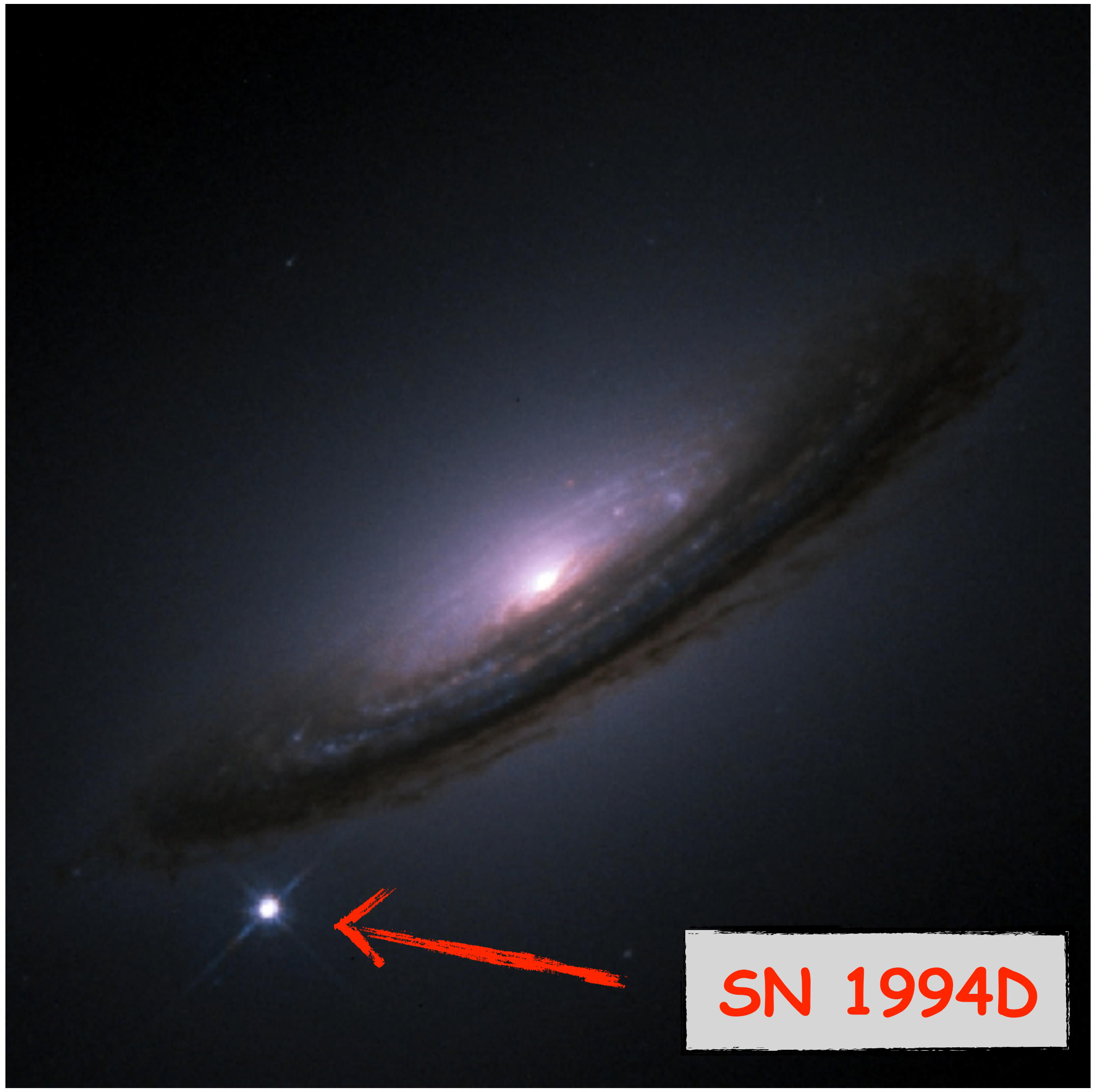
NPAC course on Astroparticles

III - ASTROPHYSICS: SUPERNOVA REMNANTS

Supernova explosions

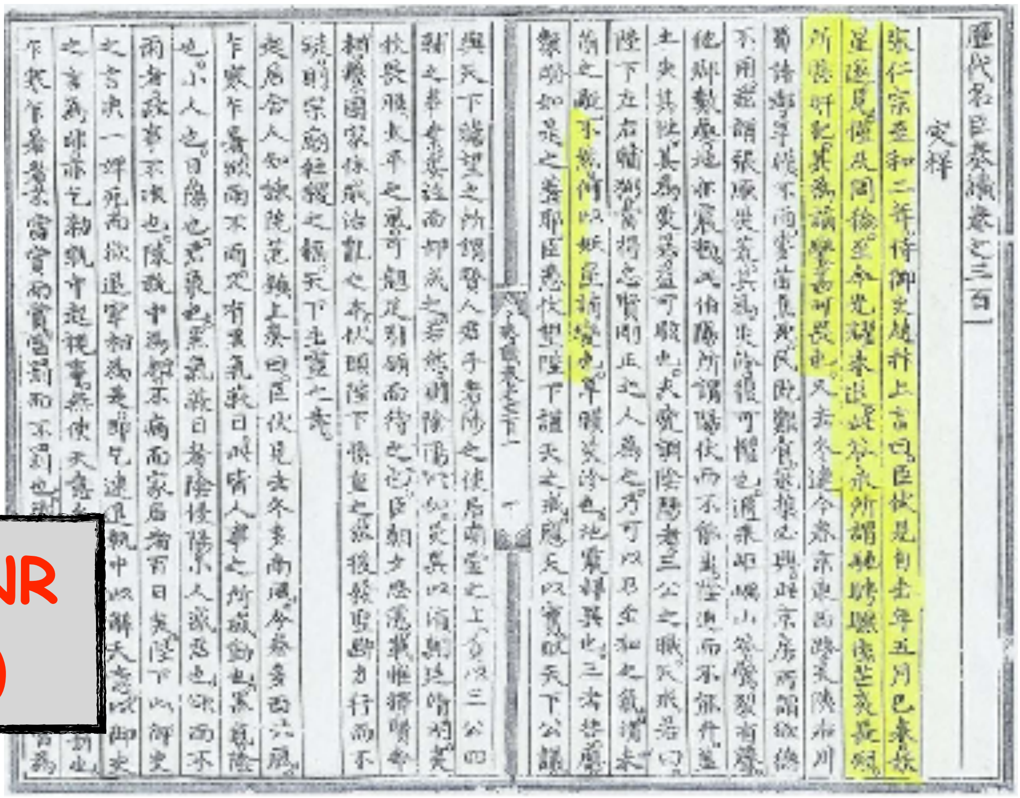
last phase of the evolution of some stars->huge increase of the star's luminosity



archaeologist G. Michanowsky studied some rock art in Bolivia and suggested that native Americans might have witnessed the explosion of the parent SN of the Vela SNR (~10-20 kyr)

Chinese astronomers (and others) reported on the appearance of bright stars

in 2016, ~6400 extragalactic SNe have been reported



Crab SNR (1059)

"Recent" supernova explosions

year	SNR	distance
185	RC W86 (?)	~3 kpc
393	RX J1713.7-3946 (?)	~1 kpc
1006	SN 1006	~2 kpc
1054	Crab	~2 kpc
1181	3C 58 (?)	~2 kpc
1572	Tycho	~2-3 kpc
1604	Kepler	~6 kpc

quite nearby SNRs

ALL recorded SNe associated with SNRs in the MW

extragalactic SNe + studies of galactic stellar population
-> ~3 SN/century in the MW

~1/200 yr -> obscured by interstellar dust

some other notable SNe

SNR	age	distance
Cas A	~350 yr	~3 kpc
G1.9+0.3	~150 yr	~8 kpc
SN 1987A	~30 yr	~50 kpc

SN not recorded: amongst the best studied SNRs (obscured?) → Cas A

SN not recorded: youngest known galactic SNR → G1.9+0.3

closest extragalactic SN (Large Magellanic Cloud) → SN 1987A

Small perturbations in a fluid: sound waves

fixed fluid element → $\frac{\mathrm{d}}{\mathrm{d}t} \longrightarrow \frac{\partial}{\partial t} + \vec{u}\cdot\nabla$ ← fixed position

mass conservation

$$\frac{\partial\varrho}{\partial t} + \nabla\cdot(\varrho\vec{v}) = 0$$

momentum conservation

$$\varrho\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\nabla P \longrightarrow \frac{\partial\vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{\nabla P}{\varrho}$$

equilibrium **perturbed**

$$\begin{array}{cccccc} \vec{v} & = & 0 & \longrightarrow & \delta\vec{v} \\ \varrho & = & \varrho_0 & \longrightarrow & \varrho_0 + \delta\varrho \\ P & = & P_0 & \longrightarrow & P_0 + \delta P \end{array}$$

small perturbations

$$\frac{\partial}{\partial t}\delta\varrho + \varrho_0\nabla\cdot\delta\vec{v} = -\nabla\cdot\left(\cancel{\delta\varrho}\,\delta\vec{v}\right)$$

$$\frac{\partial}{\partial t}\delta\vec{v} = -\cancel{\delta\vec{v}\cdot\nabla\delta\vec{v}} - \frac{1}{\varrho_0+\cancel{\delta\varrho}}\nabla\delta P$$

Small perturbations in a fluid: sound waves

$$\frac{\partial}{\partial t}\delta\varrho + \varrho_0 \nabla\cdot\vec{\delta v} = 0 \quad \rightarrow \frac{\partial}{\partial t} \rightarrow \quad \frac{\partial^2}{\partial t^2}\delta\varrho + \varrho_0 \frac{\partial}{\partial t}\nabla\cdot\vec{\delta v} = 0$$

$$\varrho_0 \frac{\partial}{\partial t}\vec{\delta v} = -\nabla\delta P \quad \rightarrow \nabla\cdot \rightarrow \quad \varrho_0 \nabla\cdot\frac{\partial}{\partial t}\vec{\delta v} = -\nabla^2\delta P$$

— subtract —>

$$\frac{\partial^2}{\partial t^2}\delta\varrho - \nabla^2\delta P = 0$$

adiabatic eq. of state

$$P = K\varrho^\gamma$$

$$\gamma = \frac{5}{3}$$

monoatomic gas

$$\delta P = \left(\frac{\partial P}{\partial\varrho}\right)_s \delta\varrho = \frac{\gamma P}{\varrho}\delta\varrho \equiv c_s^2\delta\varrho$$

constant entropy

sound speed

small perturbations

$$c_{s,0}^2 = \frac{\gamma P_0}{\varrho_0}$$

wave equation

$$\frac{\partial^2}{\partial t^2}\delta\varrho - c_{s,0}^2\nabla^2\delta\varrho = 0$$

$$\frac{\delta\varrho}{\varrho_0} = A\ e^{i\left(\vec{k}\cdot\vec{x}\pm\omega t\right)}$$

$$A \ll 1 \quad , \quad \omega^2 = k^2 c_{s,0}^2$$

The formation of shock waves

let's relax the assumption of small perturbations —> finite amplitude perturbations

