# NPAC course on Astroparticles

## Exam 2017

# Exercise #1 — Solution

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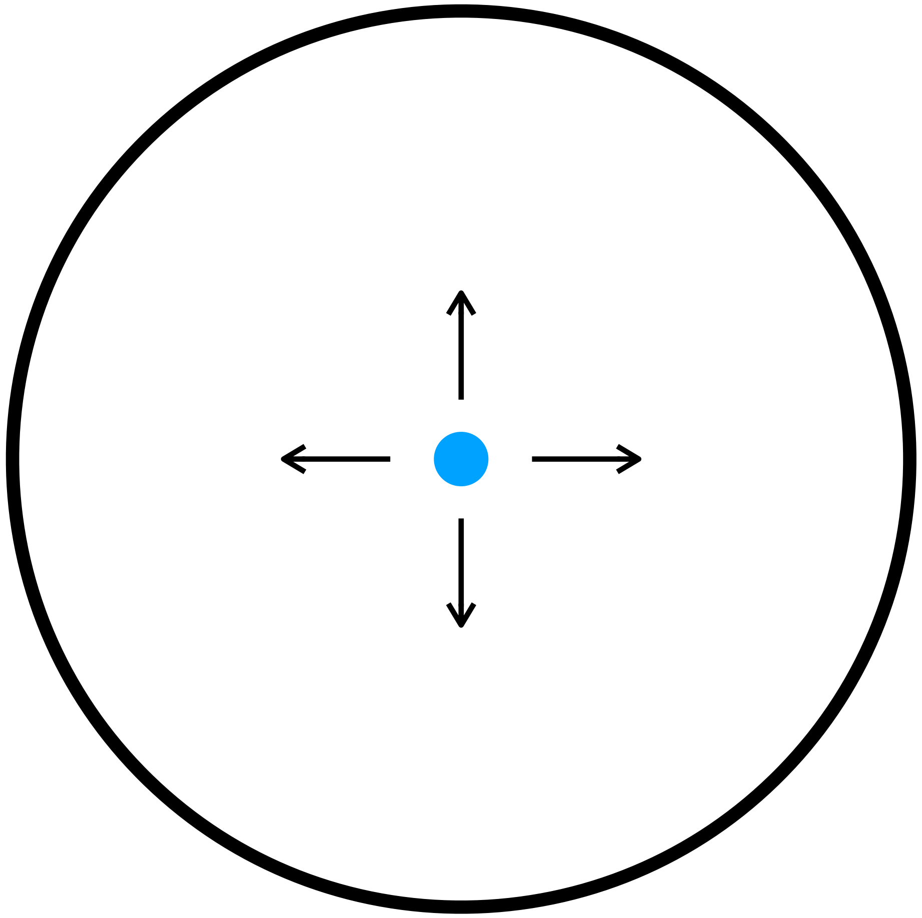

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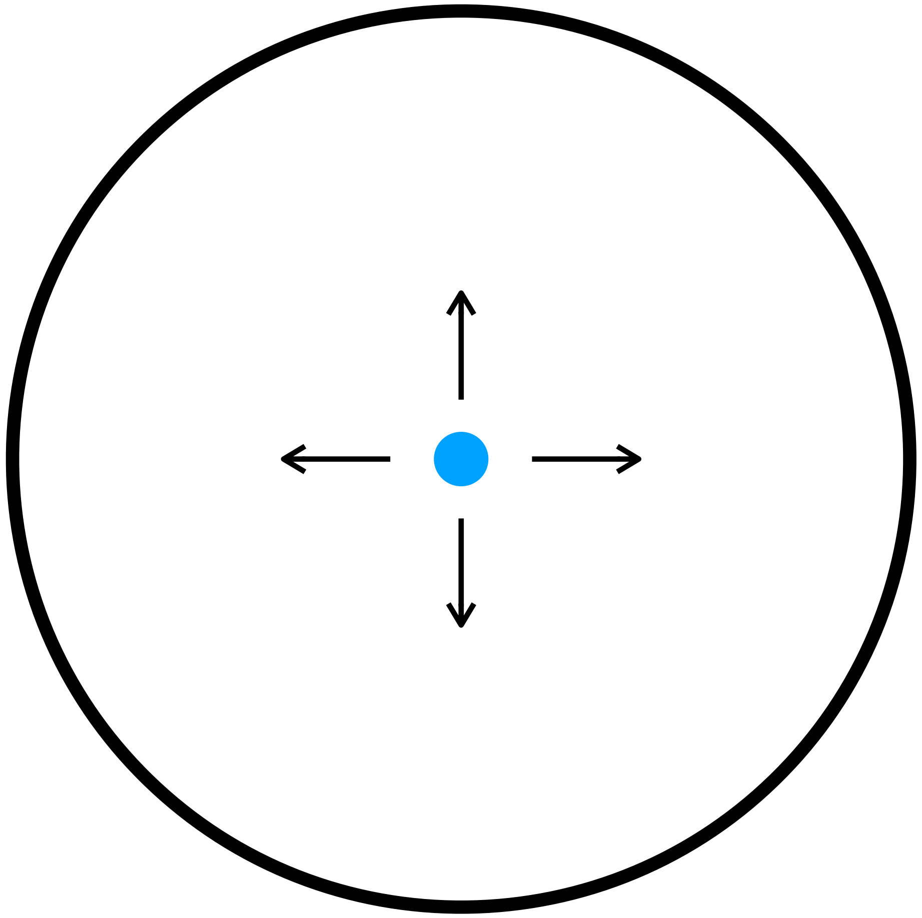

