NPAC Astroparticles Exam 2021

1 Gamma-ray emission from interstellar clouds

Interstellar clouds are dense and massive clouds of interstellar gas. They are found in the Galactic disk, and are characterised by masses in a range spanning from few tens up to millions of Solar masses. Smaller clouds tend to be denser than larger ones, according to the (very rough) scaling:

$$n_H \approx 100 \left(\frac{M_{cl}}{10^6 \ M_\odot} \right)^{-1/2} \ \text{cm}^{-3} \tag{1}$$

where n_H is the hydrogen volume density and M_{cl} the mass of the cloud. $M_\odot = 2 \times 10^{33}$ g is the mass of the Sun. Interstellar clouds are constantly bombarded by the flux of Galactic cosmic rays, and therefore they are expected to be gamma-ray sources, as a result of the decay of neutral pions produced in proton-proton interactions. The intensity of Galactic cosmic ray protons is:

$$j_{CR}(E_p) \approx 10^{-5} \left(\frac{E_p}{\text{TeV}} \right)^{-2.7} \ \text{TeV}^{-1}\text{s}^{-1}\text{cm}^{-2}\text{sr}^{-1} \tag{2}$$

where E_p is the proton kinetic energy.

Are interstellar clouds detectable by Cherenkov telescopes of current generation? Follow the steps listed below to answer this question.

1. First of all, we should check that the intensity of CRs is given by Equation 2 above also *inside* interstellar clouds. Therefore we should answer the question: can Galactic cosmic rays penetrate interstellar clouds? To answer this question:

   (a) Estimate the radius of an interstellar cloud R_{cl} as a function of its mass. Express such radius in parsecs. [Hint: assume that the cloud is spherical. The mass of the proton is $\sim 2 \times 10^{-24}$ g. 1 pc = 3×10^{18} cm.]

   (b) Estimate the proton-proton energy loss time for cosmic rays inside interstellar clouds. Express this timescale in years. [Hint: the cross section for proton-proton interactions is $\sigma_{pp} \sim 4 \times 10^{-26}$ cm^2. At each interaction, a cosmic ray proton loses a fraction $\kappa \sim 0.5$ of its energy (κ is called inelasticity). Cosmic rays move at the speed of light $c = 3 \times 10^{10}$ cm/s. 1 yr $\sim 3 \times 10^7$ s.]

   (c) Assume that the interstellar cloud is embedded in an ambient magnetic field oriented along the Galactic disk (see Fig. 1). How long it takes for a cosmic ray to cross the cloud? Assume that cosmic rays diffuse along magnetic field lines, and that their motion is described by a typical diffusion coefficient $D \sim 10^{28} (E/\text{GeV})^{0.5}$ cm^2/s. Express this diffusive crossing time in years and compare it with the proton-proton energy loss time computed above. Comment the result.

2. Estimate the total angular extension (diameter, in degrees) of an interstellar cloud of mass M_{cl} located at a distance d from the observer. Compare it with the angular resolution (point spread function) $\vartheta_{res} \sim 0.1°$ of a Cherenkov telescope of current generation. Express d in kpc. Comment the result.

3. Estimate the gamma-ray flux above a photon energy of 1 TeV $\phi_\gamma(> 1 \text{ TeV})$ (in units cm^{-2} s^{-1}) for an interstellar cloud of mass M_{cl} located at a distance d from the observer. Show that such flux is proportional to M_{cl}/d^2. [Hint: we saw in the class that if $Q_p(E_p)$ cosmic ray protons of energy E_p are injected each second within a given region, then the expected gamma-ray luminosity $Q_\gamma(E_\gamma)$ from proton-proton interactions from that region is:

   $$Q_\gamma(E_\gamma) E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p) E_p^2$$

   where E_γ is the energy of the emitted gamma-ray photons and η_π is the fraction of the cosmic ray power converted into pions:

   $$\eta_\pi = 1 - \exp\left[- \left(\frac{\tau_{res}}{\tau_{pp}} \right) \right] .$$

   where τ_{res} is the residence time in the region and τ_{pp} the energy loss time due to proton-proton interactions. How should you modify the expressions above in order to make them useful to solve this Exercise?]

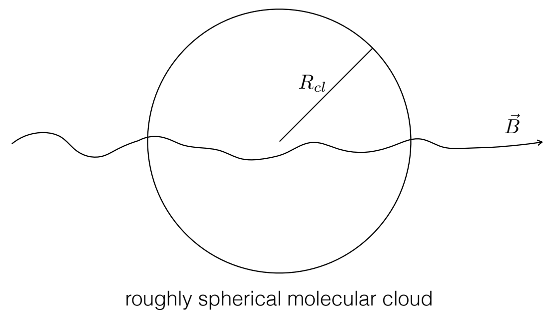

Figure 1: A roughly spherical interstellar cloud embedded in the interstellar magnetic field (see Exercise 1).

4. For which values of mass and distance are interstellar clouds detectable by Cherenkov telescopes of current generation? [Hint: remember that a *point source* is detectable by a Cherenkov telescope of current generation if its flux is larger than $\phi^P_{min}(> 1 \text{ TeV}) \approx 10^{-13}$ cm^{-2} s^{-1}. Which is the minimum flux for extended sources?]

2 Two shocks

Consider a plane, infinite, and non-radiative shock moving from left to right along the z axis with a constant velocity v_1. The current position of the shock is z_1, and the gas upstream of the shock ($z > z_1$, zone A in Fig. 2) is uniform and at rest. Consider now a second plane, infinite, and non-radiative shock located at $z_2 < z_1$, which is also moving from left to right with a constant velocity v_2. Will the second shock eventually catch the first one?

