NPAC course on Astroparticles

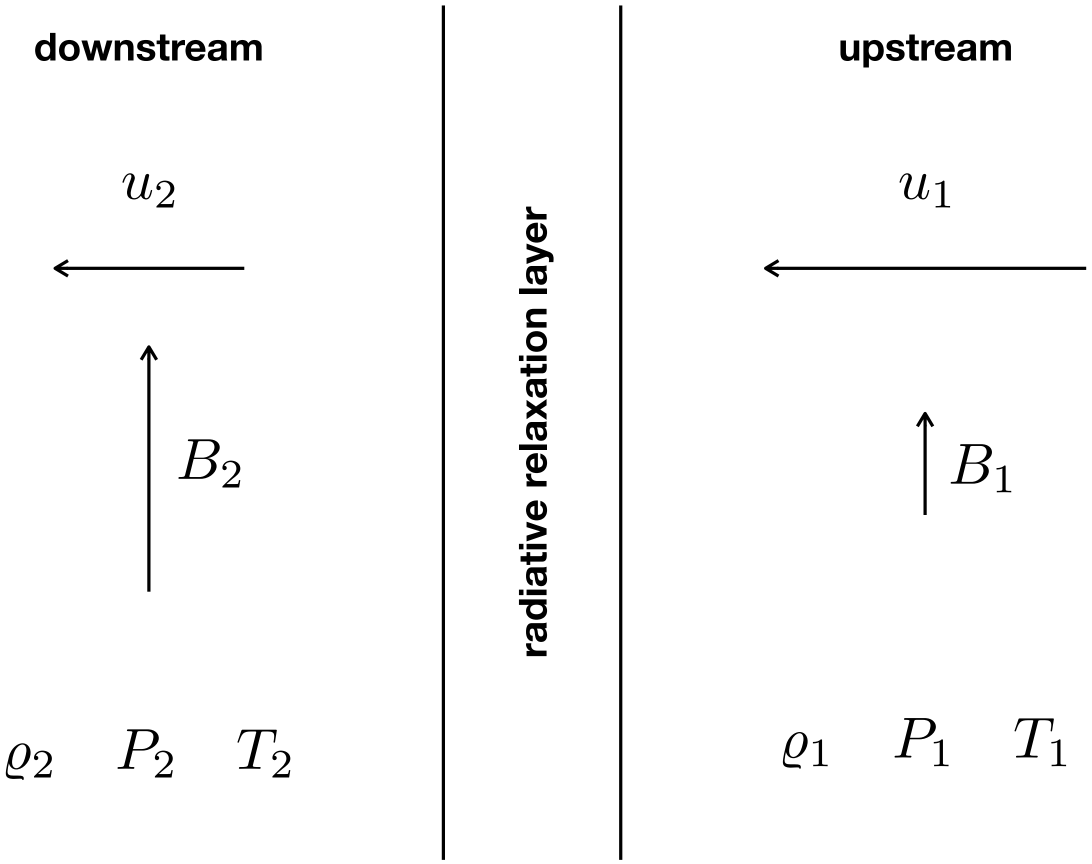
Exam 2019

Exercise #1 — Solution

Radiative isothermal shock with magnetic field

isothermal shock

$$T_2 = T_1 \longrightarrow P_i = c_s^2 \varrho_i$$



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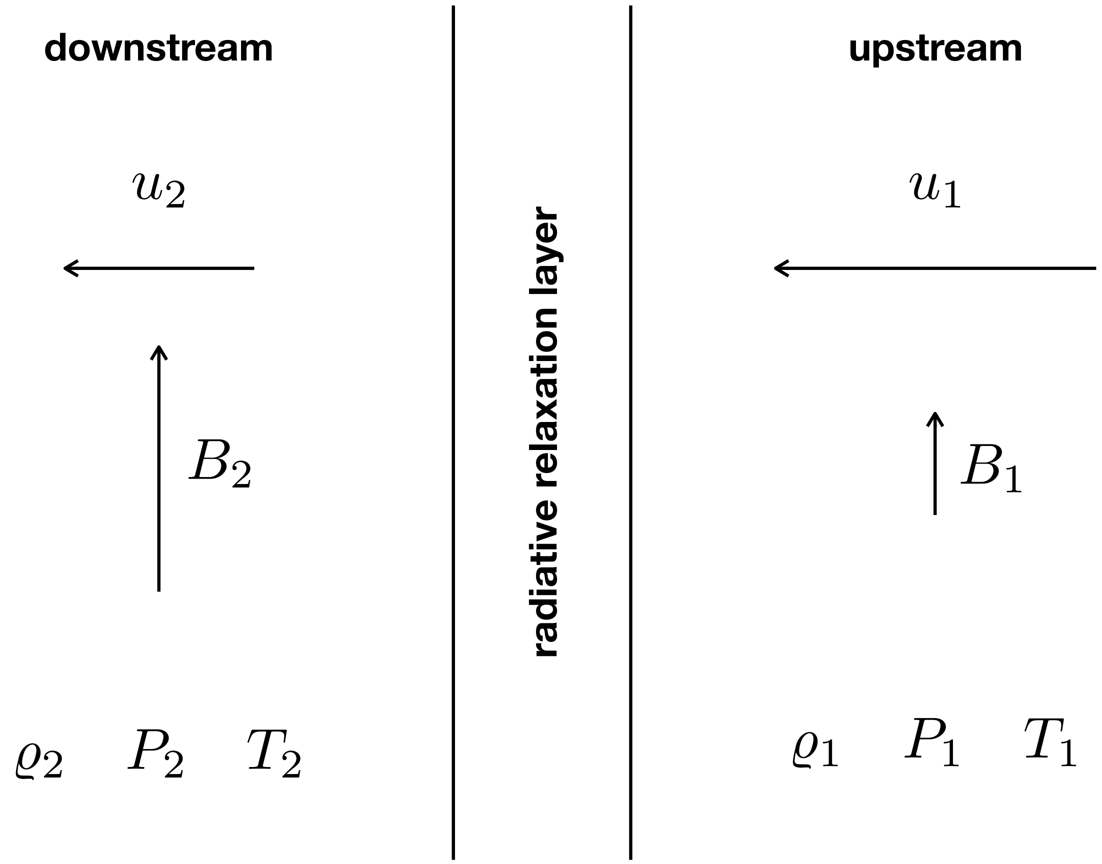

$$\begin{aligned} \varrho_1 u_1 &= \varrho_2 u_2 \\ \varrho_1 u_1^2 + P_1 &= \varrho_2 u_2^2 + P_2 \end{aligned}$$

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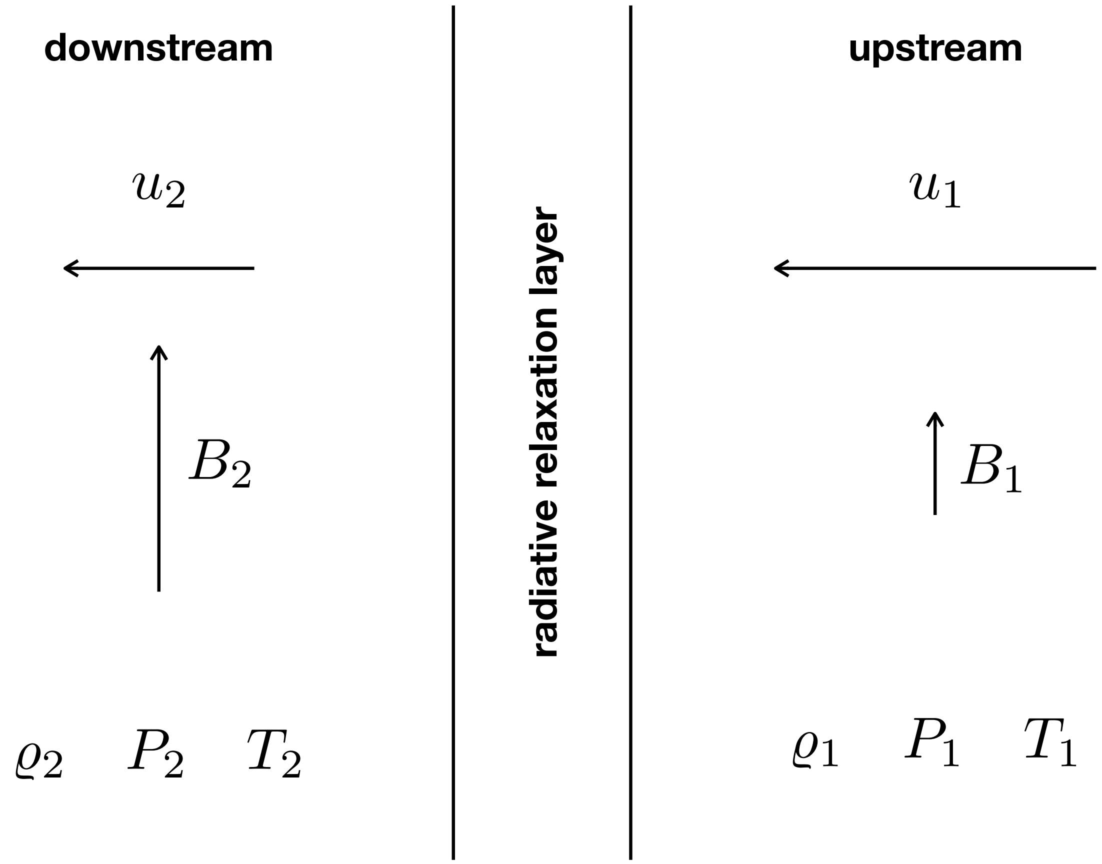