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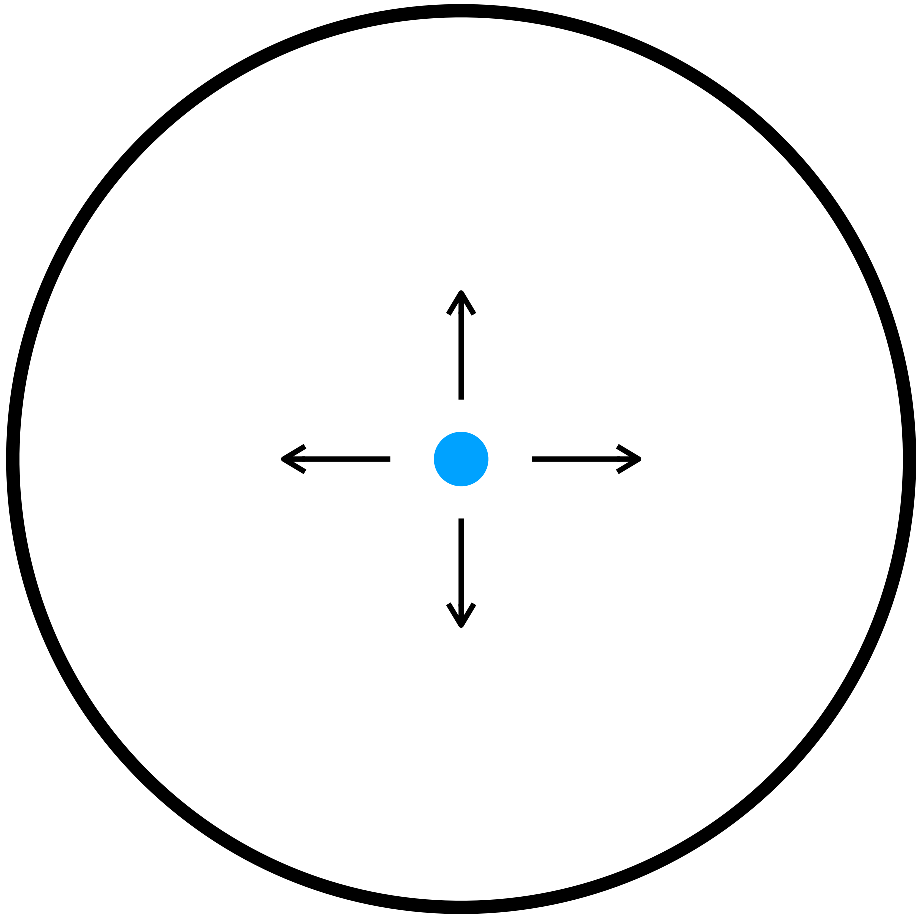

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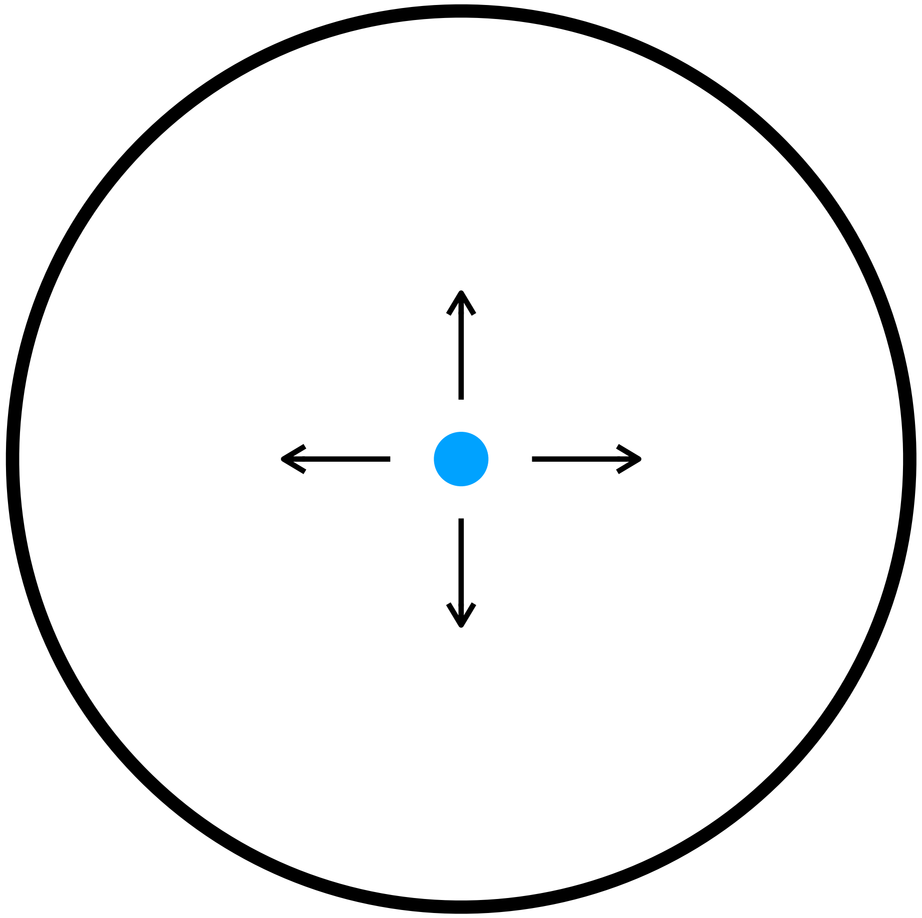

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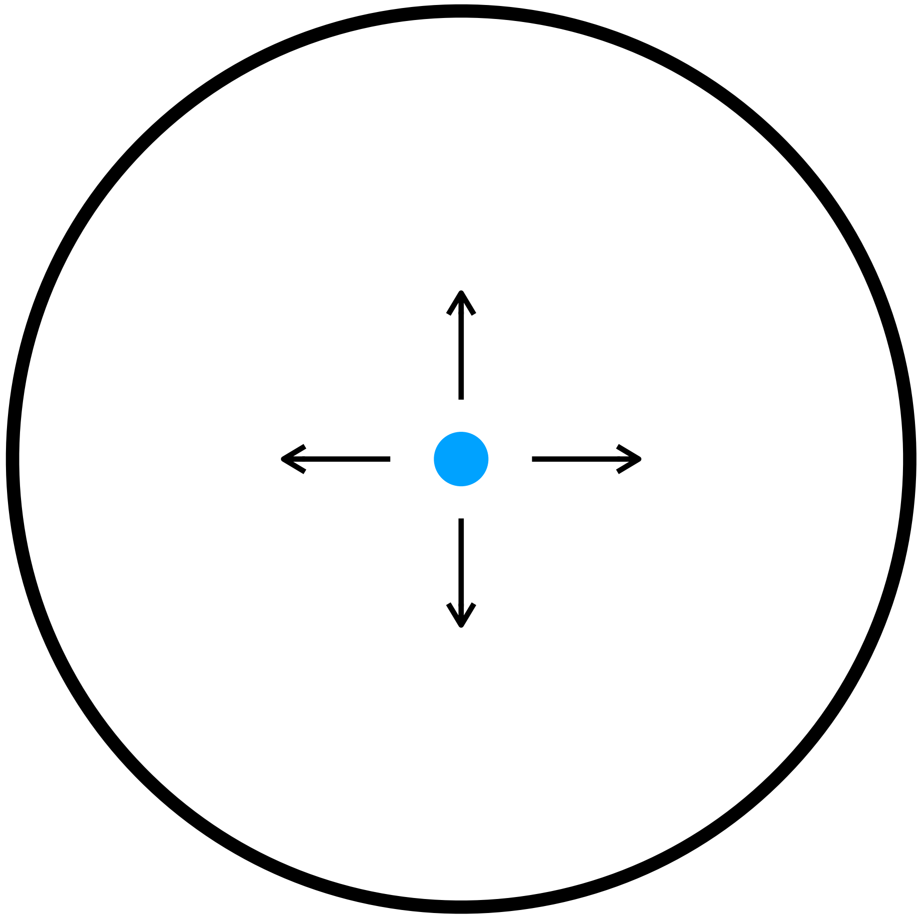