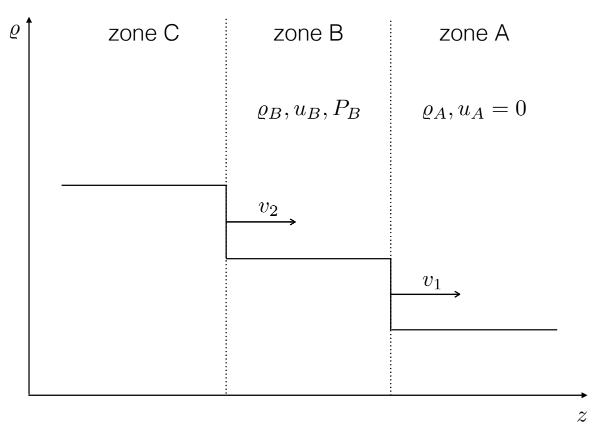

Figure 2: Setup of the problem for Exercise 2.

[Hint: there is a very simple argument that involves no calculations that can be used to answer this question. In alternative, you can do the following:

1. For simplicity, consider that the shock moving at a velocity v_1 is strong (its Mach number is $\mathcal{M}_1 \gg 1$).

2. Remember that the jump conditions for a strong shock are:

$$\frac{\varrho_d}{\varrho_u} = \frac{u_u}{u_d} = 4 \tag{3}$$

$$P_d = \frac{3}{4}\varrho_u u_u^2 \tag{4}$$

where ϱ_i and u_i are the gas density and velocity of the fluid upstream ($i = u$) and downstream ($i = d$) of the shock, respectively, and P_d is the gas pressure downstream of the shock. Such conditions are valid *in the rest frame of the shock* (where the shock is at rest). A shock must have a Mach number $\mathcal{M} = u_u/c_{s,u} > 1$, where $c_{s,u}$ is the sound speed in the upstream fluid.

3. Apply the jump conditions to the first shock, and compute the velocity that the second shock should have in order to be a shock (Mach number larger than 1).]

3 The implosion of a bubble

In the class, we studied a number of explosive phenomena. In this exercise we will study the opposite phenomenon: an implosion.

Consider an infinite and non-radiative plasma of uniform density ϱ_0 and pressure P_0. The plasma contains a spherical cavity (a bubble) of initial radius R_0. At some initial time the plasma outside of the bubble is at rest. The pressure inside the bubble is equal to 0 (the cavity is empty!), while outside of the bubble the pressure is finite (equal to P_0). Summarising: the initial density and pressure of the plasma are equal to ϱ_0 and P_0 for any $r > R_0$. On the other hand, for $r < R_0$ both density and pressure are equal to 0. Here, we have chosen the origin of the spherical coordinate system to be the centre of the bubble Due to this difference of pressure, the bubble will collapse.

The goal of this exercise is to study the spherical collapse, and determine the radius of the bubble as a function of time, $R(t)$, as well as its velocity $\dot{R}(t) = \mathrm{d}R/\mathrm{d}t$. We will chose the time coordinate so that time $t = 0$ corresponds to the end of the implosion, when $R = 0$ and the bubble disappears. Finally, we assume that the plasma in *incompressible*, i.e. the density is equal to ϱ_0 at any time and any position outside of the bubble ($r > R$). Following the steps below, you will show that as time approaches $t = 0$ the solution becomes scale free: $R = A(-t)^\alpha$. We used $-t$ instead of t in the expression because the collapse happens for times $t < 0$.

1. Use dimensional analysis to show that the problem possesses a characteristic time scale τ_0.

2. How much energy E_{tot} had to be used to create the bubble? [Hint: to create the bubble a certain volume of space had to be emptied. Therefore, the energy was used to perform a "$P\mathrm{d}V$" work.]

3. Notice that E_{tot} is a conserved quantity. It represents all the energy that has been added to the system in order to create the bubble. Therefore it will also represent the total energy of the system during the collapse. At this stage, are you allowed to find A and α using dimensional analysis? After replying, follow the next steps to find a formal derivation of these two parameters.

4. At the initial time $t = t_0$ the kinetic energy E_k of the system is 0 (the plasma is at rest) and the total energy is purely in the form of a potential energy, $E_{tot} = E_p$ (you computed such potential energy in point 2 above). During the collapse the potential energy will be gradually converted into kinetic energy as the plasma collapses. At $t = 0$ the energy will be entirely kinetic, $E_{tot} = E_k$. Can you compute the expression of the kinetic energy E_k as a function of time? [Hint: instead of finding $E_k(t)$ it is more convenient to find $E_k(R(t), \dot{R}(t))$. This can be done following these steps:]

   (a) Use the continuity equation to find an expression for the plasma velocity u for $r > R$. The expression for u will contain also R and $\dot{R}$. The continuity equation in spherical coordinates is:

$$\frac{\partial \varrho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \varrho u\right) = 0 \tag{5}$$

   Remember that the plasma density $\varrho = \varrho_0$ is a constant (both in time and space, outside of the bubble). This simplifies a lot the equation.

(b) Once you have the expression of u find the expression of the kinetic energy of the system [Hint: this involves an integration on the entire space outside of the bubble].

5. Now that you have computed E_{tot} in point 1 above, and $E_{kin}(t)$ you need to find an expression for the potential energy $E_p(t)$. Once you have found it, use the equality $E_{tot} = E_k(t) + E_p(t)$ to obtain the expression:

$$\dot{R}^2 = \frac{2P_0}{3\varrho_0}\left[\left(\frac{R_0}{R}\right)^3 - 1\right] \tag{6}$$

6. Show that $R = A(-t)^\alpha$ is *not* a solution of the equation above. However, is there a limit you can consider where this is the case? In such a limit, find A and α. Are these numbers familiar to you?