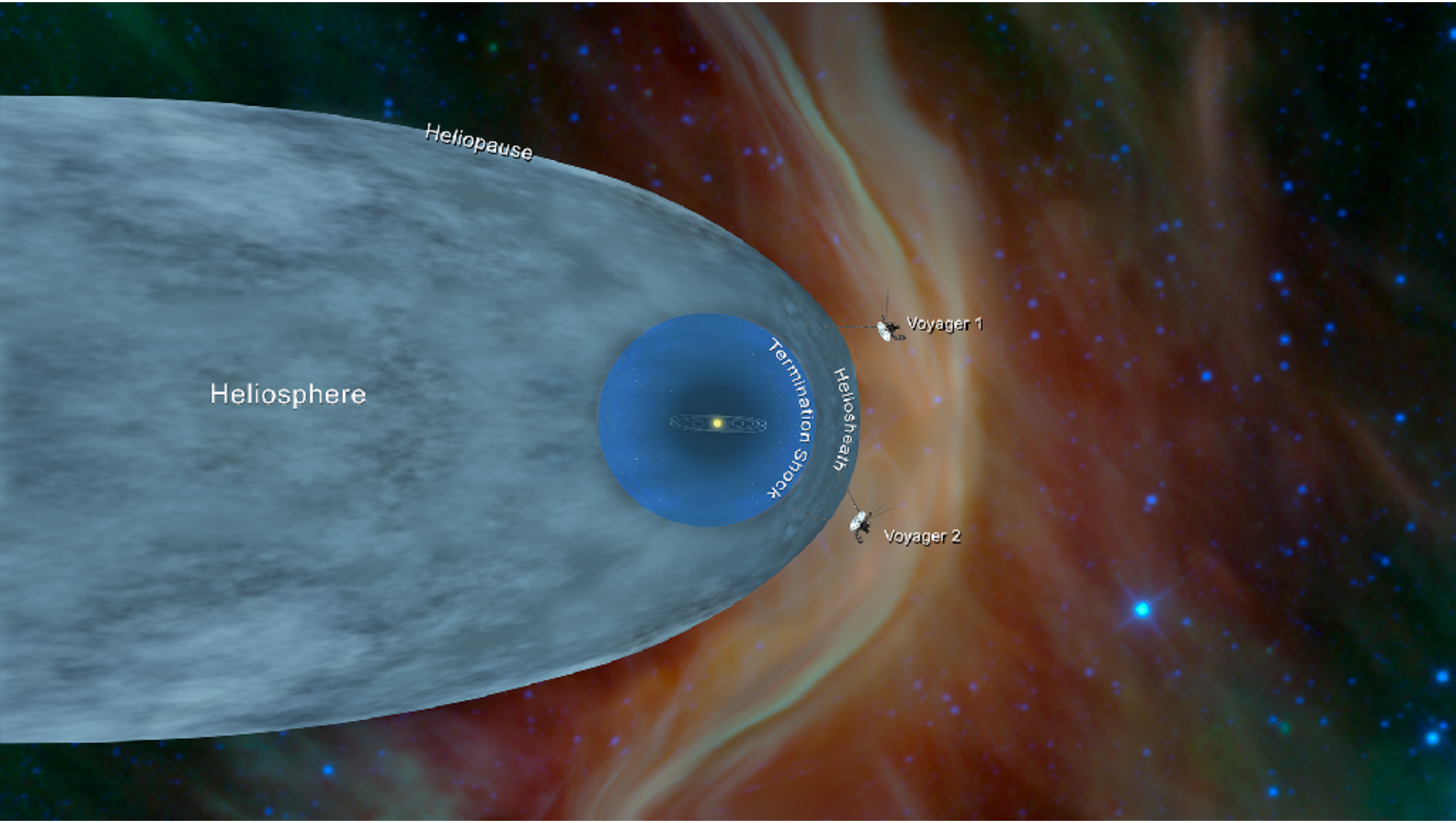
NPAC course on Astroparticles

Exercise #1 — Solution

Voyager 1 and 2: measuring the interstellar spectrum of cosmic rays!