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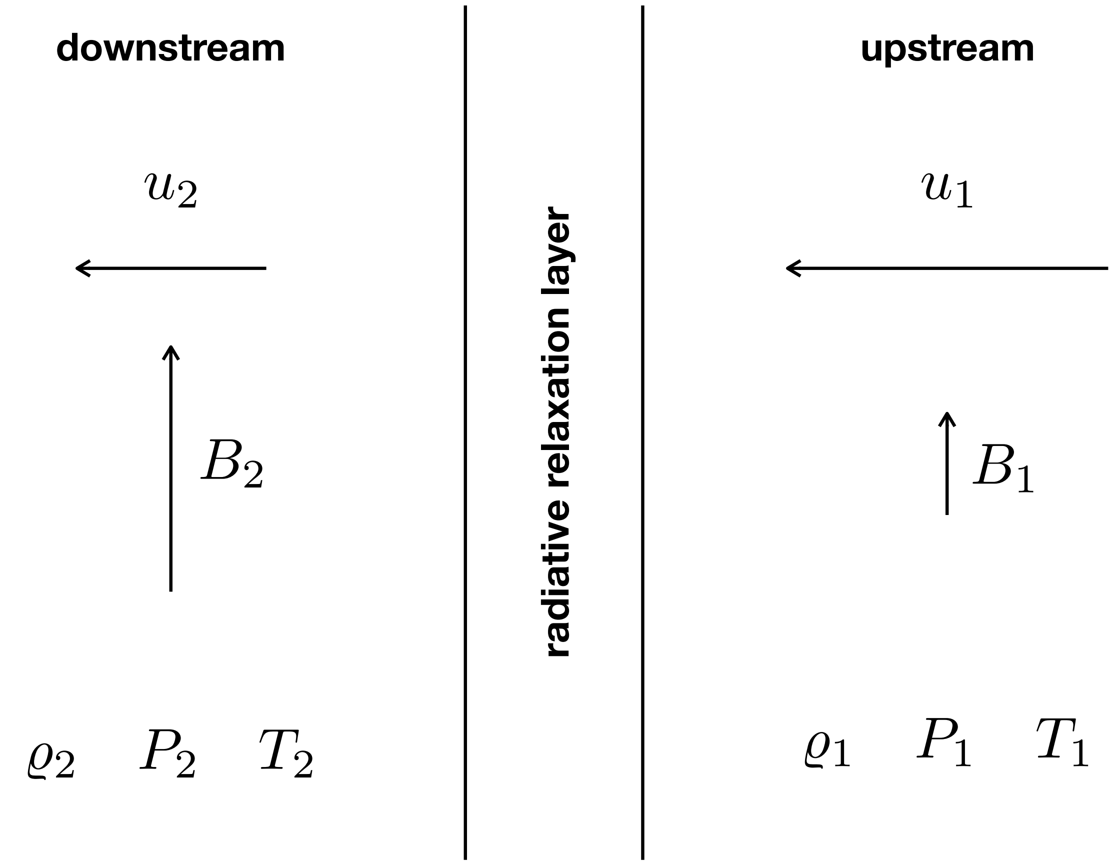

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divide by $\varrho_1 c_s^2 \rightarrow \mathcal{M}^2 + 1 = \dfrac{\mathcal{M}^2}{r} + r$

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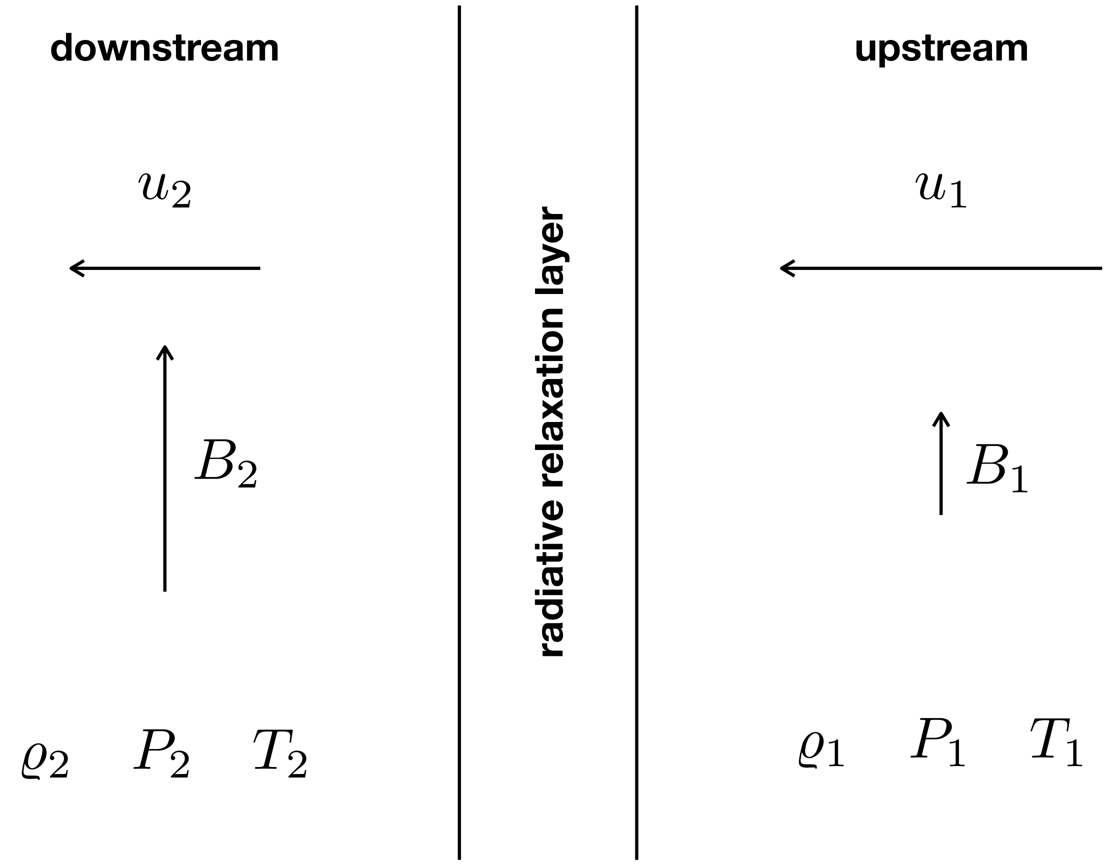

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divide by $\varrho_1 c_s^2 \rightarrow \mathcal{M}^2 + 1 = \dfrac{\mathcal{M}^2}{r} + r \rightarrow r^2 - r(\mathcal{M}^2 + 1) + \mathcal{M}^2 = 0$

