Exam (2017)

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1 The expansion of interstellar bubbles

A supernova explosion releases in an impulsive event an amount of mechanical energy E. This results in the appearance of a spherical shock expanding in the ambient interstellar medium. If the supernova explodes in an homogeneous and cold medium of mass density ϱ, and if radiative losses can be neglected (i.e. we are in the Sedov/adiabatic phase), the radius of the shock R_s scales with time t as:

$$R_s \sim \left(\frac{E}{\varrho} \right)^{1/5} t^{2/5} \quad . \tag{1}$$

1. Using dimensional analysis, derive the scaling of the shock radius with time assuming that, instead of in an impulsive event, the mechanical energy is released continuously at a rate L (L is a *luminosity* and has the dimensions of an energy per unit time). This is what happens, for example, in stellar winds from massive stars. Analogously to supernova remnants, the wind induces the formation of a spherical shock expanding in the interstellar medium. The mass swept up by the shock is concentrated in a thin shell, and a low density region (an *interstellar bubble*, in astronomical jargon) forms inside the shell.

   To derive the scaling between shock radius and time, assume that:

   - the system is adiabatic;
   - the ambient interstellar medium is homogeneous and is characterised by a mass density ϱ;
   - the shock is strong, i.e., the pressure of the interstellar medium can be neglected;
   - the rate of energy injection L is constant in time, and;
   - the mass ejected by the wind is negligible with respect to the mass of the interstellar medium swept up by the shock.

2. Consider now a system where the shock is radiative but the fluid injected by the wind is adiabatic (interstellar bubbles spend most of their life in

this phase). Show that in this case the slope η of the scaling $R_s = A \times t^\eta$ is identical to that obtained in point 1 above, while the normalization factor A is affected. [Hint: repeat what has been done in point 1 above by substituting L with an effective luminosity $L_{eff} = L - L_{rad}$, where L_{rad} is the energy radiated away from the shock per unit time. For a radiative shock L_{rad} is equal to the flux of mechanical energy flowing across the shock surface.]

3. How does the scaling change if both the shock and the injected fluid are radiative? To answer the question, consider a stellar wind injecting an amount of mass per unit time $\dot{M}$ at a velocity v_w. Both $\dot{M}$ and v_w are constant in time. [Hint: the rate of injection of mechanical energy can be expressed as $L = (1/2)\dot{M}v_w^2$. The total momentum μ_{tot} of the system is not conserved, being the integral in time of the rate of momentum injection into the system: $\mu_{tot} = \int \mathrm{d}t\dot{M}v_w = \dot{M}v_w t$.]

4. How does the scaling change if the system is adiabatic but the energy injection rate is not constant in time, and scales as $L \propto t^a$?

2 The maximum energy of protons and electrons accelerated at shock waves

Astrophysical shock waves are cosmic ray accelerators. Both protons and electrons can be accelerated in such systems. Consider the time evolution of a supernova remnant shock during the first few centuries after the supernova explosion. At these early times, the supernova remnant is in the free expansion phase. During this phase, the shock velocity $u_s \sim 3 \times 10^3$ km/s remains constant in time, and thus the shock radius scales as:

$$R_s = u_s t \quad . \tag{2}$$

Assume now that the transport of relativistic particles (both protons and electrons) close to the shock is diffusive and characterised by the Bohm diffusion coefficient:

$$D_B = \frac{1}{3} R_L c \quad , \tag{3}$$

where R_L is the particle Larmor radius and c is the speed of light. Finally, assume that the magnetic field strength B is constant both in space (upstream and downstream of the shock) and in time (B does not change during the evolution of the remnant).

1. Derive or estimate by means of a dimensional argument the expression of the acceleration time of particles at the shock [Hint: assume the shock to be plane and infinite].

2. Estimate the energy loss time of protons and electrons due to proton-proton interactions and synchrotron emission, respectively. Assume that

the density of the interstellar medium is 1 cm^{-3} and that the magnetic field strength at the shock is 30 μG. Compare the energy loss times with the duration of the free expansion phase and comment.

