix element computations
  - Extraction of the amplitude from the Feynman rules

$$i\mathcal{M} = ig_s^2 \left[\bar{v}_2\gamma^\mu u_1\right] \frac{\eta_{\mu\nu}}{s} \left[\bar{u}_3\gamma^\nu v_4\right] T^a_{c_2c_1} T^a_{c_3c_4}$$



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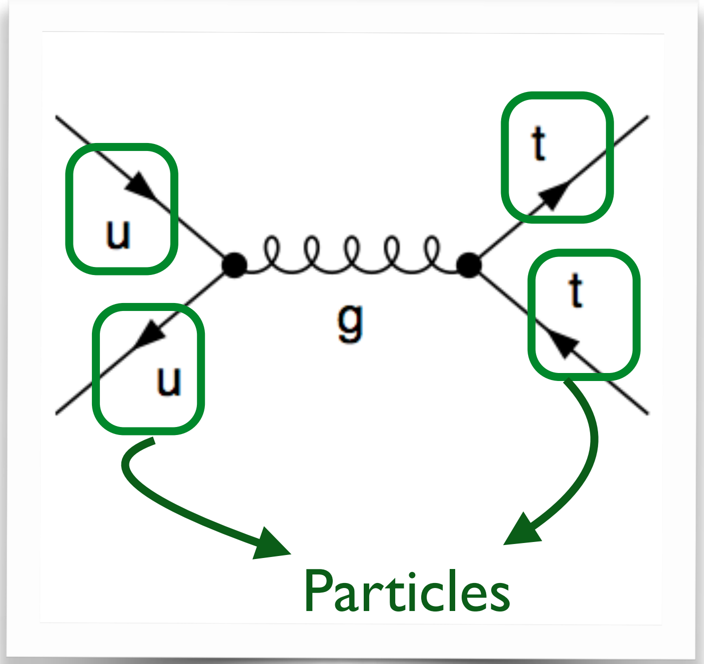

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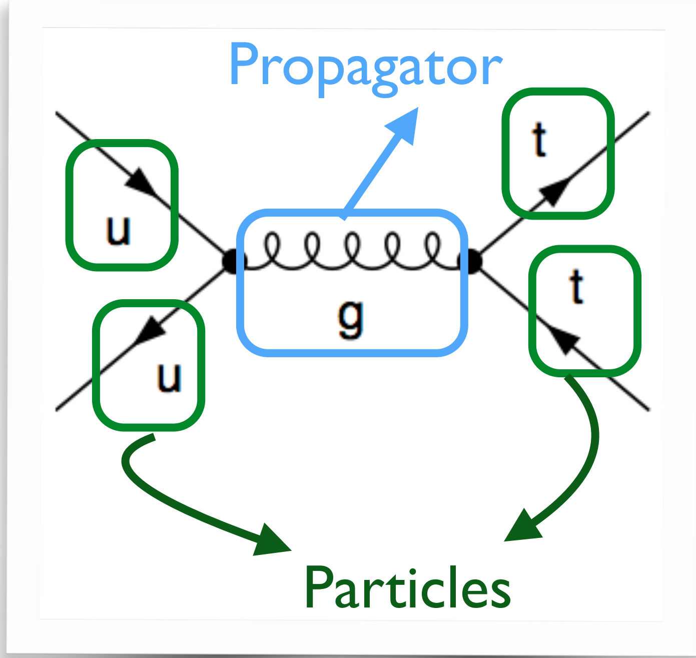

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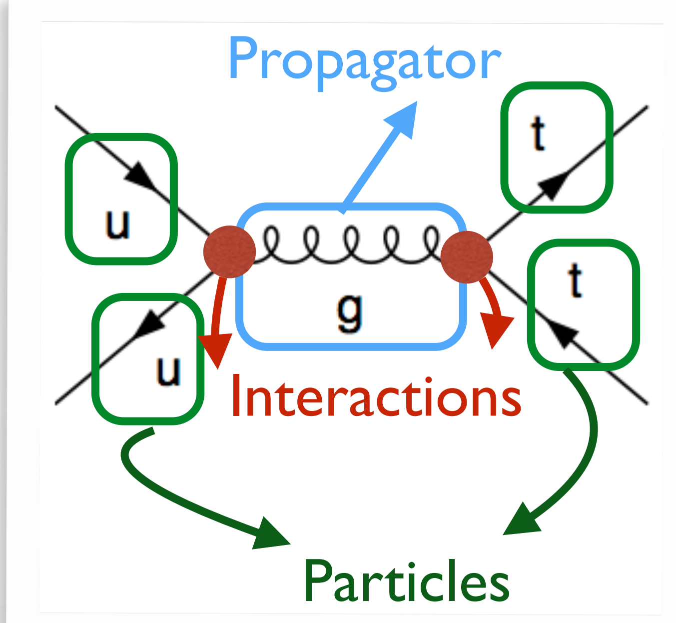

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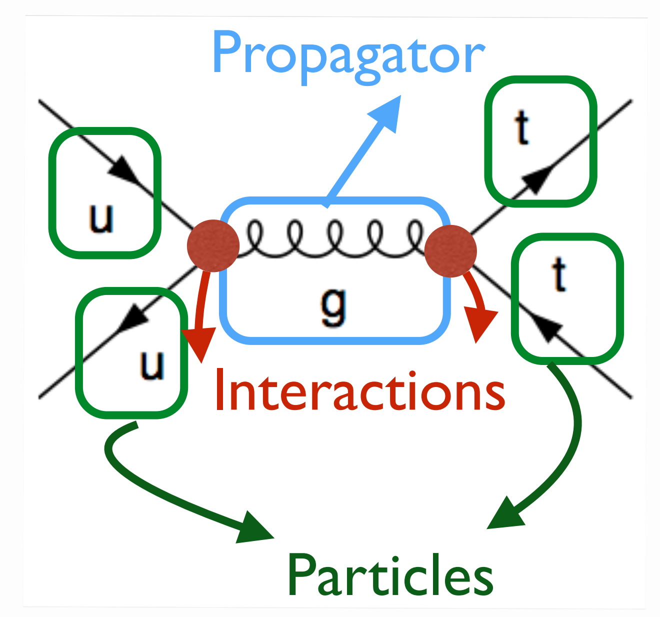

✤ Squaring with the conjugate amplitude

✤ Algebraic calculation (colour and Lorentz structures)

✤ Sum/average over the external states

$$\overline{|\mathcal{M}|^2} = \frac{1}{36}\frac{2g_s^4}{s^2}\mathrm{Tr}\left[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu\right]\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right]$$
$$= \frac{16g_s^4}{9s^2}\left[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right]$$

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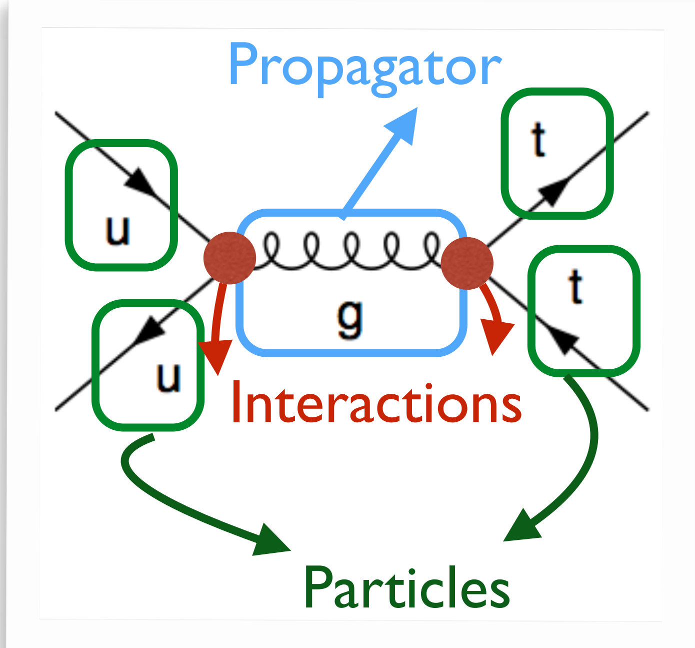


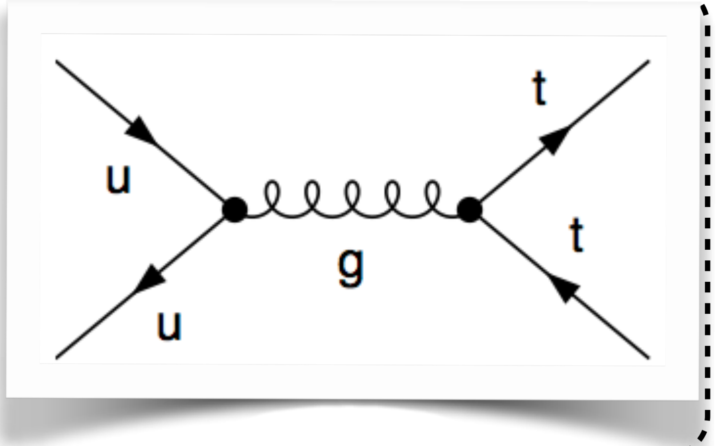
- The number of diagrams increases with the number of final-state particles
  - The complexity rises as N^2
  - Any calculation beyond 2-to-3 becomes a problem

➢ **Helicity amplitudes**

Helicity amplitudes

✦ Principle

✤ Evaluation of the amplitude for fixed external helicities
✤ Add all amplitudes (we get complex numbers)
✤ Squaring
✤ Sum/average over the external states



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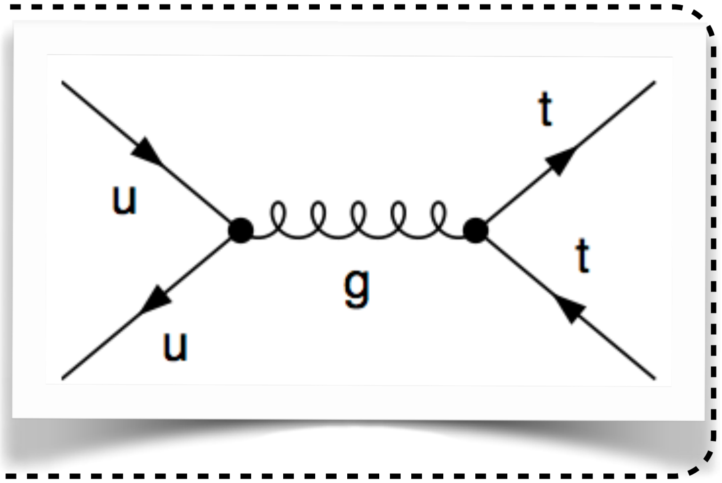

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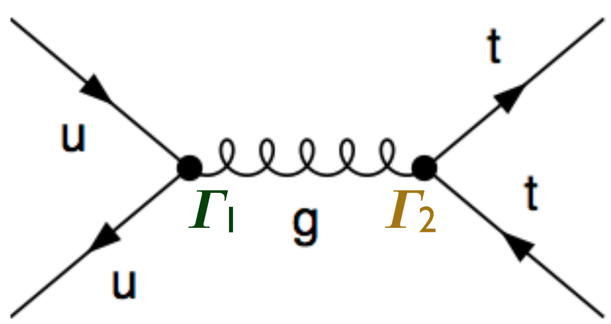

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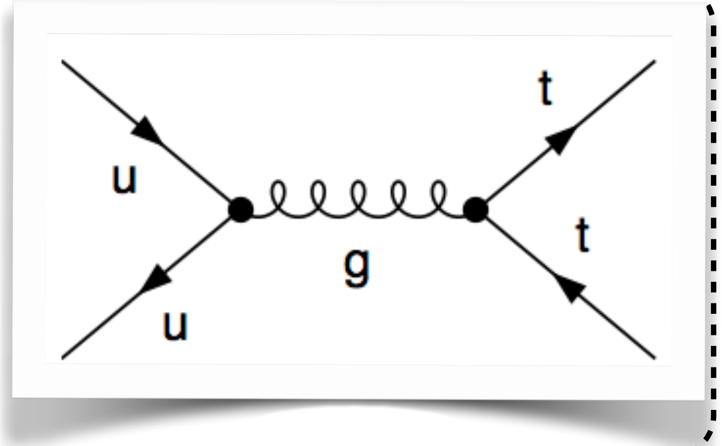

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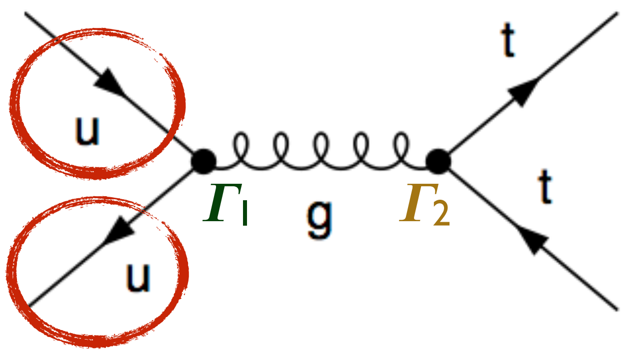

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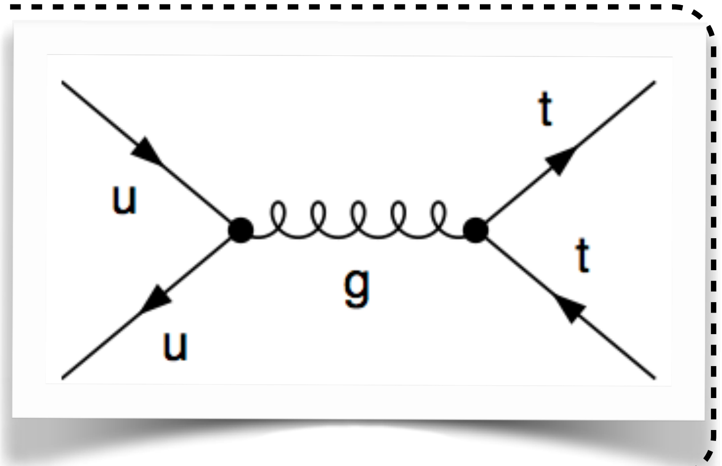

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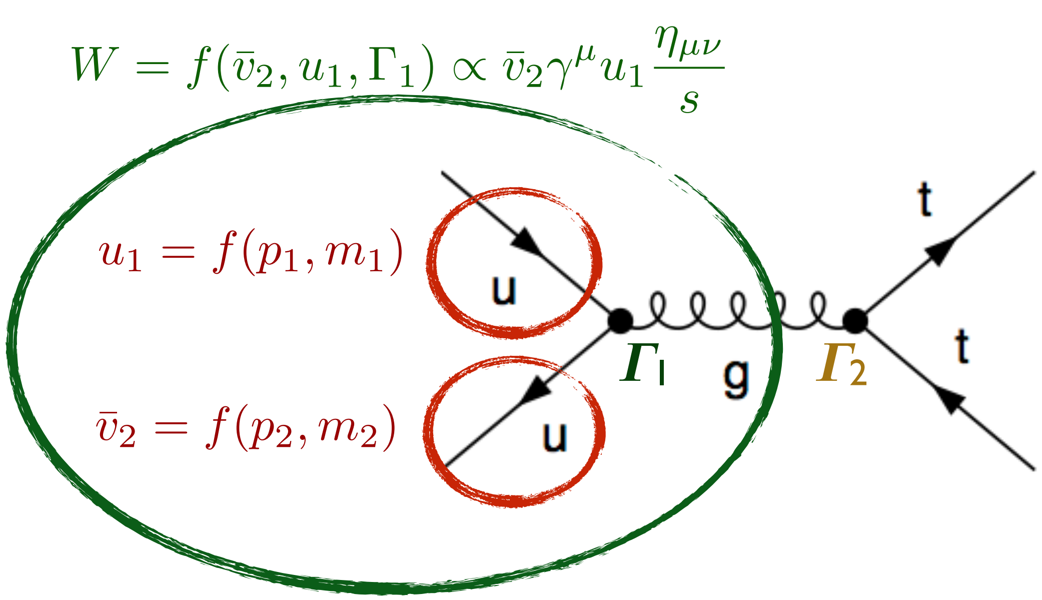

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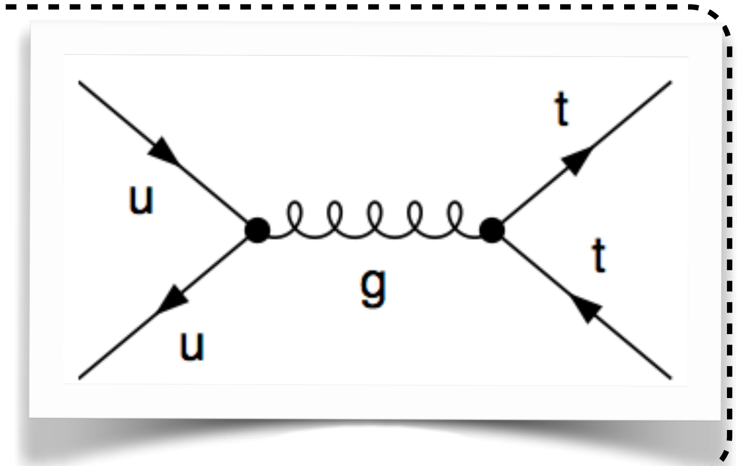

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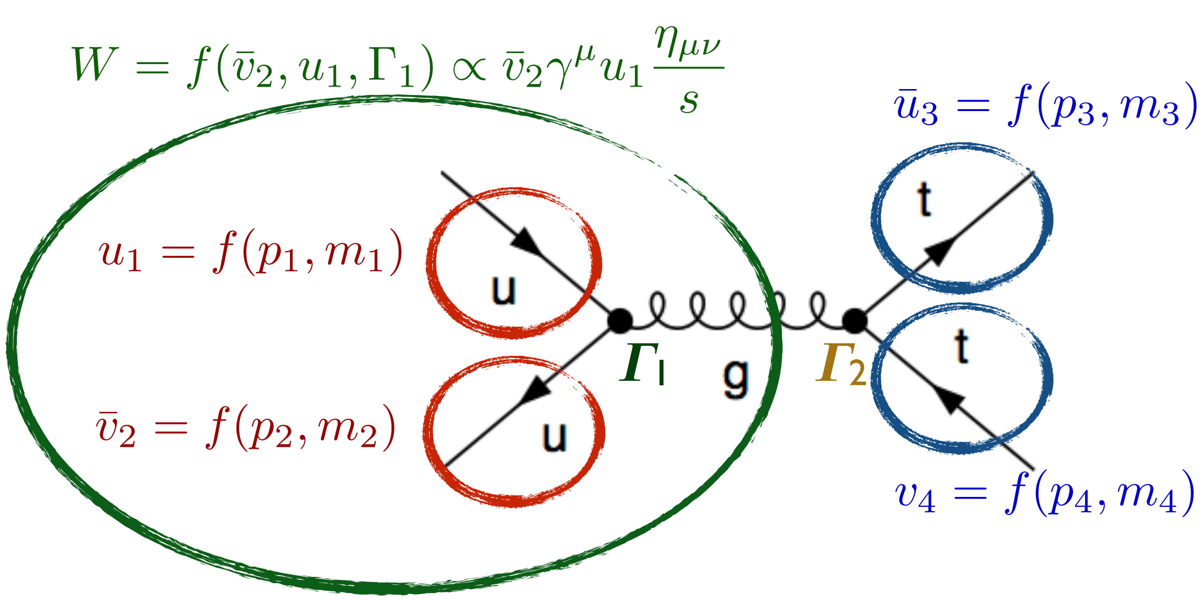

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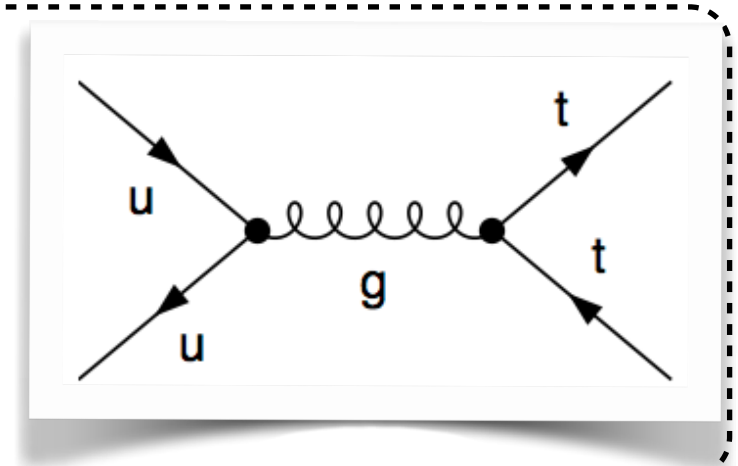

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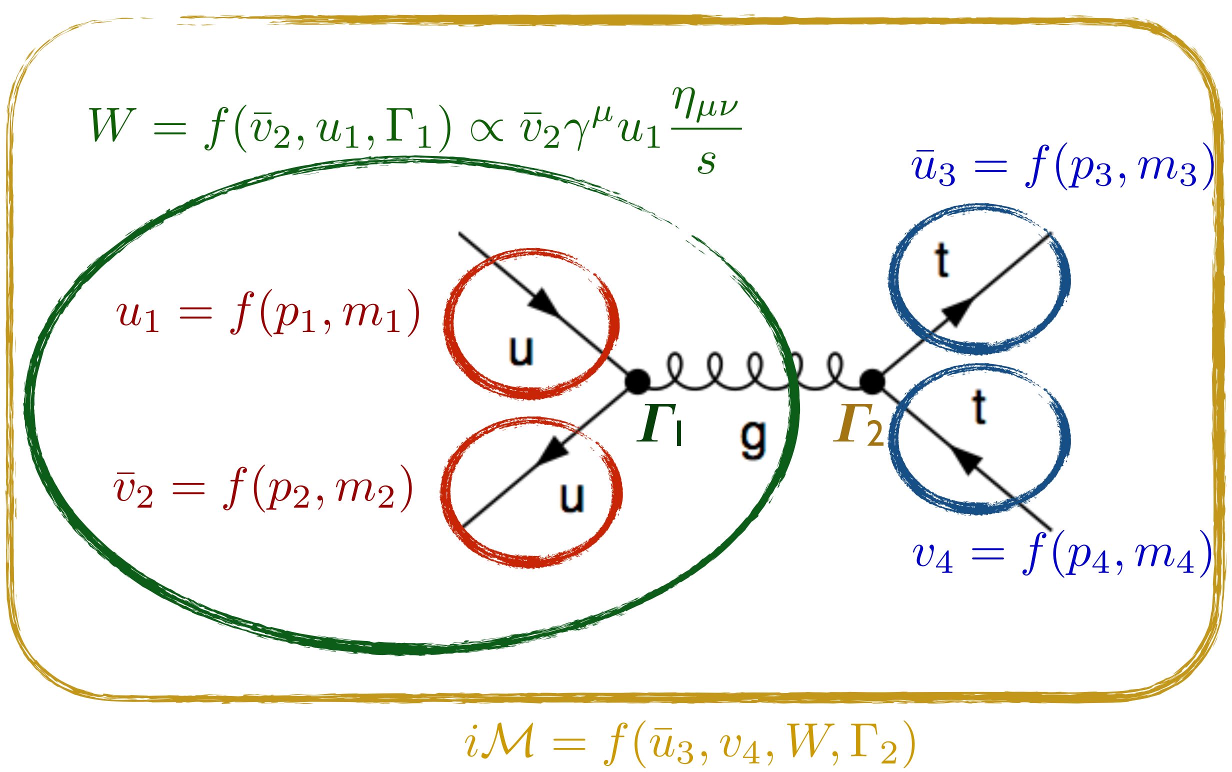

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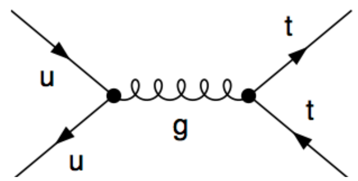
HELAS

✦ The building blocks of the amplitude are the so-called HELAS functions



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$$W = f(\bar{v}_2, u_1, \Gamma_1)$$
$$i\mathcal{M} = f(\bar{u}_3, v_4, W, \Gamma_2)$$

