NPAC Astroparticles Exam 2020

1 Shock jump conditions in the presence of a magnetic field

Consider a plane, infinite, non-radiative shock moving at a constant speed u_1 through a fully ionised plasma of density ϱ_1 and pressure P_1. To study this system, it is convenient to move to the rest frame where the shock is at rest. In this frame, the downstream plasma is slowed down to a velocity u_2, it is compressed to a density ϱ_2 and heated up to reach a pressure P_2. The shock is characterised by a Mach number $\mathcal{M} = u_1/c_s > 1$, with $c_s^2 = (5/3)P_1/\varrho_1$, and c_s is the speed of sound in the upstream medium (assumed to be a monoatomic gas).

In this exercise, we consider the case where the plasma is magnetised. For simplicity, consider the case where the magnetic field is perpendicular to the shock normal (i.e. parallel to the shock surface, see figure). Under these circumstances, the equations of mass, momentum, and energy conservation are:

$$\varrho_1 u_1 = \varrho_2 u_2 \tag{1}$$

$$\varrho_1 u_1^2 + P_1 + \frac{B_1^2}{8\pi} = \varrho_2 u_2^2 + P_2 + \frac{B_2^2}{8\pi} \tag{2}$$

$$\frac{1}{2}\varrho_1 u_1^3 + \frac{5}{2}P_1 u_1 + \frac{B_1^2}{4\pi}u_1 = \frac{1}{2}\varrho_2 u_2^3 + \frac{5}{2}P_2 u_2 + \frac{B_2^2}{4\pi}u_2 \tag{3}$$

where B_1 and B_2 represent the magnetic field strength upstream and downstream of the shock, respsctively. At this point, it is convenient to introduce three quantities, the shock compression factor $r = \varrho_2/\varrho_1 > 1$, the ratio between the magnetic and thermal pressure in the upstream plasma $\alpha = B_1^2/(8\pi P_1)$, and the Alfven speed in the upstream plasma $v_A = B_1/\sqrt{4\pi\varrho_1}$.

1. Find the relationship connecting B_2 to B_1 and eliminate B_2 from the three conservation equations. [Hint: use the theorem of magnetic flux freezing, which states that the flux of the magnetic field is conserved across a surface moving with the plasma. Express B_2 as a function of B_1 and r.]

2. Reduce the three equations above to a single equation containing the three variables r, $\mathcal{M}$ and α. [Hint: you should end up with a cubic equation in r.]

3. Show that the condition for the existence of a shock (i.e. $r > 1$) is $u_1^2 > c_s^2 + v_A^2$ (and not $u_1 > c_s$ as for shocks propagating in non-magnetised plasmas). [Hint: Notice that $r = 1$ is a solution. Show then that the cubic equation in r can be rewritten as $(r-1)(ar^2 + br + c) = 0$, where a, b, and c are functions of $\mathcal{M}$ and α. Then solve for r reminding that $r = 1$ is not a shock solution.]

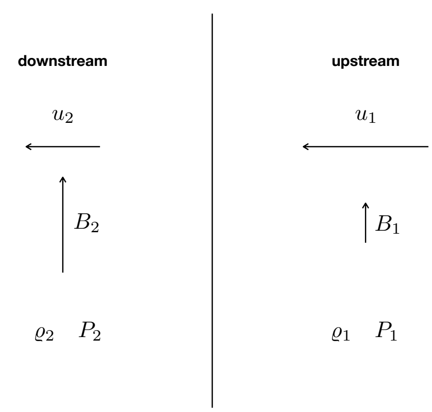

Figure 1: Shock configuration for Exercise 1.

2 Gamma rays from a starburst galaxy

Starburst galaxies are galaxies characterised by a very high rate of supernova explosions, concentrated in the galactic nucleus. The nucleus of the starburst galaxy NGC 253, located at a distance of 3.5 Mpc, has been detected by H.E.S.S. at photon energies in the range 0.3 TeV $< E_\gamma <$ 10 TeV. The measured gamma-ray spectrum can be fitted by a power law $F(E_\gamma) = F_0(E_\gamma/\text{TeV})^{-\Gamma}$ with $F_0 = 1.3 \times 10^{-13}$ cm^{-2} s^{-1} TeV^{-1} and $\Gamma = 2.4$. The nuclear region of NGC 253 is filled with a very dense gas, and this suggests that the observed gamma ray emission originates from proton-proton interactions between energetic protons and the gas.