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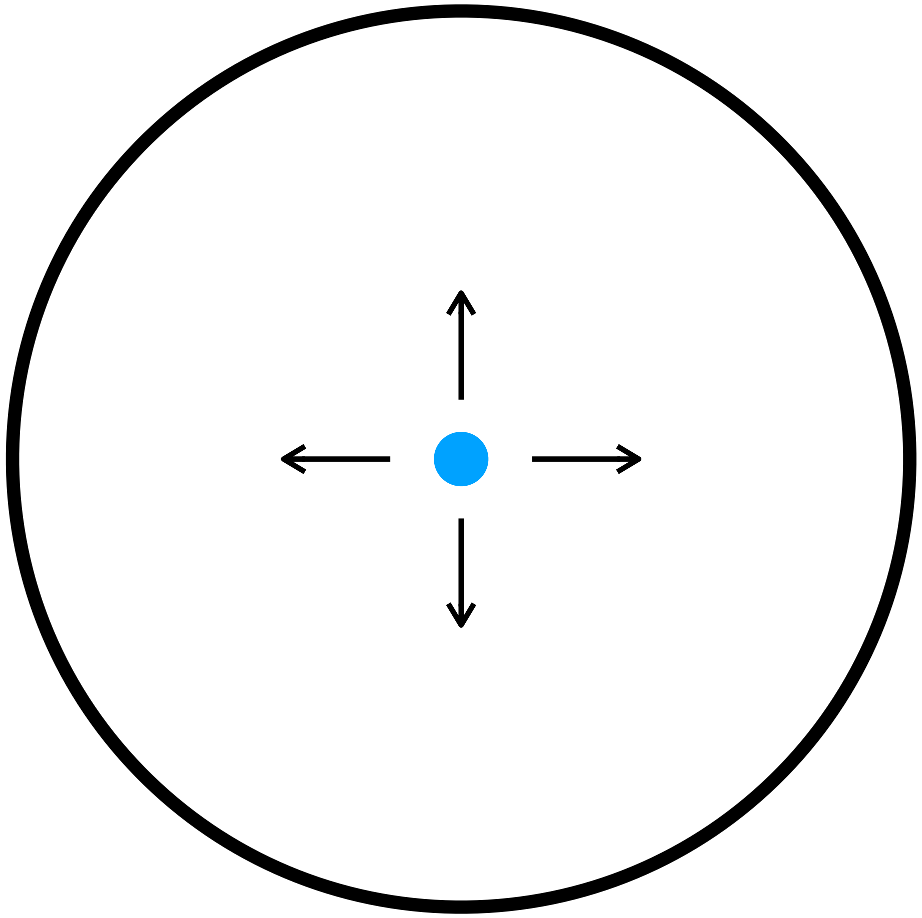

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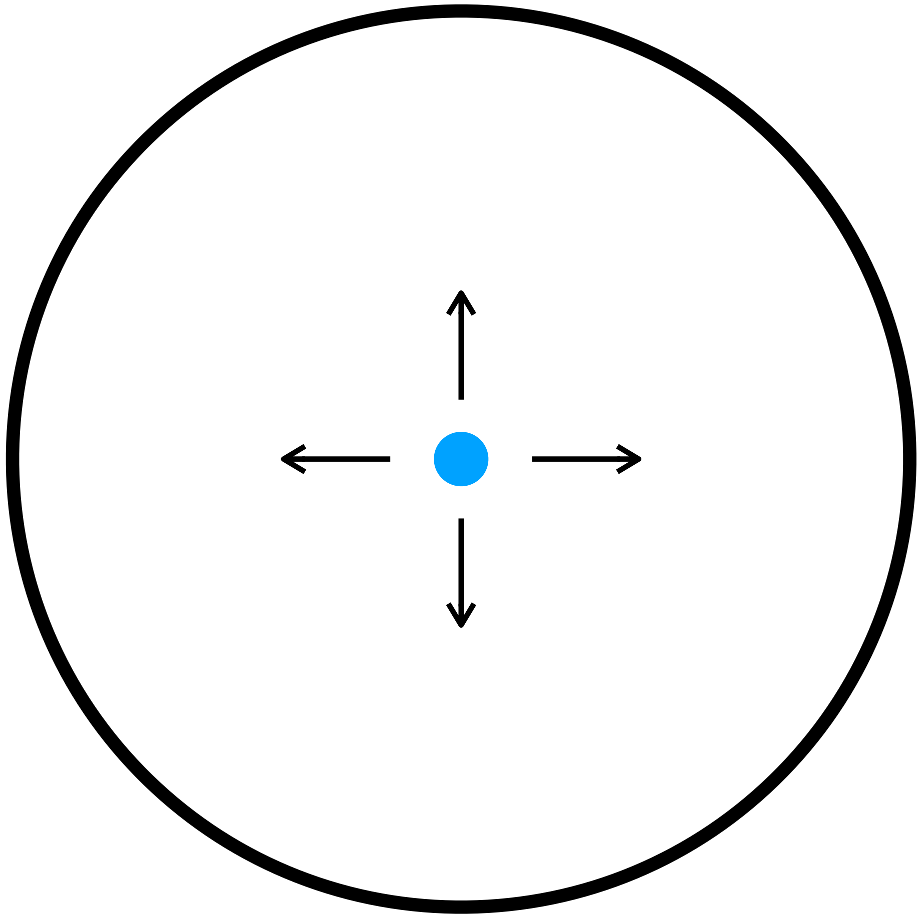

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# Exercise #2 — Solution

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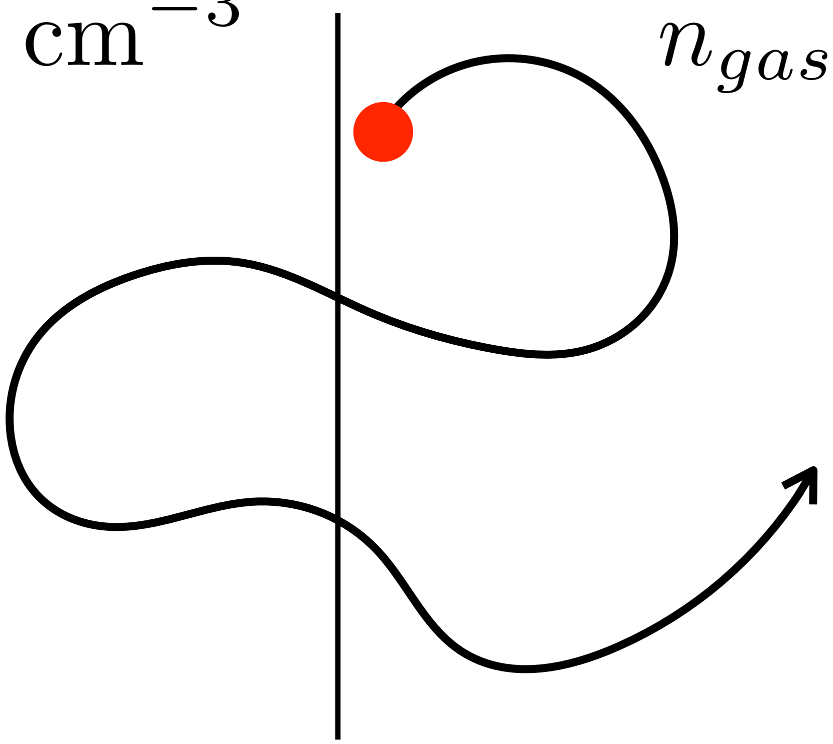