[Hints:

- The cross section for proton-proton interactions is $\sigma_{pp} \approx 4 \times 10^{-26}$ cm^2, and the inelasticity of the interaction is $\kappa \approx 0.5$.
- The power (energy per unit time) emitted by an electron of energy E in a magnetic field of strength B is

$$P = \frac{4}{3}\sigma_T c \gamma^2 \frac{B^2}{8\pi} \quad ,$$

  where $\sigma_T \sim 6.65 \times 10^{-25}$ cm^2 is the Thomson cross section, γ is the electron Lorentz factor, and c is the seed of light. The mass of the electron is 9.1×10^{-28} g.
- Some useful constants: 1 yr $= 3.1 \times 10^7$ s, 1 eV $= 1.6 \times 10^{-12}$ erg.]

3. Estimate now the maximum energy E_{max} of protons and electrons accelerated at the shock as a function of the time since the supernova explosion. Show that the behaviour of the scaling of E_{max} with time is equal for protons and electrons for times smaller than a time t_*, and different at later times. Why? What does t_* represents?

   [Hint: an approximate expression for the Larmor radius of a relativistic particle of energy E (expressed in eV) gyrating around a magnetic field of strength B (expressed in Gauss) is: $R_L \sim \frac{E(\text{eV})}{300\ B(\text{G})}$cm.]

4. Electrons accelerated at the shock emit synchrotron radiation. Show that, for times longer than t_*, the maximum energy of the synchrotron photons ϵ_s^{max} does not depend on the strength of the magnetic field at the shock, but depends on the shock velocity only. Demonstrate that the following scaling applies: $\epsilon_s^{max} \propto u_s^2$. Finally, estimate the numerical value of ϵ_s to show that the synchrotron emission extends up to the X-ray domain.

   [Hints:

   - electrons of Lorentz factor γ emit synchrotron photons of characteristic frequency:

$$\nu_s \sim \gamma^2 \frac{qB}{2\pi mc} \tag{4}$$

     where $q = 4.8 \times 10^{-10}$ statC, B is the magnetic field strength (in Gauss), $m = 9.1 \times 10^{-28}$ g is the electron mass, and c is the speed of light;
   - use the estimate made in the previous exercise of the maximum energy of electrons accelerated at shocks;
   - the value of Planck's constant is: $h = 6.63 \times 10^{-27}$ erg s.]

3 Very-high-energy gamma rays from the remnant of the supernova SN1987A

The most recent supernova explosion witnessed by human beings in the very local Universe happened in 1987 in the Large Magellanic Cloud. The Large Magellanic Cloud is a satellite galaxy of the Milky Way located at a distance of ~ 50 kpc from the Earth.

Assume that a fraction η of the supernova explosion energy $E = 10^{51}$ erg has been converted into cosmic rays of energy larger than 1 GeV, with a differential energy distribution $N(E) \propto E^{-2.4}$ and that these cosmic rays are still confined within the supernova remnant shell.

1. Estimate the supernova remnant gamma-ray luminosity above photon energies of 1 TeV as a function of η and of the number density of the ambient medium n.

2. Gamma-ray telescopes of future generation will be able to detect sources (above photon energies of 1 TeV) down to a flux level of the order of $\phi_{min} \approx 10^{-14}$ cm$^{-2}s^{-1}$. Determine the values of η and n that would result in a detection. Comment the result.

[Hint: the production rate of cosmic ray protons $Q_p(E_p)$ is connected to that of gamma rays by the expression:

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p)E_p^2$$

where $\eta_\pi \sim \tau_{res}/\tau_{pp}$ and $E_\gamma \sim 0.1 \times E_p$. τ_{res} is the residence time of protons inside the supernova remnant, and τ_{pp} is the energy loss time due to proton-proton interactions.]