$c_s \cancel{=} c_{s,0}$

supersonic

$c_s > c_{s,0}$

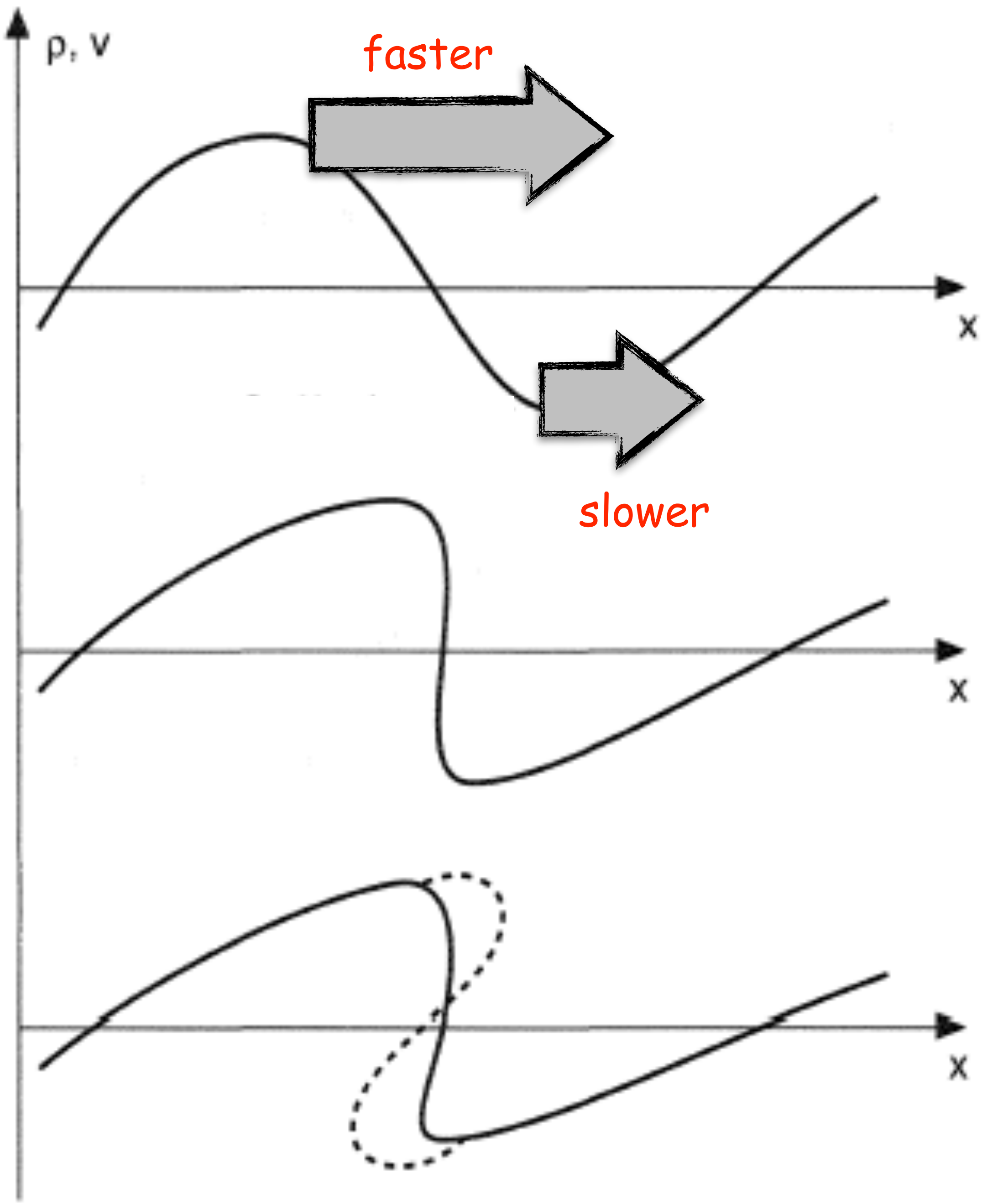
the propagation speed is faster when the density is larger

