NPAC Astroparticles Exam 2018: solutions

1 Shock waves in magnetised plasmas

1. Let's call B_1 and B_2 the strength of the magnetic field upstream and downstream of the shock, respectively. **Parallel shock**: the gas is compressed by a factor of 4 *along* magnetic field lines. A surface of area a^2 parallel to the shock surface and moving with the fluid is not affected by the passage of the shock. So, for the theorem of magnetic flux freezing we have $B_1 a^2 = B_2 a^2 \longrightarrow B_1 = B_2$. **Perpendicular shock**: in this case the compression of the gas is *across* magnetic field lines. A surface of area a^2 orthogonal to the shock surface and moving with the fluid is reduced to $a^2/4$ after the passage of the shock. So, flux freezing gives: $B_1 a^2 = B_2 a^2/4 \longrightarrow B_2 = 4B_1$.

2. The magnetic field upstream of the shock has a component parallel to the shock normal equal to $B_1 \cos\vartheta_1$, and a component orthogonal to it equal to $B_2 \sin\vartheta_1$. The parallel component is not affected by the shock passage, while the perpendicular one increases by a factor of 4:

$$B_2 \cos\vartheta_2 = B_1 \cos\vartheta_1 \quad (1)$$

$$B_2 \sin\vartheta_2 = 4B1 \sin\vartheta_1 \quad (2)$$

From these expressions one can easily see that:

$$B_2 = \sqrt{(B_2\cos\vartheta_2)^2 + (B_2\sin\vartheta_2)^2} = B_1\sqrt{1 + 15\cos^2\vartheta_1} \quad (3)$$

$$\tan\vartheta_2 = 4\tan\vartheta_1 \quad (4)$$

2 High energy cosmic ray electrons

1. The energy density of the magnetic field is:

$$\omega_B = \frac{B^2}{8\pi} \sim 4\times10^{-13}\mathrm{erg/cm^3} \sim 0.2\ \mathrm{eV/cm^3} \quad (5)$$

which gives an energy loss time:

$$\tau_L \sim \frac{E}{\frac{\mathrm{d}E}{\mathrm{d}t}}|_{E=20\ \mathrm{TeV}} \sim 3\times10^4\mathrm{yr} \quad (6)$$

2. In a time τ_L electrons diffuse a length l_d along magnetic field lines:

$$l_d \sim \sqrt{D(20\ \mathrm{TeV})\tau_L} \sim 10^2\mathrm{pc} \quad (7)$$

3. We don't expect to receive electrons if the age of the source t_a is larger than τ_L, so we must impose $t_a < \tau_L$. The maximum distance that cosmic ray electrons can travel *along magnetic field lines* is thus $l_d \sim \sqrt{D(20\ \mathrm{TeV})t_a} < \sqrt{D(20\ \mathrm{TeV})\tau_L}$. The actual (geometrical) distance of the source will be smaller than l_d (magnetic field lines may be curved). The fraction of the Galaxy from which we can receive electrons is:

$$\phi \ll \left(\frac{l_d^2}{R_{MW}^2}\right) \sim 10^{-4} \quad (8)$$

where R_{MW} is the radius of the Galactic disk. The inequality $\ll$ holds because nearby sources may be magnetically disconnected from us (i.e. magnetic field lines leaving the source never reach the Earth).

3 The gamma-ray emission from the Galactic centre clouds

1. The gas density of the cloud is:

$$n_{cl} = \frac{M_{cl}}{\pi R_{cl}^2 h_{cl} m_p} \sim 3\times10^2\mathrm{cm^{-3}} \gg n_{ISM} \quad (9)$$

2. The energy loss time due to proton-proton interactions is:

$$\tau_{pp} = (\sigma_{pp} c n_{cl} \kappa)^{-1} \sim 2 \times 10^5 \text{yr} \tag{10}$$

3. The typical residence time in the cloud is:

$$\tau_{res}(E) \sim \frac{R_{cl}^2}{D(E)} \sim 4 \times 10^4 \left(\frac{E}{3 \text{ TeV}} \right)^{-0.3} \text{yr} \ . \tag{11}$$

Protons of energy $E > 3$ TeV are responsible for the gamma ray emission of photons of energy $E_\gamma > 0.3$ TeV, which implies that $\tau_{res}(E_p) \ll \tau_{pp}$ for all relevant energies. Cosmic ray protons leave the cloud without losing energy.

4. Since $\tau_{res}(E) \ll \tau_{pp}$ we can use the approximation: $\eta_\pi \sim \tau_{res}/\tau_{pp}$. Thus, we can write:

$$Q_p(E_p) \propto Q_\gamma(E_\gamma) \left(\frac{E_\gamma}{E_p} \right)^2 \tau_{res}(E_p)^{-1} \propto E_p^{-2} \tag{12}$$

5. The condition $\tau_{pp} \gg \tau_{res}(E_p) \gg t_a$ implies that all the cosmic ray protons injected by the impulsive sources are still within the cloud. In this case the appropriate expression to compute the gamma ray emission is:

$$Q_\gamma(E_\gamma) E_\gamma^2 = \frac{1}{3\tau_{pp}} N_p(E_p) E_p^2 \longrightarrow N_p(E_p) \propto E_p^{-2.3} \ . \tag{13}$$

The spectrum of cosmic rays can be derived from gamma-ray observations:

$$N_p(E_p) E_p = 3 \ \tau_{pp} Q_\gamma(E_\gamma) \frac{E_\gamma^2}{E_p} = 12\pi \tau_{pp} d^2 F_\gamma(E_\gamma) \frac{E_\gamma^2}{E_p} = 12\pi \tau_{pp} d^2 F_0 10^{0.3} \left(\frac{E_p}{E_0} \right)^{-1.3} E_0 \tag{14}$$

where d is the distance to the Galactic centre and we defined F_0 and $E_0 = 1$ TeV such as $F_\gamma(E_\gamma) = F_0(E_\gamma/E_0)^{-2.3}$. The total energy in form of cosmic rays is then:

$$W_{CR}(> 1 \text{ GeV}) = \int_{1 \text{ GeV}}^{\infty} \text{d}E_p E_p N_p(E_p) = 12\pi \tau_{pp} d^2 F_0 10^{0.3} E_0^2 \int_{1 \text{ GeV}}^{\infty} \frac{\text{d}E_p}{E_0} \left(\frac{E_p}{E_0} \right)^{-1.3} \sim 5 \times 10^{49} \text{erg} \tag{15}$$

which is about 5% of the typical explosion energy of a supernova.

4 Detonation waves

1. The time-independent physical quantities involved in the problem are ϵ and ϱ. Using dimensional analysis we can write:

$$[R_s] = [\epsilon]^\alpha \, [\varrho]^\beta \, [t]^\gamma \tag{16}$$

where $[x]$ indicates the dimensions of x. The solution is then $R_s \sim \epsilon^{1/2} t$. Note that this does not depend on the ambient density!

2. At any given time, the energy of the system is given by:

$$E = E_0 + \epsilon \varrho \frac{4\pi}{3} R_s^3 \tag{17}$$

which for $E \gg E_0$ (and neglecting numerical factors of order unity) reduces to $E \propto \epsilon \varrho R_s^3$. By using the well known scaling $R_s \sim (E/\varrho)^{1/5} t^{2/5}$ one can recover the solution obtained in point 1 above.

3. Also in this case the solution is $R_s \propto t$ because the dependence on the density cancels out. This can be seen by repeating what was done in point 2 above.