Let us now describe in a simplified way the nuclear region of NGC 253. Assume that it is a spherical region of radius $R_{nucl} \sim 300$ pc, filled with a gas of density $n_{gas} \sim 600$ cm^{-3}. Assume that energetic protons are produced at the centre of the spherical region at a rate $Q_p(E_p)$ [number of particles per unit time per unit energy]. They then diffuse in the turbulent magnetic field and eventually escape the region. The nuclear regions of starburst galaxies are very turbulent, and therefore the spatial diffusion coefficient of energetic particles is expected to be much smaller than that in the Milky Way. Here, we will consider a diffusion coefficient *along* magnetic field lines equal to $D = 10^{28}(E_p/\text{TeV})^{0.3}$ cm^2/s. The setup of the problem is illustrated in Figure 2.

1. Compute the energy loss time τ_{pp} of cosmic ray protons due to proton-proton interactions suffered in the nuclear region. Express this time scale in years. [Hint: remember that the cross section for proton-proton interactions is roughly independent on particle energy and is equal to $\sigma_{pp} \sim 4 \times 10^{-26}$ cm^2. During each interaction, a cosmic ray proton loses a fraction $\kappa \sim 0.5$ of its energy (κ is called *inelasticity*). Cosmic rays move at the speed of light $c = 3 \times 10^{10}$ cm/s. 1 yr $\sim 3 \times 10^7$ s.]

2. Constrain the diffusive escape time τ_{esc} of energetic protons from the nuclear region. [Hint: remember that D is the diffusion coefficient *along* magnetic field lines. Since the topology of the magnetic field lines is unknown only a constrain can be obtained for τ_{esc}. 1 pc $= 3 \times 10^{18}$ cm.]

3. The spectrum of the energetic protons responsible for the observed gamma ray emission is connected to the observed gamma ray spectrum by the well known equation:

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p)E_p^2 \qquad \text{where} \qquad \eta_\pi = 1 - \exp\left[-\left(\tau_{esc}/\tau_{pp}\right)\right]$$

   where $Q_\gamma(E_\gamma)$ is the rate at which gamma-ray photons of energy E_γ are emitted from the nuclear region and η_π is the fraction of the cosmic ray power converted into pions. Show that $Q_p(E_p) \propto E_p^{-\alpha}$ and find the value of α. [Hint: can you simplify the expression for η_π?].

4. Compute the total power W_p injected in the nuclear region in form of energetic protons. Assume that the spectrum of protons extends as a single power law down to energies of $E_p \sim 1$ GeV. [Hint: 1 eV $= 1.6 \times 10^{-12}$ erg].

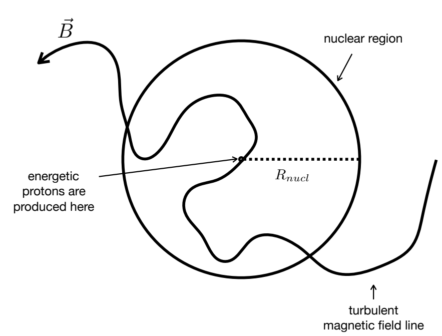

Figure 2: Setup of the problem for Exercise 2.

3 Radiation-pressure dominated bubbles

Consider a star cluster (a group of stars concentrated in a very small region of space) emitting energy in form of photons at a constant rate L. L is a luminosity and has the units of an energy per unit time. We assume the star cluster to be point like, located at $R = 0$, and surrounded by a gas of uniform mass density ϱ. Under certain circumstances, the radiation pressure exerted by the photons onto the surrounding gas can create a cavity around the cluster. This is because the matter is pushed away from the star cluster and accumulates in a very thin spherical shell of radius $R = R_s$. The shell expands supersonically and, as in the case of supernova remnants, is bounded by a spherical shock wave. The expansion velocity of the shock is u_s.

If radiation pressure is the dominant force acting on the shell, a scale free solution of the problem can be found: $R_s \sim At^{\alpha}$ and $u_s = \alpha R_s/t$. Here, t is the time since the star cluster began to emit photons. We will assume that the shell radiates photons (i.e. the energy of the system is not conserved), that the density in the cavity is so low that photons are not absorbed there, and that the shell is optically thick (i.e. all the photons emitted by the stars are absorbed in the shell). Remember that if a photon flux F (energy per unit surface per unit time) is absorbed by a gas, the momentum transferred to the gas per unit surface and unit time is F/c, where c is the speed of light. Finally, assume that the expanding spherical shock is strong (i.e. you can neglect the pressure of the gas surrounding the star cluster).

1. Find the rate $\dot{\mu}$ at which momentum is transferred to the entire shell, and show that such quantity does not depend on time.

2. Estimate A and α.

3. Call E_k the kinetic energy of the shell, and E_{rad} the total energy radiated by the stars since $t = 0$. Show that, while E_k increases with time, the ratio E_k/E_{rad} decreases and tends to zero for late times.

4. Estimate A and α considering a situation where the density of the plasma around the star cluster is not uniform, but scales as $\varrho = \varrho_0 R^{-2}$, where R is the distance from the cluster.