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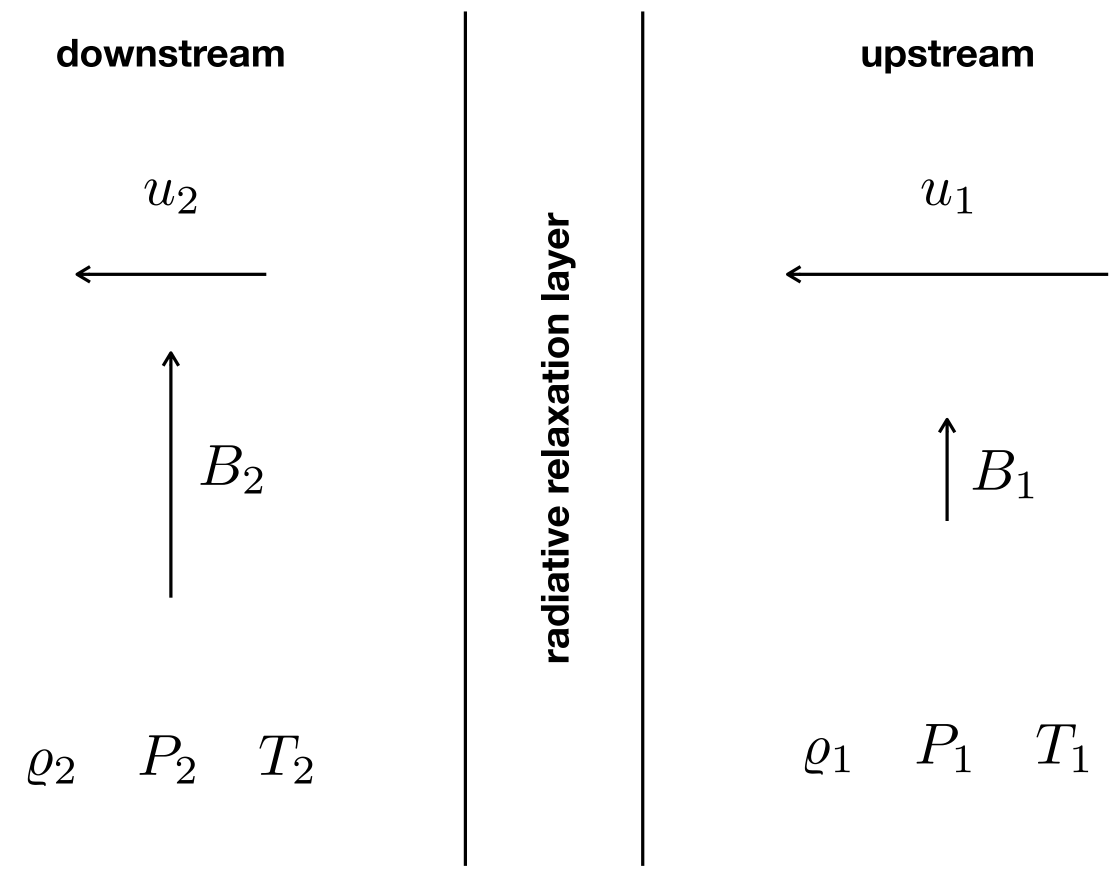

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$$\rightarrow (r-1)(r-\mathcal{M}^2) = 0$$

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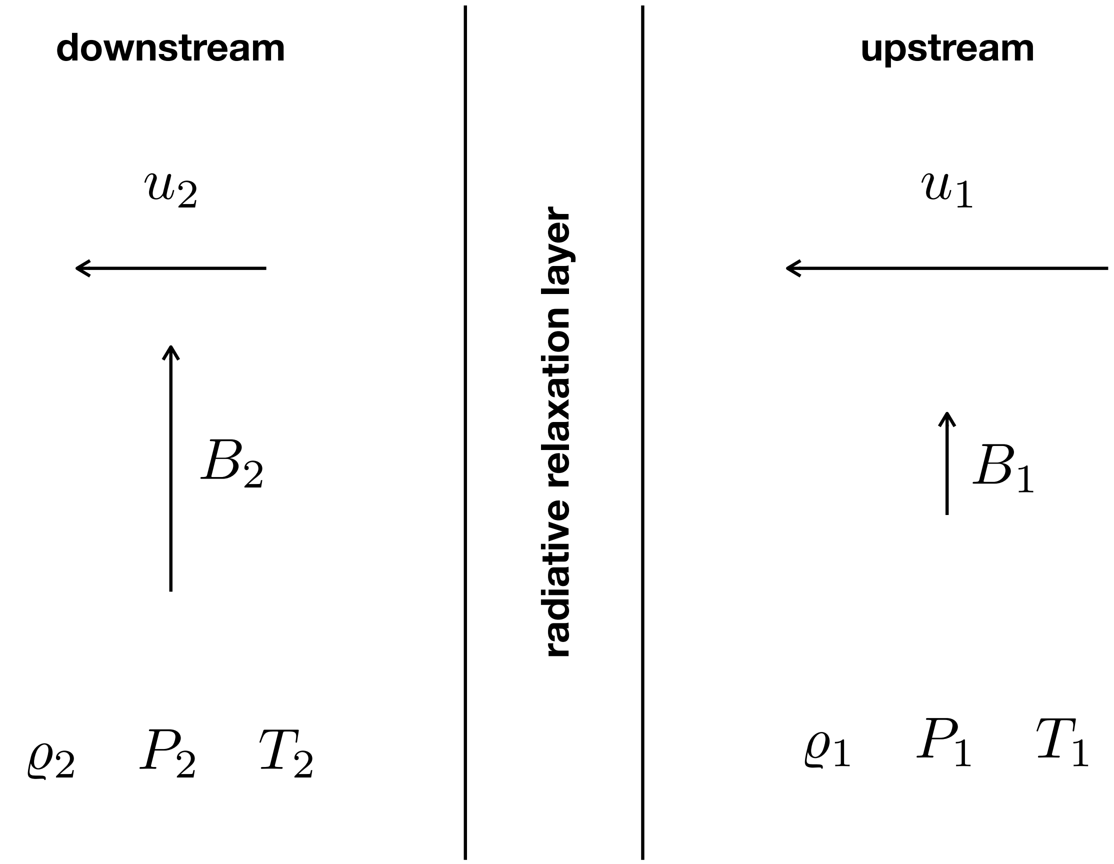

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divide by $\varrho_1 c_s^2 \rightarrow \mathcal{M}^2 + 1 = \frac{\mathcal{M}^2}{r} + r \rightarrow r^2 - r(\mathcal{M}^2 + 1) + \mathcal{M}^2 = 0$

$$\rightarrow \cancel{(r-1)}(r - \mathcal{M}^2) = 0 \longrightarrow \boxed{r = \mathcal{M}^2}$$

for $\mathcal{M} \longrightarrow \infty$

$$\varrho_2 = r\varrho_1 = \mathcal{M}^2 \varrho_1 \longrightarrow \infty$$

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$$\varrho_2 u_2^2 \propto \mathcal{M}^{-2} \longrightarrow 0$$

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During the class one of you asked: is this correct? One could write:

$$\varrho_2 u_2^2 = (\varrho_1 u_1) u_2 = \varrho_1 \frac{u_1^2}{\mathcal{M}^2} = \varrho_1 c_s^2$$

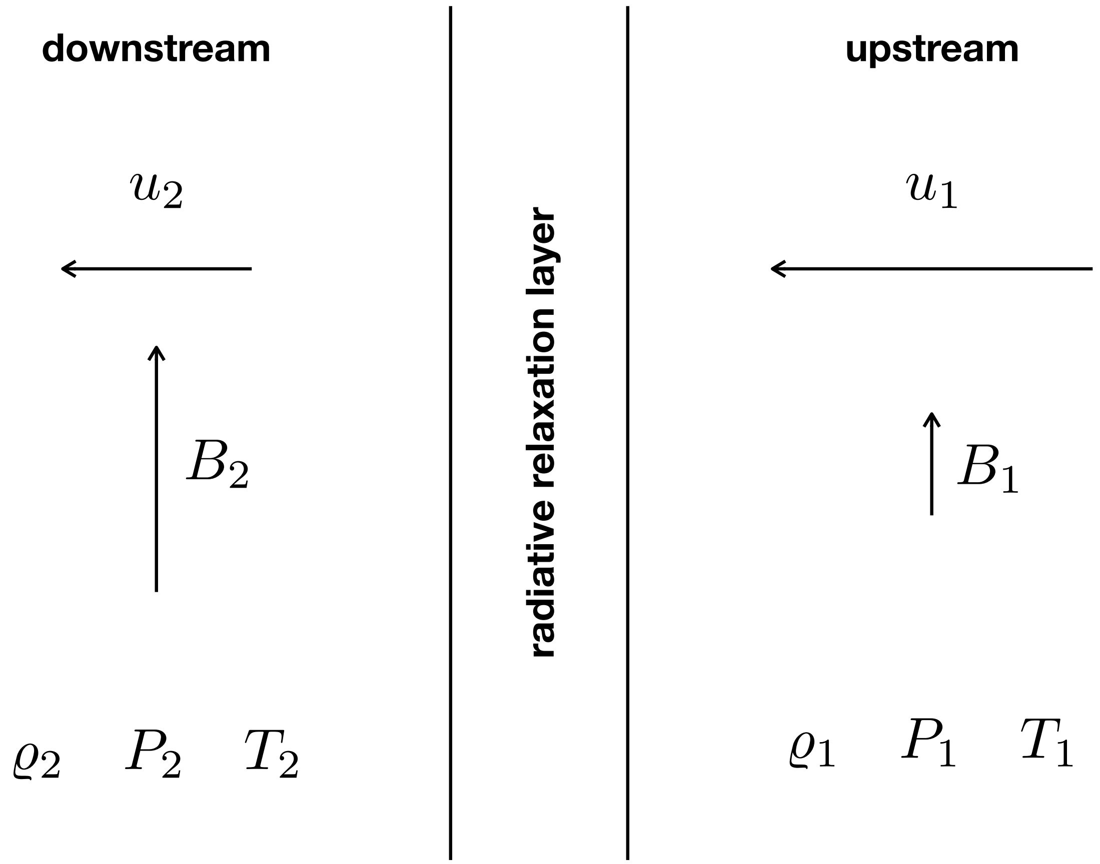
Which is not zero! (?)