- HELAS ≡ HELicity Amplitude Subroutine
- One specific routine for each Lorentz structure (Γ_i)
- Not generic for any model
  - ★ SM [Murayama,Watanabe & Hagiwara (KEK-91-11)]
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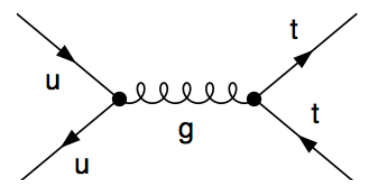

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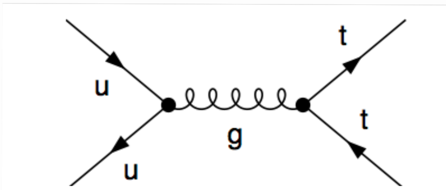

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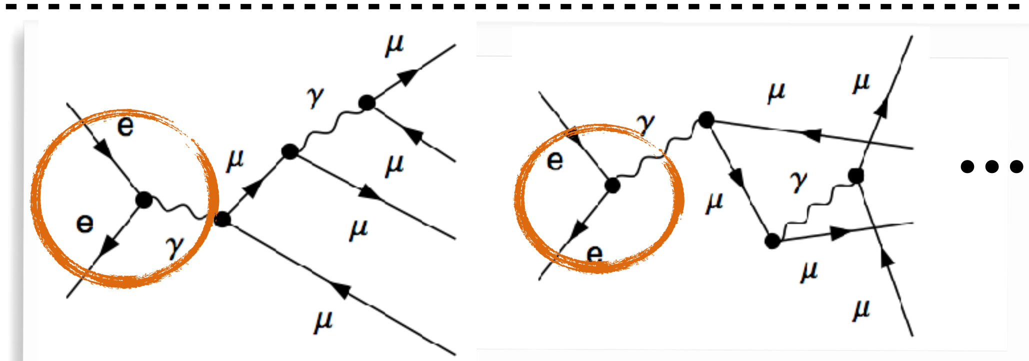
✦ Recycling: reusing pieces across diagrams

- Gain in computing time

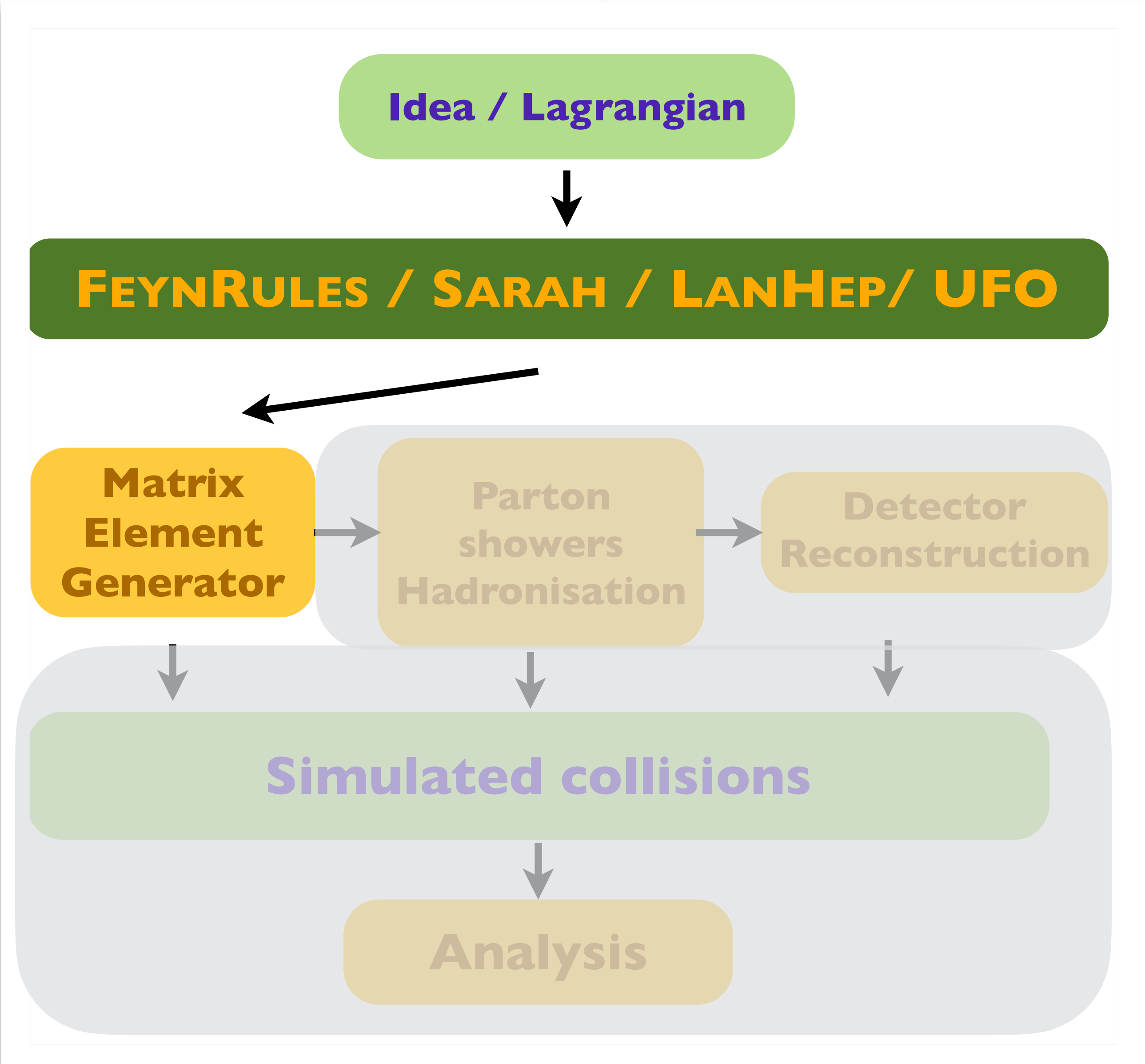


Comparison

	For M diags	For N particles	$2 \to 6$ example
Analytical	M^2	$(N!)^2$	10^9
Helicity	M	$N!\ 2^N$	10^7
Recycling	M	$(N-1)!\ 2^{N-1}$	$5\text{x}10^5$

Back to the simulation chain

✦ Tools connecting an idea to simulated collisions

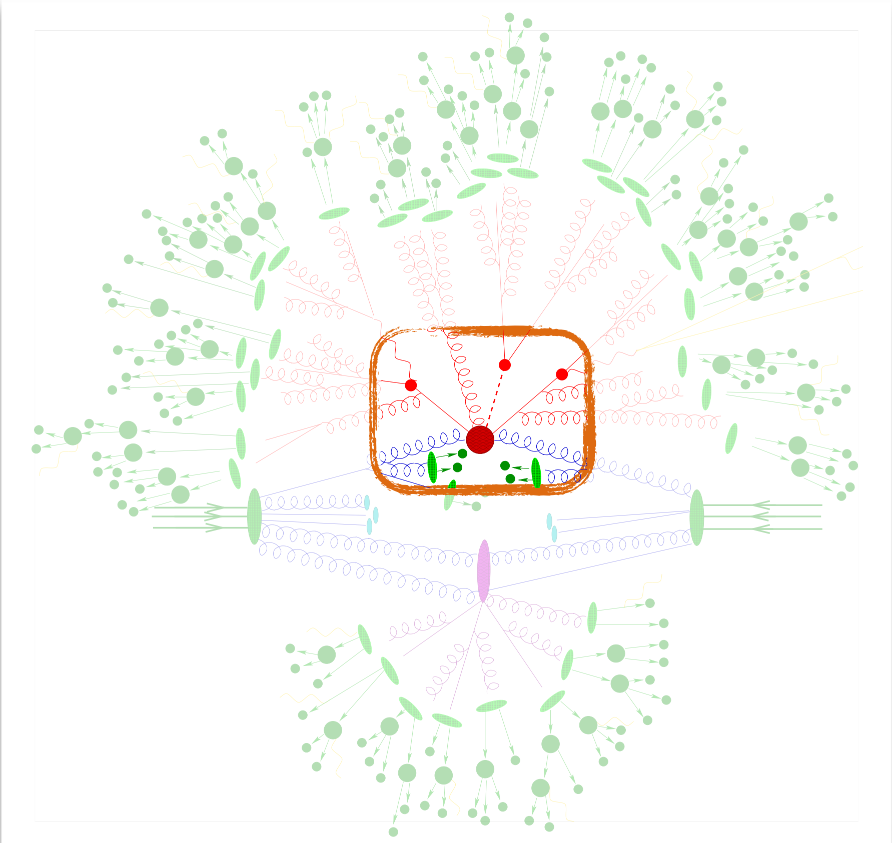


✤ Hard scattering process

★ Feynman diagram / amplitude generation

★ <u>Monte Carlo integration</u>

★ Event generation



Observable calculations

✦ The QCD factorisation theorem

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- The evaluation of any observable requires the integral calculation
- The phase space ↝ highly-dimensional integral (*3n-2* integrals ≡ *n*-body final state)
  - ★ The phase space structure ↝ analytical calculations hopeless
- The integrand is a very peaked function (propagators)

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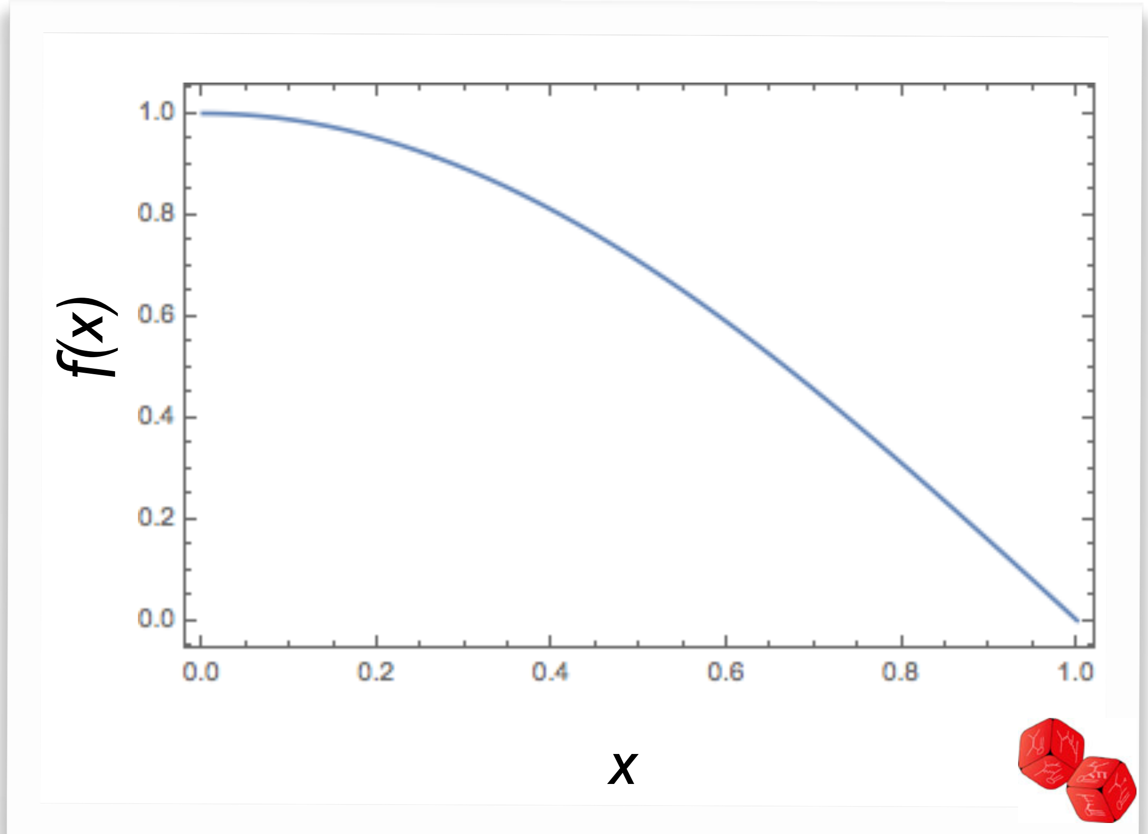
General and flexible numerical methods

Monte Carlo integration: the method

✦ The 1D example: evaluate the integral *I*

$$I = \int_a^b \mathrm{d}x \; f(x)$$



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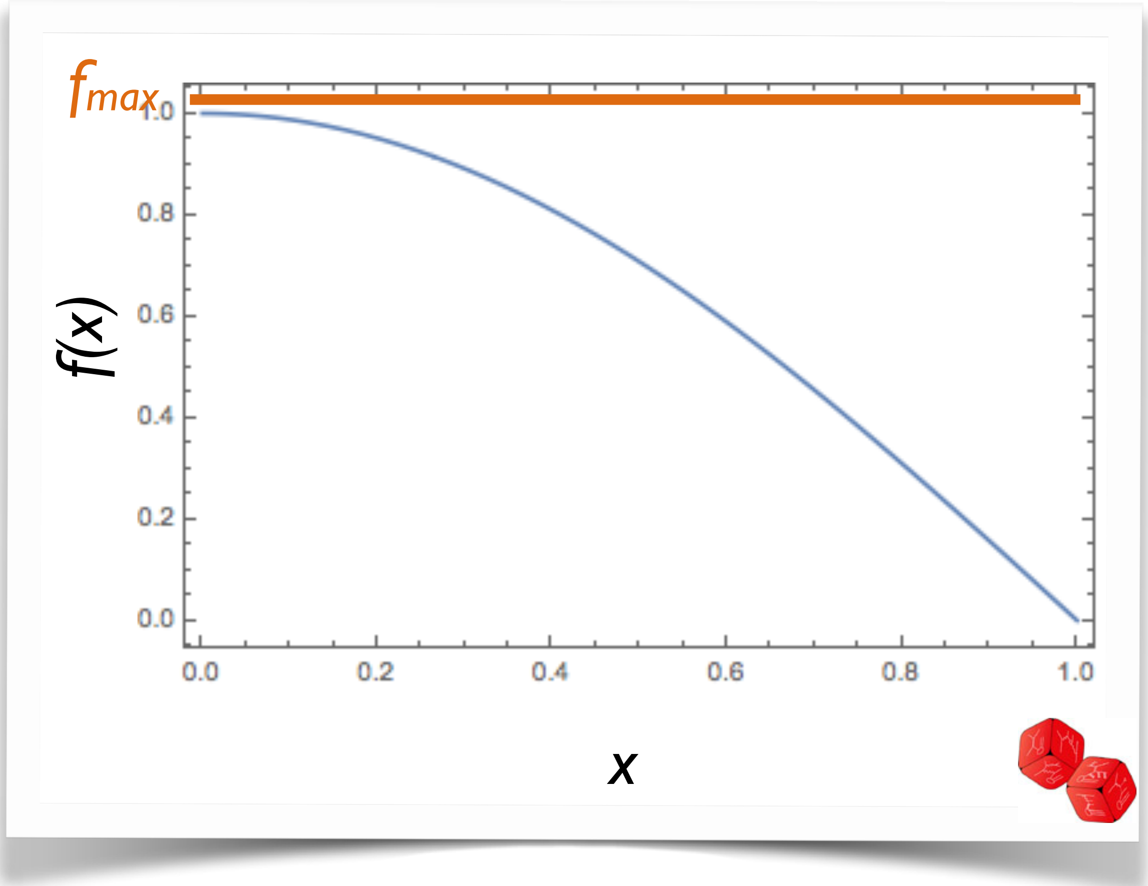

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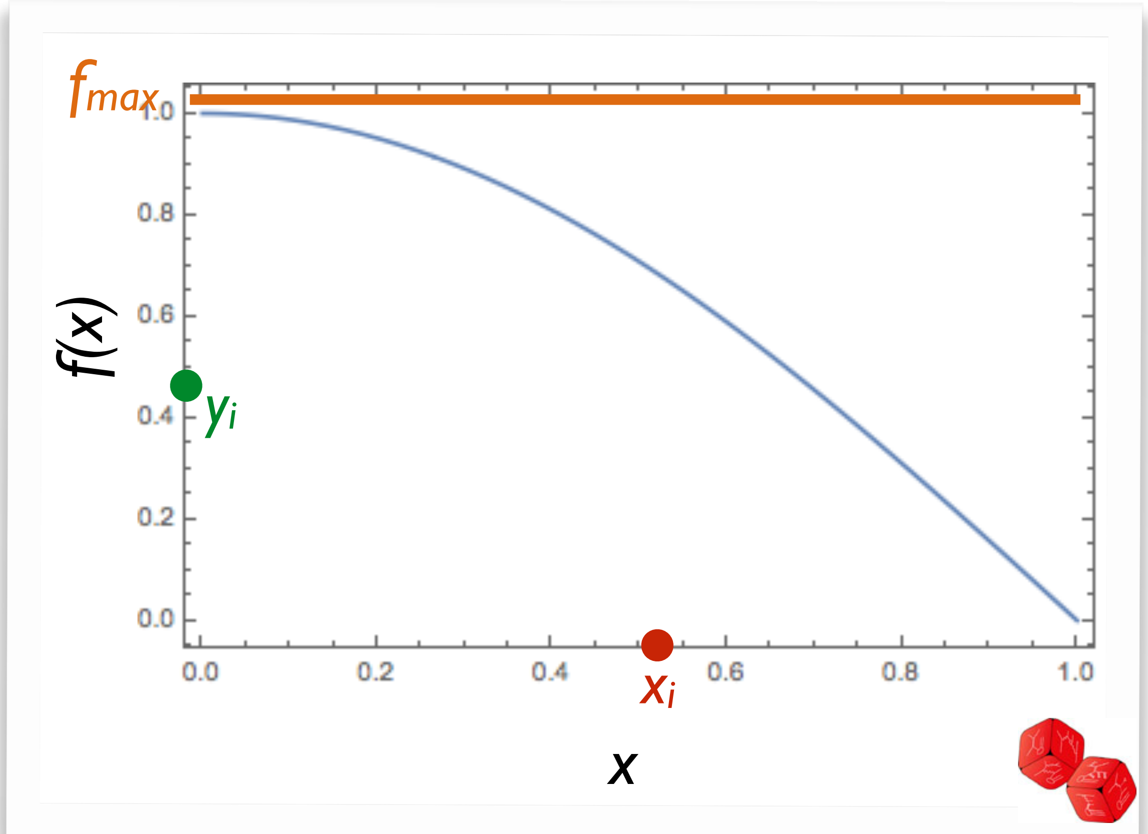

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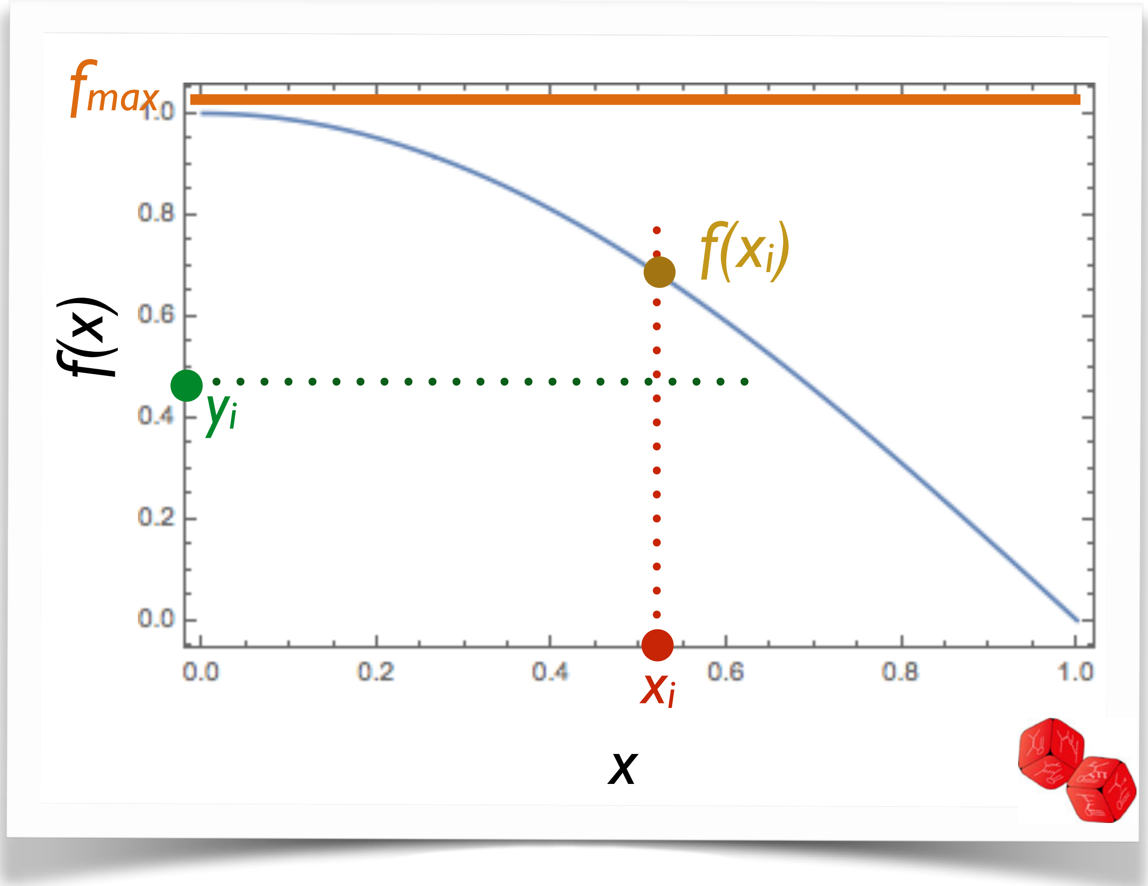

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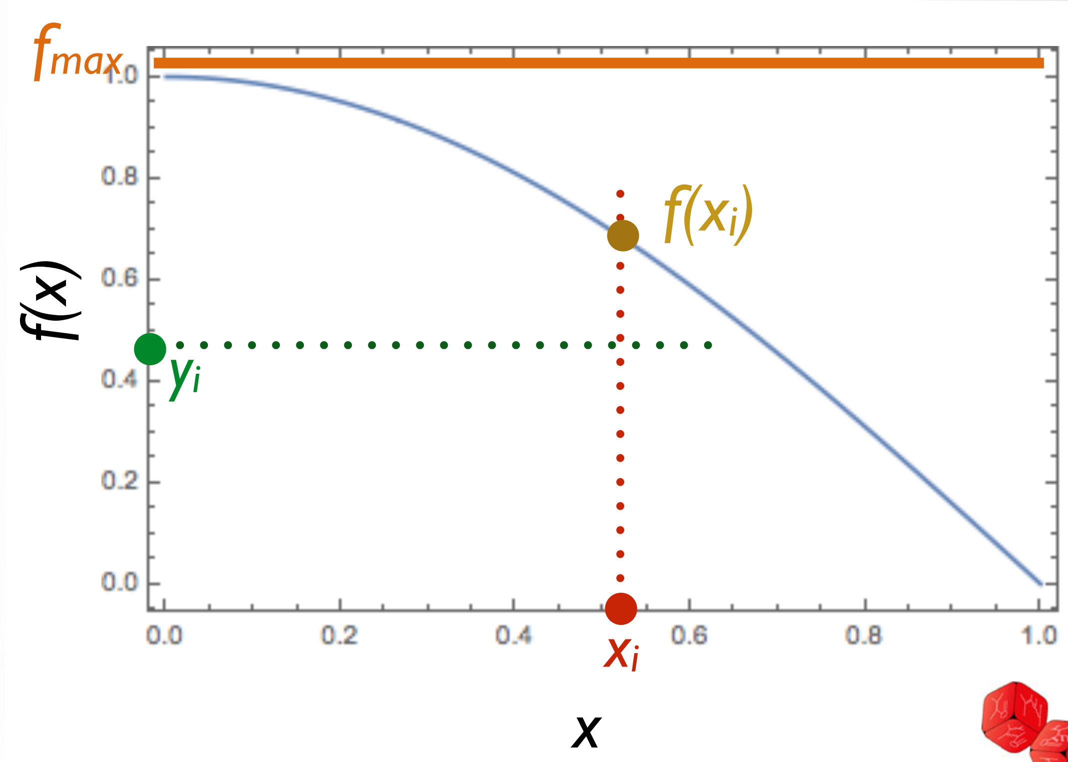
4. Evaluate the integral

$$I_N = \frac{N_{\text{accepted}}}{N_{\text{total}}} \mathcal{V}$$



Monte Carlo integration: the error

✦ The mean value theorem

- ✤ If *f(x)* is continuous:

$$\exists \xi \in [a,b] : I = \int_a^b \mathrm{d}x \ f(x) = (b-a) f(\xi) = (b-a)\langle f \rangle$$