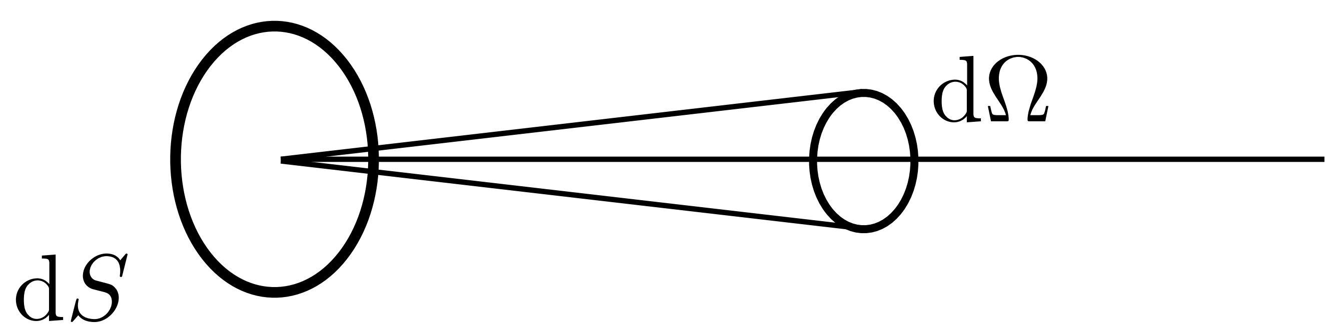
Cosmic ray <u>intensity</u> measured by Voyager 1 and 2 in the local interstellar medium

$$
\begin{aligned}
j(E) &= A\left(\frac{E}{E_0}\right)^{0.1} & E < E_0 \sim 1\ \mathrm{GeV} \\
&= A\left(\frac{E}{E_0}\right)^{-2.7} & E > E_0
\end{aligned}
$$

Intensity is what is measured by instruments

$$[j] = [A] = \mathrm{eV}^{-1}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$$

Number of particles of a given energy (eV^{-1}) passing through a unit surface orthogonal to the arrival direction of particles (cm^{-2}) each second (s^{-1}) and per unit solid angle (sr^{-1})



It is more convenient to think in terms of the CR number density n(E)

$$[n(E)] = \mathrm{eV}^{-1}\mathrm{cm}^{-3}$$

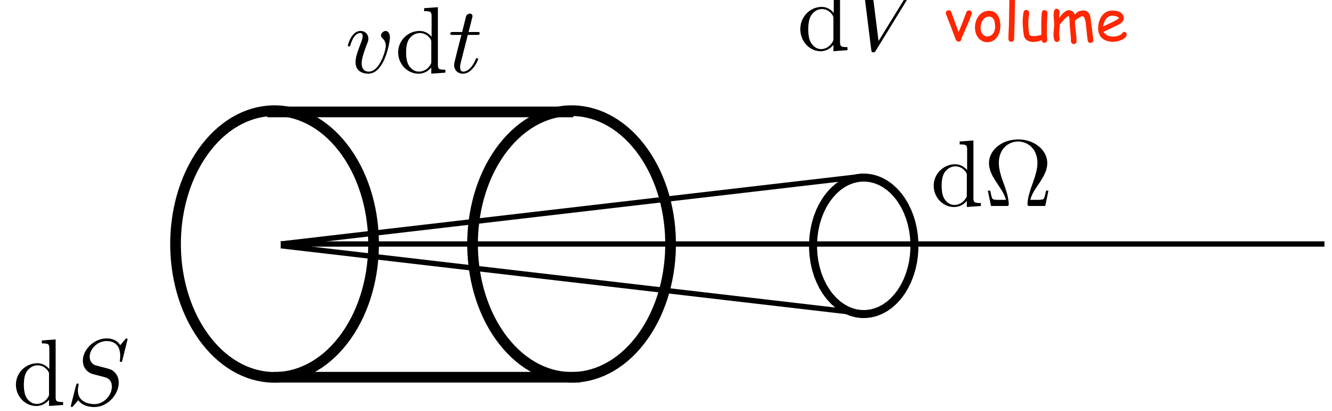
Why? Because we will need to compute the CR energy density:

$$w_{CR} = \int \mathrm{d}E\ E\ n(E) \qquad [w_{CR}] = \mathrm{eV\ cm}^{-3}$$

How do we go from j to n?