↓

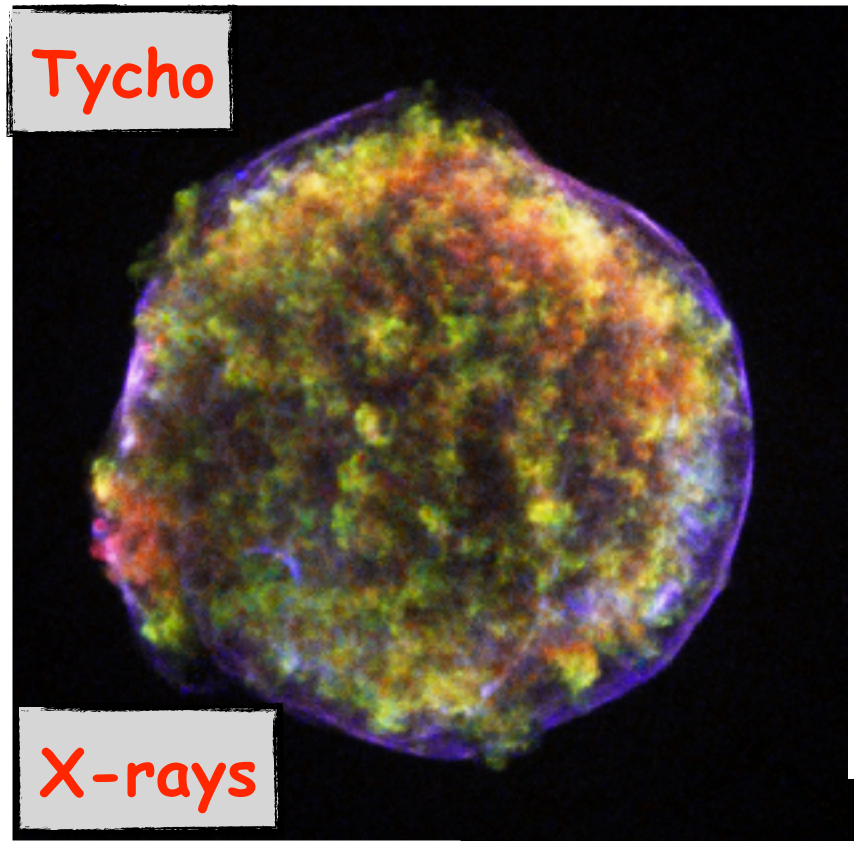
steepening of the wave

↓

formation of a discontinuity

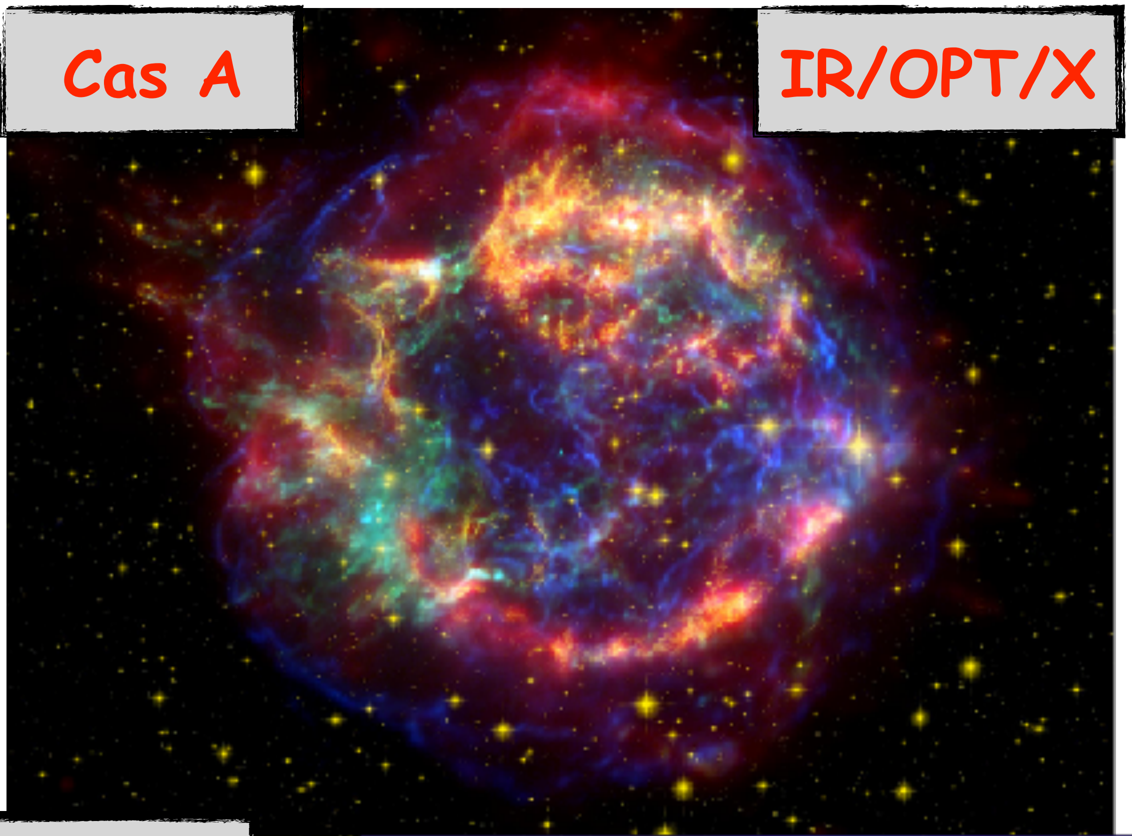


Supernova remnant shocks



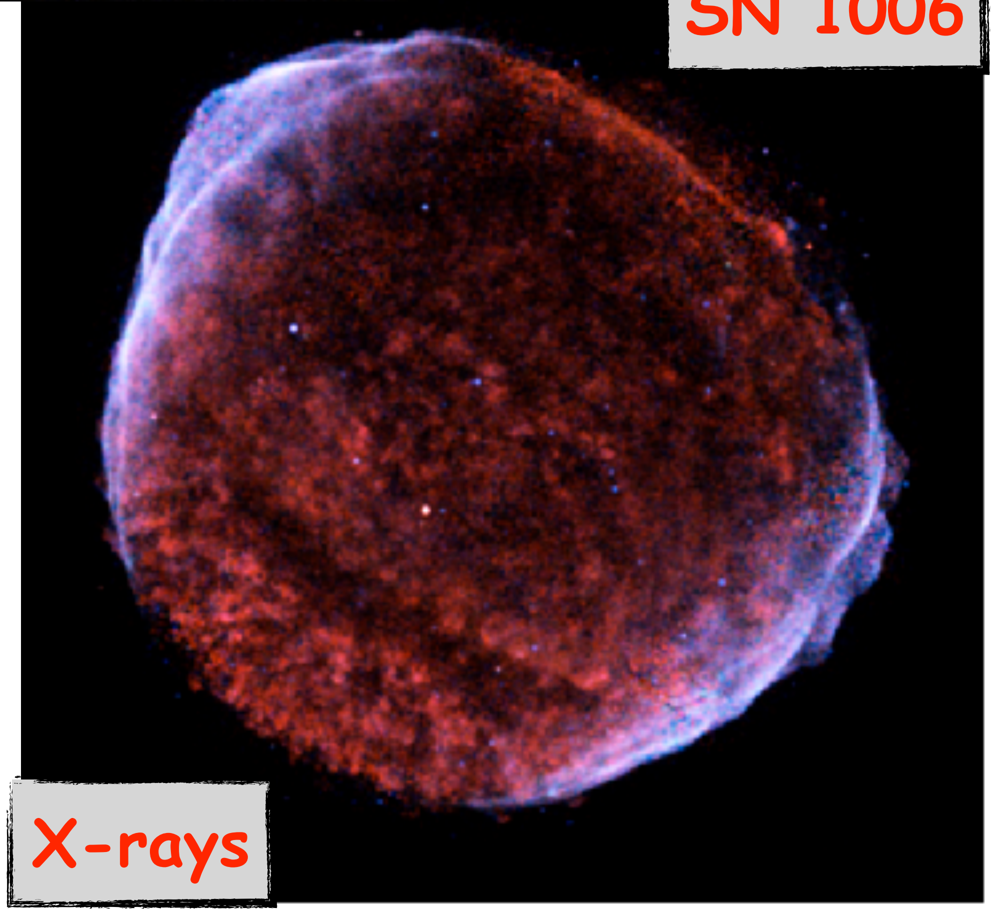
Tycho

X-rays



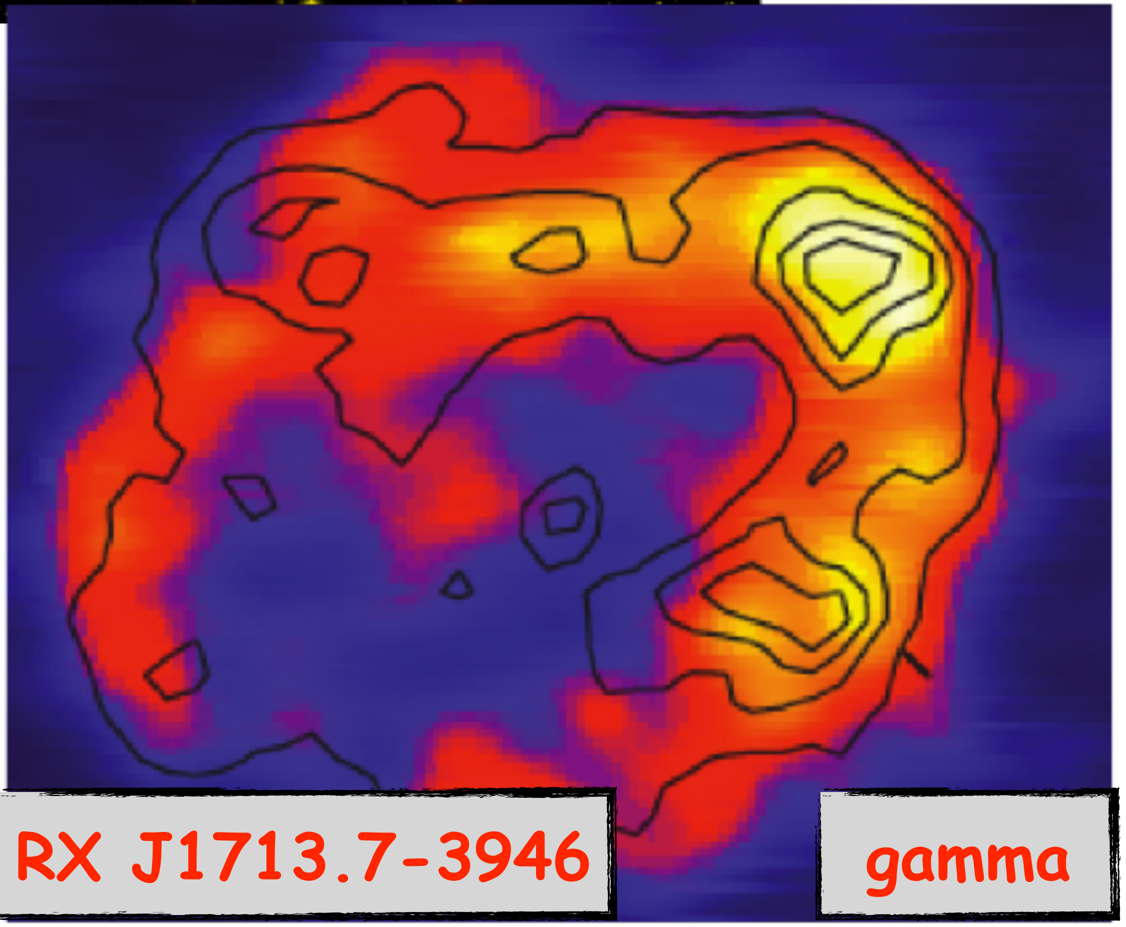
Cas A

IR/OPT/X



SN 1006

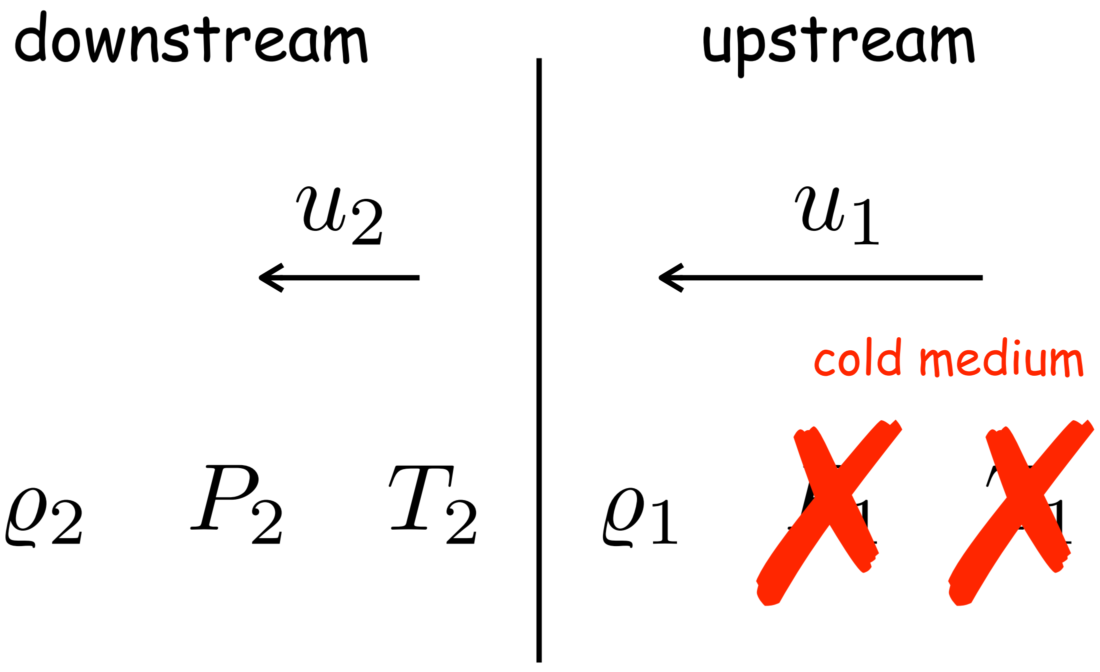
X-rays



RX J1713.7-3946

gamma

Shock waves: conservation laws



shock rest frame

Mach number

$$\mathcal{M} = \frac{u_1}{c_s}$$

strong shock

$$\mathcal{M}^2 \gg 1 \longrightarrow \frac{\varrho_1 u_1^2}{\gamma P_1} \gg 1$$

mass $$\varrho_1 u_1 = \varrho_2 u_2$$

momentum $$\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2$$

$$\left.\right\} \quad 1 + \cancel{\frac{P_1}{\varrho_1 u_1^2}} = \frac{u_2}{u_1} + \frac{P_2}{\varrho_1 u_1^2}$$

energy $$\frac{1}{2}\varrho_1 u_1^3 + \cancel{\varrho_1 u_1 \left(\epsilon_1 + \frac{P_1}{\varrho_1}\right)} = \frac{1}{2}\varrho_2 u_2^3 + \varrho_2 u_2 \underbrace{\left(\epsilon_2 + \frac{P_2}{\varrho_2}\right)}$$

internal energy per unit mass

specific enthalpy

Jump (Rankine-Hugoniot) conditions

compression factor $r = \frac{\varrho_2}{\varrho_1} = \frac{u_1}{u_2}$

ideal gas $P_2 = (\gamma - 1)\epsilon_2 \varrho_2$

$$\begin{cases} \frac{P_2}{\varrho_1 u_1^2} = 1 - \frac{1}{r} \quad \rightarrow P_2 = \frac{2}{\gamma+1}\varrho_1 u_1^2 \xrightarrow[\gamma\,=\,5/3]{} \frac{3}{4}\varrho_1 u_1^2 \\ \frac{1}{2}\varrho_1 u_1^3 = \frac{1}{2}\varrho_2 u_2^3 + \varrho_2 u_2 \left(\epsilon_2 + \frac{P_2}{\varrho_2}\right) \rightarrow \varrho_1 u_1^3 = \varrho_2 u_2^3 + \frac{2\gamma}{\gamma-1} P_2 u_2 \end{cases}$$

ram-pressure —> thermal pressure

$$\rightarrow \frac{u_1^2}{u_2^2} = 1 + \frac{2\gamma}{\gamma-1}\left(\frac{P_2}{\varrho_1 u_1^2}\right)\left(\frac{u_1}{u_2}\right) \quad \rightarrow r^2 - 1 = \frac{2\gamma}{\gamma-1}(r-1)$$

r = 1 -> unphysical solution

$$\rightarrow r = \frac{2\gamma}{\gamma-1} - 1 = \frac{\gamma+1}{\gamma-1} \xrightarrow[\gamma\,=\,5/3]{} 4$$

gas compression

Strong shocks

- compress moderately the gas

$$r = \frac{\varrho_2}{\varrho_1} = \frac{\gamma+1}{\gamma-1} \xrightarrow[\gamma = 5/3]{} 4$$

- make the downstream gas subsonic

$$\mathcal{M}_2^2 = \frac{u_2^2}{c_{s,2}^2} = \frac{\varrho_2 u_2^2}{\gamma P_2} = \frac{\gamma-1}{2\gamma} \xrightarrow[\gamma = 5/3]{} \frac{1}{5} < 1$$

- heat significantly the gas

$$P_2 = \frac{2}{\gamma+1}\varrho_1 u_1^2 \equiv \frac{\varrho_2}{m_p} kT_2$$

typical SNR shock speed

$$kT_2 = \frac{2(\gamma-1)}{(\gamma+1)^2} m_p u_1^2 \xrightarrow[\gamma = 5/3]{} \frac{3}{16} m_p u_1^2 \sim 2 \left(\frac{u_1}{1000 \ \mathrm{km/s}}\right)^2 \ \mathrm{keV}$$

thermal energy

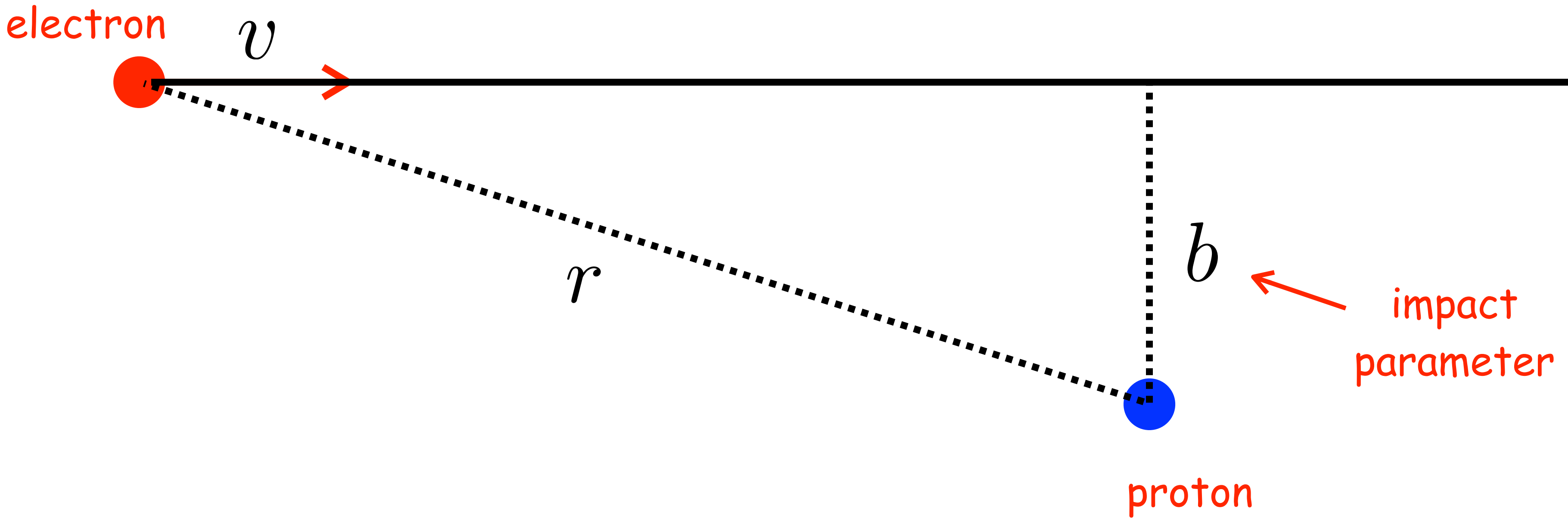
kinetic energy

hot plasma!!!