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✤ We can approximate $\langle f \rangle$ by an averaged sum, so that:

$$I = \int_a^b \mathrm{d}x \ f(x) \ \approx I_N = \frac{b-a}{N} \sum_{n=1}^{N} f(x_n)$$

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✦ The error is given by the variance (that can be calculated)

$$V = (b-a) \int_a^b \mathrm{d}x \ f^2(x) - I^2 \ \approx V_N = \frac{(b-a)^2}{N} \sum_{n=1}^N f^2(x_n) - I_N^2$$

✤ Independent from the number of dimensions

Discretising an integral

✦ Integrals are evaluated as averaged sums over randomly chosen points

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✦ Result

$$I = I_N \pm \sqrt{\frac{V_N}{N}}$$

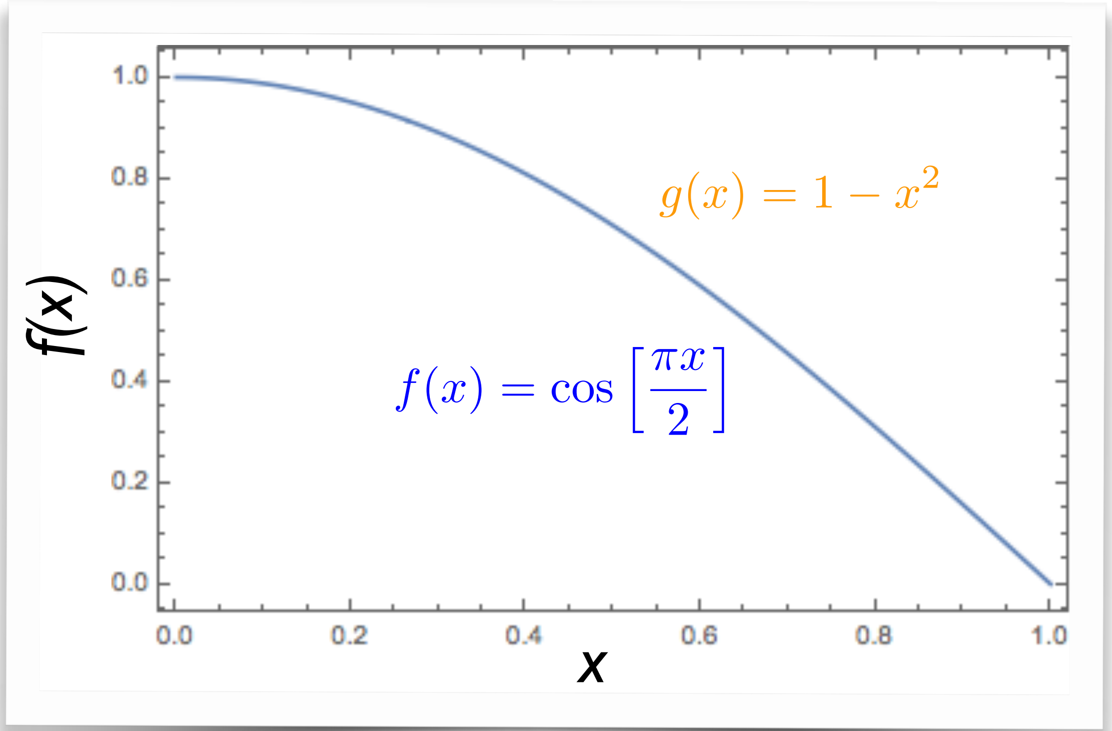
- The error can easily be estimated
- The error is independent from the number of dimensions
- Improvement possible by minimising V_N
- Ideal case: $f(x)$ = cst $(V=V_N=0)$
  - ★ Change of variables to flatten the integrand

Importance sampling: a practical example

✦ Integral to calculate

$$I = \int_0^1 \mathrm{d}x \ \cos\left[\frac{\pi x}{2}\right] = \frac{2}{\pi} \approx 0.6366$$

$$I_N = 0.637 \pm \frac{0.307}{\sqrt{N}}$$



✦ Remarks

- Convergence is slow
- Precision ⇒ large N
- Strength: scalability with n_{dim}

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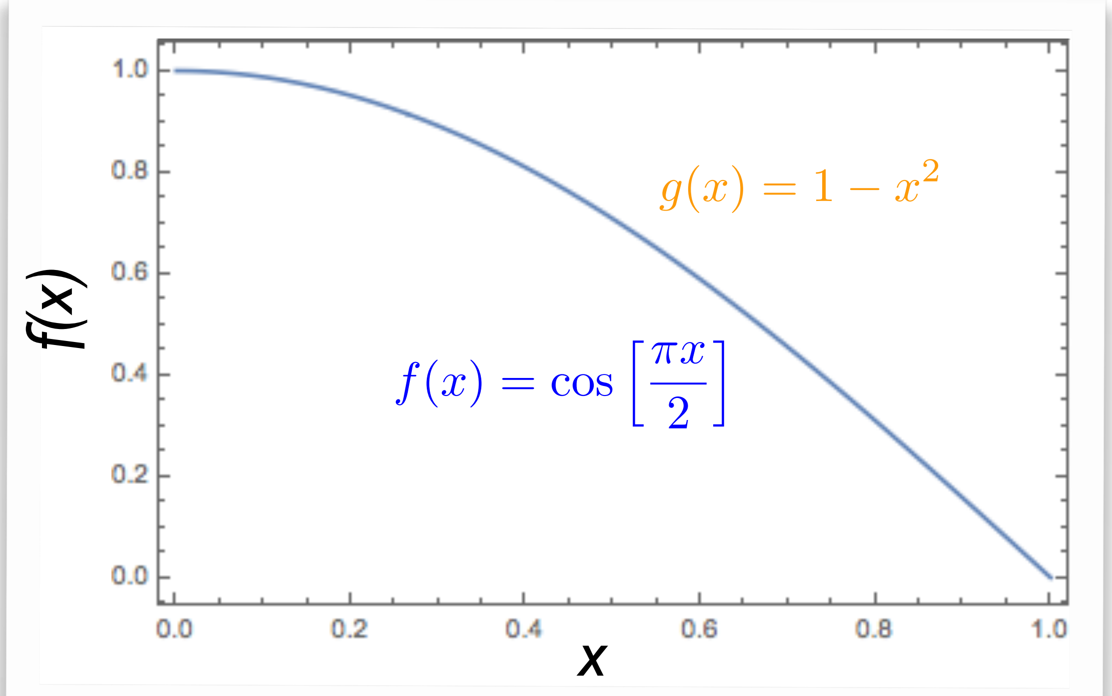

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- The ratio $f(x)/g(x) \simeq 1$ so that this product is roughly constant (ideal case)

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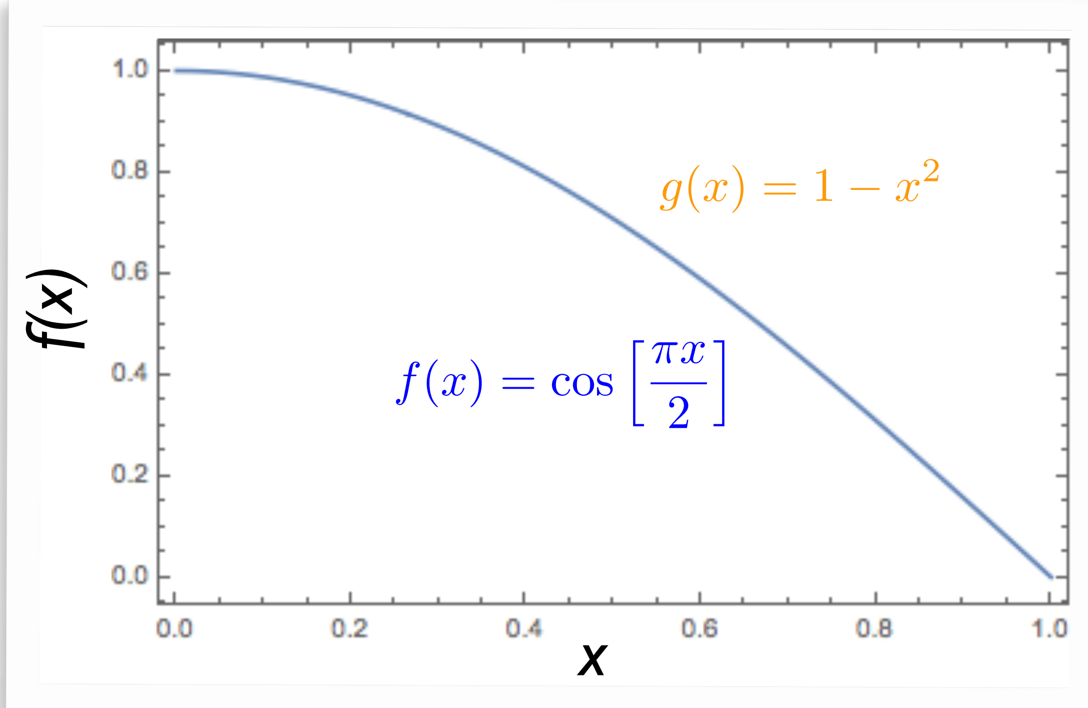

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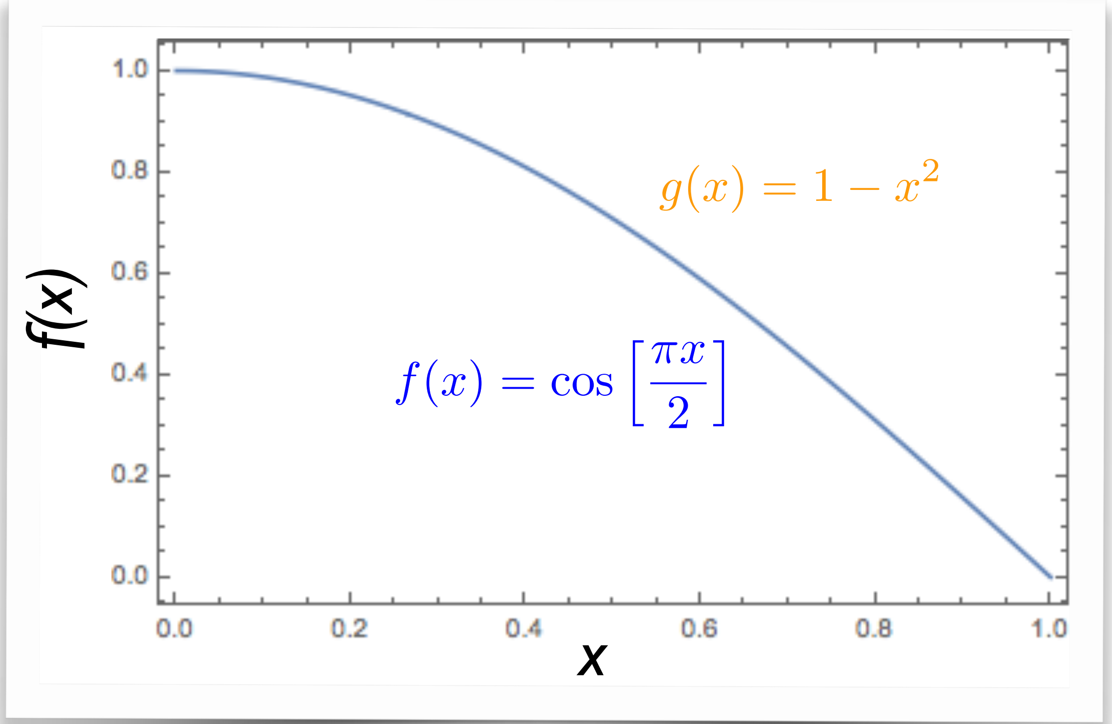

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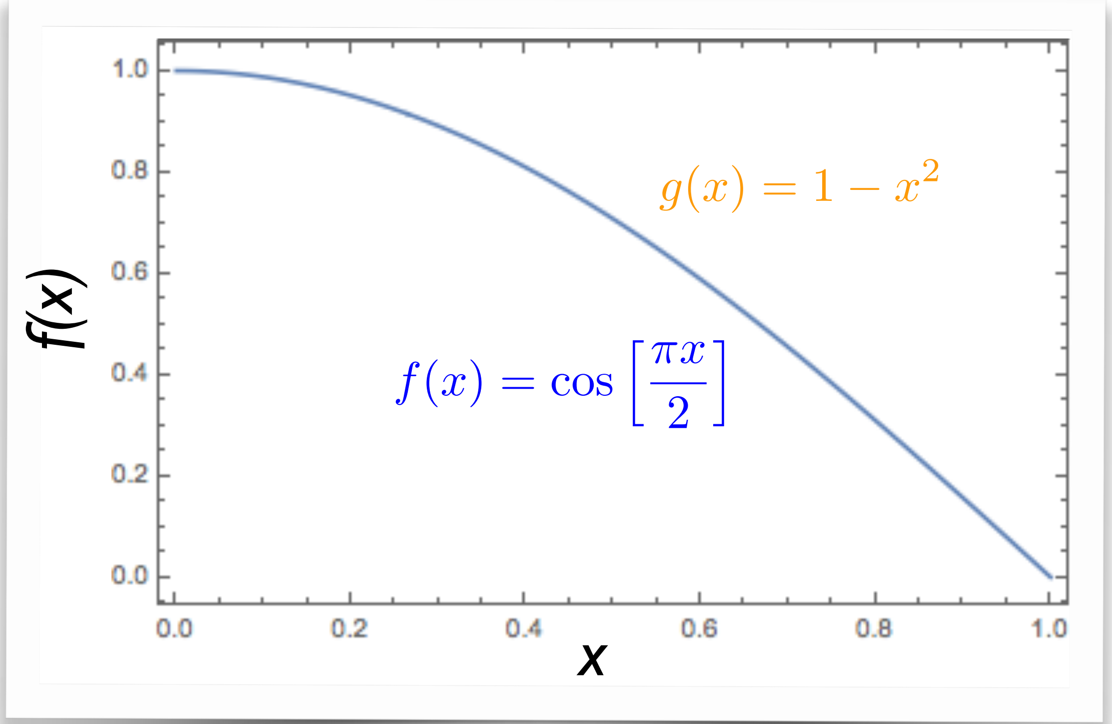

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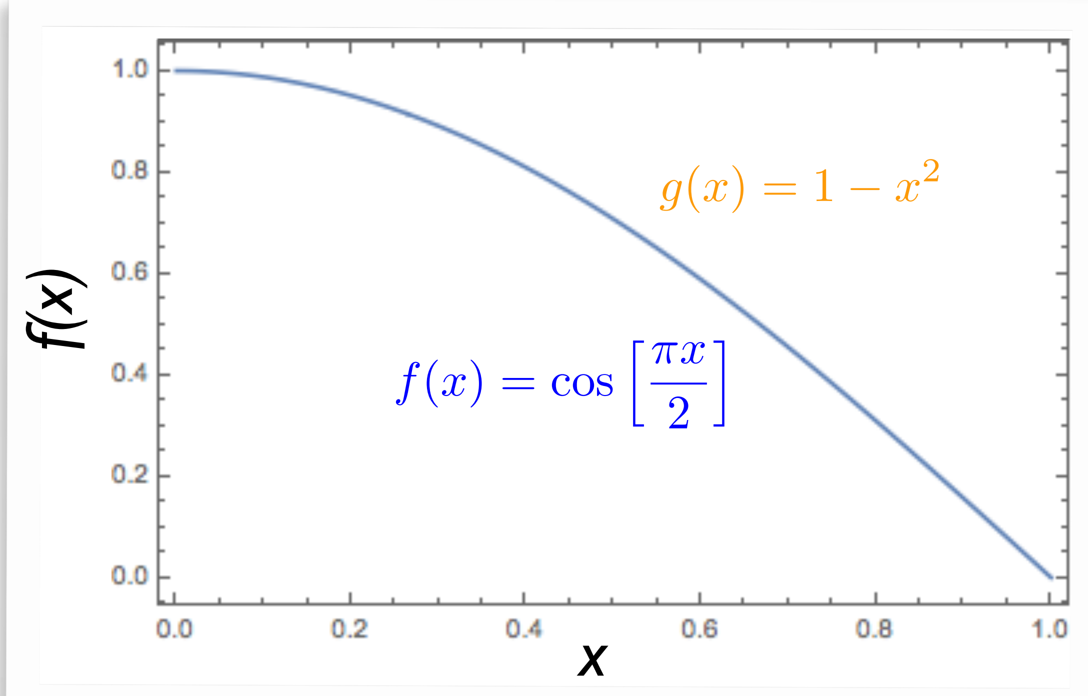

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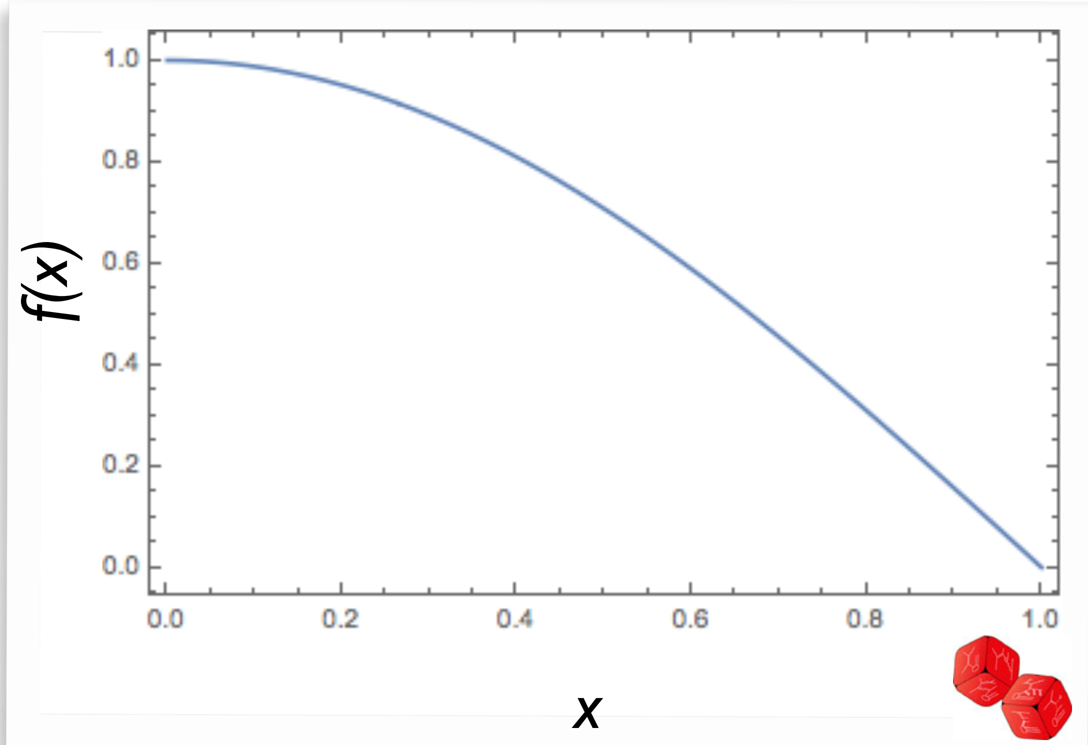
➢ **Faster convergence**

Importance sampling in action

✦ The 1D example: evaluate the integral *I*

$$I = \int_a^b \mathrm{d}x \; f(x)$$



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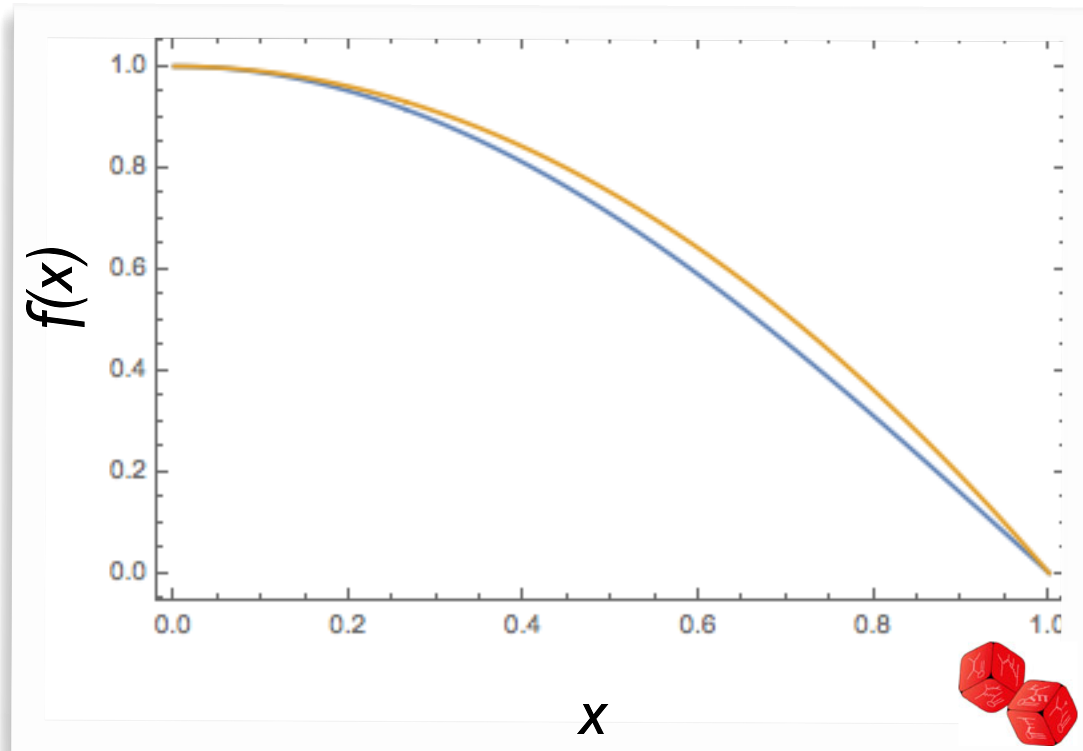
$$I = \int_a^b \mathrm{d}x \; f(x)$$

1. Find *g(x) so that g(x)* > *f(x)* $\forall\, x \in [a,b]$



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   - ★ pick a random point x_i distributed as $g(x)$
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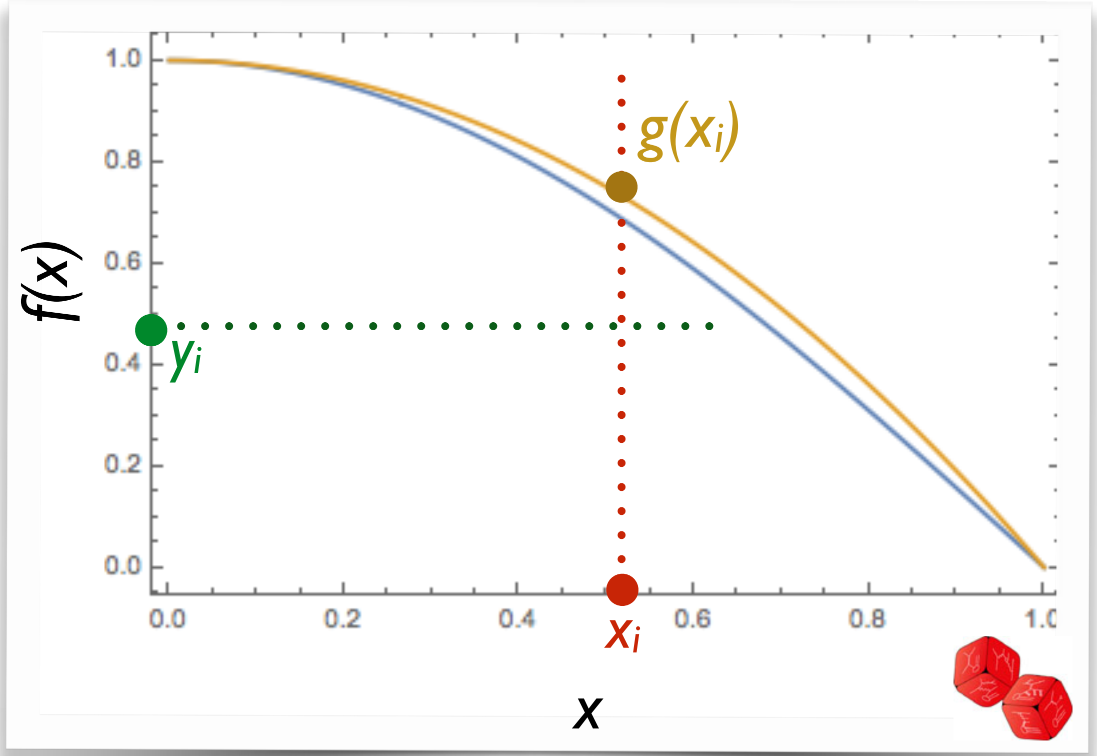