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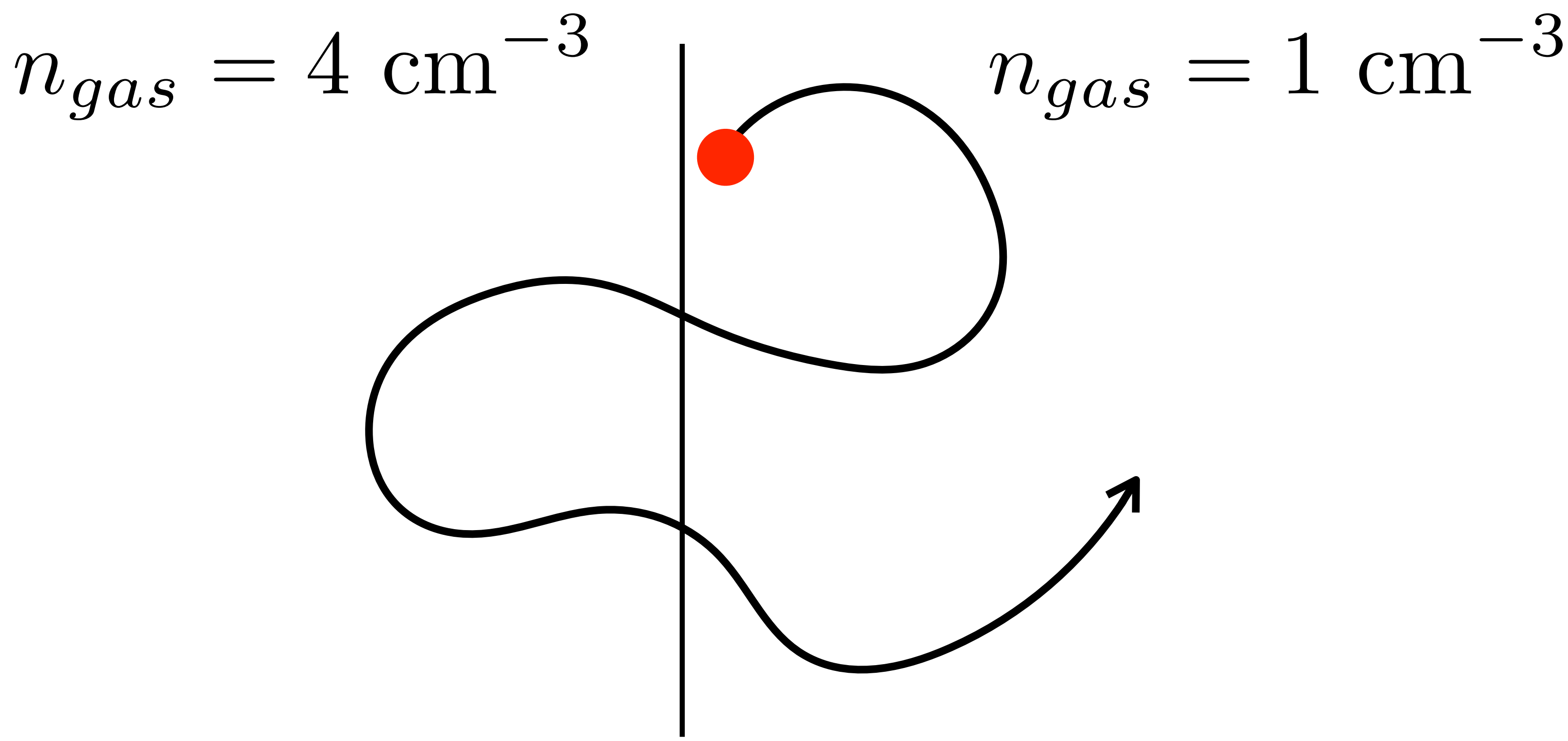


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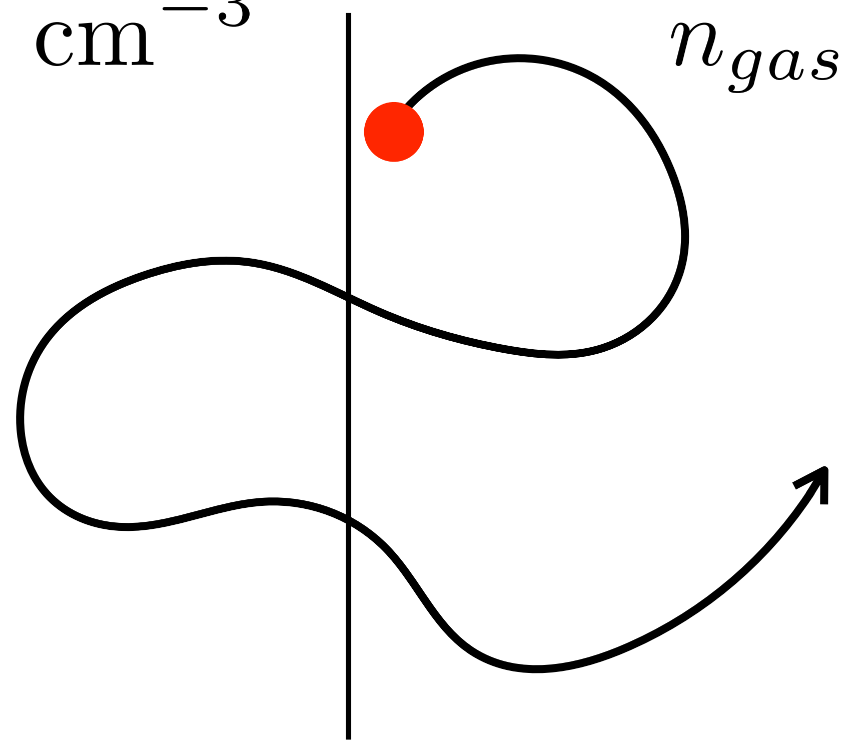

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Comment: CR protons do not lose energy, CR electrons lose energy only if their energy is larger r than 50 TeV