Emission from hot plasmas: thermal Bremsstrahlung

Radiation from a thermal electron-proton plasma

$$v = \sqrt{\frac{3kT}{m}} \longrightarrow v_e = \left(\frac{m_p}{m_e}\right)^{1/2} v_p \gg v_p$$



$$F = \frac{e^2}{r^2}$$

power emitted by an accelerated charge

$$P = \frac{2e^2}{3c^3} a^2$$

$$\begin{cases} r \approx b \rightarrow a \approx \frac{e^2}{m_e b^2} \\ r \gg b \rightarrow a \approx 0 \end{cases}$$

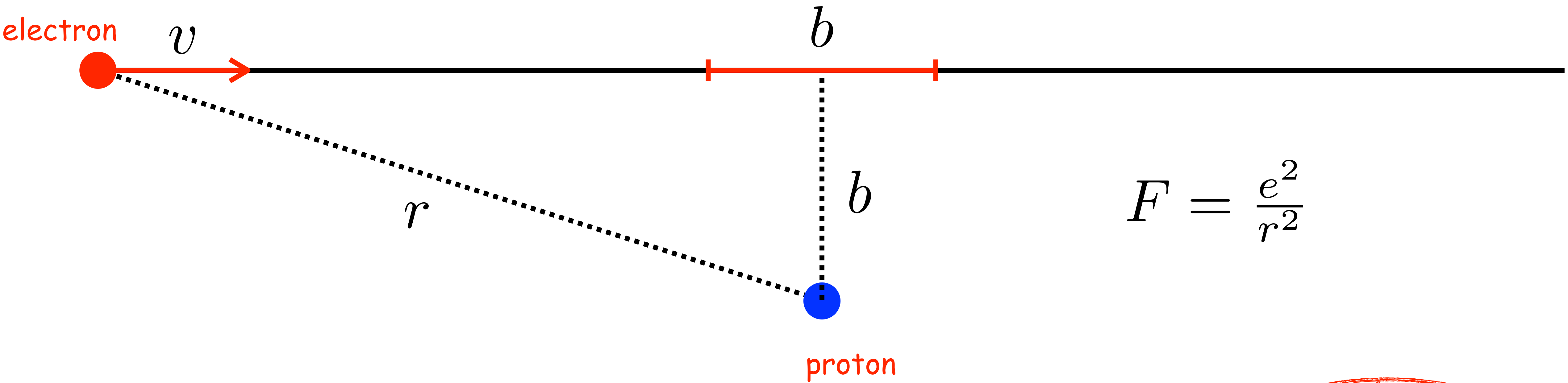
Thermal Bremsstrahlung

very brutal approximations...

characteristic time for the interaction

$$\tau \approx \frac{b}{v} \longrightarrow \omega \approx \frac{1}{\tau} = \frac{v}{b}$$

characteristic frequency of the emitted radiation



$$F = \frac{e^2}{r^2}$$

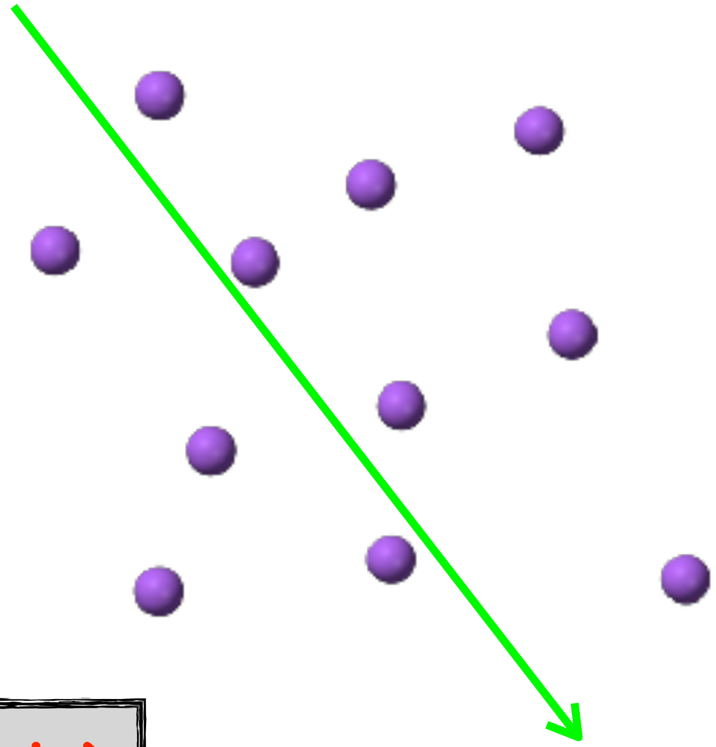
power emitted by an accelerated charge

$$P = \frac{2e^2}{3c^3}a^2$$

$$\begin{cases} r \approx b \rightarrow \boxed{a \approx \dfrac{e^2}{m_e b^2}} \\ r \gg b \rightarrow a \approx 0 \end{cases}$$

Thermal Bremsstrahlung

rough estimate of the impact parameter b



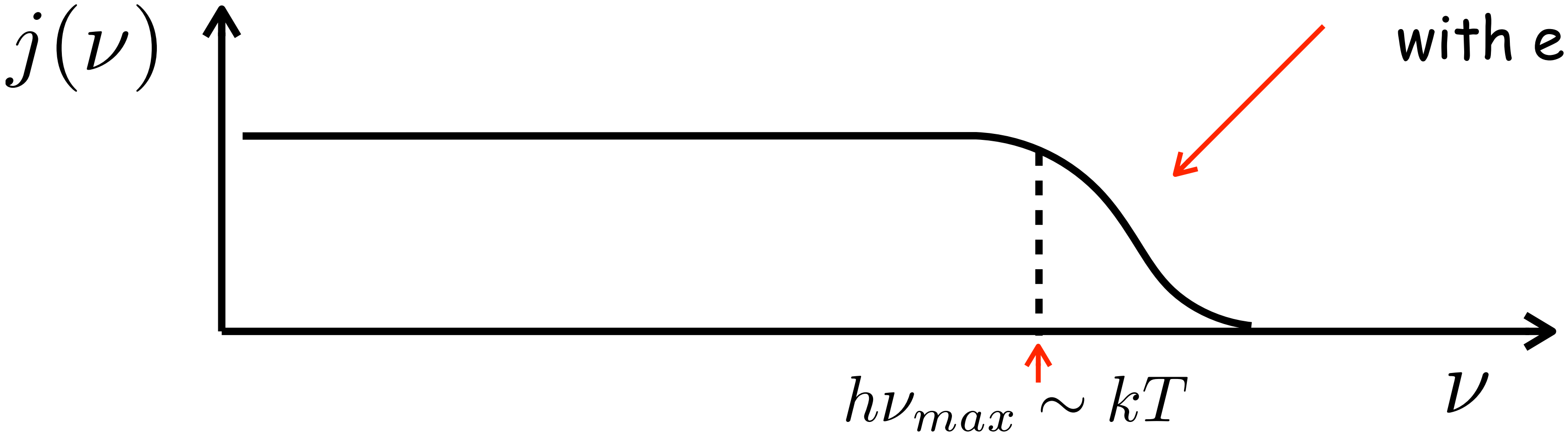
plasma proton density -> n_p

mean distance between protons -> $l_p \sim n_p^{-1/3} \approx b$

emissivity (power per unit frequency per unit solid angle)

ω -> ν = ω/2π

$$j(\nu) \approx \frac{n_e P}{(4\pi)} \frac{2\pi}{\omega} = \frac{n_e n_p e^6}{3c^3 m_e^2} \left(\frac{m_e}{kT}\right)^{1/2}$$



suppression -> no electrons with energy >> kT

Thermal Bremsstrahlung

exact solution

$$j(\nu) = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{n_e n_p e^6}{m_e^2 c^3} \left(\frac{m_e}{kT}\right)^{1/2} e^{-\frac{h\nu}{kT}}\,\bar{g}_{ff}$$

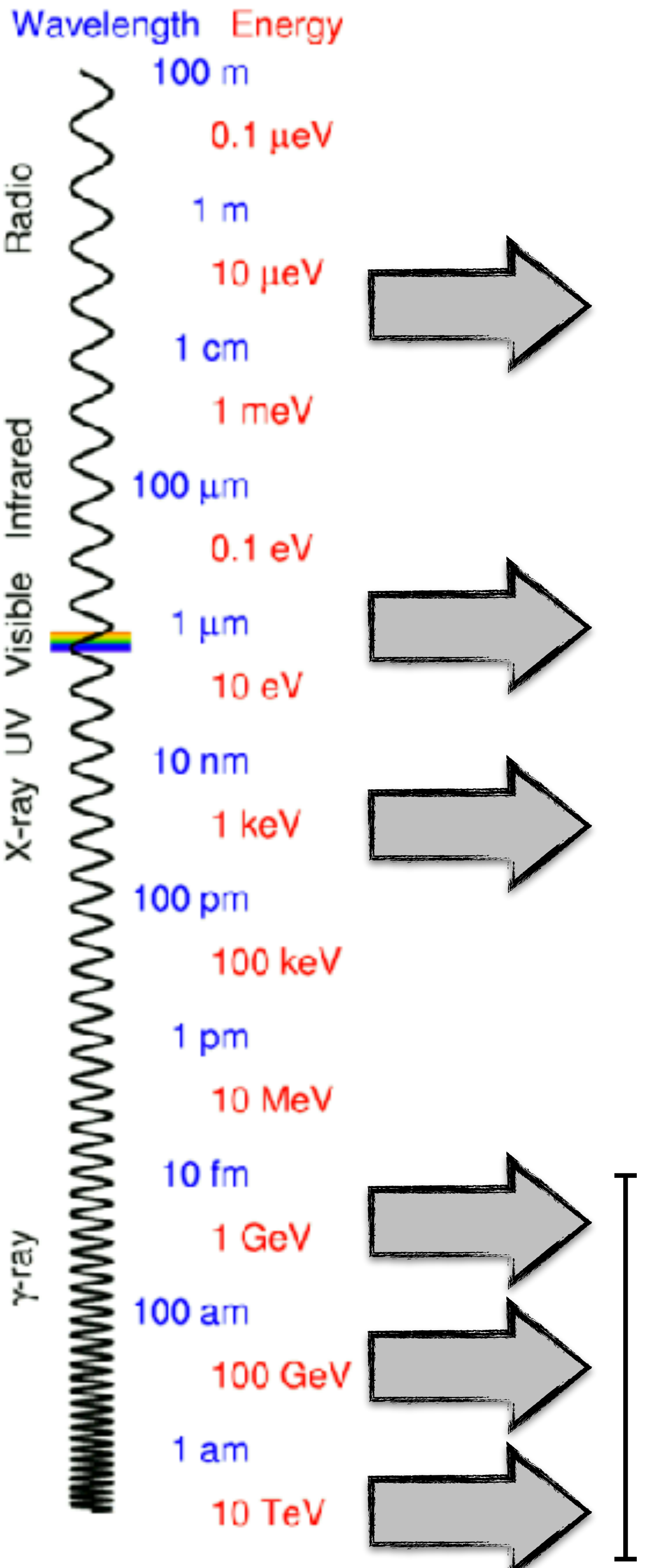
$$j = \int \mathrm{d}\nu j_\nu = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{n_e n_p e^6}{m_e^2 c^3} \frac{(m_e kT)^{1/2}}{h} \bar{g}_{ff}$$

in numbers (cgs units)

$$j(\nu) = 5.4 \times 10^{-39} Z^2 n_e n_i T^{-1/2} e^{-\frac{h\nu}{kT}}\,\bar{g}_{ff}\ \mathrm{erg/s/cm^3/Hz/sr}$$

$$j = 1.1 \times 10^{-28} Z^2 n_e n_i T^{1/2} \bar{g}_{ff}\ \mathrm{erg/s/cm^3/sr}$$

