NPAC Astroparticles Exam 2019

1 The role of the magnetic field at radiative isothermal shocks

Consider an infinite, plane, and radiative shock as in Figure 1. In the shock frame, the upstream gas moves from right to left at a velocity u_1, and is characterised by a density ϱ_1, a pressure P_1, and a temperature T_1. The gas is heated at the shock transition and then it radiates energy in a "radiative relaxation layer" (RRL). As a consequence, the gas temperature drops within the RRL and reaches an equilibrium value T_2 in the downstream region. Moreover, in the downstream region, the gas moves away from the shock at a velocity u_2, and is characterised by a density ϱ_2 and a pressure P_2. In this exercise, we will consider an *isothermal* shock, which is a particular case of a radiative shocks, where $T_1 = T_2$. The conditions to be satisfied to conserve mass and momentum across the shock are:

$$\varrho_1 u_1 = \varrho_2 u_2 \tag{1}$$

$$\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2 \tag{2}$$

where $P_i = c_s^2 \varrho_i$ $(i = 1, 2)$ is the isothermal equation of state of the gas, and $c_s = constant$ the isothermal sound speed.

1. Solve the system of equations 1 and 2 and find an expression for the shock compression factor $r \equiv \varrho_2/\varrho_1$ as a function of the shock Mach number $\mathcal{M}$. Show that $\varrho_2 \to \infty$ and $u_2 \to 0$ for $\mathcal{M} \to \infty$. What happens to the term $\varrho_2 u_2^2$ for $\mathcal{M} \to \infty$?

2. Assume now that a magnetic field B_1 is present upstream of the shock. For simplicity, take the vector $\vec{B}_1$ to be perpendicular to the shock normal and consider the case of a strong shock ($\mathcal{M} \gg 1$) which moves at a velocity much larger than the Alfvén speed $u_1 \gg B_1/\sqrt{4\pi\varrho_1}$. Show that, in this case, in the limit $\mathcal{M} \to \infty$ the gas density downstream of the shock does not diverge, but has a finite value. Find the expressions for ϱ_2 and u_2. [Hint: Add to Eq. 2 the contribution from the magnetic pressure, and ignore the small terms on both sides of the equation. Remember how a magnetic field is compressed at a perpendicular shock.]

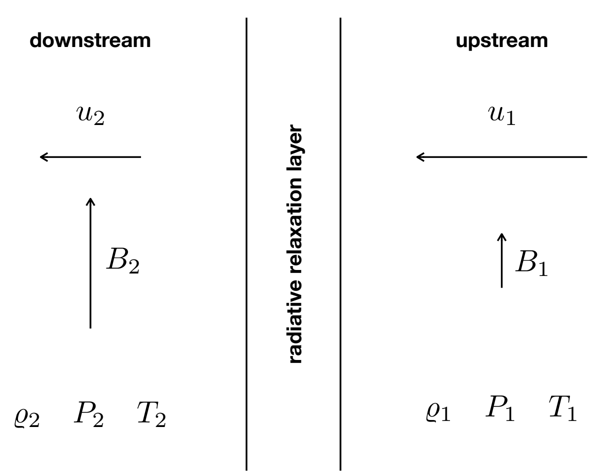

Figure 1: Shock configuration for Exercise 1.

2 Gamma ray emission from the Coma cluster of galaxies

Clusters of galaxies are very large, roughly spherical structures containing hundreds of galaxies. Their are cosmological objects, in the sense that their age is comparable to the age of the Universe. The Coma cluster, located at a distance of $d \sim 100$ Mpc from the Earth, is probably the best studied among these objects. Its radius is of the order of $R \approx 1$ Mpc. Its total mass is equal to $M_{tot} \approx 10^{15} M_{\odot}$, and is largely dominated by dark matter. About 10% of the total mass is in form of a diffuse gas, called intracluster gas, that fills the entire volume of the cluster.

It has been suggested that clusters of galaxies might be gamma-ray sources, due to the interactions between relativistic protons and the intracluster gas. The relativistic protons can be accelerated, for example, by the supernova remnants contained within the galaxies that form the cluster. In this exercise we will explore this scenario.

1. Compute the density of the intracluster gas n_{IC} in cm^{-3}, assuming that it is homogeneously distributed within a spherical volume of radius R. [Hint: assume the gas to be composed by atomic hydrogen only. The proton mass is $m_p \sim 2 \times 10^{-24}$ g, the solar mass is $1 M_{\odot} \sim 2 \times 10^{33}$ g, and a parsec is 1 pc $\sim 3 \times 10^{18}$ cm.]

2. Compute the energy loss time τ_{pp} for relativistic protons undergoing inelastic proton-proton interactions with the intracluster gas. Compare it with the age of the Universe. Comment the result. [Hint: the cross section for proton-proton interactions is $\sigma_{pp} \sim 4 \times 10^{-26}$ cm^2, and the inelasticity of the process is $\kappa \sim 0.5$. The age of the Universe is of the order of $\approx 10^{10}$ yr. 1 yr = 3×10^7 s. The speed of light is $c \sim 3 \times 10^{10}$ cm/s.]

3. It is known from observations that a turbulent magnetic field at the $\approx \mu$G level is present in the intracluster medium. Therefore, relativistic protons are expected to diffuse in space in a way similar to cosmic rays in our Galaxy. Theoretical estimates suggest that the particle diffusion coefficient in the intracluster medium may be of the order of $D \sim 10^{28} (E_p/\text{GeV})^{1/3}$ cm^2/s where E_p is the energy of the proton. Estimate the diffusive escape time $\tau_{esc}(E_p)$ of relativistic protons from the Coma cluster and compare it with the age of the Universe. Comment the result.