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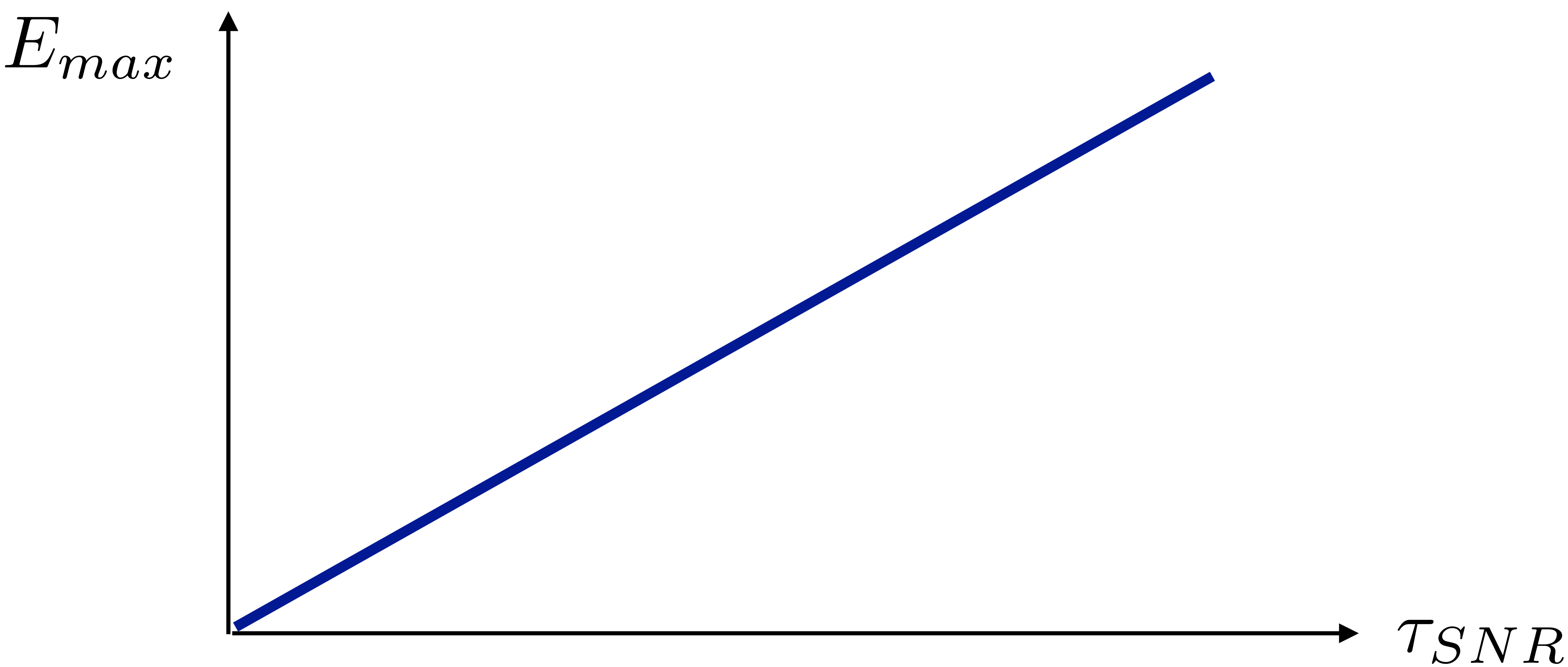

Electrons of energy smaller than 50 TeV behave exactly as protons. A different behaviour is expected at larger energies, where one should equate the age of the system to the energy loss time.

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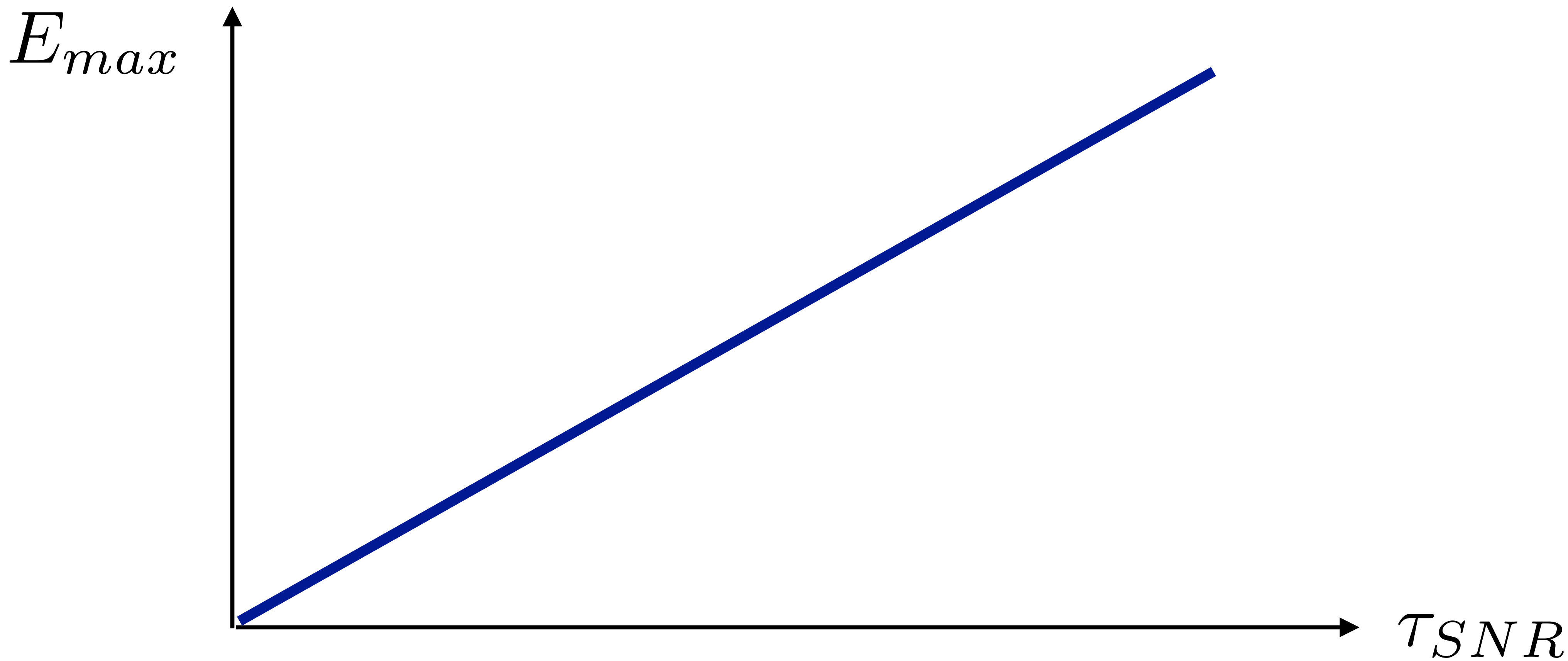