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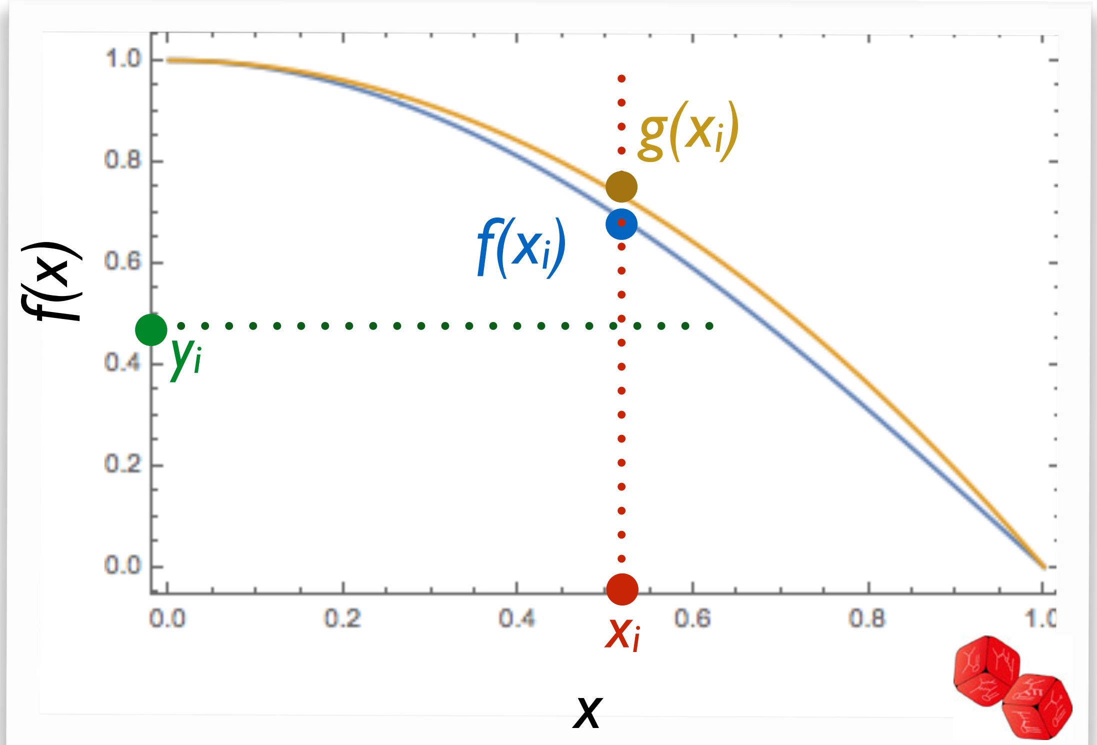

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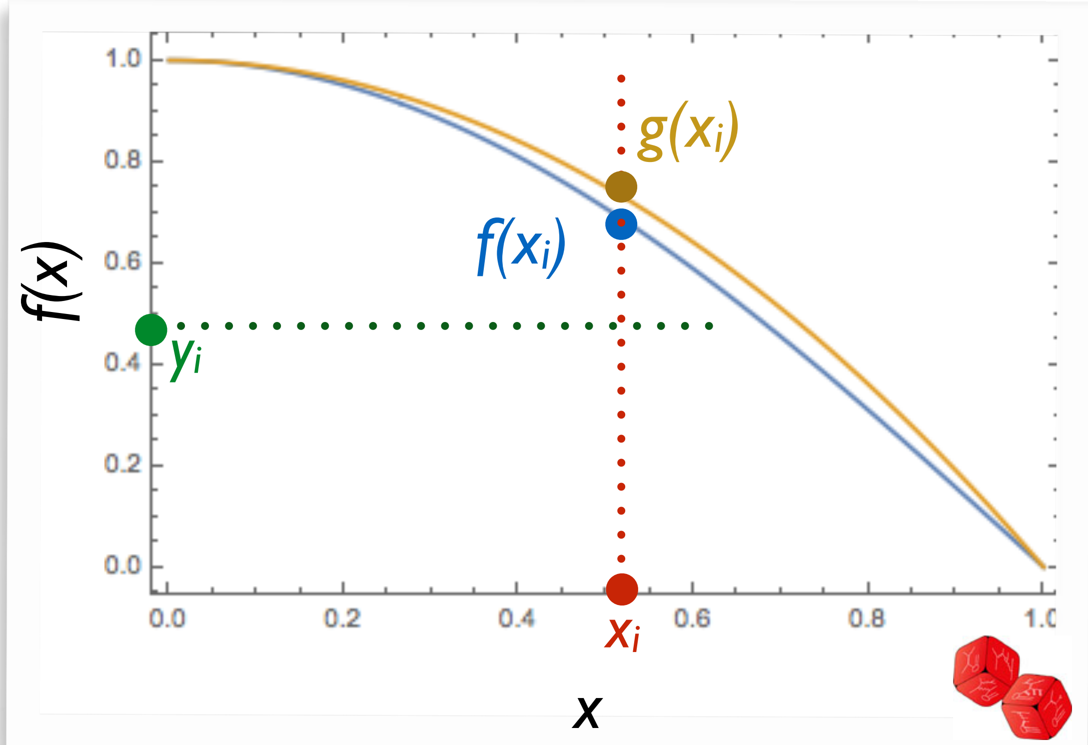

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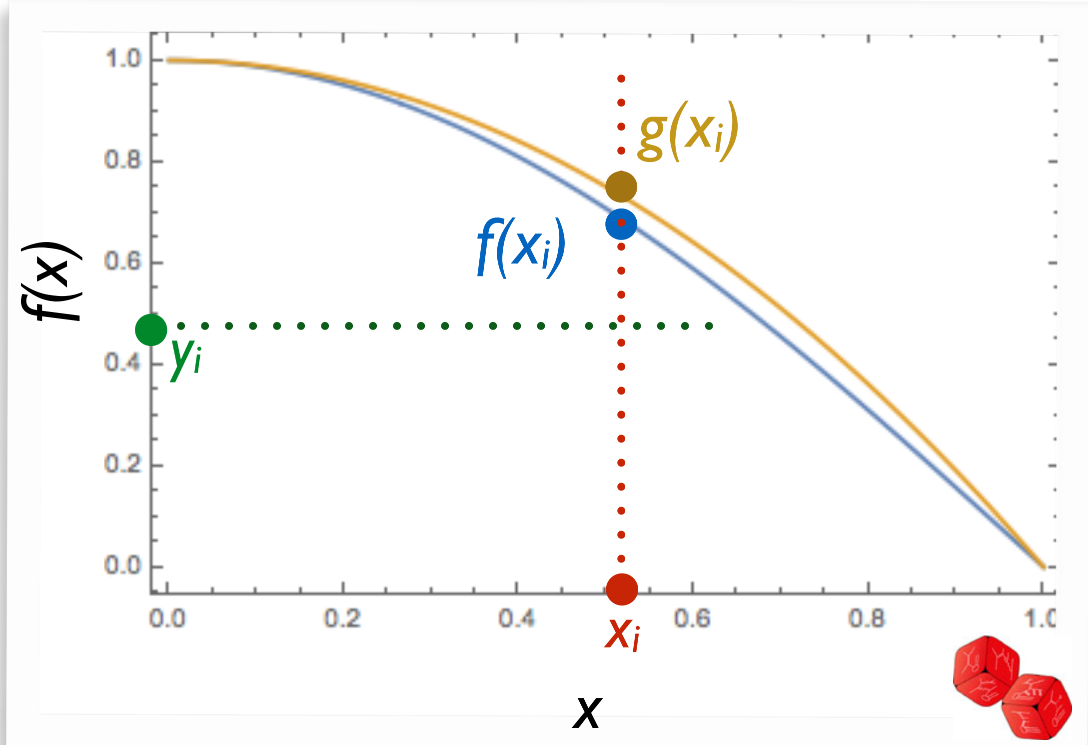

Improved efficiency

Problem of a peaked integrand

✦ QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b \mathrm{d}\Phi_n \mathbf{f_{a/p_1}}(x_a, \mu_F)\ \mathbf{f_{b/p_2}}(x_b, \mu_F)\ \left|\mathcal{M}\right|^{\mathbf{2}} \mathcal{O}_\omega(\Phi_n)$$

- For each point, we have a weight given by $\mathbf{f_{a/p_1}}(x_a, \mu_F)\ \mathbf{f_{b/p_2}}(x_b, \mu_F)\ \left|\mathcal{M}\right|^{\mathbf{2}}$
- Interpretation: each momentum configuration yields a weight

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✤ Interpretation: each momentum configuration yields a weight

✦ Problem: the integral is peaked (~ propagators)

✤ Random phase space points: very little chance to contribute

★ Few points carry the bulk of the integral

✤ Flattening the integrand ~ change of variables (importance sampling)

★ Need for some knowledge about the integrand

Problem of a peaked integrand

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✦ Construction of an approximative function of the integrand

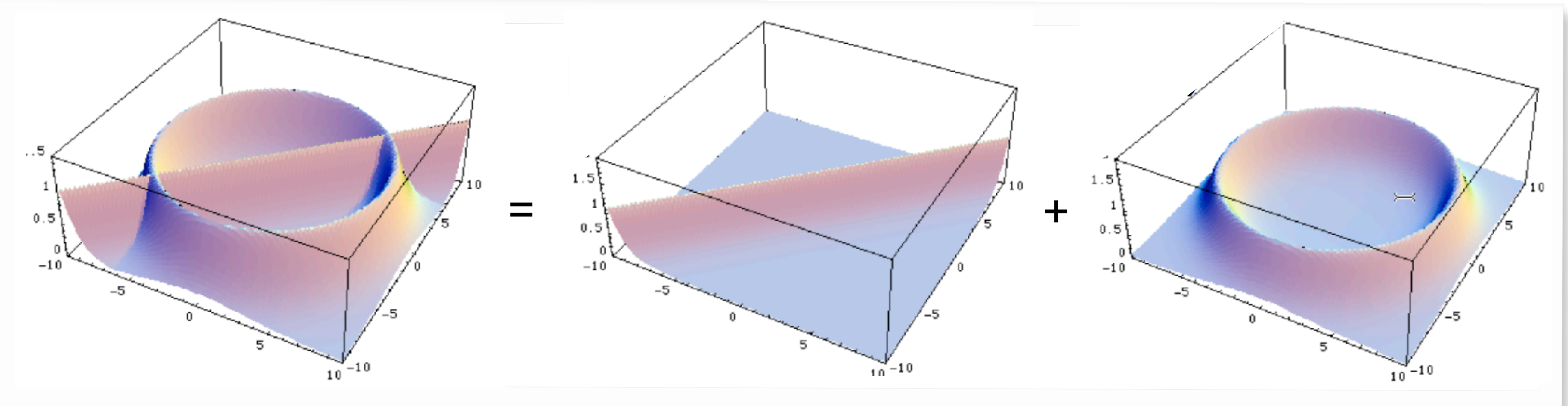
- Division of the integration domain in sub-domains (variable bin-size)
  - ★ Adjustment: identical variance in each bin
  - ★ Minimisation of the overall variance
- More bins where the integrand fluctuates more
  - ★ This binned function provides an approximation of the integrand *g(x)*

Multi-channel integration

✦ Separation of the integral among different channels

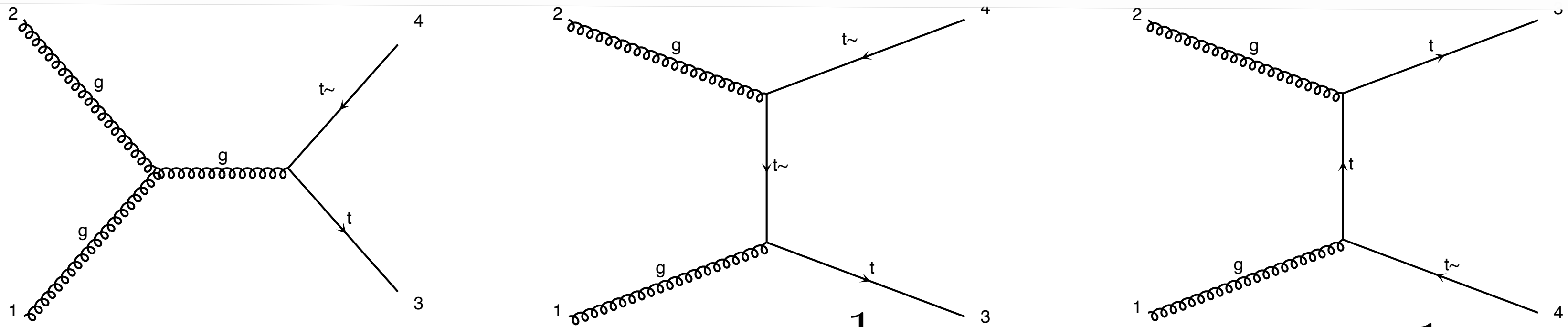
$$g(x) = \sum_{\text{channels}} \alpha_i g_i(x) \quad \text{with} \quad \sum_{\text{channels}} \alpha_i = 1$$

✤ Each channel takes care of one peak of the integrand at a time



Multi-channel integration: an example

✦ Top-antitop production: 3 diagrams



$$\mathcal{A}_s \propto \frac{1}{s} \qquad \mathcal{A}_t \propto \frac{1}{t - m_t^2} \qquad \mathcal{A}_u \propto \frac{1}{u - m_t^2}$$

$$g(x) = \sum_{\text{channels}} \alpha_i g_i(x)$$

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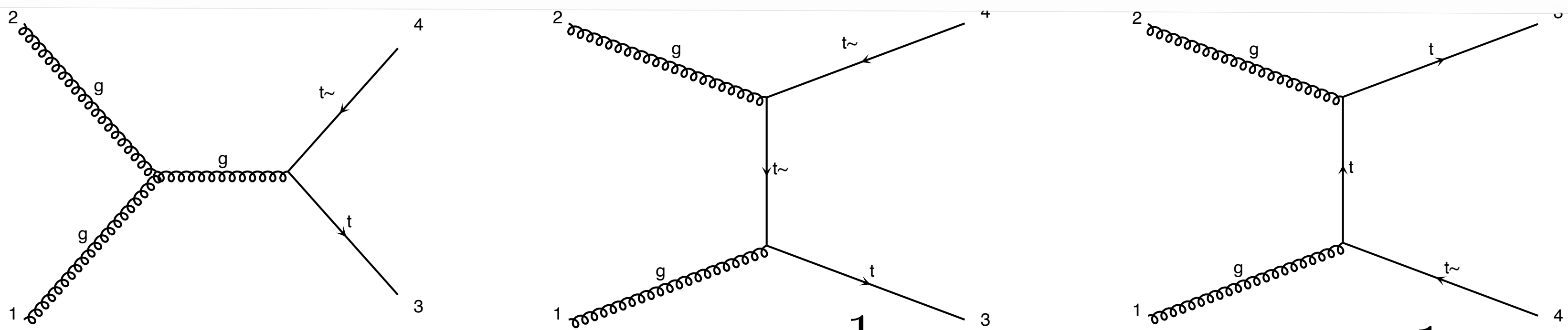

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✤ Three different pole structures

$$I = \int \mathrm{d}\Phi_2 \, \big|\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u\big|^2 = \sum_{i=s,t,u} \int \mathrm{d}\Phi_2 \, \frac{\overbrace{\big|\mathcal{A}_i\big|^2}^{g_i(\Phi)}}{\underbrace{\big|\mathcal{A}_s\big|^2 + \big|\mathcal{A}_t\big|^2 + \big|\mathcal{A}_u\big|^2}_{g(\Phi)}} \underbrace{\big|\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u\big|^2}_{f(\Phi)}$$

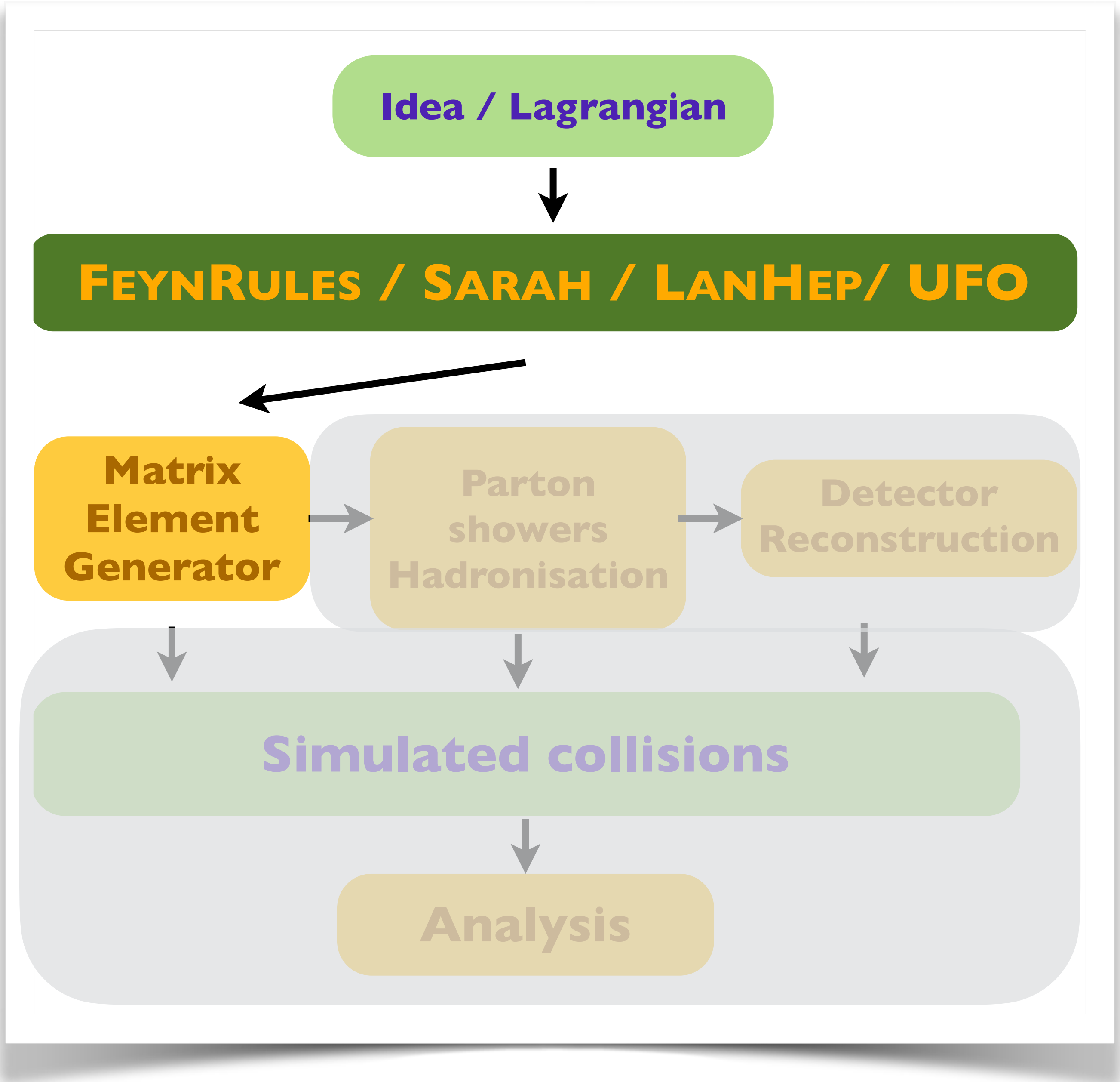
★ $f(\Phi) \,/\, g(\Phi) \simeq 1$

★ The integration of one single diagram is easy (the pole structure is known)

★ Multi-channeling on the basis of the different diagrams

Back to the simulation chain

✦ Tools connecting an idea to simulated collisions

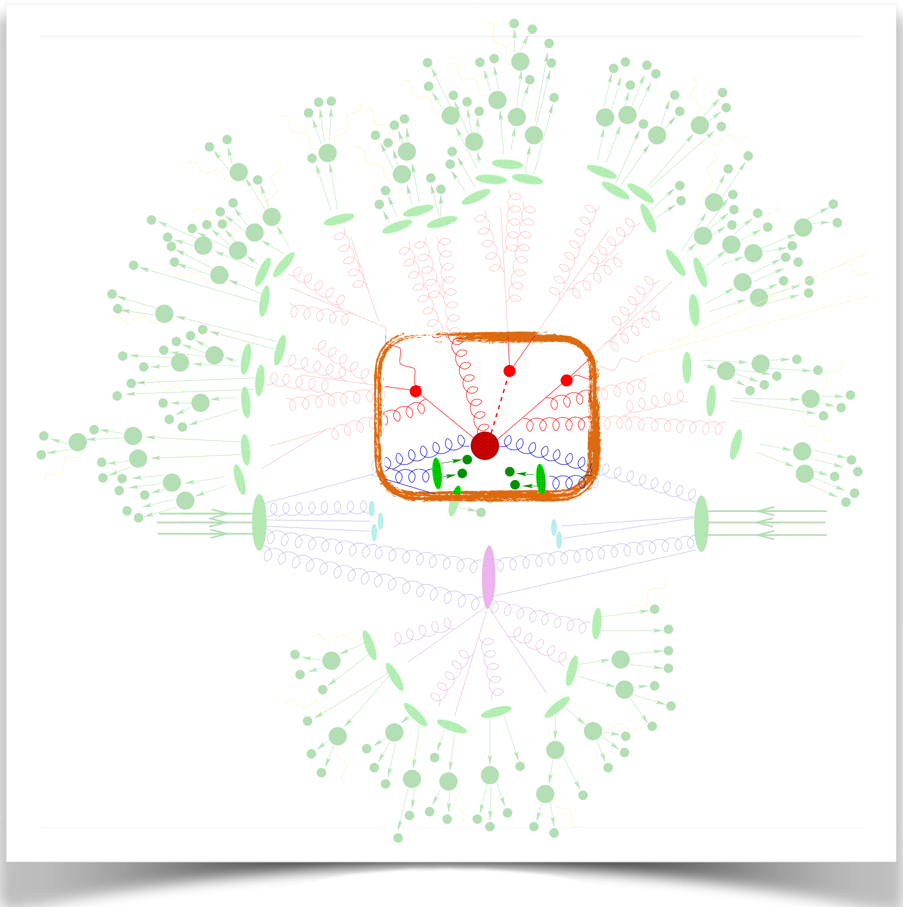


✤ Hard scattering process

★ Feynman diagram / amplitude generation

★ Monte Carlo integration

★ <u>Event generation</u>



Weighted and unweighted events

- Accepted points ↝ event generation
  - One point ≡ one event
  - Integrand value ↝ event weight
  - Not all events are equal