The electromagnetic spectrum



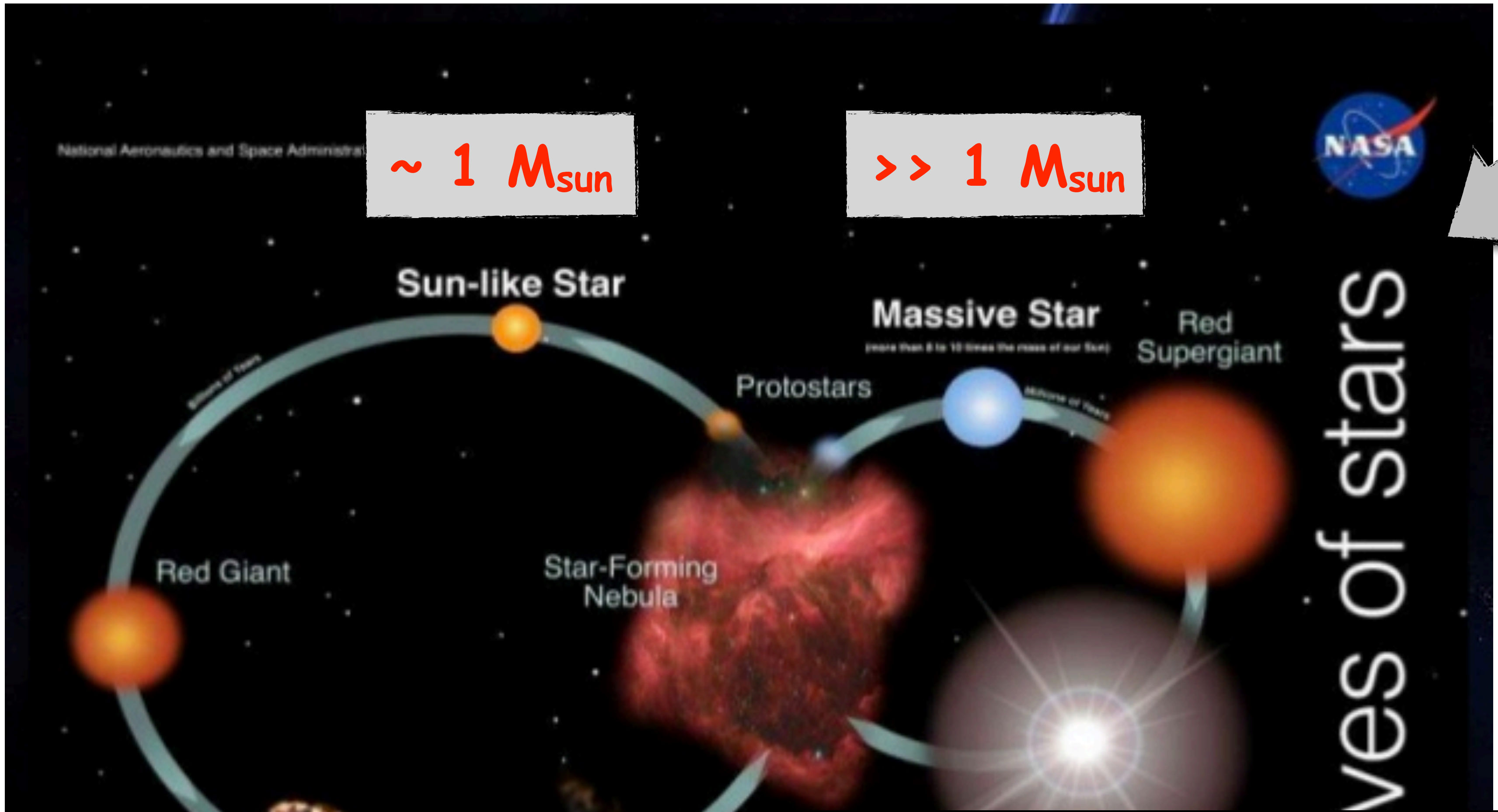
21 cm line -> atomic neutral Hydrogen (HI)

visible light -> stars

thermal emission from hot plasmas (millions of K)

interactions of cosmic ray protons with matter

Thermonuclear & core-collapse supernovae



~ 1 M_{sun}

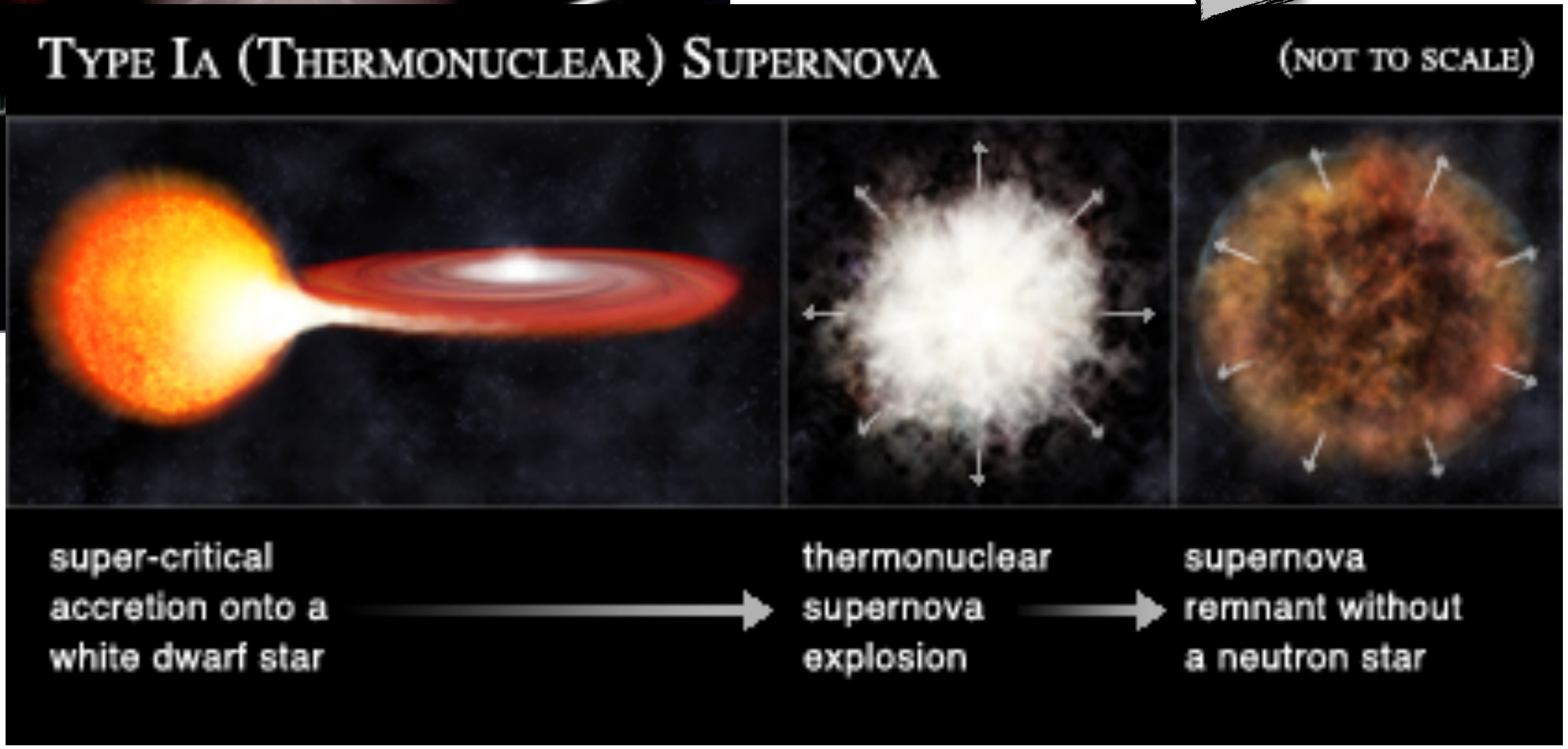
>> 1 M_{sun}

core-collapse

$$M_{ej} \sim \text{several } M_\odot$$

$$M_{ej} \sim 1.4 M_\odot$$

thermonuclear



explosion energy

$$E_{SN} \sim 10^{51} \text{erg}$$

Dynamical evolution of supernova remnants
Free expansion phase

an amount of matter M_{ej} is ejected with velocity v_0 and kinetic energy E_{SN}

$$E_{SN} = \frac{1}{2} M_{ej} v_0^2 \longrightarrow v_0 = 10000 \left(\frac{E_{SN}}{10^{51} \ \mathrm{erg}} \right)^{1/2} \left(\frac{M_{ej}}{M_\odot} \right)^{-1/2} \quad \mathrm{km/s}$$

sound speed in the ISM

$$c_s = \left(\gamma \frac{kT}{m_p} \right)^{1/2} \approx 10 \left(\frac{T}{10^4 \ \mathrm{K}} \right)^{1/2} \ \mathrm{km/s}$$

$$\mathcal{M} \approx 100 \quad \Longrightarrow \quad \boxed{\text{shock wave}}$$

shock wave

$$\begin{cases} v_s = v_0 \\ R_s = v_0 t \end{cases}$$

as long as

density of the interstellar medium

$$M_{ej} \gg M_{sw} = \frac{4\pi}{3} R_s^3 \varrho_0$$

mass of swept up medium

Dynamical evolution of supernova remnants
Free expansion phase

density of the interstellar medium

$$\begin{cases} v_s &= v_0 \\ R_s &= v_0 t \end{cases} \quad \text{as long as} \quad M_{ej} \ll M_{sw} = \frac{4\pi}{3} R_s^3 \varrho_0$$

mass of swept up medium

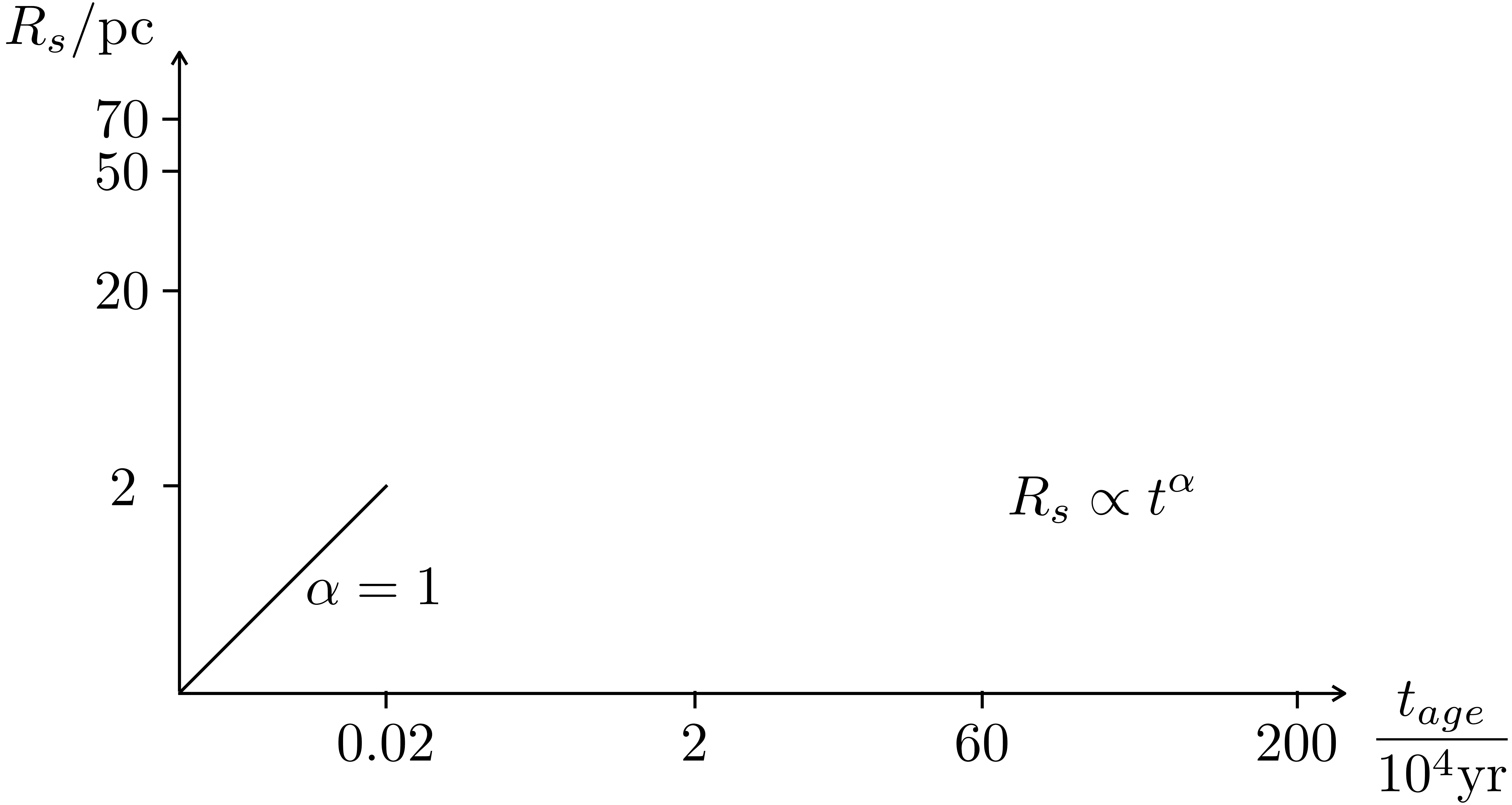
$$M_{ej} \approx M_{sw} \longrightarrow R_{ej} \sim 2 \left(\frac{M_{ej}}{M_\odot} \right)^{1/3} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-1/3} \mathrm{pc}$$

$$v_{ej} \approx \frac{R_{ej}}{v_0} \sim 2 \times 10^2 \left(\frac{M_{ej}}{M_\odot} \right)^{5/6} \left(\frac{E_{SN}}{10^{51} \mathrm{erg}} \right)^{-1/2} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-1/3} \mathrm{yr}$$