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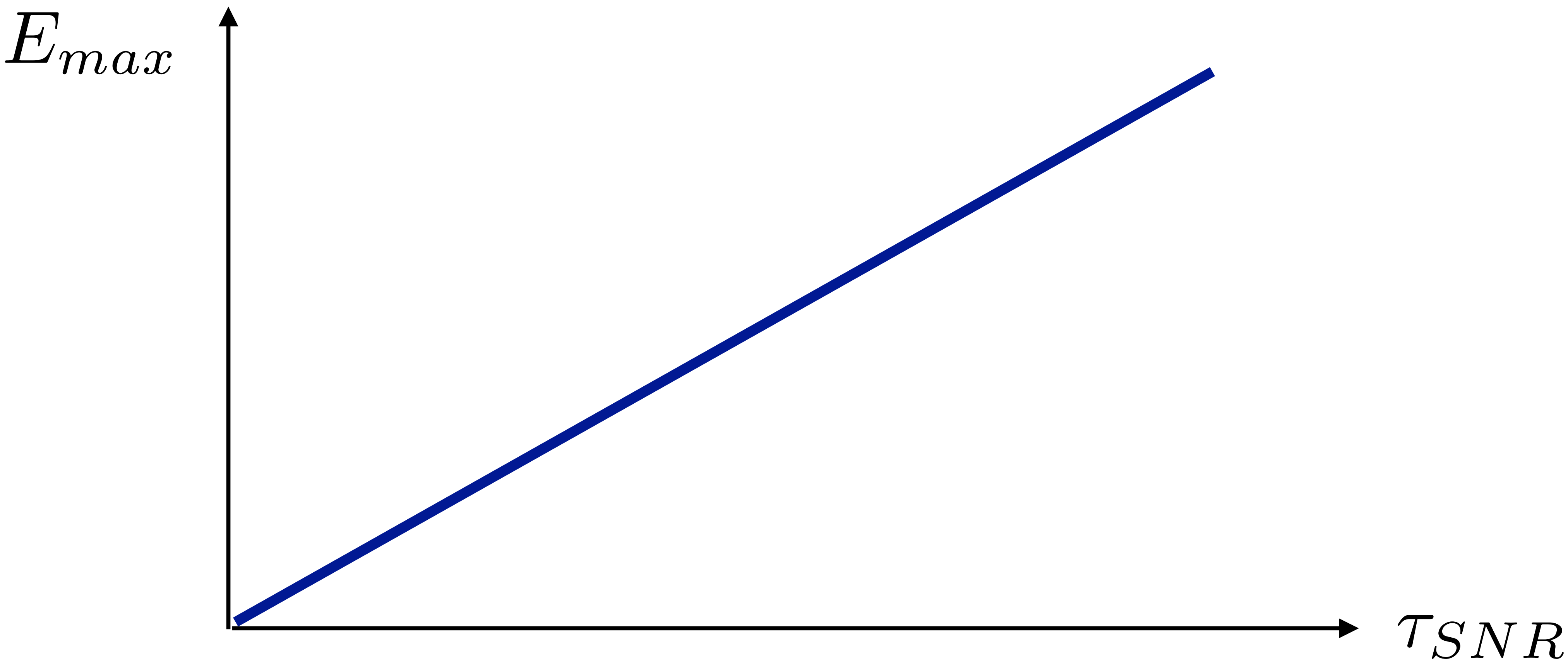

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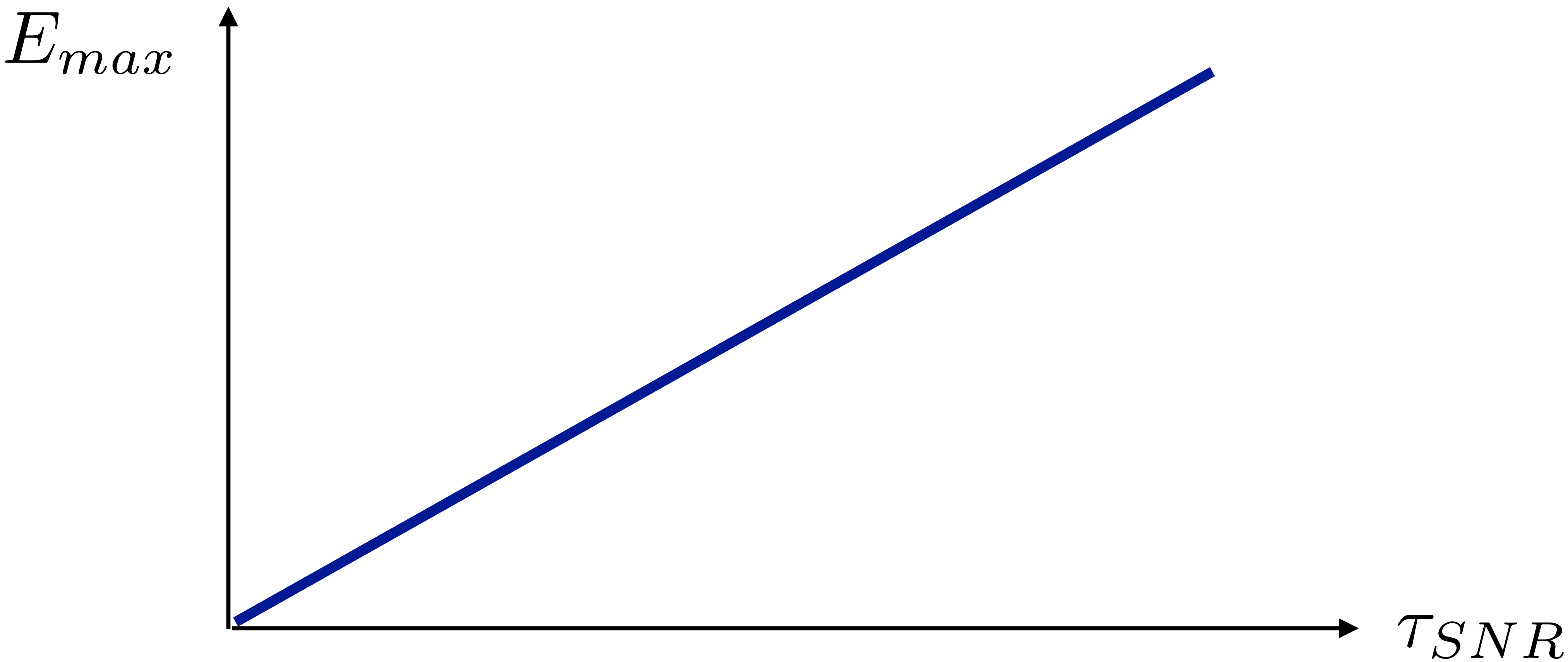