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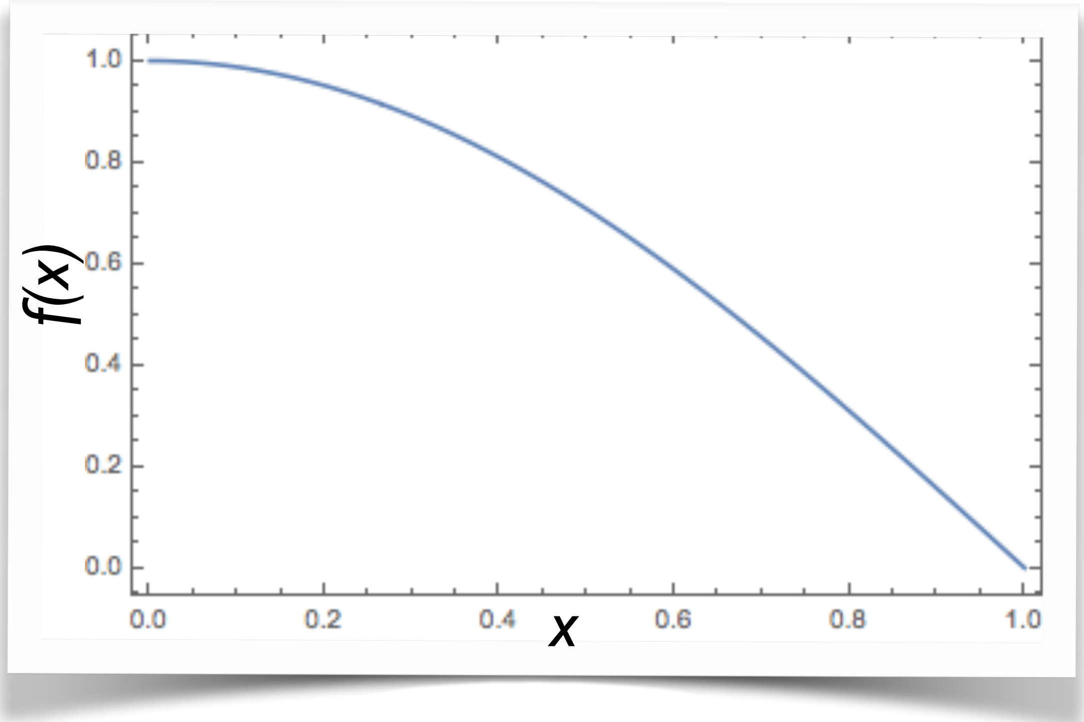

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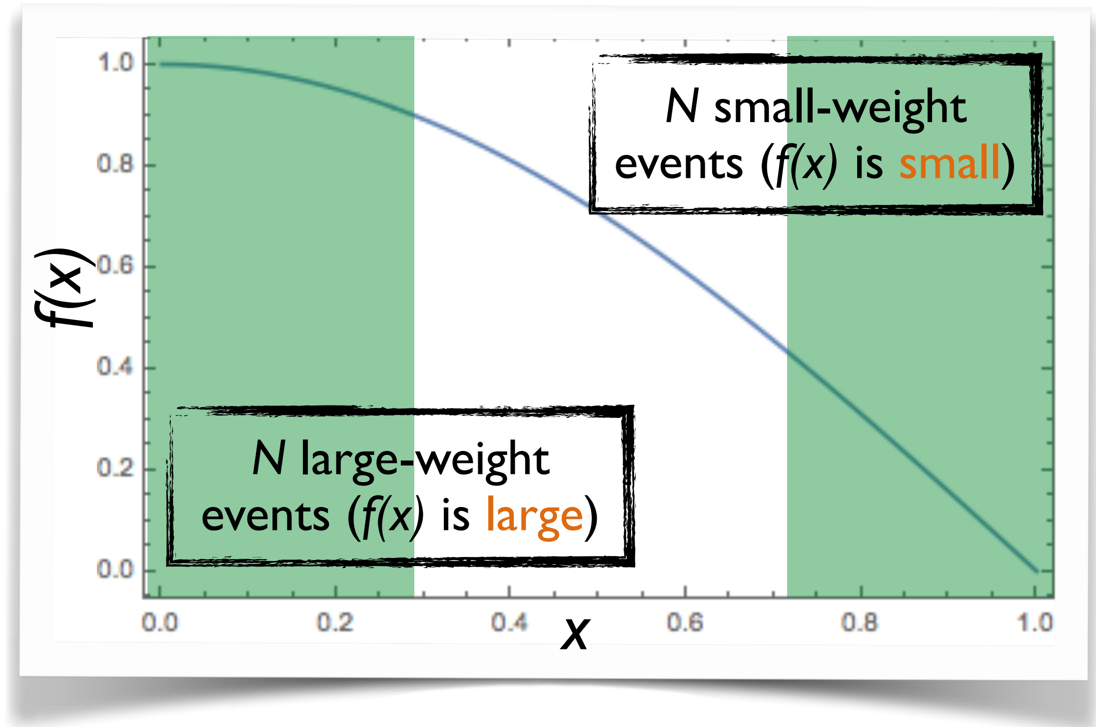

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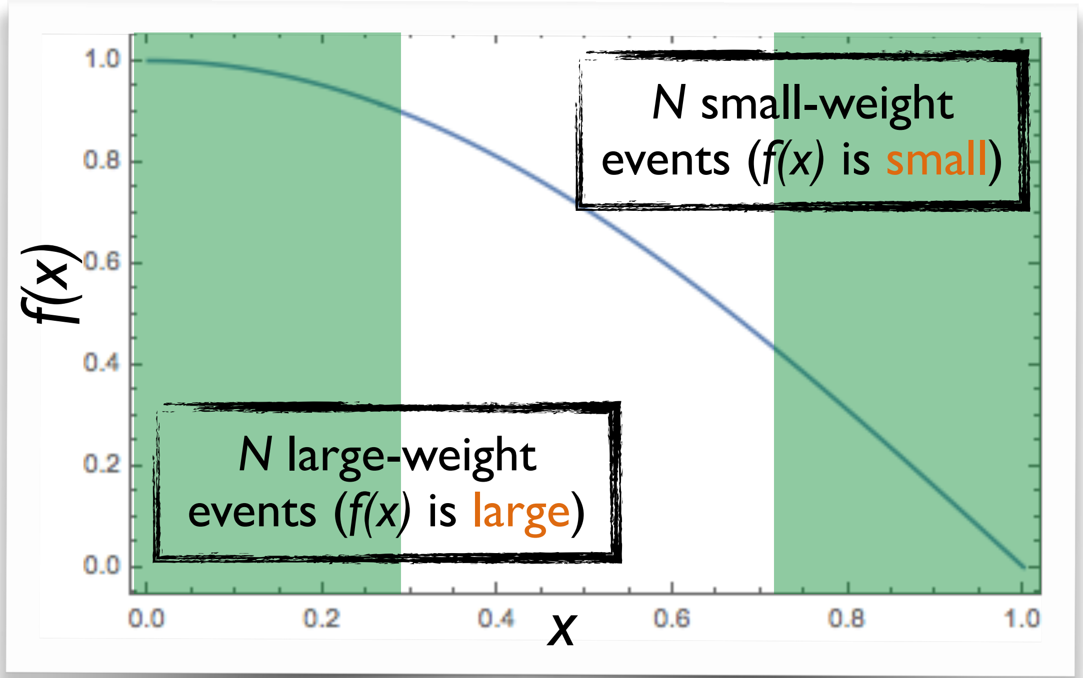

✦ Enforcing equal-weight events

- ✤ Distributed as occurring in nature
- ✤ All events are equal
- ✤ Weight value: recovering the total rate

Unweighted events

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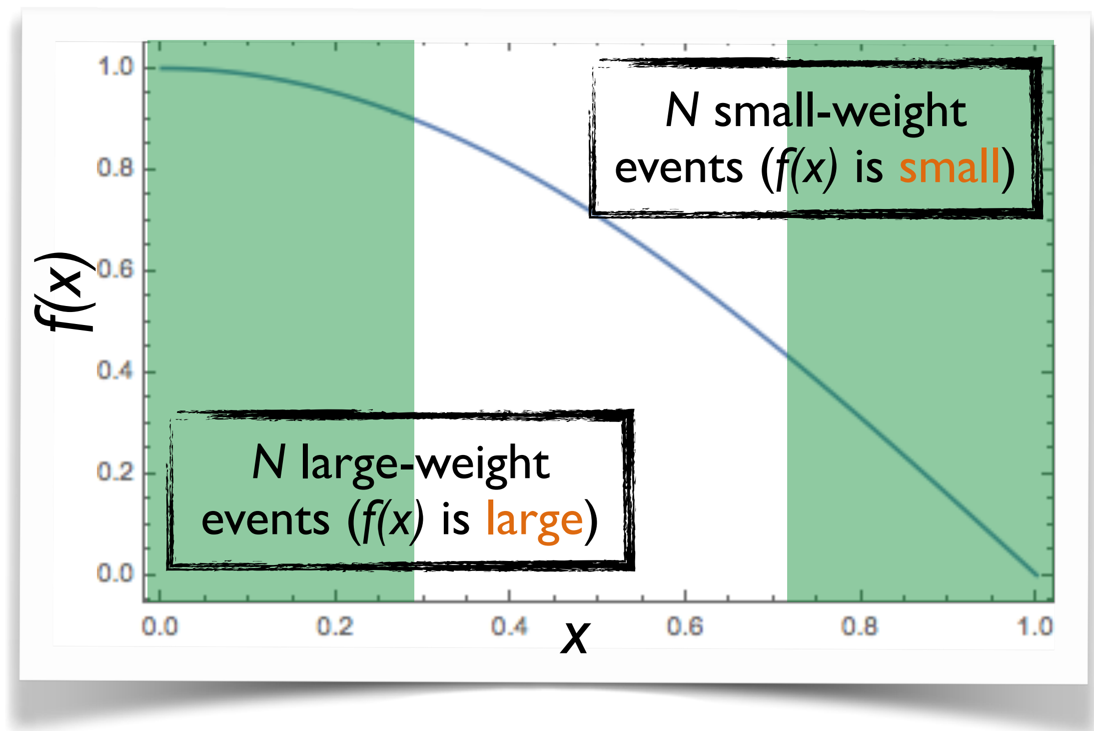

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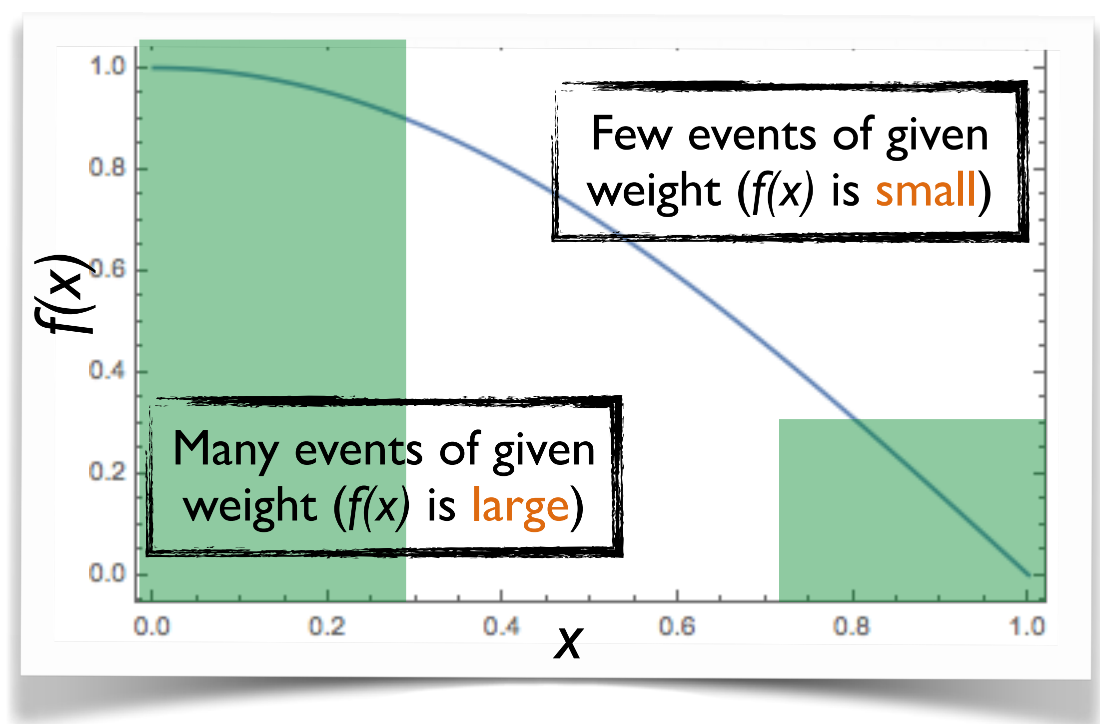
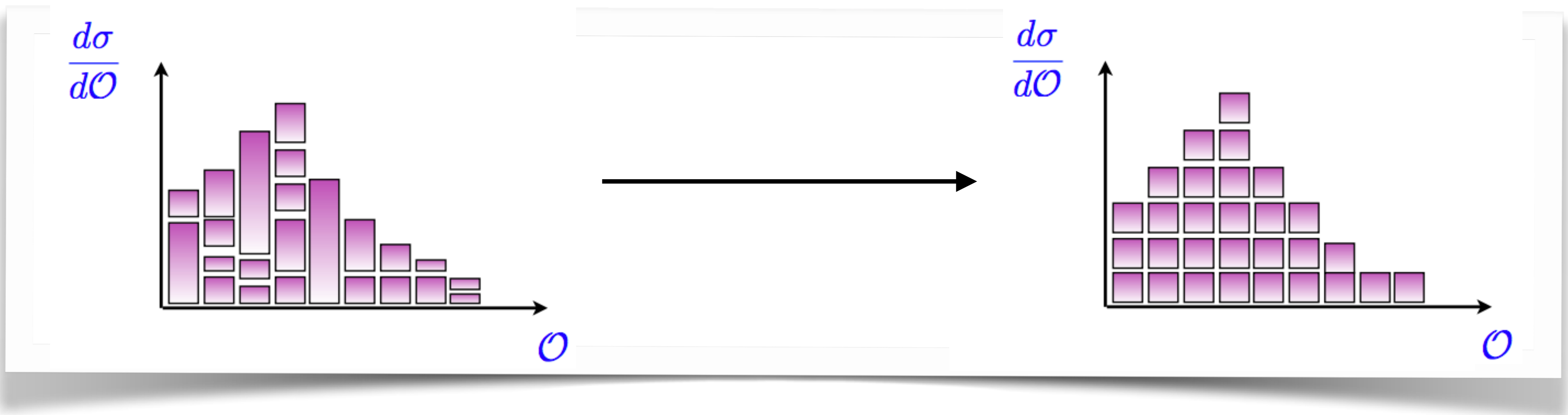


Unweighted events in practice

✦ Principle

- ✤ We calculate the maximum weight during the MC integration phase $\omega_{\max}$
- ✤ The desired number of events and the total rate yield the average weight $\langle\omega\rangle$
- ✤ Each generated event is accepted with a probability $\omega(\Phi)$ / ω_{max}
- ✤ Each event is assigned the weight $\langle\omega\rangle$



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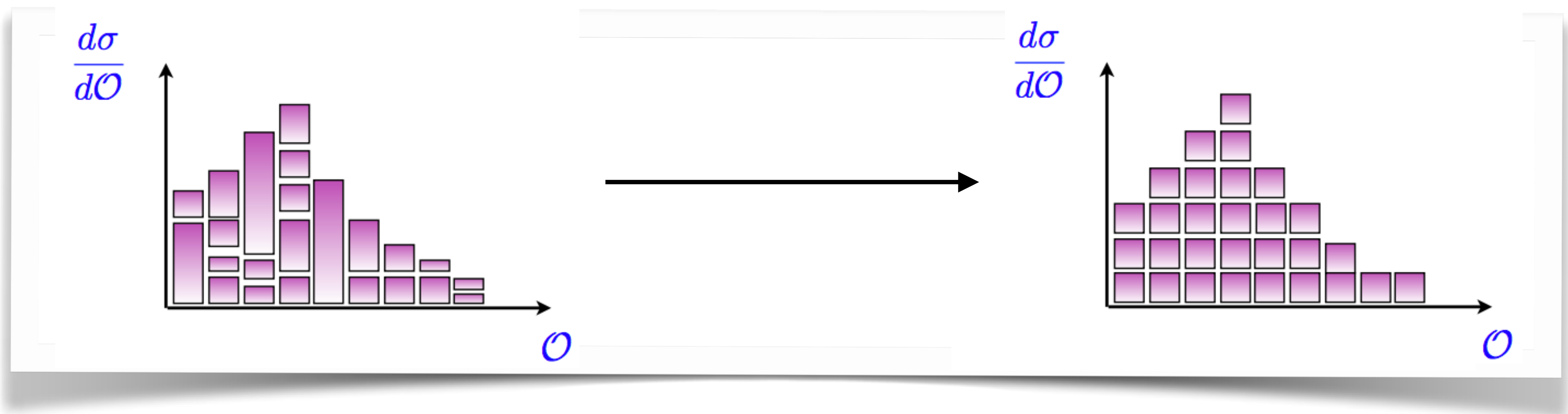


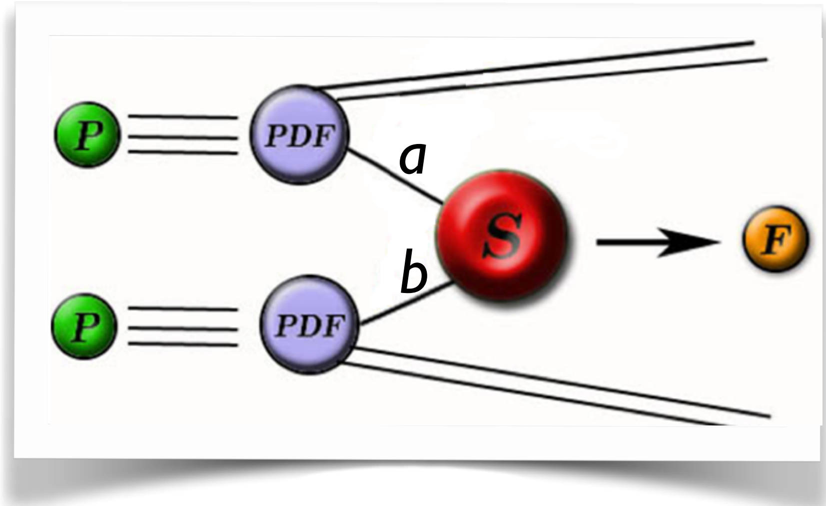
✦ Requirements

- The integrand has to be bounded from above ($\omega_{\max}$ must exist)
- The integrand has to be positive-definite (can however be by-passed)

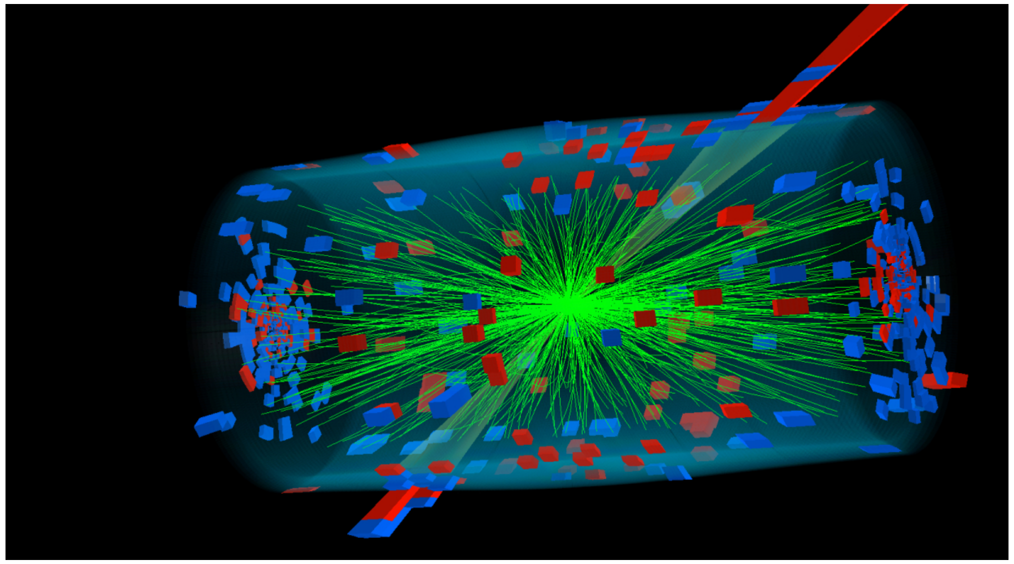
Summary so far

✦ Matrix elements can be generated automatically for any model

✦ Monte Carlo integration
- ✤ Cross section
- ✤ Unweighted events distributed as in nature
- ✤ Any observable can be extracted
- ✤ Selection cuts can be imposed



✦ Automated codes exist
- ✤ MADGRAPH5_aMC@NLO, SHERPA, WHIZARD, *etc.*



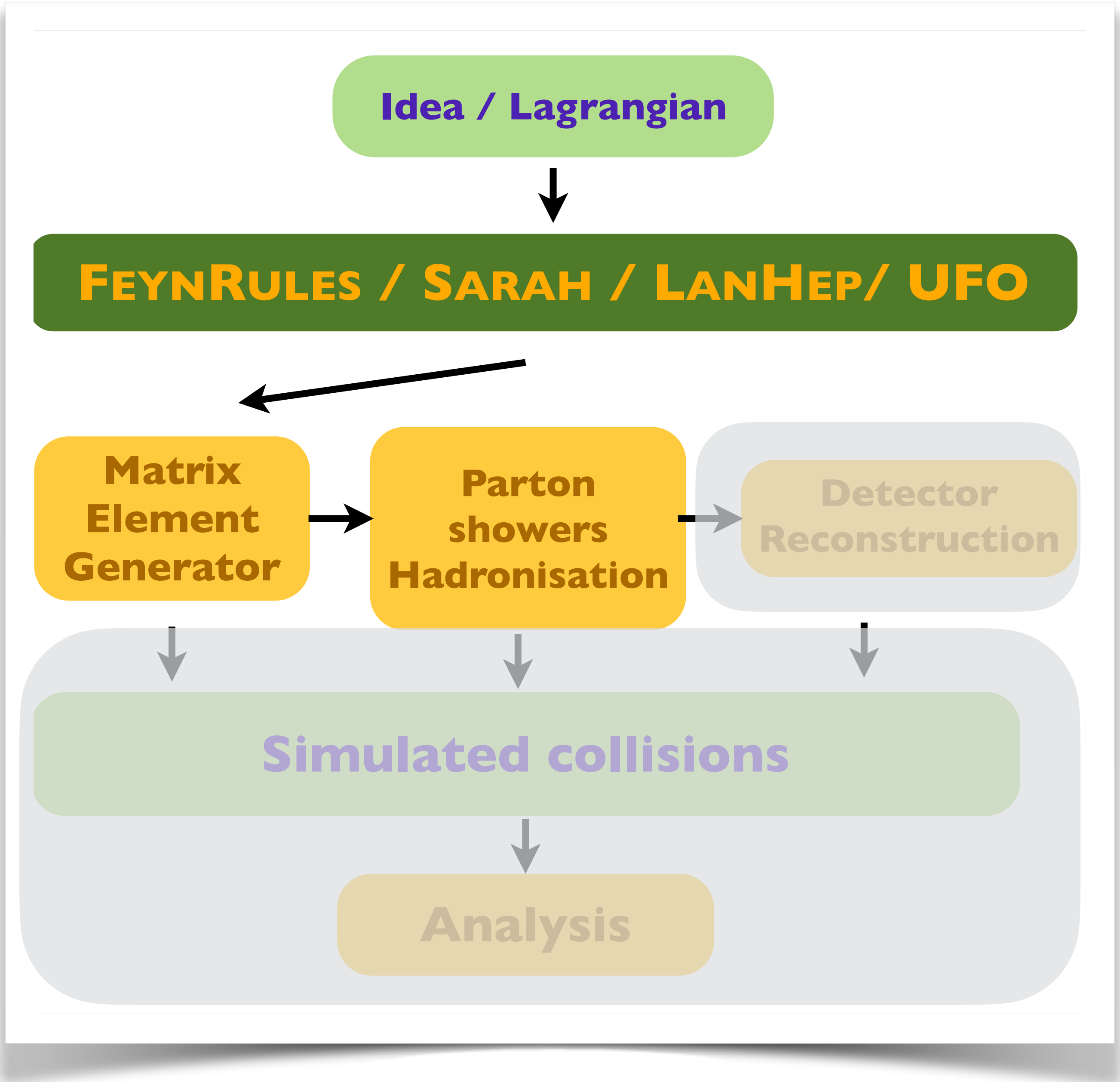
Still far from a realistic LHC collision…

Outline

1. The Standard Model of particle physics and Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
4. **Parton showers, hadronisation & underlying event**
5. Summary