Where is the trick? Well, in fact you can proceed further and write:

$$\varrho_2 u_2 = ... = \varrho_1 c_s^2 = P_1 \ll \varrho_1 u_1^2$$

Where we used the condition for a strong shock for the last inequality. In the limit of infinite Mach number one sees that P_1 goes to zero.

The role of magnetic field



$$
\begin{aligned}
\varrho_1 u_1 &= \varrho_2 u_2 \\
\varrho_1 u_1^2 + P_1 + \frac{B_1^2}{8\pi} &= \varrho_2 u_2^2 + P_2 + \frac{B_2^2}{8\pi}
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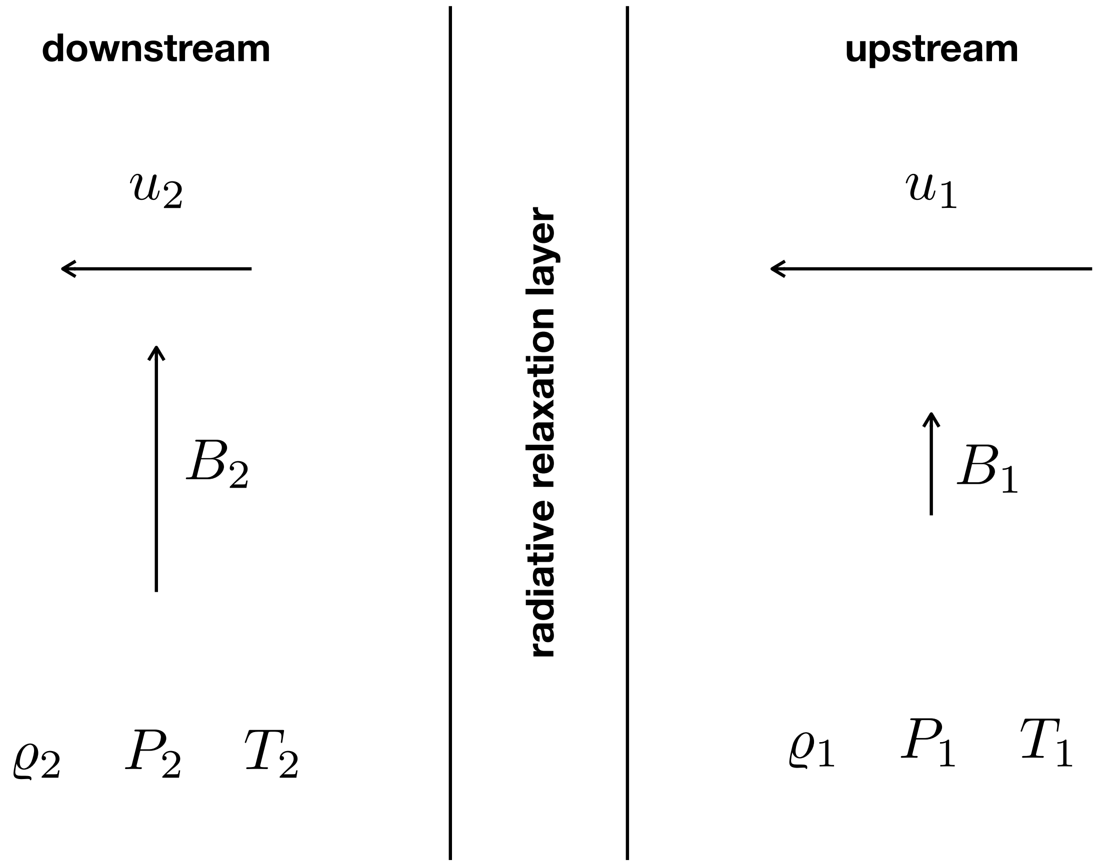

$$\mathcal{M} \gg 1$$

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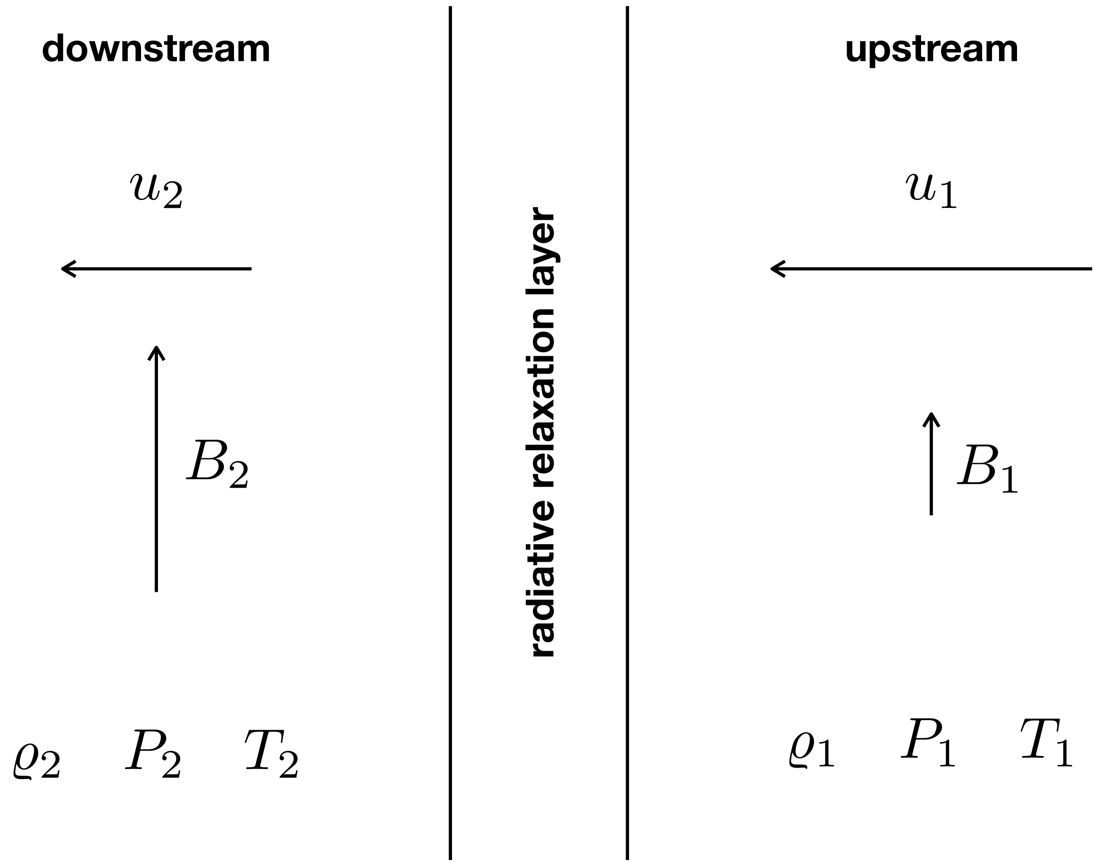