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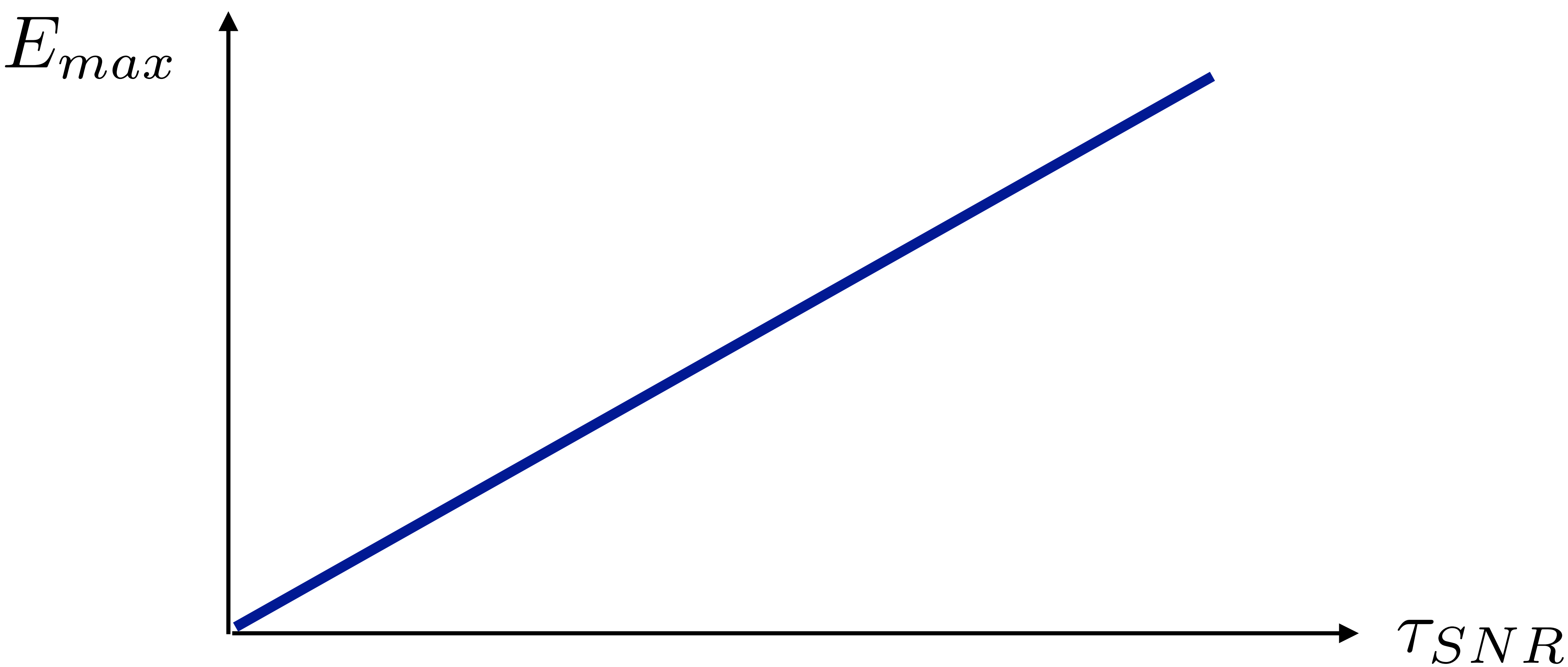

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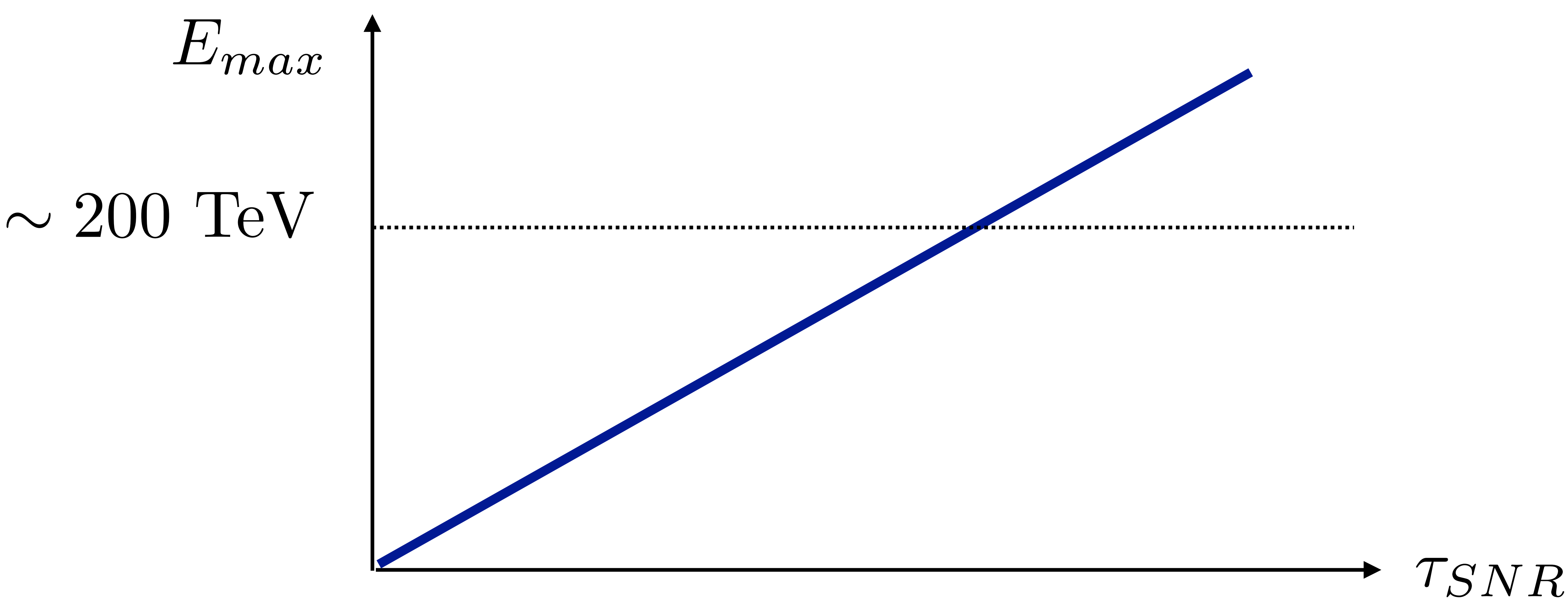