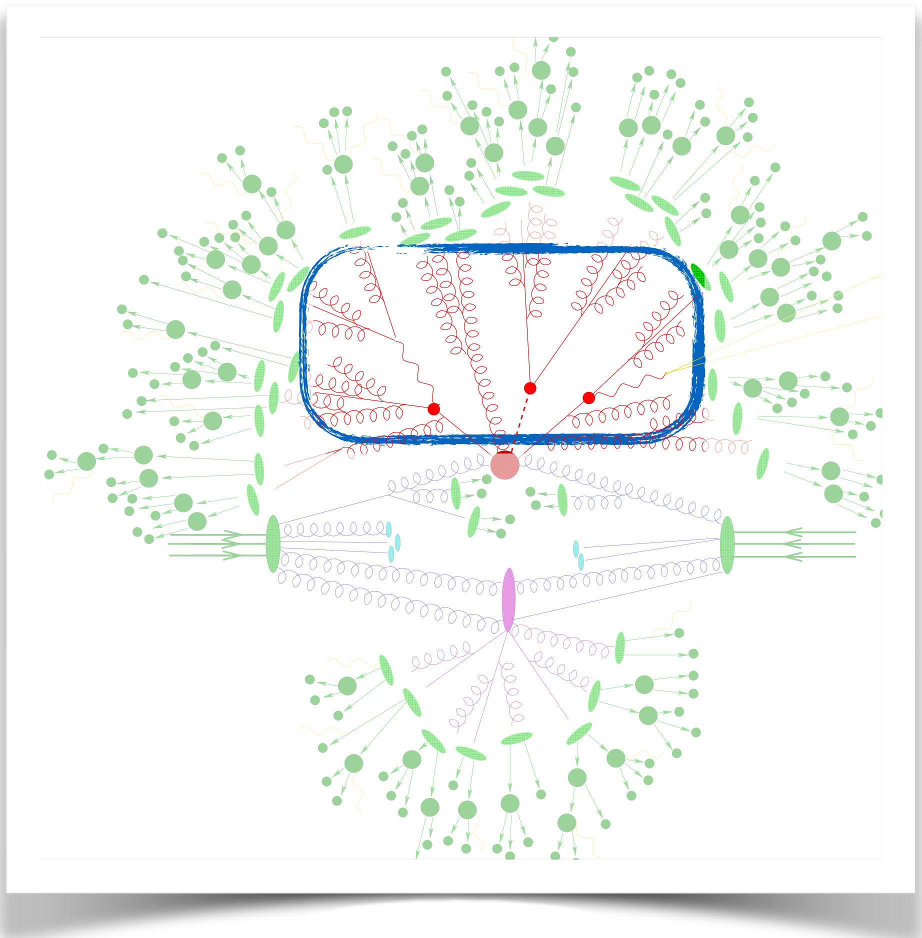
The simulation chain - step 3

✦ Tools connecting an idea to simulated collisions



✤ QCD environment

★ Parton showering
★ Hadronisation
★ Underlying event



Parton evolution - generalities

- ✦ Accelerated charges radiate
  - ✤ Large momentum transfers ≡ lot of radiation

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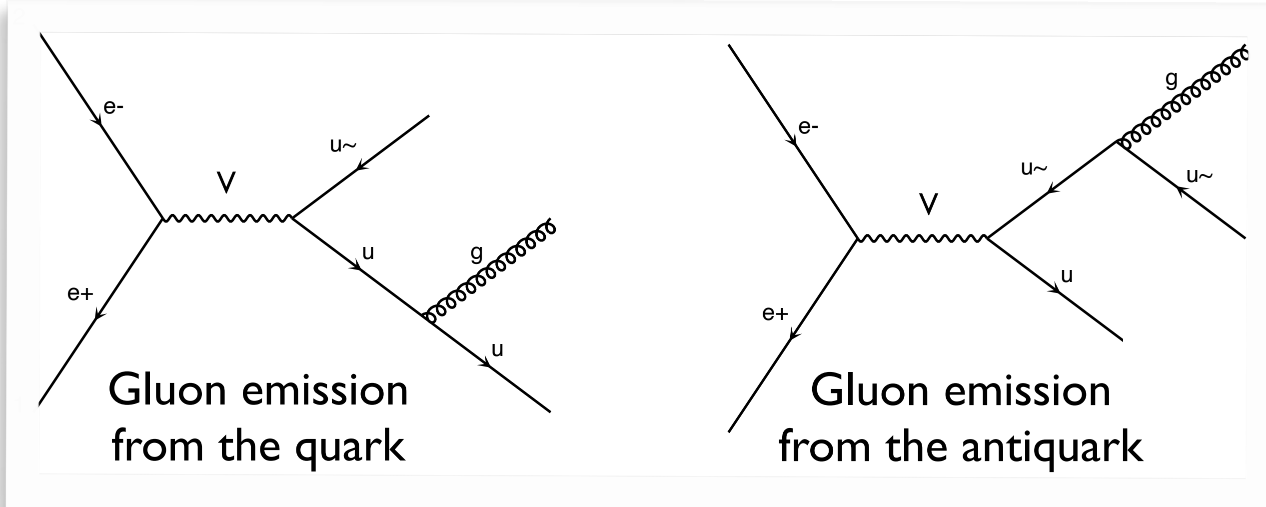
- ✦ Highly energetic coloured particles radiate
  - ✤ Each parton is dressed with an arbitrary number of partons (multiple radiation)
    - ➢ Radiated partons also radiate
  - ✤ One ends up with a cascade of radiations ➢ parton showers

QCD factorisation in the infrared

✦ Toy process: $e^+ e^- \to u \bar{u} g$



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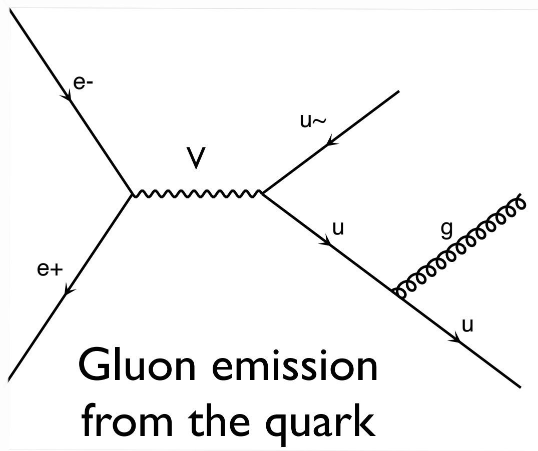

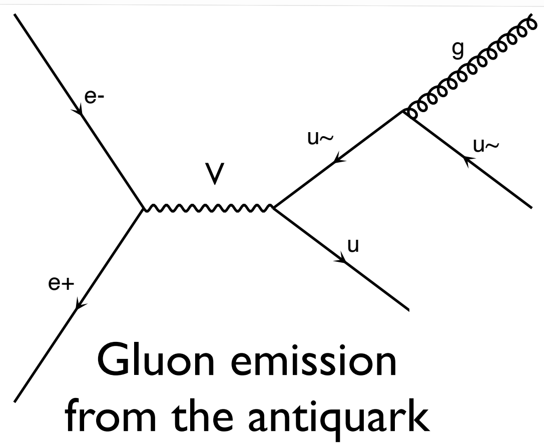

$$\mathrm{d}\sigma_{2\to 3} \propto C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta} \frac{1+z^2}{1-z}\, \mathrm{d}\cos\theta\, \mathrm{d}z$$

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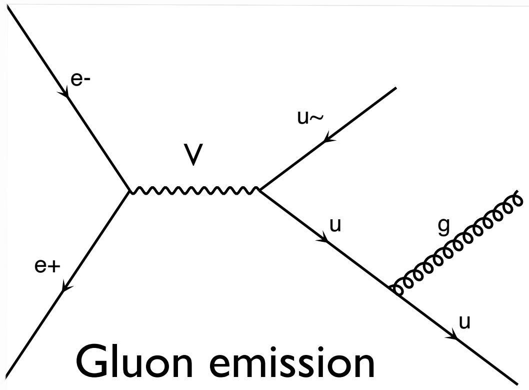

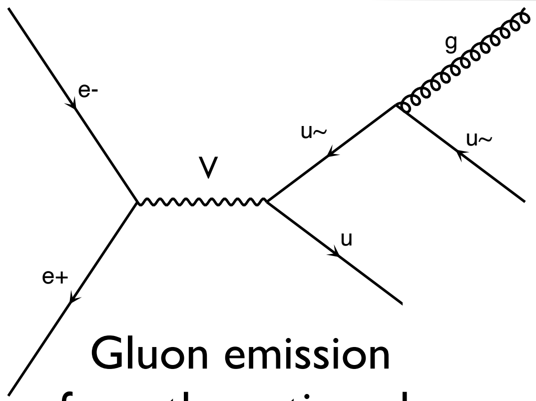

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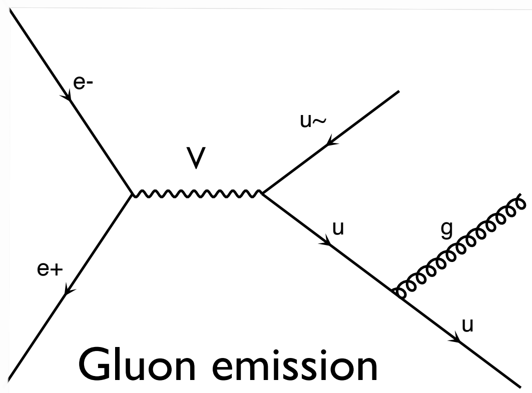
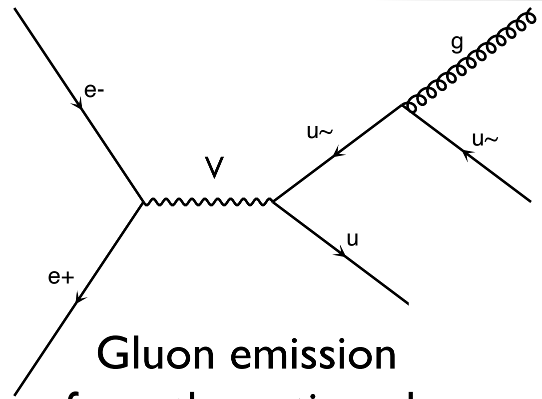


Gluon emission from the quark



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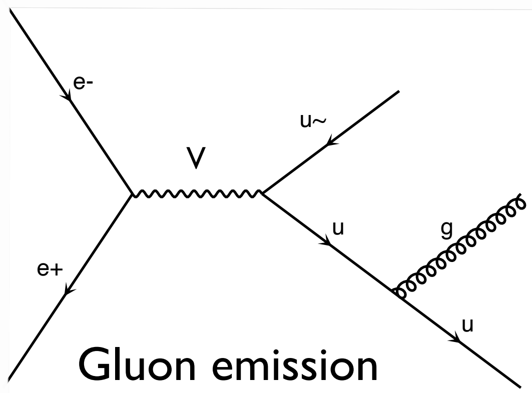

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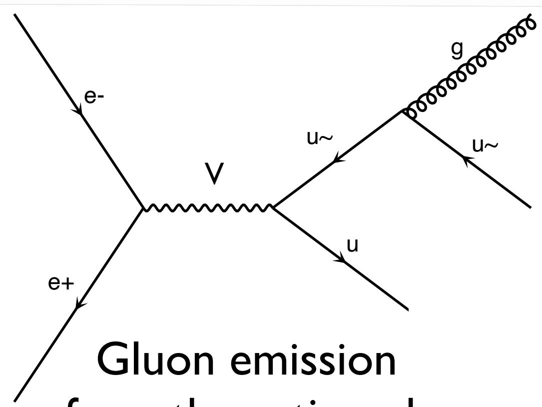

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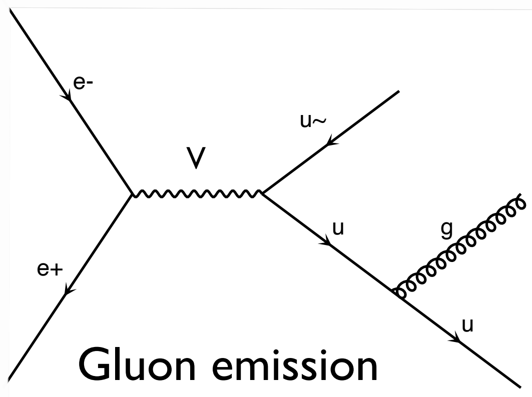

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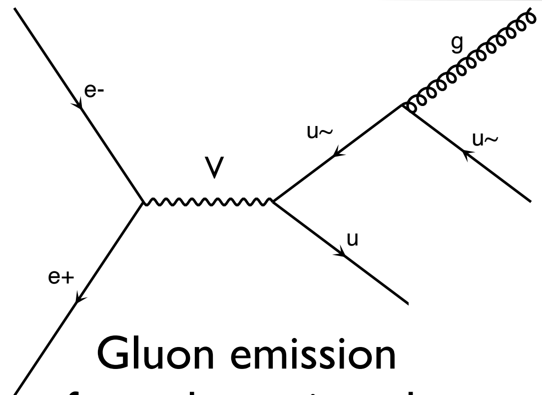

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✦ QCD radiation can be factorised (in the soft and collinear limit)

$$d\sigma_{2\to3} \propto \sigma_{2\to2} \sum_{i=q,\bar{q}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} \frac{1+z^2}{1-z} \, dz$$

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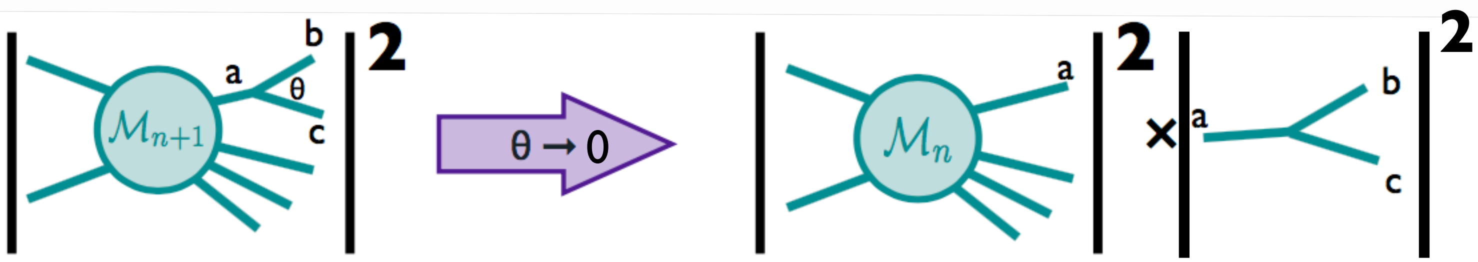

$$d\sigma_{n+1} \propto \sigma_n \sum_{\text{partons}} \frac{dt}{t}\, dz\, \frac{\alpha_s(t)}{2\pi} P_{ab}(z) \qquad \text{with} \qquad \frac{d\theta^2}{\theta^2} = \frac{dp_T}{p_T^2} = \frac{dq^2}{q^2} \equiv \frac{dt}{t}$$

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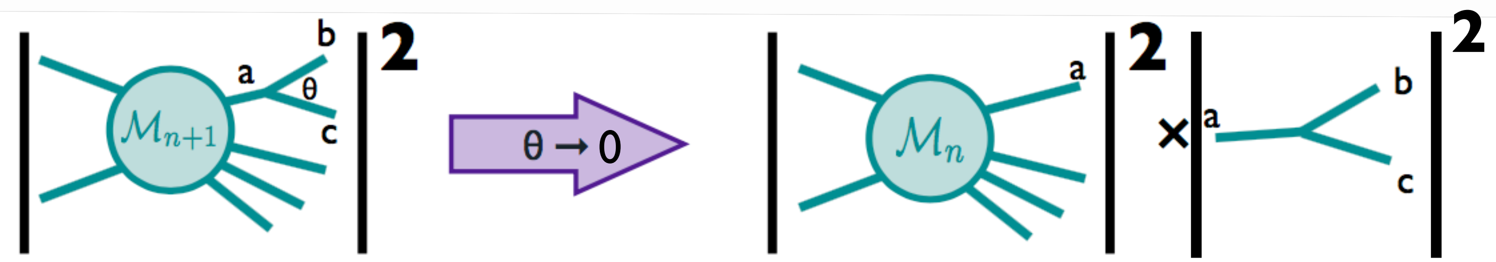

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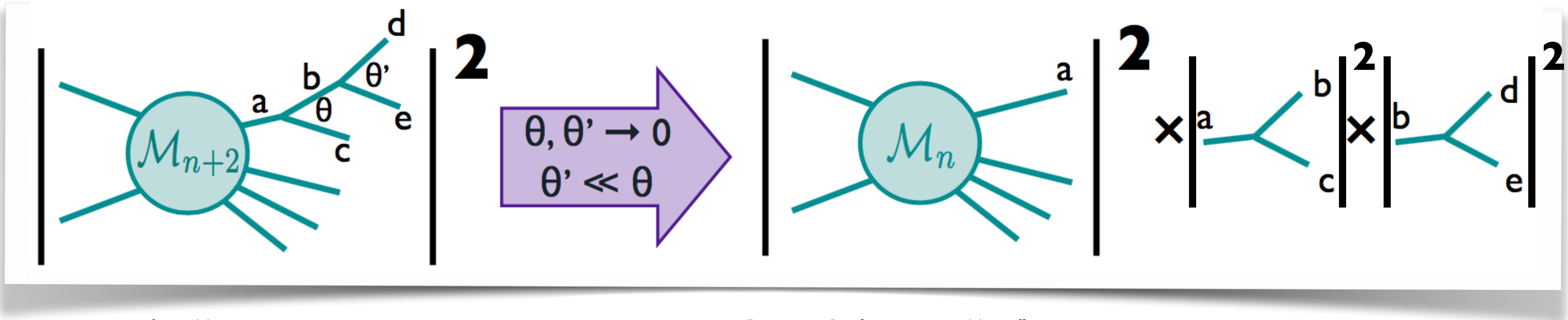
- The strong coupling is evaluated at the scale t
  - ★ t is the evolution variable (hardness of the branching, vanishes in the collinear limit)
  - ★ t controls the collinear behaviour
- $P_{ab}(z)$ consists in the QCD splitting kernels
  - ★ z controls the soft behaviour
  - ★ Universal resummation of their higher-order corrections

Further generalisation: multiple emission

✦ We iteratively add 2 emissions



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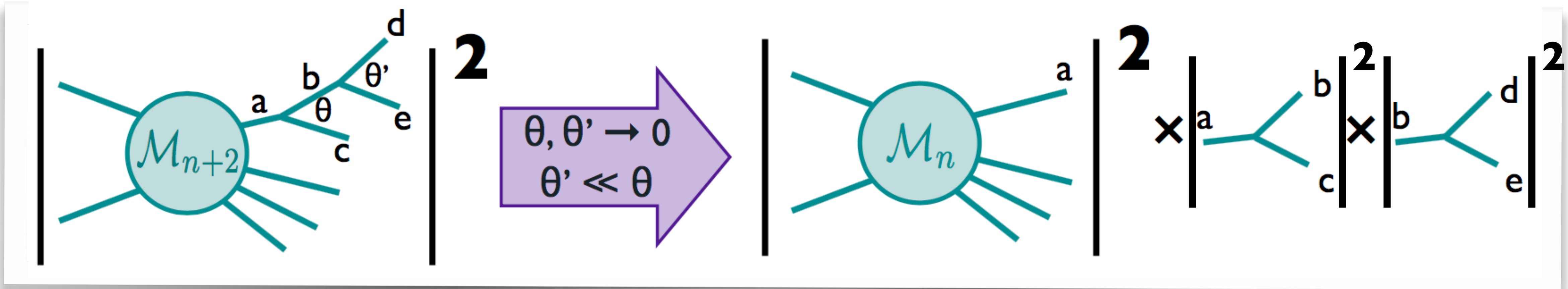

- $\theta' \ll \theta$: successive emission are ordered (or $t \ll t'$)
- Leading contribution to the $(n+2)$-body matrix element

$$\begin{aligned} \mathrm{d}\sigma_{n+2} &\propto \sigma_{n+1} \sum_{a,b} \frac{\mathrm{d}t}{t}\, \mathrm{d}z\, \frac{\alpha_s(t)}{2\pi}\, P_{ab}(z) \\ &\propto \sigma_n \sum_{a,b,b'} \left[\frac{\mathrm{d}t}{t}\, \mathrm{d}z\, \frac{\alpha_s(t)}{2\pi}\, P_{ab}(z)\right] \times \left[\frac{\mathrm{d}t'}{t'}\, \mathrm{d}z'\, \frac{\alpha_s(t')}{2\pi}\, P_{bb'}(z')\right] \end{aligned}$$

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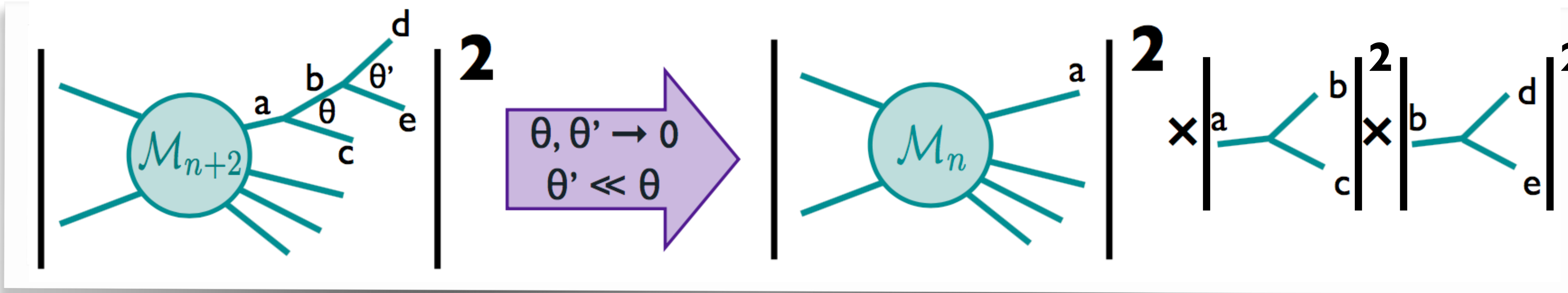

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✦ Iterative sequence of ordered emissions

- The $n+1$ emission independent of the history ~ Markov chain (no interferences)
- Leading contribution to the (n+k)-emission configuration: $\theta_1 \gg \theta_2 \gg \theta_3 \gg \ldots$

No-emission probability

- ✦ Parton showers: building a radiation history
  - ✤ A parton branches at *t*
    - ↝ It did no do it before
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- Derivation of the Sudakov form factor
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$$
\begin{aligned}
1 &= P_{\rm no\ emission}(t+{\rm d}t) + P_{\rm emission}(t+{\rm d}t)\\
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- Solution (probability a parton does not radiate between t_1 and t_2):

$$\Delta_a(t_1,t_2) \equiv P_{\text{no emission}}(t_1,t_2) = \exp\left[-\int_{t_1}^{t_2}\frac{\mathrm{d}t}{t}\sum_b\int \mathrm{d}z\,\frac{\alpha_s(t)}{2\pi}P_{ab}(z)\right]$$

Parton showers: the algorithm

✦ Splitting kernels and the Sudakov yield an evolution equation

$$\phi_a(t,t_0) = \Delta_a(t,t_0) + \sum_b \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \mathrm{d}z\ \Delta(t,t') \frac{\alpha_s(t')}{2\pi} P_{ab}(z)\ \phi_b(t', z t_0) \phi_c(t', (1-z) t_0)$$

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✦ The parton shower algorithm

- Start: a parton a at a scale t_0
- We generate an emission scale t_I according to the Sudakov probability $\Delta_a(t_0,t_I)$
  - ★ If $t_I < t_{cut}$, the algorithm stops ($t_{cut} \equiv$ breaking down of perturbative QCD)
  - ★ If $t_I > t_{cut}$, we generate z_I according to $P_{ab}(z)$ ⇝ one extra final-state parton
- Iteration until stops for all partons

Limitations and improvements

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  - ★ Multiparton matrix element merging