$$\mathcal{M} \gg 1 \qquad \mathcal{M}_A \gg 1$$

$$u_1 \gg V_A = \frac{B_1}{\sqrt{4\pi\varrho_1}}$$

$$\varrho_1 u_1 = \varrho_2 u_2$$

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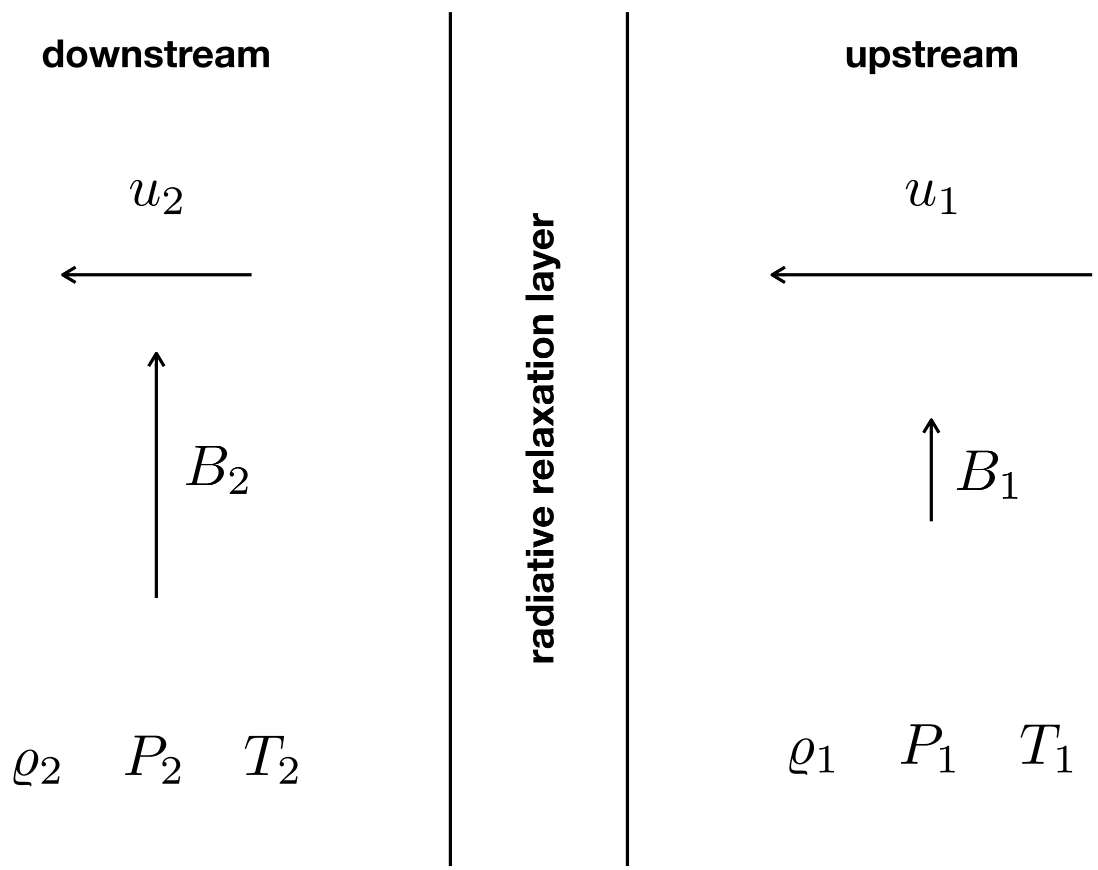

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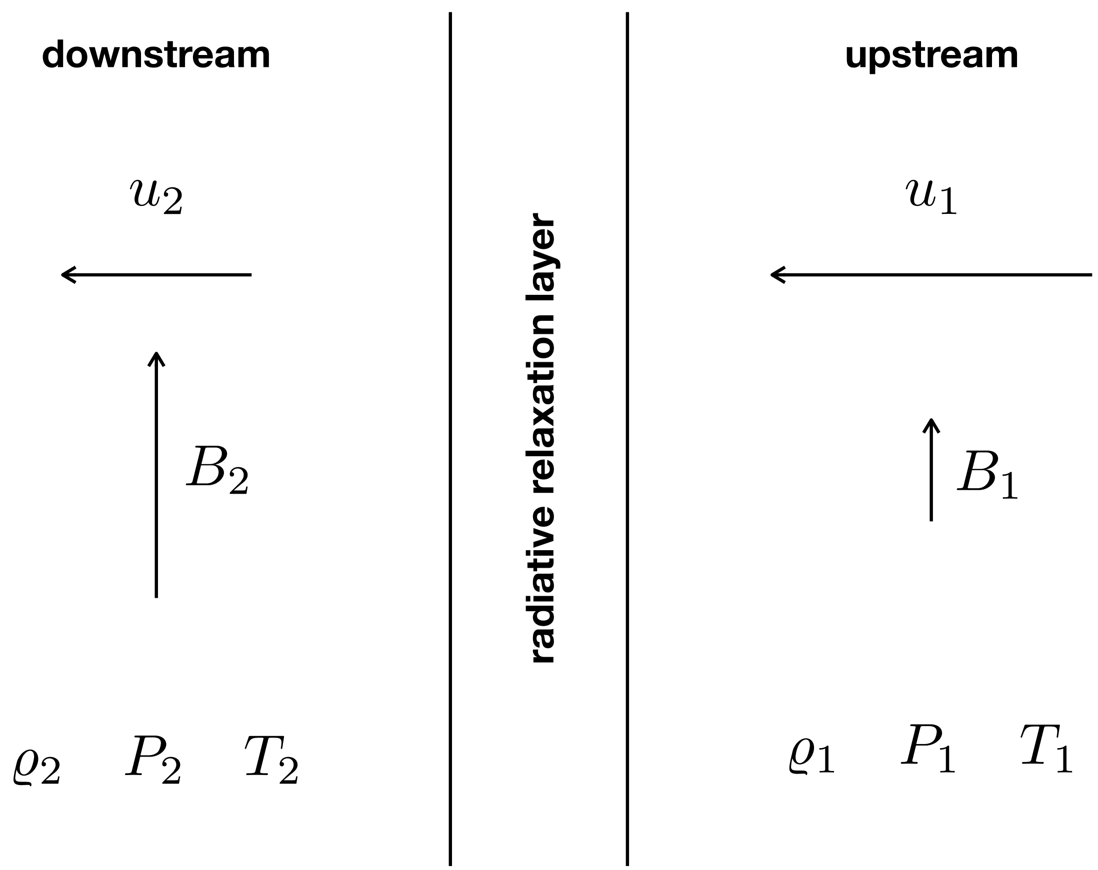

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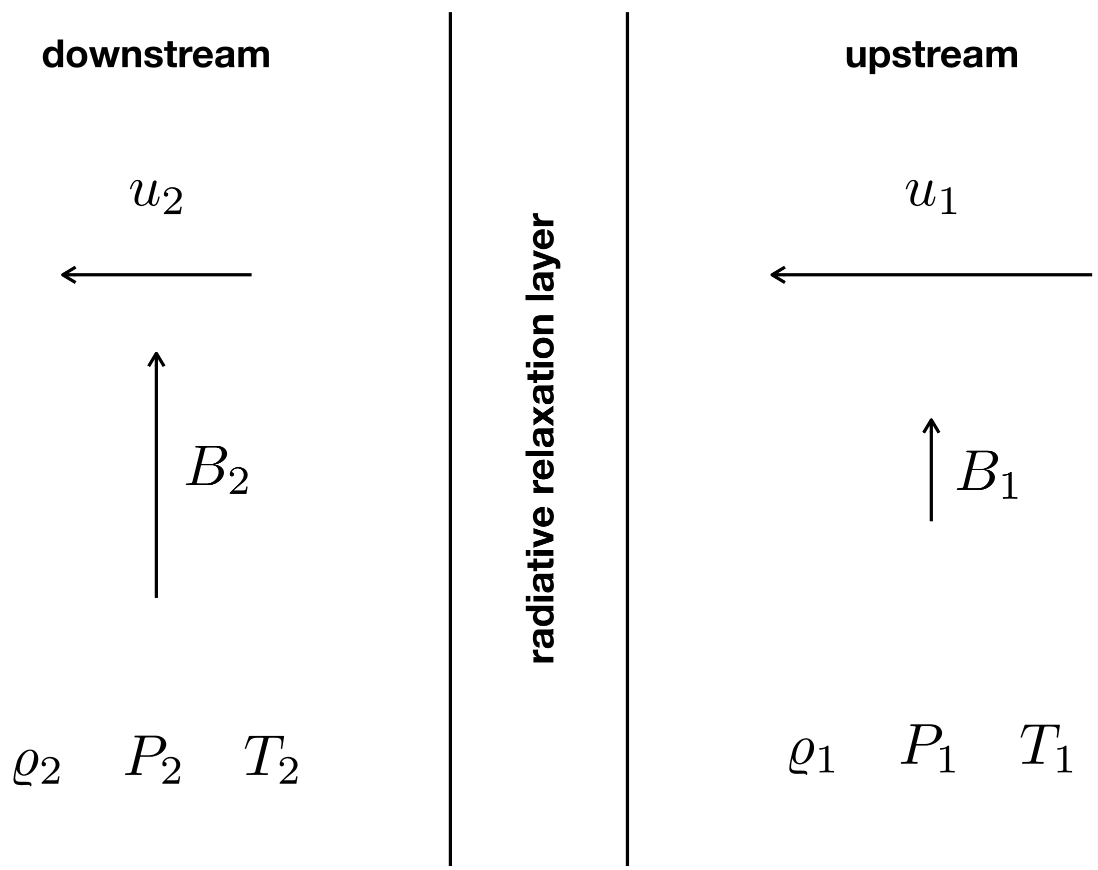
Is there a maximum value for r?