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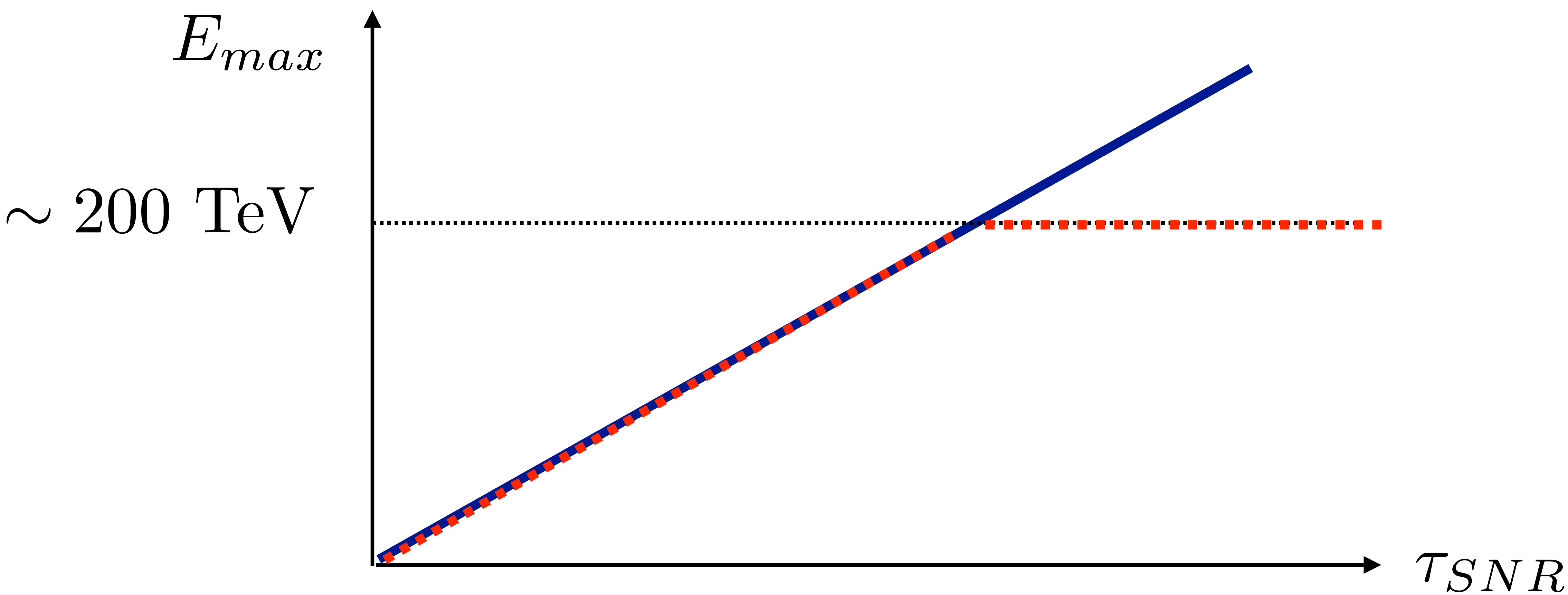

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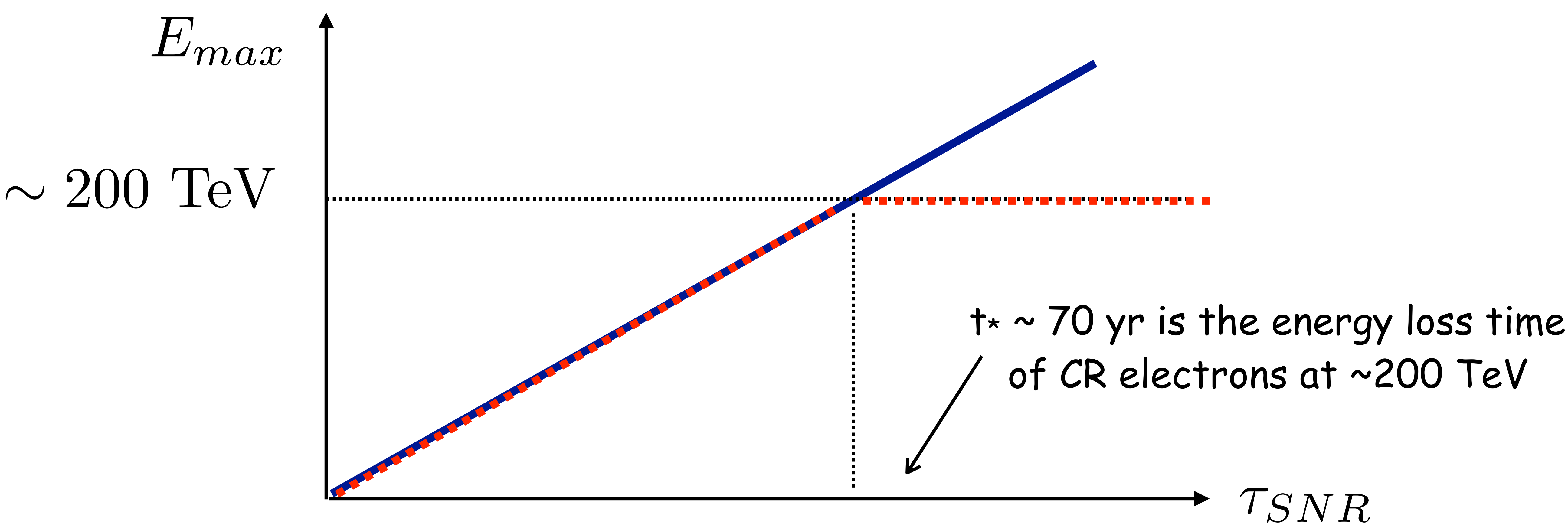

# Maximum energy of protons and electrons accelerated at shocks - 4

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# Exercise #3 — Solution

# SN1987A in gamma rays — 1

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p)E_p^2$$

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$$Q_\gamma = \frac{100}{3} \frac{N_p(E_p)}{\tau_{pp}} \longleftarrow Q_\gamma(E_\gamma)E_\gamma^2 = \frac{N_p(E_p)}{3\tau_{pp}} E_p^2$$

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$$E_\gamma \sim \frac{E_p}{10}$$

## Normalization

$$W_{CR} = \eta W_{SN} = \int_{1\ \mathrm{GeV}} \mathrm{d}E_p E_p N_p(E_p) = ... \longrightarrow N_* = \frac{0.4}{10^{1.2}} \frac{\eta W_{SN}}{E_*^2}$$

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$$\Phi(> 1\ \mathrm{TeV}) = \int_{1\ \mathrm{TeV}} \mathrm{d}E_\gamma Q_\gamma(E_\gamma) = ... = 3.2 \times 10^{32} \left( \frac{\eta}{0.1} \right) \left( \frac{n_{gas}}{\mathrm{cm}^{-3}} \right)\ s^{-1}$$

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## Flux

$$\phi = \frac{\Phi}{4\pi d^2} \sim 10^{-15} \left( \frac{\eta}{0.1} \right) \left( \frac{n_{gas}}{\mathrm{cm}^{-3}} \right) \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Is it detectable?

$$\phi > \phi_{min} \rightarrow \left(\frac{\eta}{0.1}\right)\left(\frac{n_{gas}}{\mathrm{cm}^{-3}}\right) > 10$$

Only for densities significantly larger than the typical one in the interstellar medium
(the efficiency $\eta$ must be of course smaller than 1)