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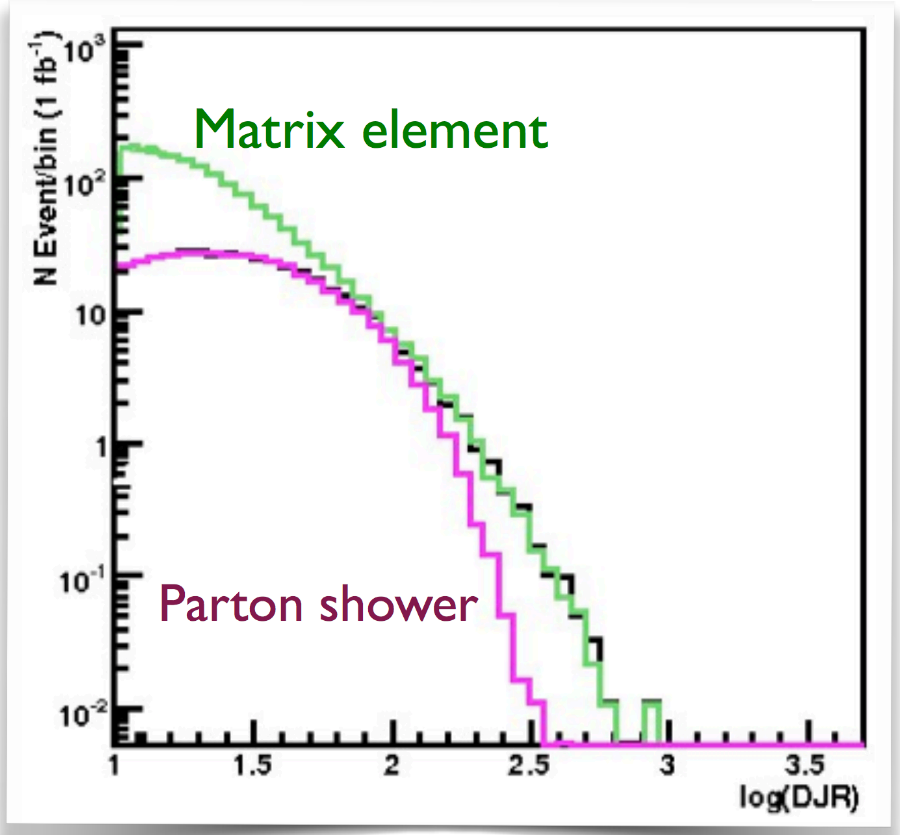

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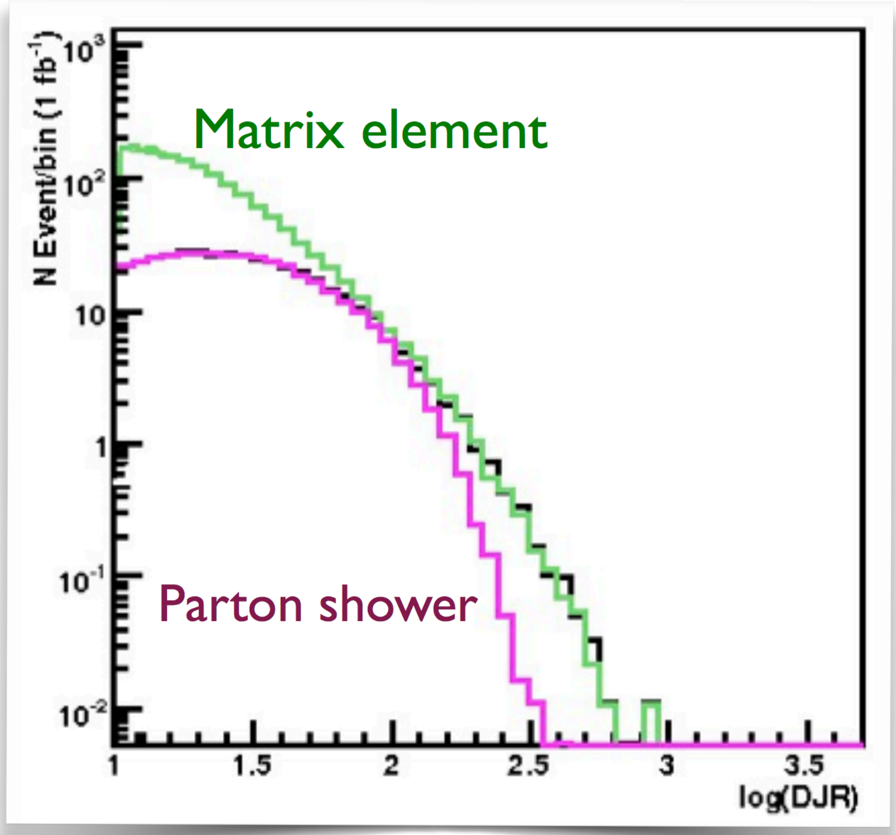

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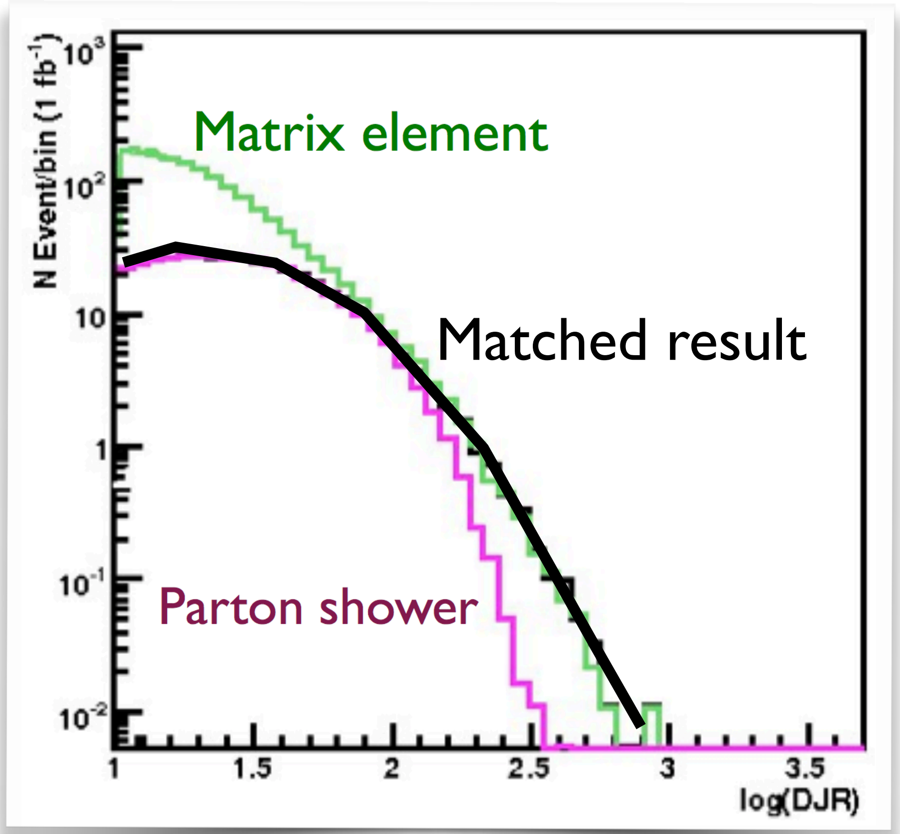

- ✦ Matching prescription: the best of both worlds
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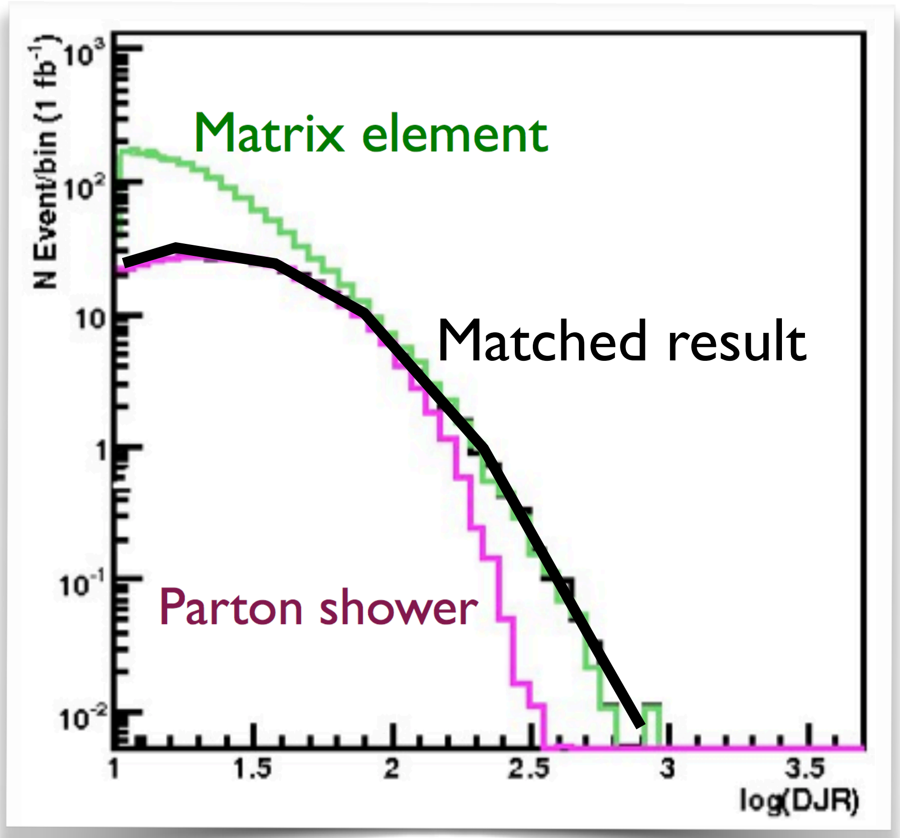

✦ Matching prescription: the best of both worlds

- The matrix elements control hard radiation
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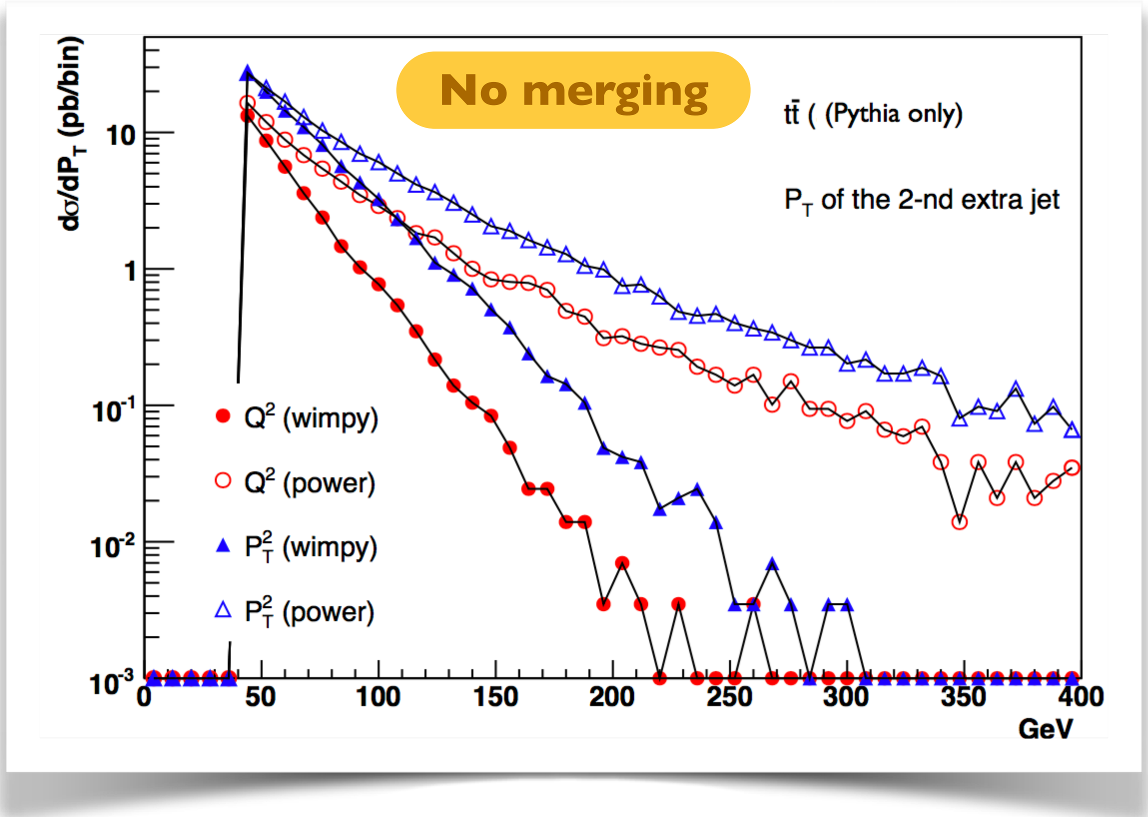
✦ Multipartonic matrix element merging (prescription)

- Matrix elements containing 0, 1, 2, … *N* extra partons
- Parton showering of each event
- Removal of any double counting

Matching / merging at work

✦ Different shower configuration leads to different predictions

✤ Loss of predictive power (especially in the hard regime)



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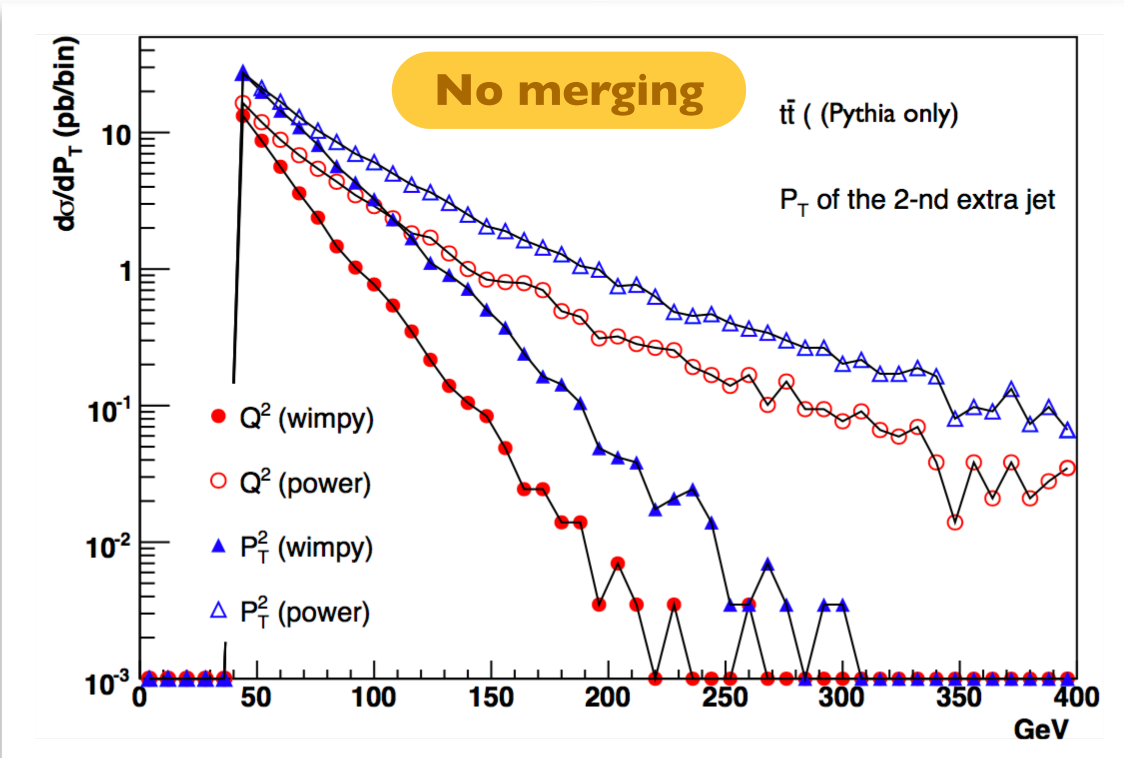
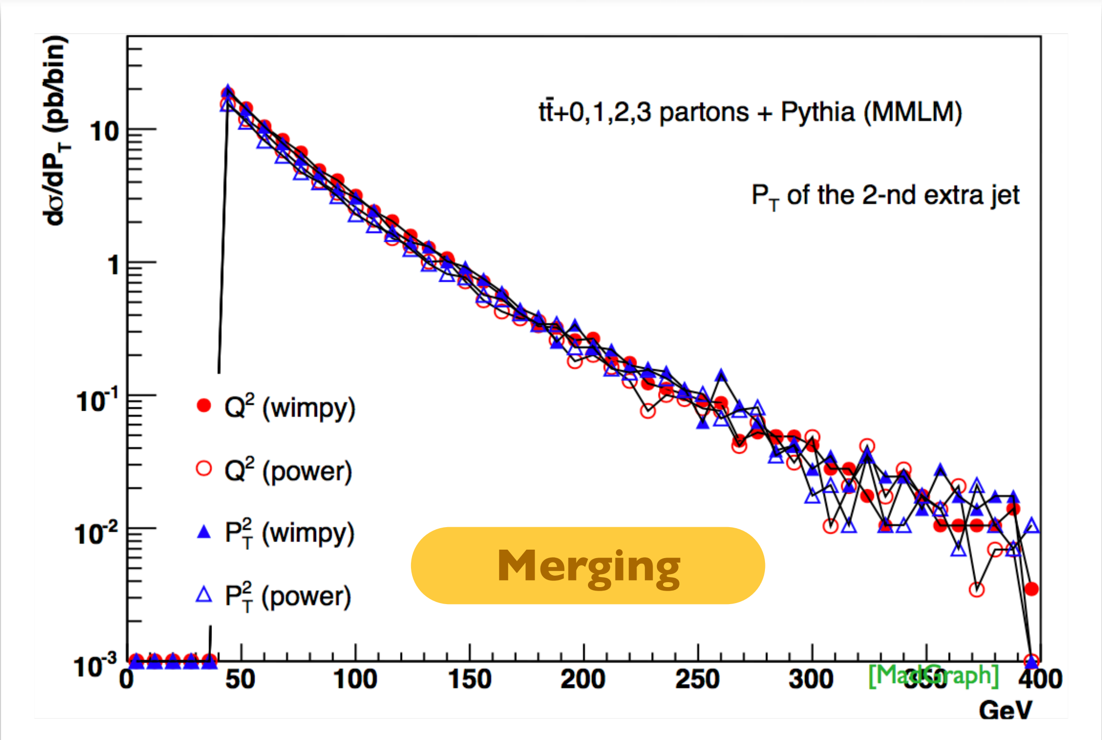
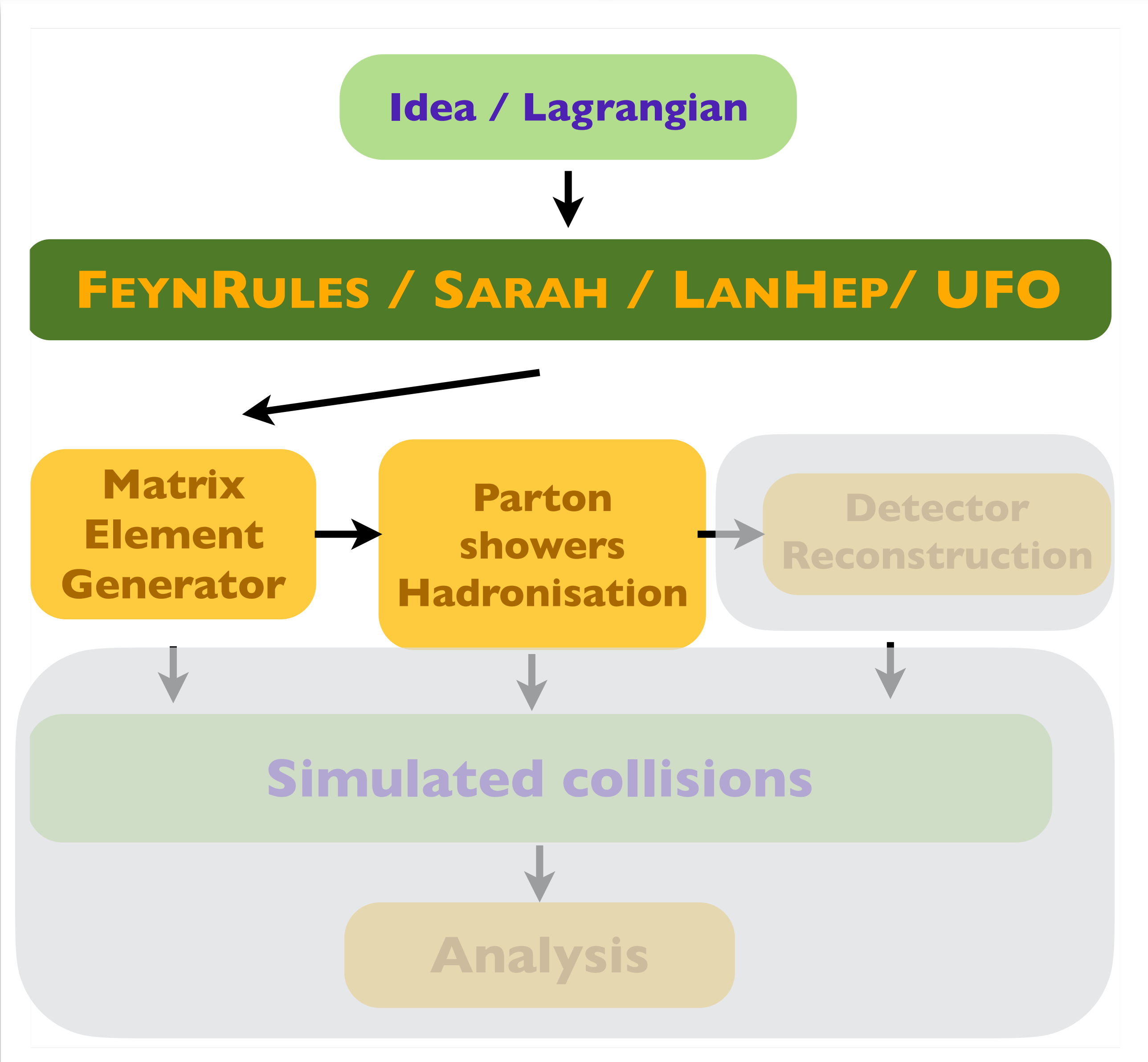


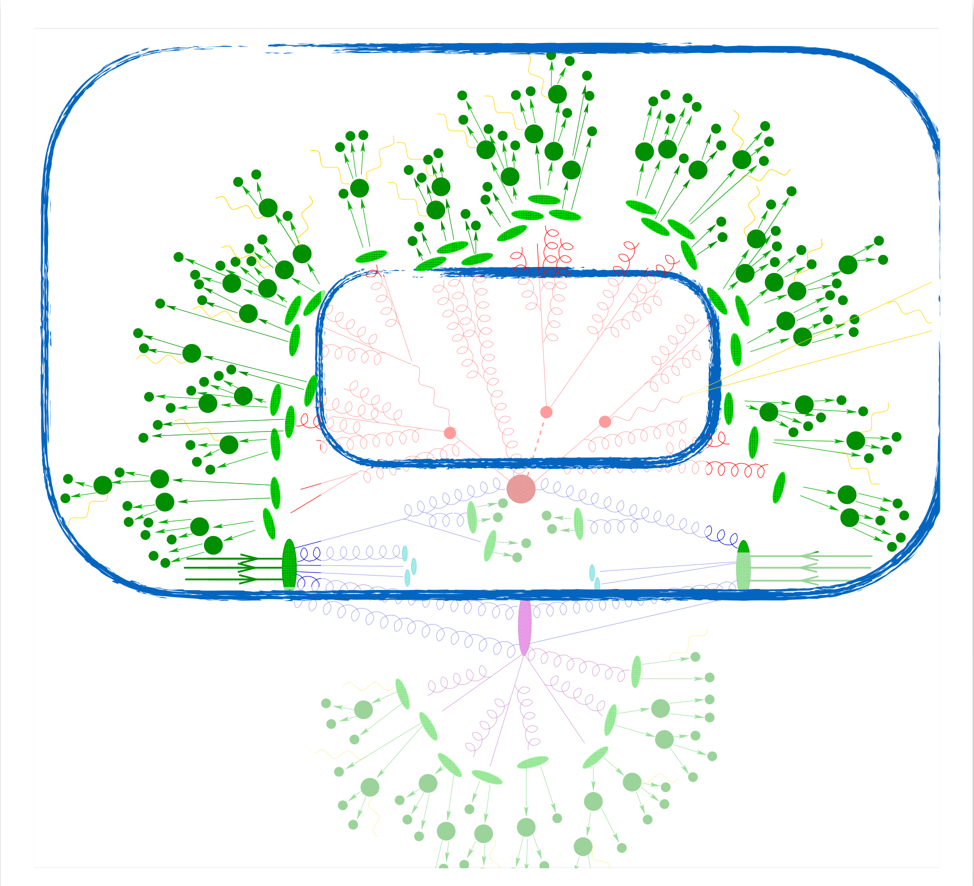


The simulation chain - step 4

✦ Tools connecting an idea to simulated collisions



✤ QCD environment
- ★ Parton showering
- ★ Hadronisation (and hadron decays)
- ★ Underlying event