Time evolution of a SNR

$$M_{sh}/M_\odot \approx \quad 1$$

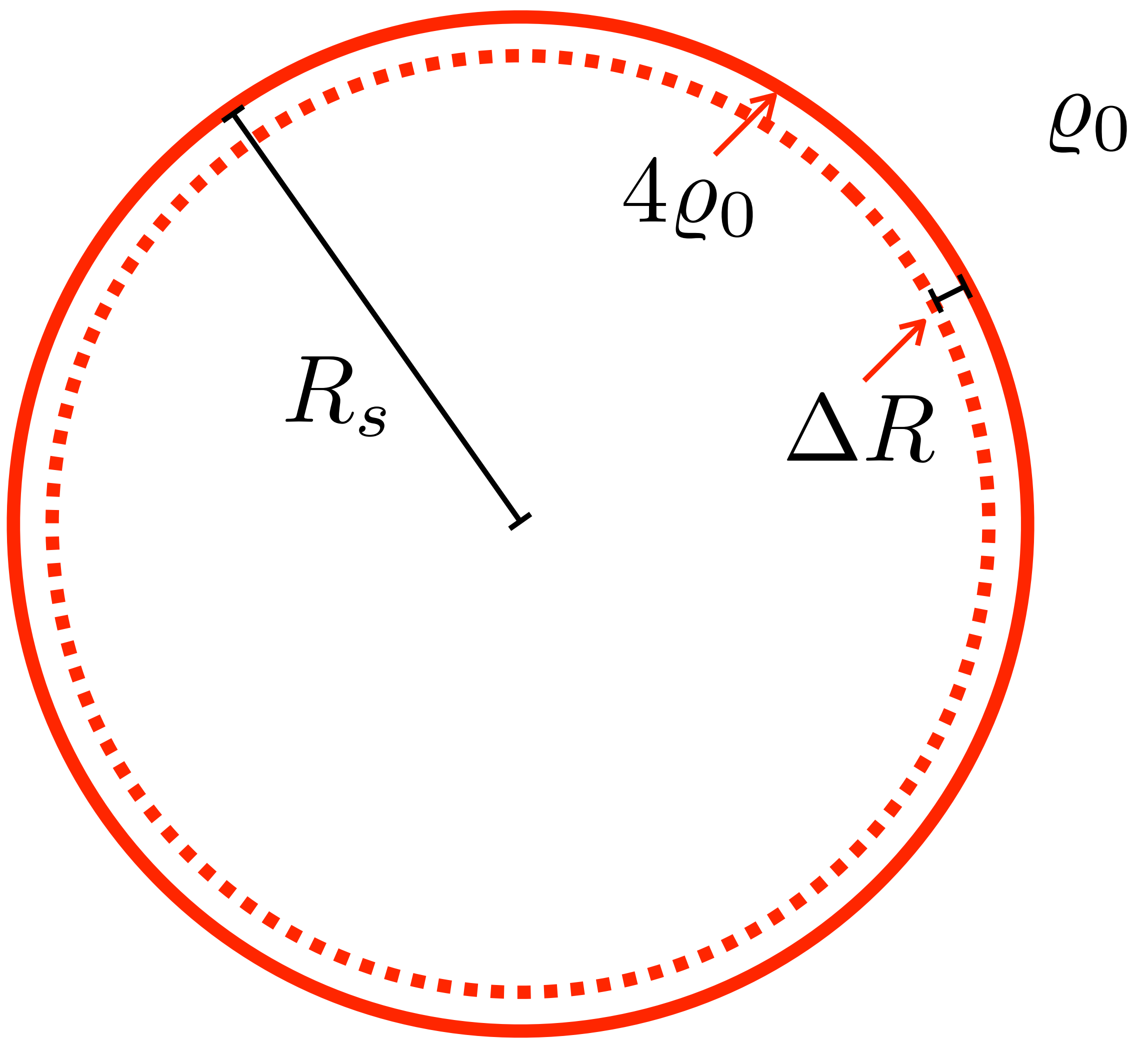
$$\frac{v_s}{10^3\ \mathrm{km/s}} \approx \quad 10$$



The thin shell approximation

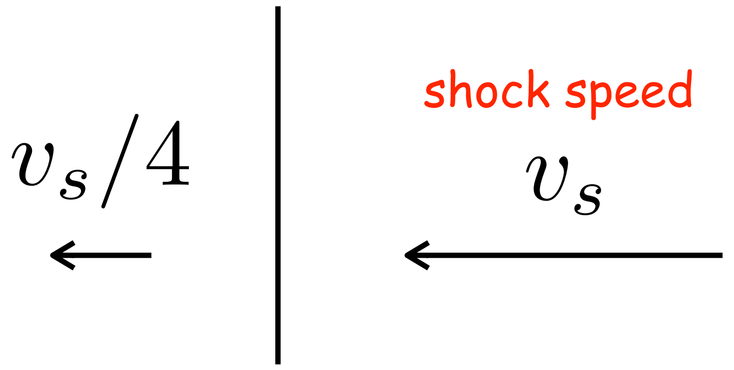
$M_{sw} \gg M_{ej}$

SNR shock

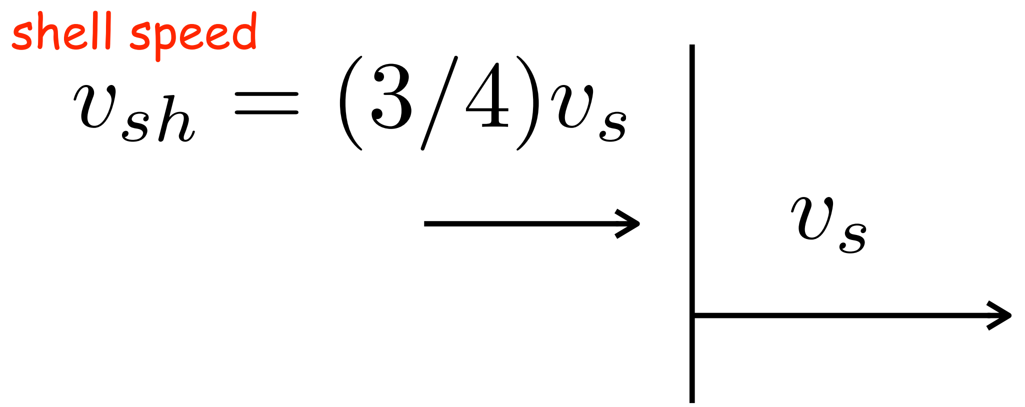


velocity of the shell

shock rest frame



lab rest frame



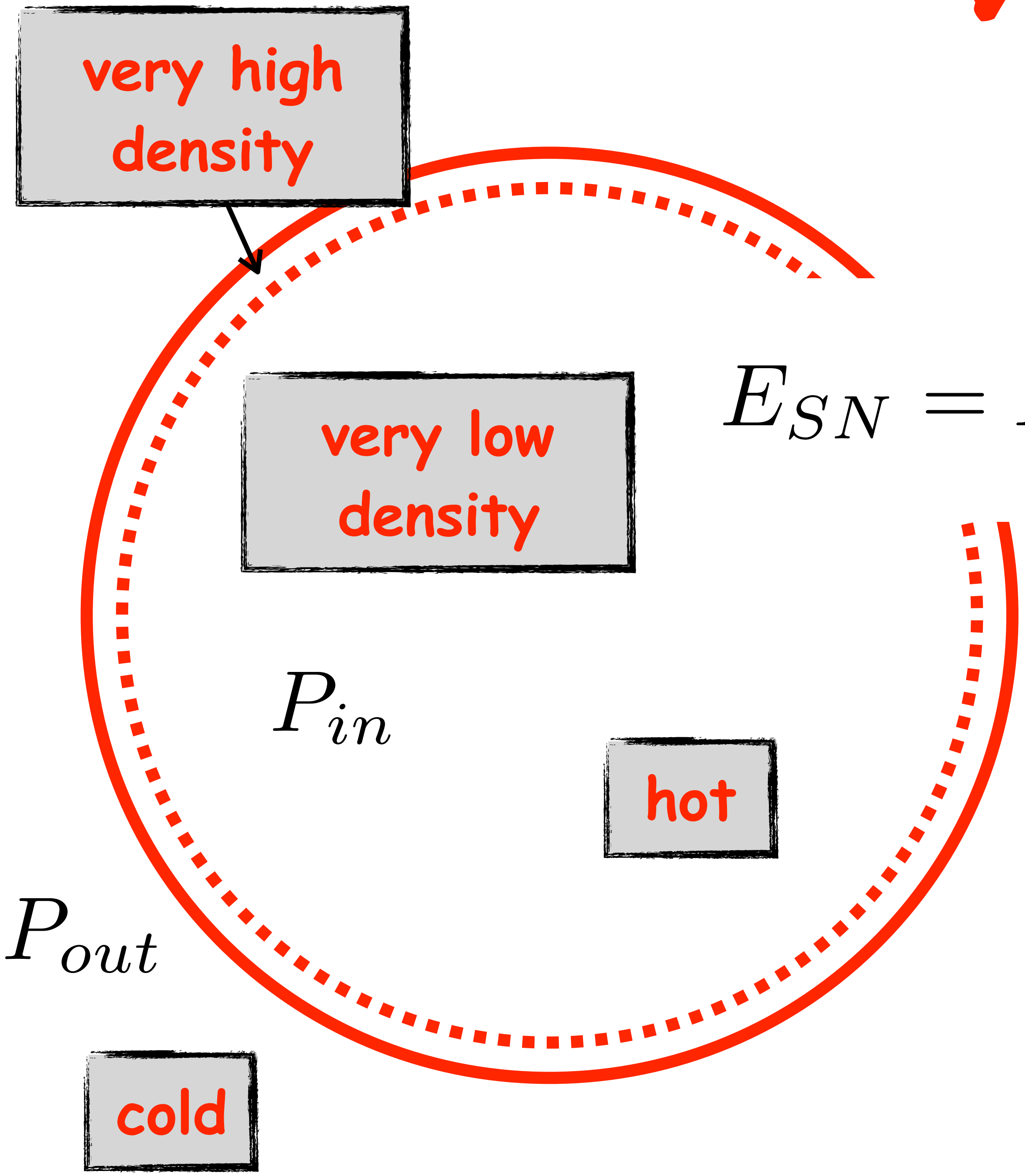
$$\frac{4\pi}{3} R_s^3 \varrho_0 = 4\pi R_s^2 \Delta R (4\varrho_0) \quad \longrightarrow \quad \frac{\Delta R}{R_s} = \frac{1}{12} \approx 0.08$$

thin shell

The thin shell approximation

the mass is concentrated in an infinitesimally small shell behind the shock

mass inside the shell $$M = \cancel{M}_{ej} + 4\pi \int_0^{R_s} \mathrm{d}R \; R^2 \varrho_0 \approx \frac{4\pi}{3} R_s^3 \varrho_0$$



energy conservation

$$E_{SN} = E_k + E_{th} = \frac{1}{2} M v_{sh}^2 + \frac{P_{in}}{\gamma - 1} \frac{4\pi}{3} R_s^3$$

momentum conservation

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(M v_{sh} \right) = 4\pi R_s^2 \left(P_{in} - \cancel{P_{out}} \right)$$

strong shock

Solution of the equations

Let's search for power law solutions...