The role of magnetic field



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$$\propto \frac{1}{r} \qquad \propto r \qquad \propto r^2$$

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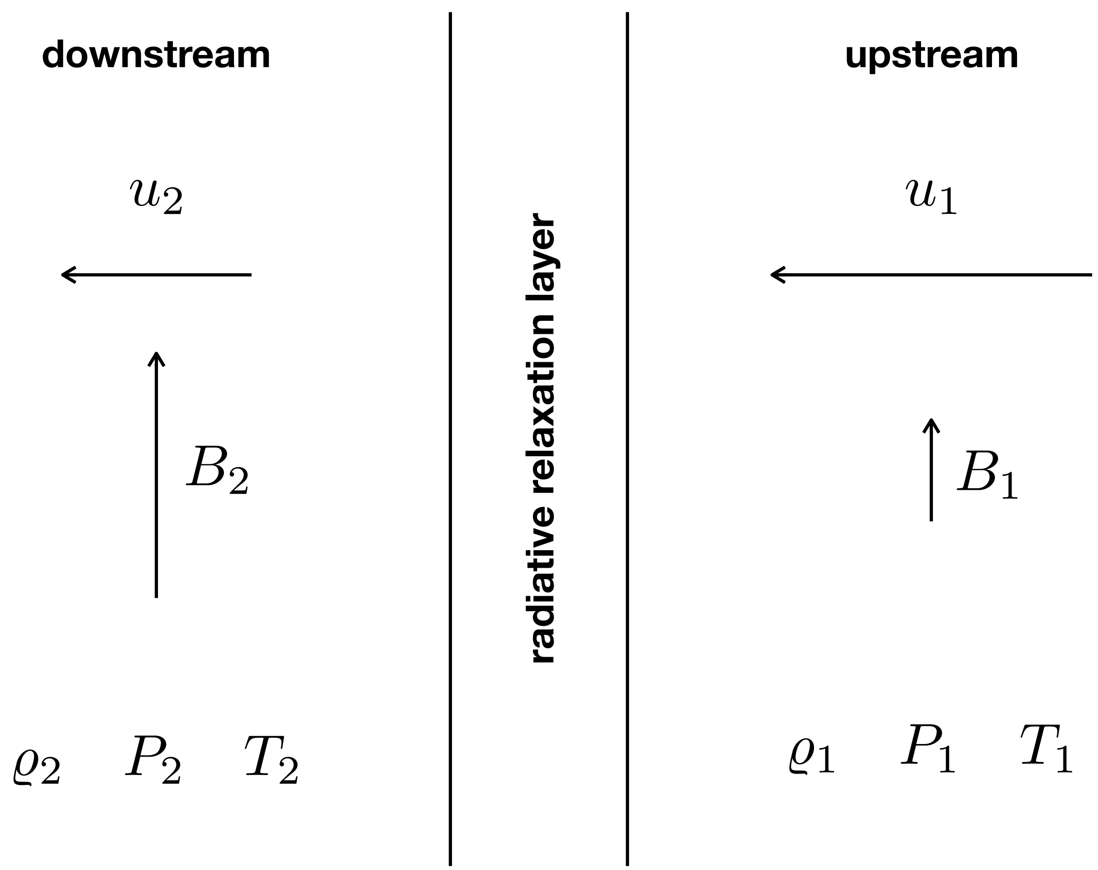

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Is there a maximum value for r?

$$\varrho_1 u_1^2 = r^2 \frac{B_1^2}{8\pi} \longrightarrow r^2 = 2 \frac{4\pi \varrho_1}{B_1^2} u_1^2$$

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Let's manipulate the second equation by dividing it by ϱ_1

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u_1^2 + c_s^2 + \frac{V_A^2}{2} = \frac{u_1^2}{r} + r c_s^2 + r^2 \frac{V_A^2}{2}
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$$u_1^2 \left(\frac{r-1}{r} \right) + c_s^2 (1-r) + \frac{V_A^2}{2}(1-r^2) = 0$$

r = 1 is a solution (not a shock solution!) —> so let's divide by (r-1)

$$\frac{u_1^2}{r} - c_s^2 - \frac{V_A^2}{2}(r+1) = 0$$

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The only positive solution is:

$$\boxed{r = \sqrt{2}\frac{u_1}{V_A} = \sqrt{2}\mathcal{M}_A}$$

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For a non-magnetised plasma r —> infinity when M —> infinity.
For a magnetised plasma there is, in general, a limit to r even when M —> infinity.
The only case when r diverge is for M —> infinity **AND** M_A —> infinity.
But this is the case of an unmagnetised plasma!

Exercise #2 — Solution

Density of the intracluster medium

$$M_{tot} \sim 10^{15} M_{\odot} \longrightarrow M_{IC} \sim 0.1 \times M_{tot} \sim 10^{14} M_{\odot}$$

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Energetic protons do not have time to cool nor to escape! —> they accumulate in clusters for an Hubble time

Gamma rays

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p)E_p^2$$

$$\eta_\pi = 1 - \exp\left(-\frac{\tau_{res}}{\tau_{pp}}\right)$$

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age of the Universe!
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$$Q_\gamma(E_\gamma) = \frac{\eta_\pi}{3} Q_0 \left(\frac{E_p}{E_0}\right)^{-2.1} \left(\frac{E_p}{E_\gamma}\right)^2$$

1 TeV

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1 TeV

$$\Phi_\gamma(> 1\ \mathrm{TeV}) = \frac{100\eta_\pi}{3} Q_0 \int_{1\ \mathrm{TeV}} \mathrm{d}E_\gamma \left(\frac{E_p}{E_0}\right)^{-2.1}$$

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age of the Universe!
~10^{10} yr

$$\eta_\pi = 1 - \exp\left(-\frac{\tau_{res}}{\tau_{pp}}\right) \longrightarrow \eta_\pi \sim \frac{\tau_{res}}{\tau_{pp}} \approx 0.2$$

$$Q_\gamma(E_\gamma) = \frac{\eta_\pi}{3} Q_0 \left(\frac{E_p}{E_0}\right)^{-2.1} \left(\frac{E_p}{E_\gamma}\right)^2 = \frac{100\eta_\pi}{3} Q_0 \left(\frac{E_p}{E_0}\right)^{-2.1}$$

1 TeV

$$\Phi_\gamma(> 1 \text{ TeV}) = \frac{100\eta_\pi}{3} Q_0 \int_{1 \text{ TeV}} \mathrm{d}E_\gamma \left(\frac{E_p}{E_0}\right)^{-2.1}$$

$$= \frac{100\eta_\pi}{3} Q_0 \int_{1 \text{ TeV}} \mathrm{d}E_\gamma \left(\frac{10\ E_\gamma}{E_0}\right)^{-2.1}$$

The Coma cluster is an extended source

$$\vartheta_{Coma} = \frac{2R}{d}$$

So the sensitivity becomes:

$$\phi_{ext} = \phi_{min} \frac{\vartheta_{Coma}}{\vartheta_{min}}$$

At this point Q_0 can be determined by equating

$$\frac{\Phi_\gamma}{4\pi d^2} = \phi_{ext}$$

Is this plausible?

Once Q_0 is determined we can estimate the power of CR sources in the cluster

$$P_{CR} = \int_{1\ \mathrm{GeV}} \mathrm{d}E_p Q(E_p) E_p$$

What is the CR power of all the galaxies in the Come cluster?

Milky Way —> $$P_{MW} \sim 10^{41} \mathrm{erg/s}$$

Number of galaxies: 2000, but much smaller than the MW, so we expect

$$P_{CR} \ll P_{MW} \times 2000$$

In fact, if you put numbers in, you get the opposite, so the answer is: NO.