Hadronisation

✦ Generalities

- ✤ Perturbative QCD breaks down at scales around 1 GeV
- ✤ Non-perturbative models: from partons to hadrons
  - ★ Cannot be computed from first principles

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✤ The cluster model [Webber (NPB`84)]

✦ Hadron decays

✤ Thousands of different channels

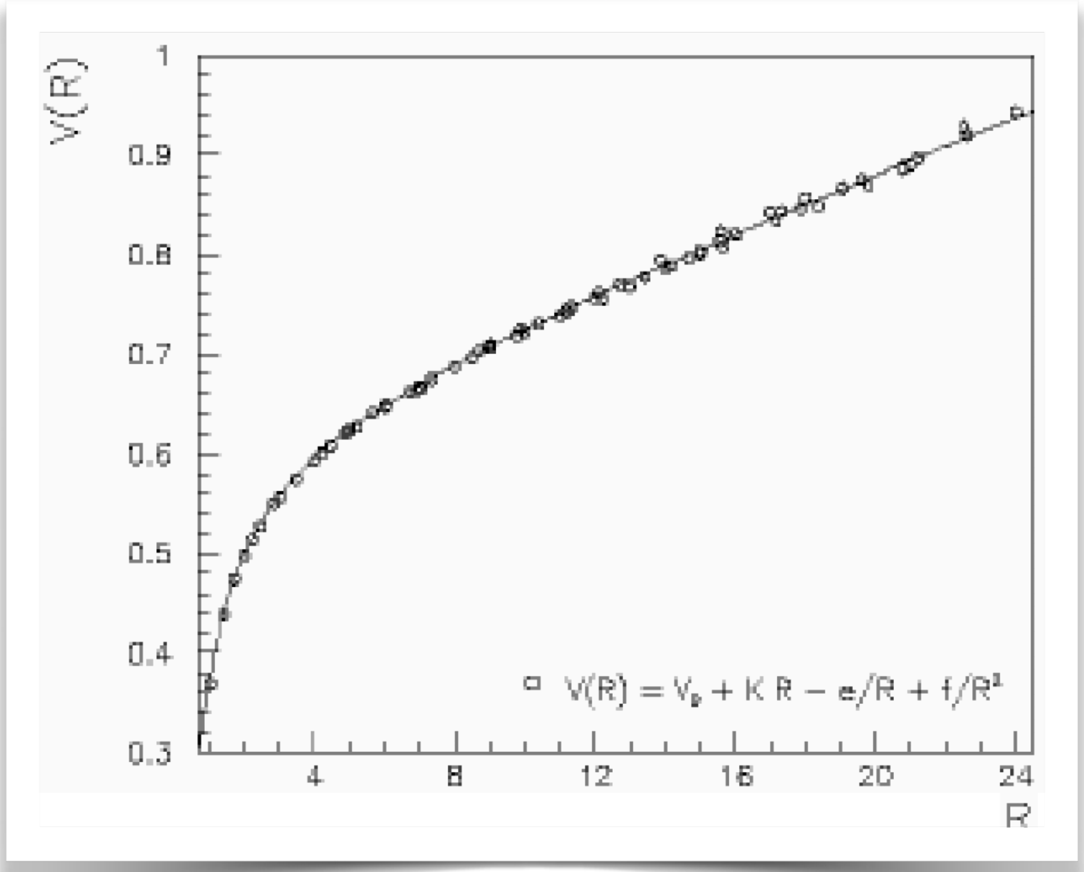
✤ Based on form factors

✤ Large uncertainties (the sum of the branching fractions may not be 1)

✤ Significant impact on the event shape

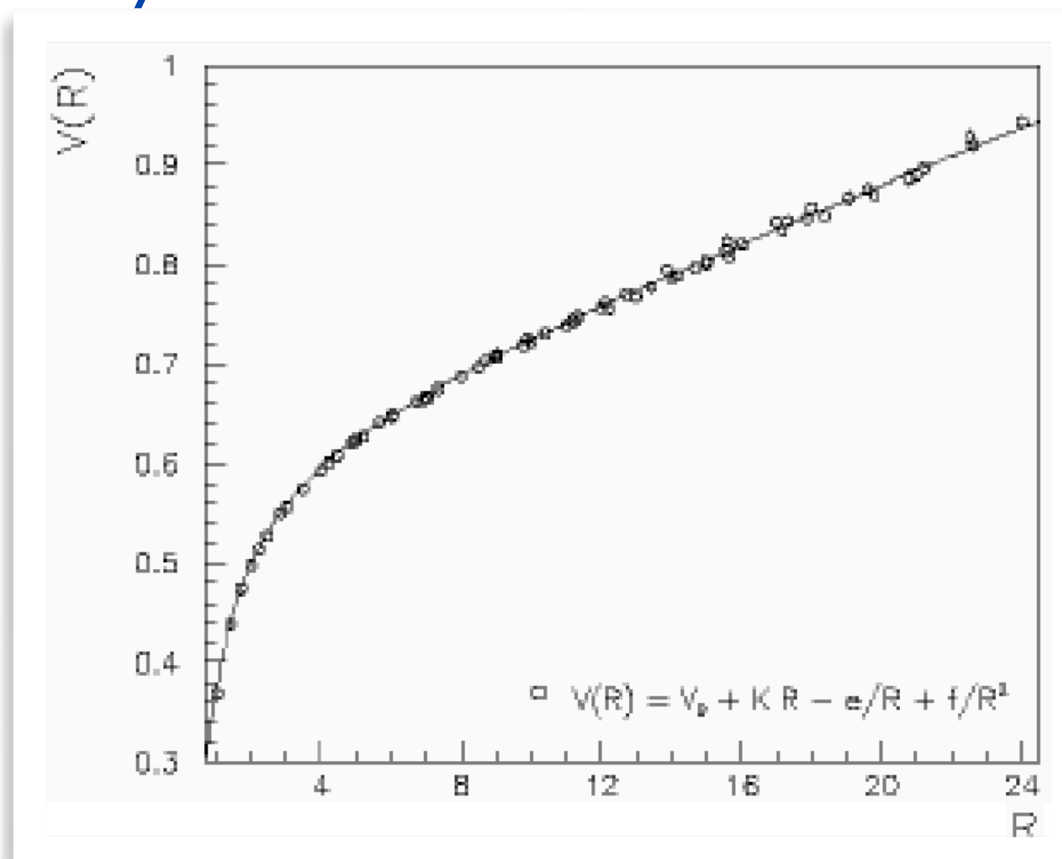
The Lund string model

- The QCD potential of a $q\bar{q}$ pair grows linearly with the distance
  - As calculated by lattice QCD
  - As calculated from quarkonia masses



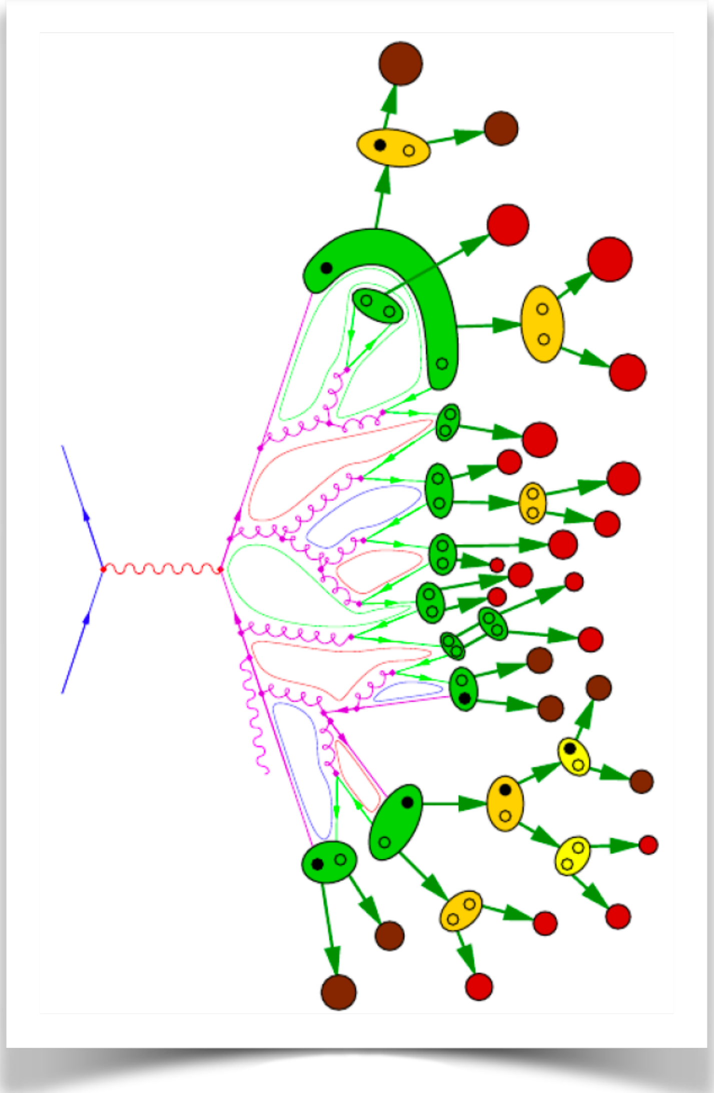
The Lund string model

- The QCD potential of a $q\bar{q}$ pair grows linearly with the distance
  - As calculated by lattice QCD
  - As calculated from quarkonia masses
- Linear growth ~ uniform string tension
  - Connects the colour charges
  - At large distance
    - More favorable to create a new pair
    - The string is broken into hadrons



The cluster model

- Relies on pre-confinement
  - Colour singlets formation
    - From partons close in phase space
    - Non-perturbative gluons splitting in $q\bar{q}$ pairs
  - Clustering of colour-adjacent partons



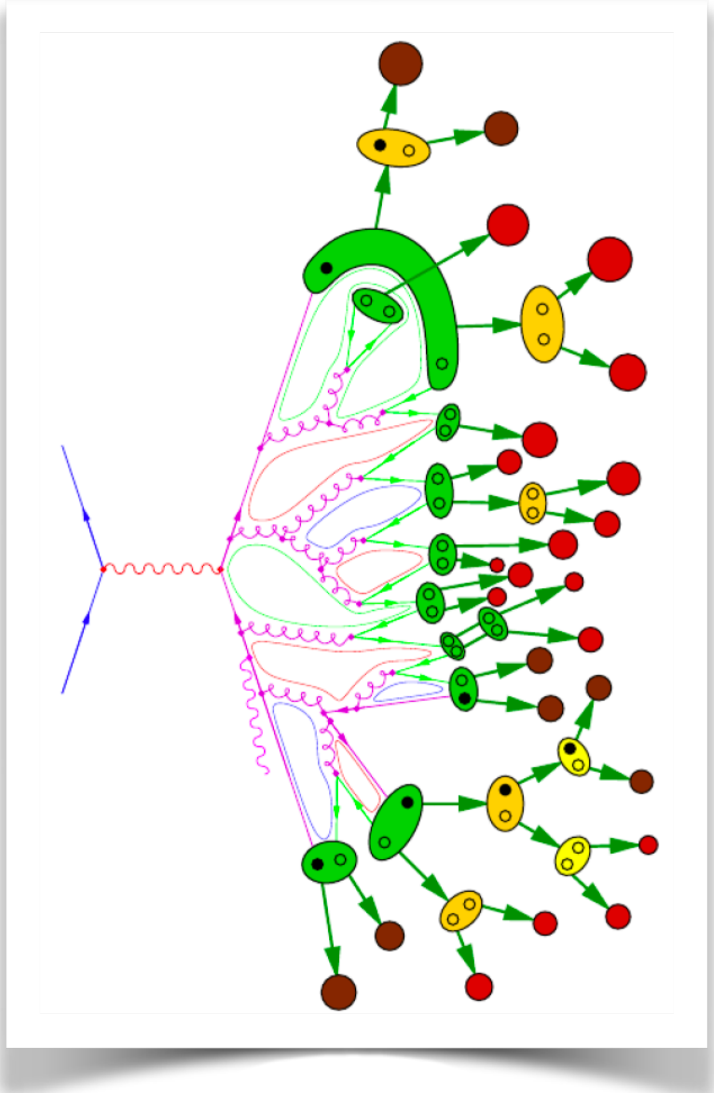
The cluster model

✦ Relies on pre-confinement

- ✤ Colour singlets formation
  - ↝ From partons close in phase space
  - ↝ Non-perturbative gluons splitting in $q\bar{q}$ pairs
- ✤ Clustering of colour-adjacent partons

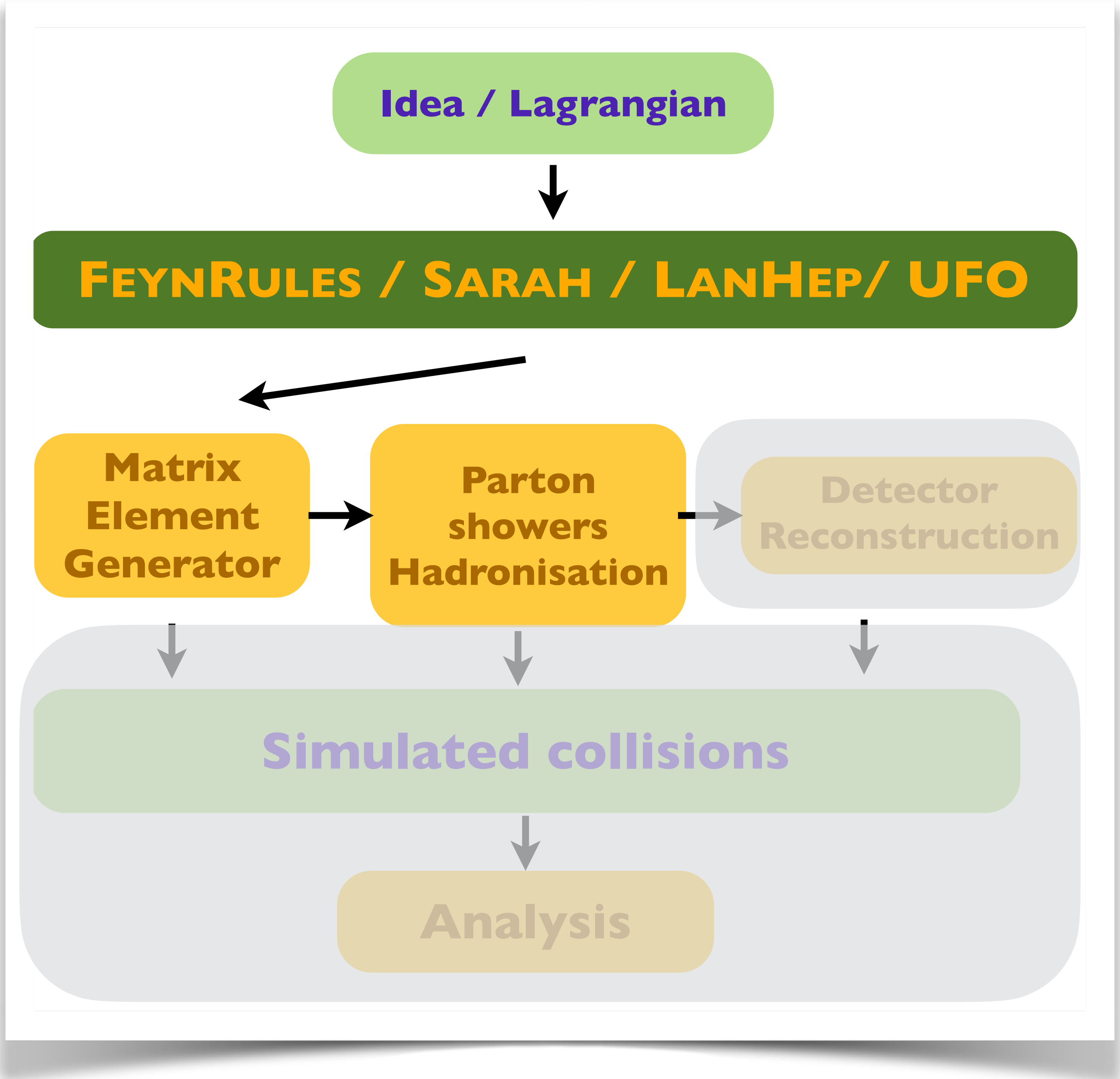
✦ Heavy clusters decay into light clusters

✦ Clusters decay into / radiate hadrons



The simulation chain - step 5

✦ Tools connecting an idea to simulated collisions

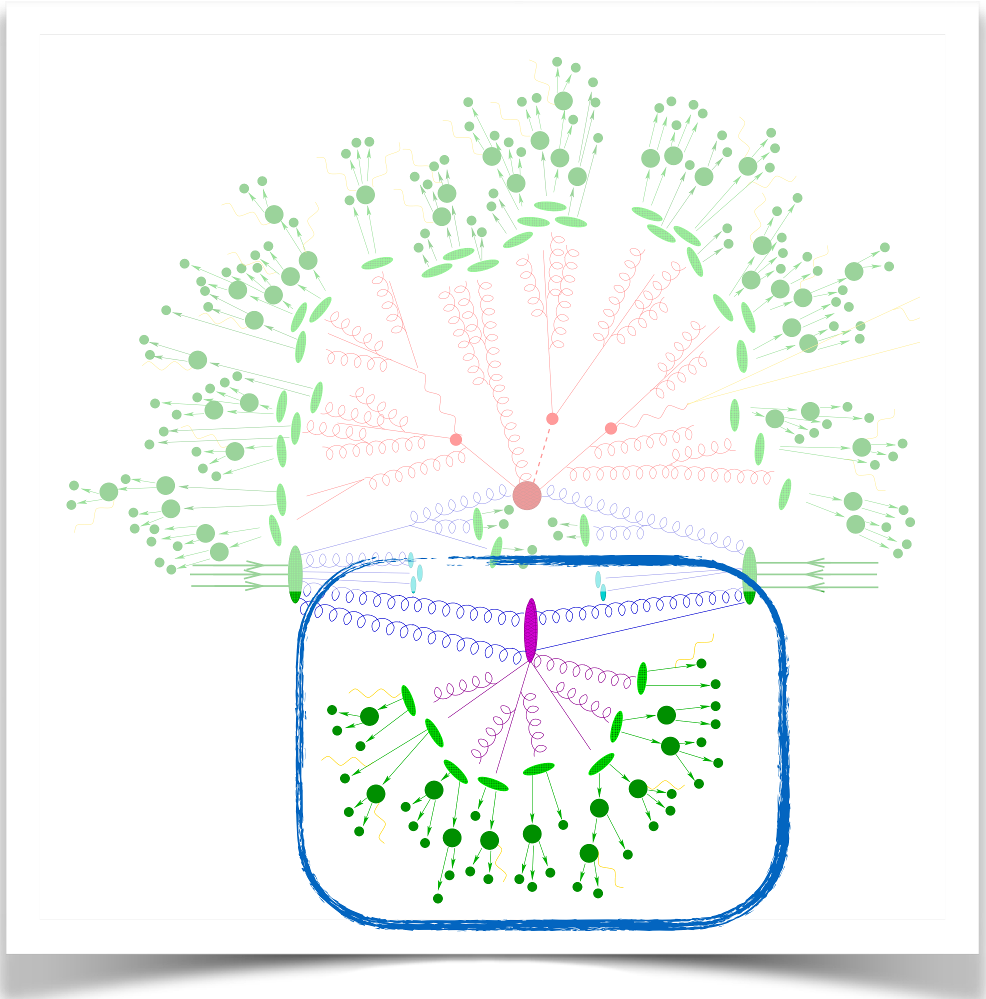


✤ QCD environment

★ Parton showering

★ Hadronisation (and hadron decays)

★ Underlying event



Underlying events

✦ Generalities: *n* independent secondary interactions

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- ✤ *n* follows a Poisson distribution with mean given by

$$\langle n \rangle = \frac{\sigma(p_T > p_T^{\rm min})}{\sigma_{\rm incl}}$$

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✤ Extra interactions ≡ perturbative collision ⊕ non-perturbative form factor

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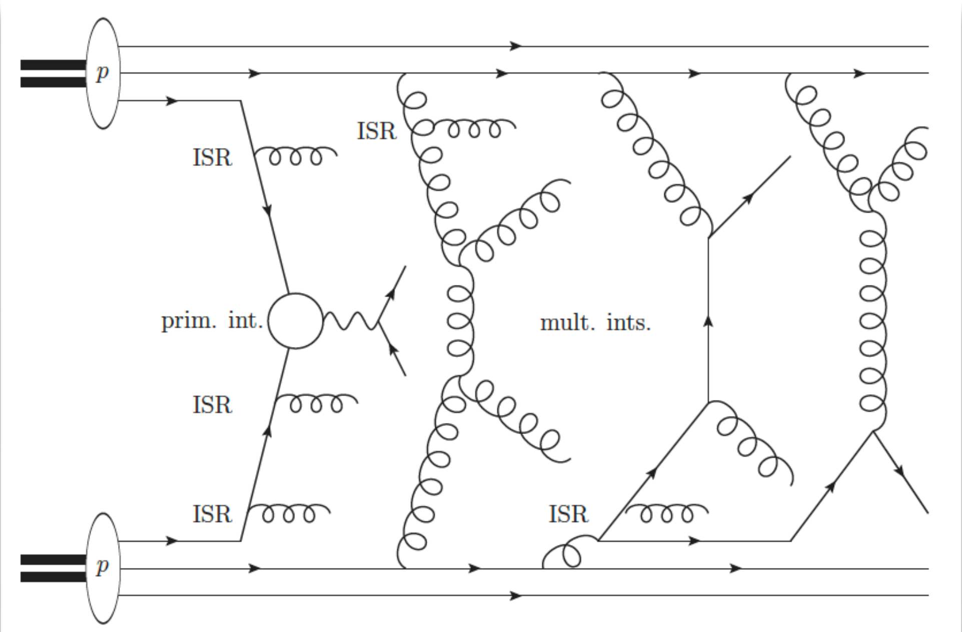
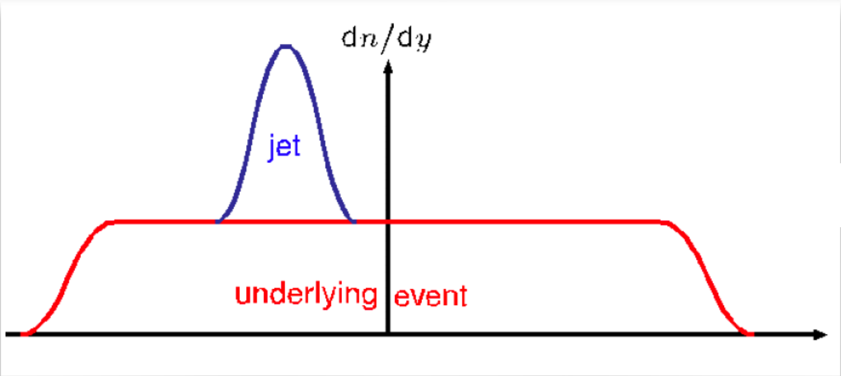
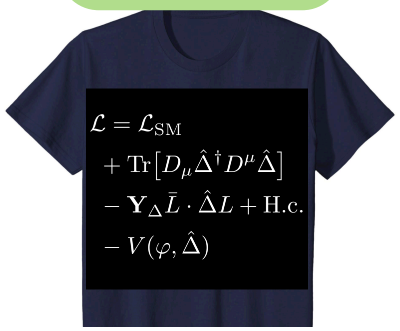




Outline

Summary

Idea
Lagrangian

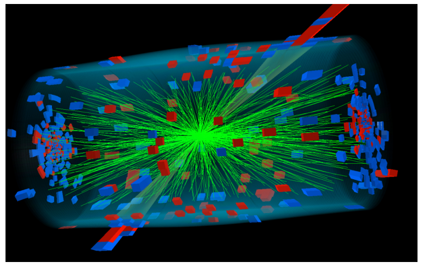


Chain of tools

Simulated collisions

Other tools

Event analysis



✦ Collider event simulation is a complex process

✤ Nature allows us to break it into factorisable pieces ↝ simulations step-by-step

★ Hard scattering

★ Parton showers

★ Hadronisation

★ Underlying event

★ Detector simulation (not covered here)

Automated tools to handle each piece, for any model