4. If $Q_p(E_p) = Q_0 (E_p/\text{TeV})^{-2.1}$ represents the number of protons of energy E_p injected each second in the intracluster medium (the units of Q_0 are TeV^{-1} s^{-1}), then the gamma-ray luminosity $Q_{\gamma}(E_{\gamma})$ resulting from the interactions between relativistic protons and gas is:

$$Q_{\gamma}(E_{\gamma}) E_{\gamma}^2 = \frac{\eta_{\pi}}{3} Q_p(E_p) E_p^2$$

   where η_{π} is:

$$\eta_{\pi} = 1 - \exp\left[- \left(\frac{\tau_{res}}{\tau_{pp}} \right) \right] .$$

   where τ_{res} is the characteristic residence time in the cluster. For which values of Q_0 the Coma cluster would be detectable by Cherenkov telescopes of current generation? [Hint: what is the appropriate value for τ_{res}? Remember that the sensitivity of Cherenkov telescopes of current generation for point sources of gamma rays is $\phi_{min}(> 1 \text{ TeV}) \sim 10^{-13}$ cm^{-2} s^{-1}, and that their angular resolution is of the order of $\vartheta_{min} \sim 0.1^{\circ}$ (diameter). What is the apparent angular extension of the Coma cluster?]

5. Are the values of Q_0 obtained above plausible? [Hint: remember that our Galaxy produces 10^{41} erg/s in form of relativistic protons. The total number of galaxies in the Coma cluster is ~ 2000, and their average mass is much smaller than the mass of the Milky Way.]

3 Cosmic ray diffusion and the "paradox" of superluminal motions

The transport of cosmic rays in a turbulent magnetic field can be described as a diffusion process characterised by a spatial diffusion coefficient D. The diffusion length of cosmic rays $l_d(t)$ is the typical spatial displacement of cosmic rays after a time t. The physical interpretation of l_d, for one-dimensional diffusion, is the following: if a given number of cosmic rays are injected at a time $t = 0$ at a position $x = 0$, after a time t the cosmic rays will occupy roughly uniformly the region defined by $-l_d < x < l_d$. At this point, we can define an *effective* velocity $v_d = l_d/t$ to describe the rate or expansion of the region occupied by cosmic rays.

1. Write the expression for l_d. Use dimensional analysis if you do not remember it. Show that for short times one gets $v_d > c$, where $c = 3 \times 10^{10}$ cm/s is the speed of light. Define t_* as the characteristic time when $v_d = c$.

2. Nothing can propagate faster than c, so something must be wrong in the reasoning made above. Can you solve the (apparent) paradox? [Hint: do you remember the definition of diffusion coefficient?]

4 The Trinity test: the first atomic explosion

The first atomic bomb exploded on July 16th in the desert of New Mexico as part of the Manhattan project. The energy released in the explosion was of course a military secret. In 1947 some pictures of that atomic explosion were published by the *Life* magazine (see Fig. 2), and the British physicist Geoffrey Taylor used them to estimate the explosion energy. Taylor published his results in 1950, and the US army was annoyed by that.

1. Estimate the explosion energy by means of the information contained in Figure 2, which shows a picture of the shock wave that resulted from the explosion. Assume the shock wave to be spherical, and estimate its radius from the 100 m scale provided in the picture, which was taken 0.025 seconds after the explosion. Assume the expanding shock to be in the *adiabatic* phase. The only additional information needed to solve the problem is the density of the air, which is roughly equal to 1.2×10^{-3} g/cm^3. [Hint: atomic bombs and supernova remnants behave exactly in the same way. Energies of bombs are measured in Kilotons. 1 Kiloton = 4×10^{19} erg.]

2. Can you prove, a posteriori, that the explosion was not, at the time the picture was taken, in the free expansion phase? [Hint: the weight of the bomb was 214 tons.]

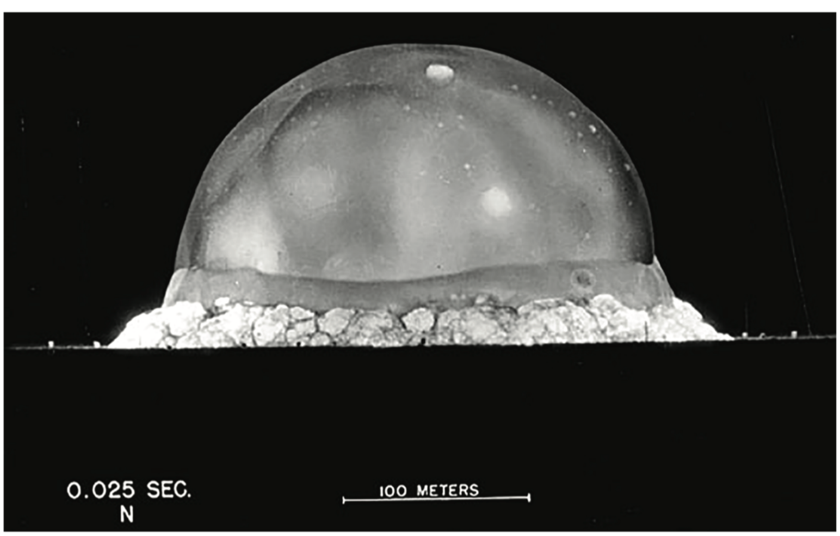

Figure 2: Picture of the first atomic explosion ever: the Trinity test.