Exercise #3 — Solution

Superluminal diffusion

relevant physical quantities l_d D t

Superluminal diffusion

relevant physical quantities $l_d \quad D \quad t$

diffusion length $l_d \sim \sqrt{D \times t}$

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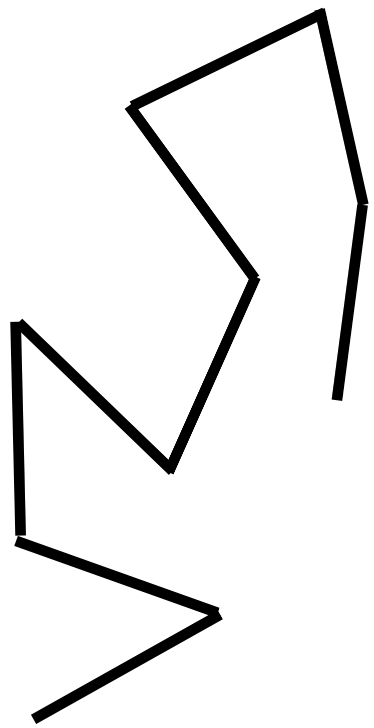
Superluminal diffusion



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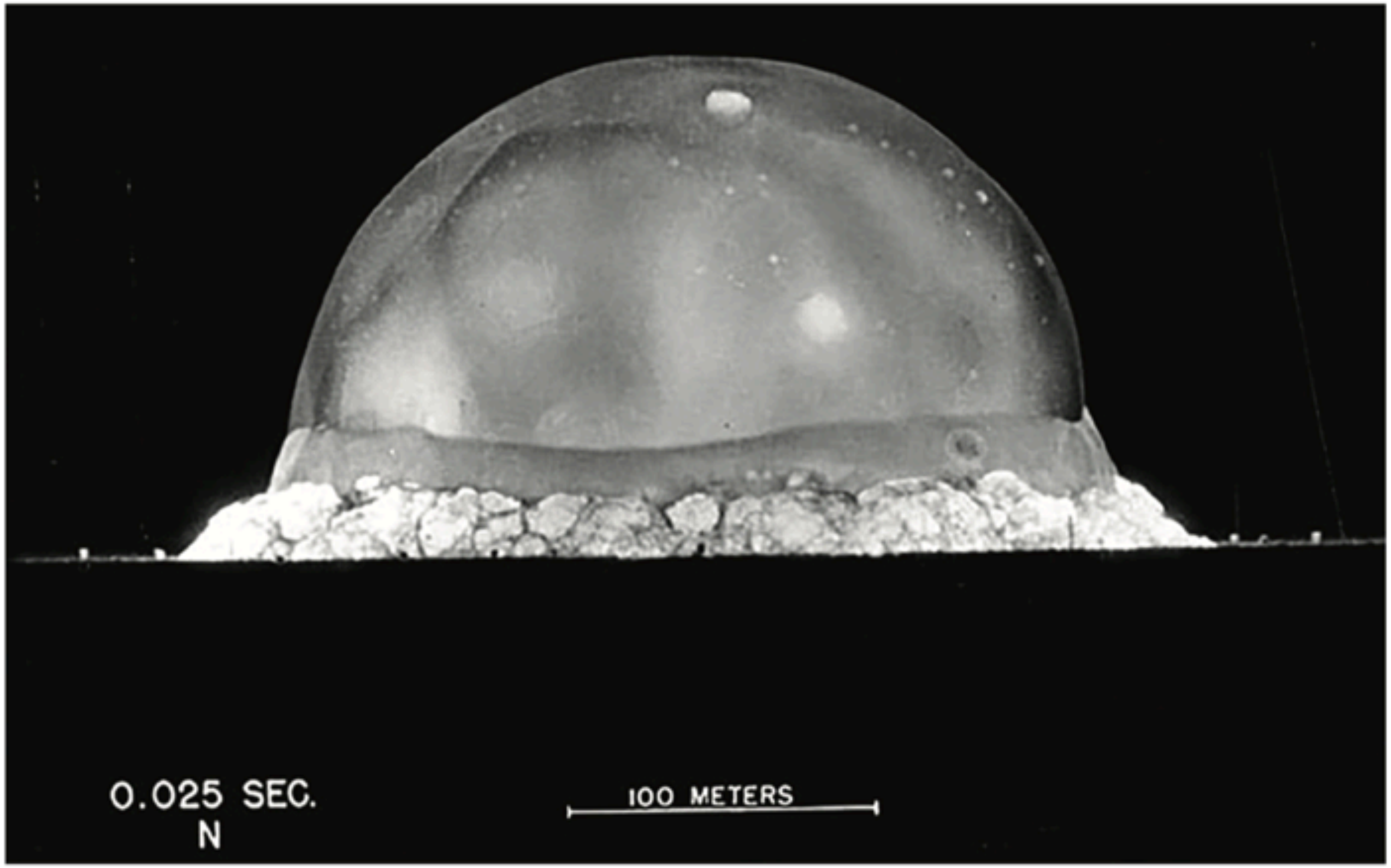
scattering time!!! —> diffusion is not a good description before t*