$$R_s = A\ t^{\alpha} \longrightarrow v_s = \frac{\mathrm{d}R_s}{\mathrm{d}t} = \frac{\alpha R_s}{t} \propto t^{\alpha-1}$$

$$M = \frac{4\pi}{3}\varrho_0 R_s^3 \qquad \gamma = \frac{5}{3} \qquad v_{sh} = \frac{3}{4} v_s$$

momentum

$$\frac{\varrho_0}{4}\frac{\mathrm{d}}{\mathrm{d}t}\left(R_s^3 v_s\right) = R_s^2 P_{in} \quad \rightarrow P_{in} = \frac{(4\alpha-1)}{4\alpha}\varrho_0 v_s^2$$

energy

$$E_{SN} = \frac{3\pi}{8}\varrho_0 R_s^3 v_s^2 + 2\pi R_s^3 P_{in}$$

$$\longrightarrow E_{SN} = \frac{\pi}{8}\varrho_0 A^5 \alpha(19\alpha-4)\ t^{5\alpha-2}$$

The Sedov (adiabatic) phase

adiabatic SNR

$$E_{SN} = \text{const} \to E_{SN} \propto t^{5\alpha-2} \to \alpha = \frac{2}{5}$$

$$\to A = \left(\frac{50}{9\pi} \frac{E_{SN}}{\varrho_0} \right)^{1/5}$$

during the Sedov phase $\frac{E_k}{E_{SN}} = \frac{1}{3}$ $\frac{E_{th}}{E_{SN}} = \frac{2}{3}$

$$\begin{cases} R_s \sim 5 \left(\frac{E_{SN}}{10^{51} \ \text{erg}} \right)^{1/5} \left(\frac{n_{ISM}}{\text{cm}^{-3}} \right)^{-1/5} \left(\frac{t}{\text{kyr}} \right)^{2/5} \ \text{pc} \\ u_s \sim 2 \times 10^3 \left(\frac{E_{SN}}{10^{51} \ \text{erg}} \right)^{1/5} \left(\frac{n_{ISM}}{\text{cm}^{-3}} \right)^{-1/5} \left(\frac{t}{\text{kyr}} \right)^{-3/5} \ \text{km/s} \end{cases}$$

Duration of the Sedov phase

Bremsstrahlung +
(mainly) lines emissions

cooling function

$$10^5 \lesssim T \lesssim 10^{7.5} \longrightarrow \Lambda \approx 2 \times 10^{-19} T^{-1/2} \mathrm{erg\ cm^3 s^{-1}}$$

cooling time

$n = n_e \sim n_i \sim 4\ n_{ISM}$

$$\tau_c \sim \frac{\epsilon_{th}}{n_i n_e \Lambda} = \frac{3nkT}{n^2 \Lambda} \approx 10^6 \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}}\right)^{-1} \left(\frac{v_s}{1000\ \mathrm{km/s}}\right)^3 \ \mathrm{yr}$$

shock heating

$$T \sim 2 \times 10^7 \left(\frac{u_s}{1000\ \mathrm{km/s}}\right)^2 \ \mathrm{K}$$

the shell becomes radiative at an age:

$$\tau_c \approx t_{age} \longrightarrow t_{age} \approx 2 \times 10^4 \left(\frac{E}{10^{51}\mathrm{erg}}\right)^{3/14} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}}\right)^{-4/7} \ \mathrm{yr}$$

Duration of the Sedov phase

$$t_{ad} \approx 2 \times 10^4 \left(\frac{E_{SN}}{10^{51}\mathrm{erg}} \right)^{3/14} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-4/7} \mathrm{yr}$$

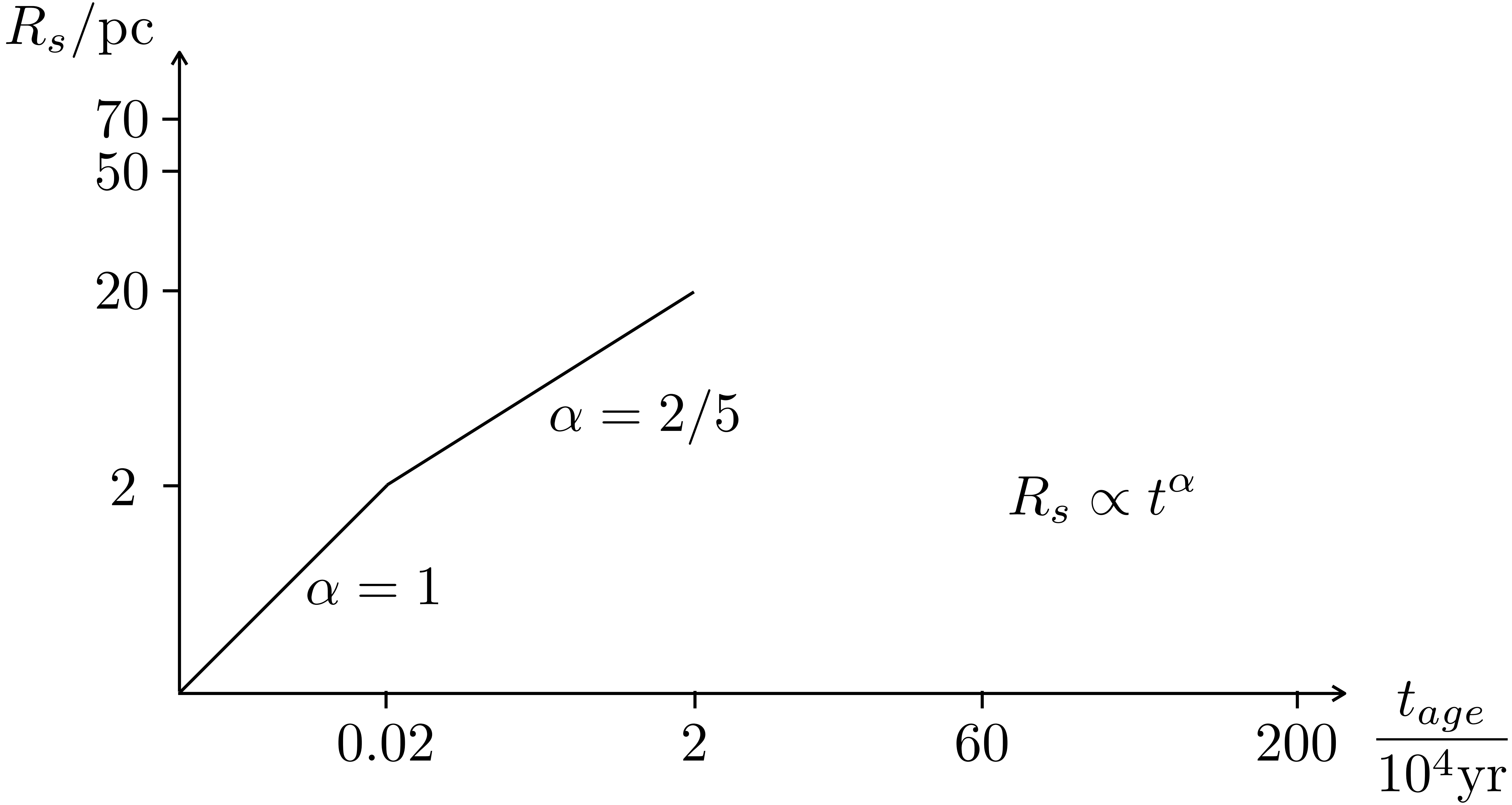
$$R_{ad} \approx 20 \left(\frac{E_{SN}}{10^{51}\mathrm{erg}} \right)^{2/7} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-3/7} \mathrm{pc}$$

$$v_{ad} \approx 3 \times 10^2 \left(\frac{E_{SN}}{10^{51}\mathrm{erg}} \right)^{1/14} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{1/7} \mathrm{km/s}$$

Time evolution of a SNR

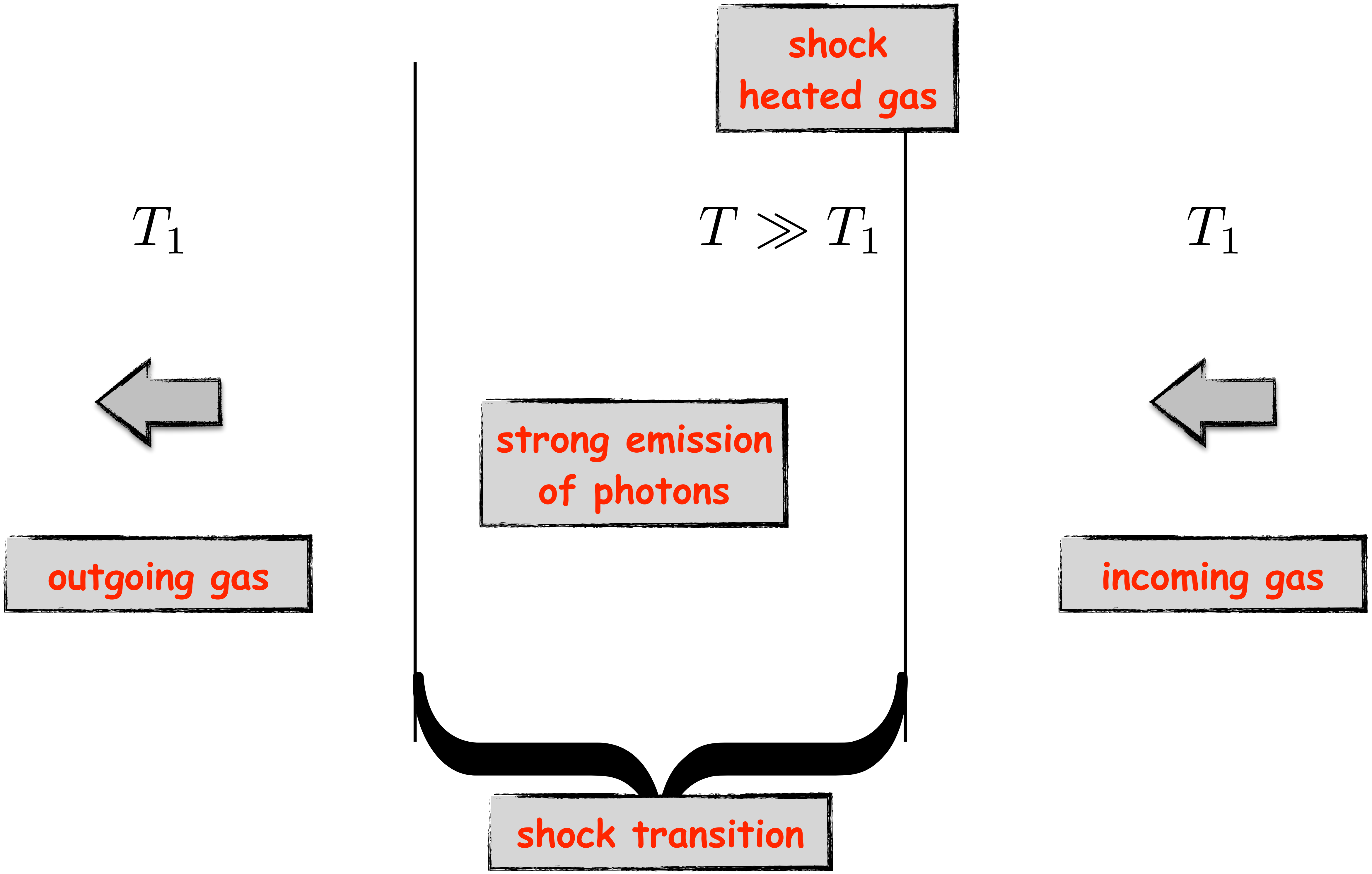
$$M_{sh}/M_\odot \approx \quad 1 \qquad\qquad 10^3$$