Exercise #4 — Solution

The Trinity test



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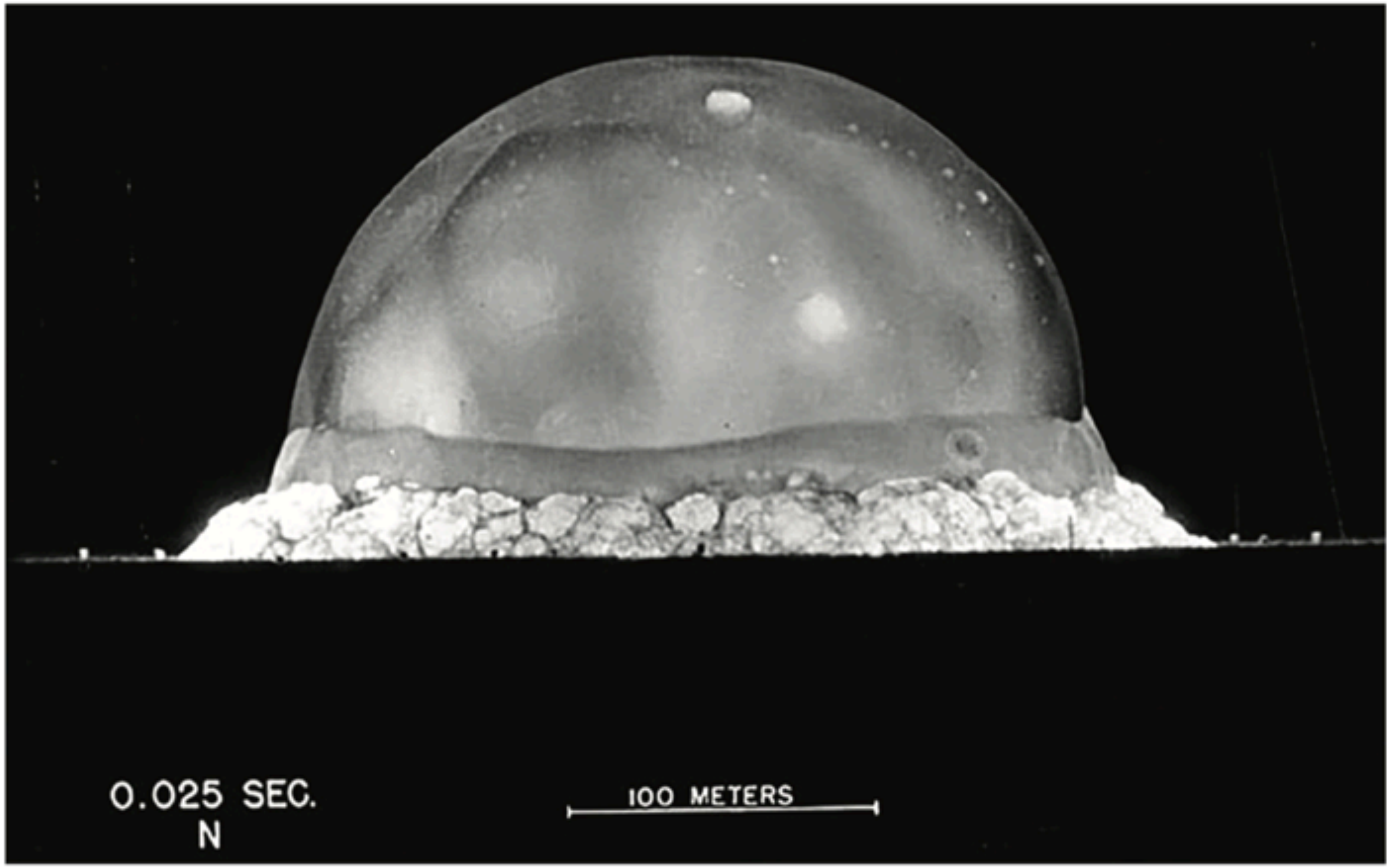
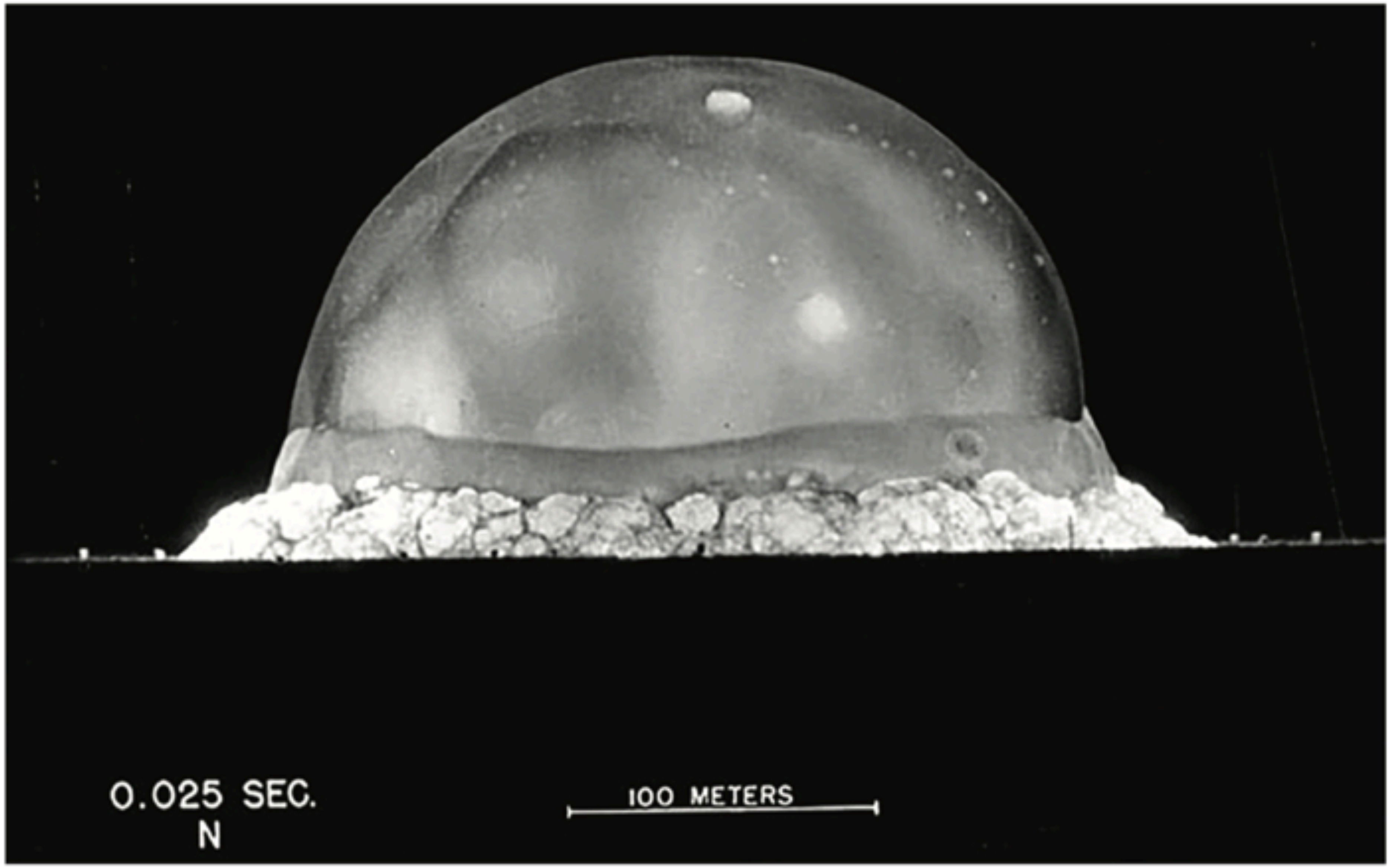


Measure roughly the radius of the shock...

The Trinity test



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Assuming that the shock is in the Sedov phase $$R_s \sim \left(\frac{E}{\varrho}\right)^{1/5} t^{2/5}$$

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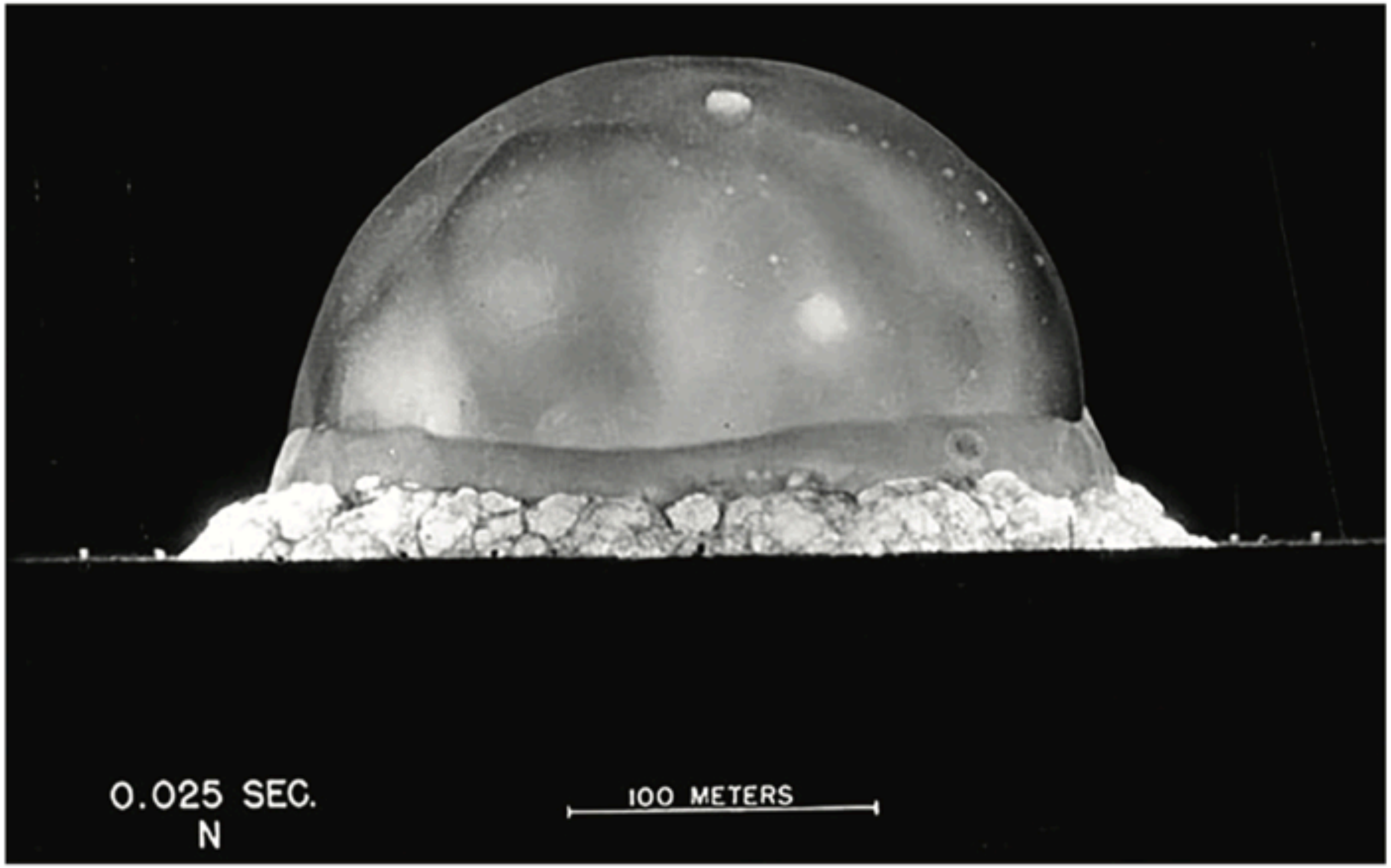

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Assuming that the shock is in the Sedov phase

$$R_s \sim \left(\frac{E}{\varrho}\right)^{1/5} t^{2/5}$$

Everything is given except for E —> the “official” estimate is ~20 kiloTon

Free expansion phase?

Mass of the bomb —> $M_{bomb} = 214 \text{ tons}$

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We are NOT in the free expansion phase!