$$\frac{v_s}{10^3\ \mathrm{km/s}} \approx \quad 10 \qquad\qquad 0.3$$



Radiative (isothermal) shocks

as a particular case we consider a shock where radiative losses are so effective to have $T_2 = T_1$ (isothermal approximation)



shock heated gas

T_1

$T \gg T_1$

T_1

strong emission of photons

outgoing gas

incoming gas

shock transition

Radiative (isothermal) shocks

as a particular case we consider a shock where radiative losses are so effective to have $T_2 = T_1$ (isothermal approximation)

isothermal $P \propto \varrho \longrightarrow P = c_s^2 \varrho$ $\qquad c_s = constant$

mass $\varrho_1 u_1 = \varrho_2 u_2$

momentum $\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2$

$$\left.\begin{aligned} \varrho_1 u_1 &= \varrho_2 u_2 \\ \varrho_1 u_1^2 + P_1 &= \varrho_2 u_2^2 + P_2 \end{aligned}\right\} \quad 1 + \cancel{\frac{P_1}{\varrho_1 u_1^2}} = \frac{u_2}{u_1} + \frac{P_2}{\varrho_1 u_1^2}$$

strong shock

$$u_2^2 = u_2 u_1 + c_s^2 = 0 \longrightarrow u_2 = \frac{u_1}{2}\left[1 \pm \sqrt{1 - \frac{4c_s^2}{u_1^2}}\right]$$

$$\longrightarrow \frac{u_1}{2}\left[1 \pm \left(1 - \frac{2c_s^2}{u_1^2}\right)\right] \begin{array}{l} \nearrow \; u_1 - \dfrac{2c_s^2}{u_1} \sim \cancel{u_1} \\ \searrow \; c_s^2/u_1 \end{array}$$

unphysical

Radiative (isothermal) shocks

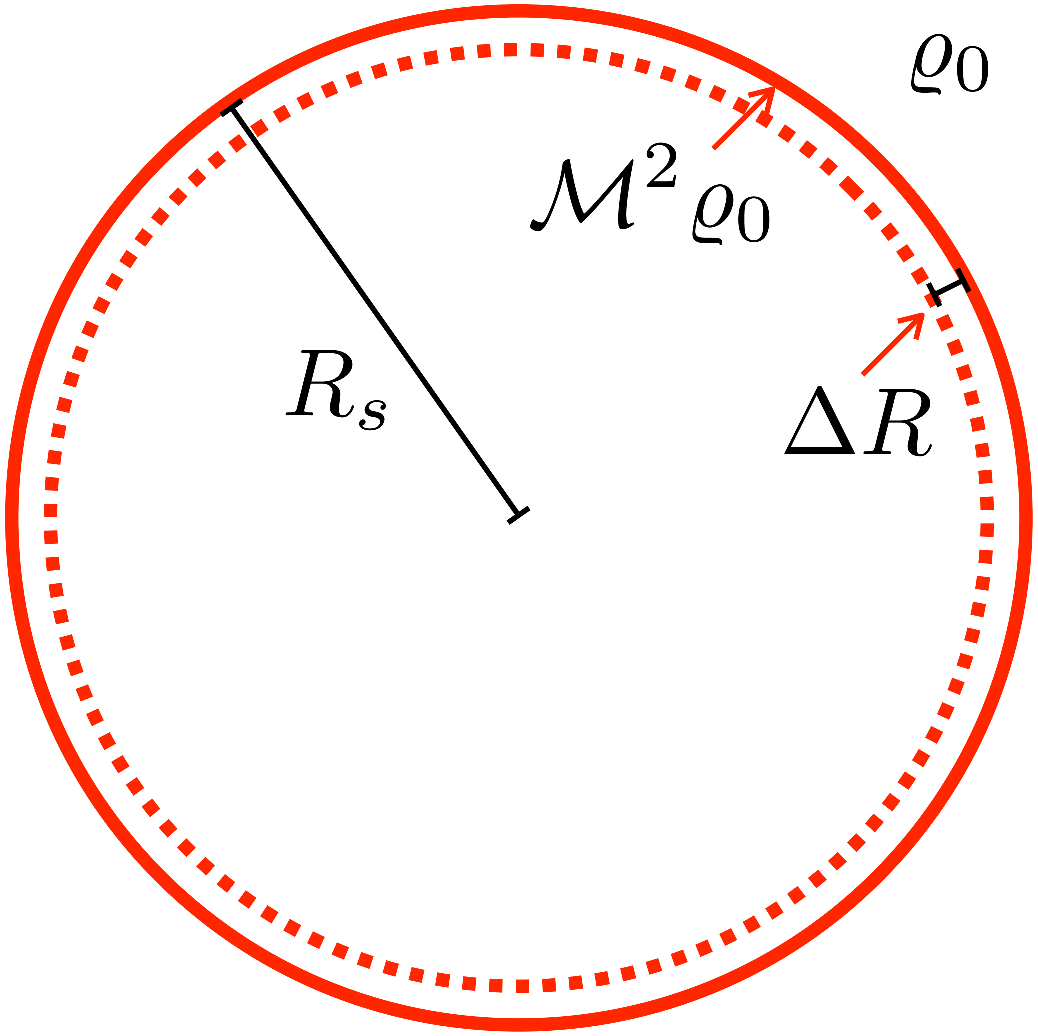
summary

$$u_2 = \frac{c_s^2}{u_1} = \frac{u_1}{\mathcal{M}^2} \xrightarrow{\mathcal{M} \to \infty} 0$$

$$r = \frac{\varrho_2}{\varrho_1} = \frac{u_1}{u_2} = \mathcal{M}^2 \xrightarrow[\mathcal{M} \to \infty]{} \infty$$

for strong shocks a very thin and dense shell forms, matter in the shell moves roughly at the same velocity of the shock -> the thin shell approximation is even more justified for radiative shocks!

The pressure driven snowplough phase



at first, the dense shell cools, while the interior does not (due to its very low density) -> all the energy dissipated at the shock is radiated away, and the SNR interior cools adiabatically

$$P_{in} V^{\gamma} = const \to P_{in} \propto R_s^{-3\gamma}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(M v_s) = 4\pi R_s^2 P_{in}$$

$$R_s \propto t^{\alpha} \longrightarrow \alpha = \frac{2}{7}$$

Momentum conserving snowplough

$$n_{in} = \epsilon \; n_{sh}$$

density in the
SNR interior

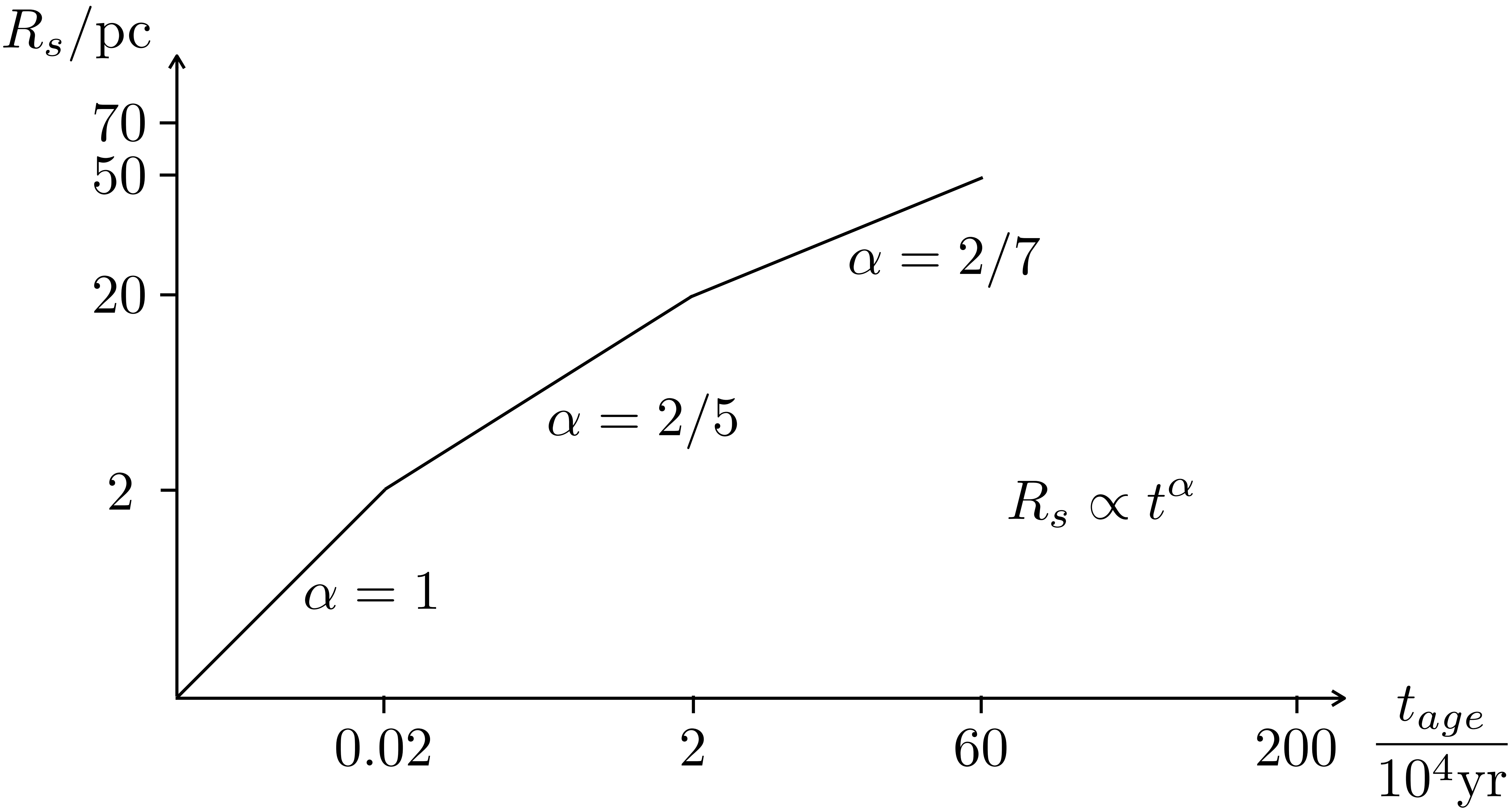
<< 1

density in the shell

the SNR interior cools much later than the SNR shell. according to the numerical simulations of Cioffi et al. 1988 the pressure driven snowplough lasts few tens of t_{ad}

Time evolution of a SNR

$M_{sh}/M_\odot \approx$	1	10^3	10^4
$\frac{v_s}{10^3 \text{ km/s}} \approx$	10	0.3	2×10^{-2}



Momentum conserving snowplough

$$n_{in} = \epsilon \; n_{sh}$$

density in the SNR interior

<< 1

density in the shell

the SNR interior cools much later than the SNR shell. according to the numerical simulations of Cioffi et al. 1988 the pressure driven snowplough lasts few tens of t_{ad}

once the interior cools, the motion of the shell is determined by momentum conservation

$$M v_s = const \Rightarrow M v_s = \frac{4\pi}{3} R_s^3 \varrho_0 v_s = \frac{4\pi}{3} \varrho_0 A^4 t^{4\alpha-1} \longrightarrow \alpha = \frac{1}{4}$$

$$\begin{cases} R_s \propto t^{1/4} \\ v_s \propto t^{-3/4} \end{cases}$$

the SNR dissolves in the ISM when the shock Mach number becomes ~1 (v_s~ c_s~ 10 km/s)

Time evolution of a SNR

$M_{sh}/M_\odot \approx$	1	10^3	10^4	3×10^4
$\frac{v_s}{10^3 \text{ km/s}} \approx$	10	0.3	2×10^{-2}	10^{-2}