$$j = \frac{\mathrm{d}n}{\mathrm{d}E\ \mathrm{d}t\ \mathrm{d}S\ \mathrm{d}\Omega} \rightarrow \frac{j}{v} = \frac{\mathrm{d}n}{\mathrm{d}E\ \boxed{v\ \mathrm{d}t\ \mathrm{d}S}\ \mathrm{d}\Omega} = \frac{1}{4\pi}\frac{\mathrm{d}n}{\mathrm{d}E\ \mathrm{d}V} = \frac{n}{4\pi}$$

$\mathrm{d}V$ volume



$$n(E) = \frac{4\pi}{v} j(E)$$

Let us now compute the CR energy density

$$w_{CR} = \int \mathrm{d}E \; E \; n(E) = 4\pi \int \mathrm{d}E \frac{E}{v(E)} j(E)$$

Trick: split the integral $\displaystyle\int_{E_{min}}^{E_{max}} \mathrm{d}E ... = \int_{E_{min}}^{E_0} \mathrm{d}E ... + \int_{E_0}^{E_{max}} \mathrm{d}E ...$

$$E > E_0 \sim 1 \text{ GeV} \quad \rightarrow \quad E = \frac{1}{2} m v^2 \rightarrow v = \left(\frac{2E}{m} \right)^{1/2}$$

$$E < E_) \sim 1 \text{ GeV} \quad \rightarrow \quad v = c$$

$$
\begin{aligned}
j(E) &= A\left(\frac{E}{E_0}\right)^{0.1} \qquad E < E_0 \sim 1\ \mathrm{GeV} \\
&= A\left(\frac{E}{E_0}\right)^{-2.7} \qquad E > E_0
\end{aligned}
$$

$$
w_{CR} = \int \mathrm{d}E\ E\ n(E) = 4\pi \int \mathrm{d}E \frac{E}{v(E)} j(E)
$$

$$
= 4\pi A \left[\int_{E_{min}}^{E_0} \mathrm{d}E \boxed{\left(\frac{m}{2E}\right)^{1/2} E} \left(\frac{E}{E_0}\right)^{0.1} + \int_{E_0}^{E_{max}} \mathrm{d}E \frac{E}{c} \left(\frac{E}{E_0}\right)^{-2.7} \right]
$$

$$
m = \frac{E_0}{c^2} \qquad\qquad \rightarrow \frac{1}{\sqrt{2}} \left(\frac{E}{E_0}\right)^{-1/2} \frac{E}{c}
$$

$$
\begin{aligned}
j(E) &= A\left(\frac{E}{E_0}\right)^{0.1} \qquad E < E_0 \sim 1\ \mathrm{GeV} \\
&= A\left(\frac{E}{E_0}\right)^{-2.7} \qquad E > E_0
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$$
= 4\pi A \left[\int_{E_{min}}^{E_0} \mathrm{d}E \left(\frac{m}{2E}\right)^{1/2} E \left(\frac{E}{E_0}\right)^{0.1} + \int_{E_0}^{E_{max}} \mathrm{d}E \frac{E}{c} \left(\frac{E}{E_0}\right)^{-2.7} \right]
$$

$$
= \frac{4\pi A E_0^2}{c} \left[\int_{E_{min}}^{E_0} \mathrm{d}\left(\frac{E}{E_0}\right) \frac{1}{\sqrt{2}} \left(\frac{E}{E_0}\right)^{0.6} + \int_{E_0}^{E_{max}} \mathrm{d}\left(\frac{E}{E_0}\right) \left(\frac{E}{E_0}\right)^{-1.7} \right]
$$

$$
E/E_0 = x
$$

$$w_{CR} = \frac{4\pi A E_0}{c}\left[\frac{1}{\sqrt{2}}\int_{x_{min}}^{1} \mathrm{d}x\; x^{0.6} + \int_{1}^{x_{max}} \mathrm{d}x\; x^{-1.7}\right]$$

$\swarrow$ $1\ \mathrm{eV/cm^3}$

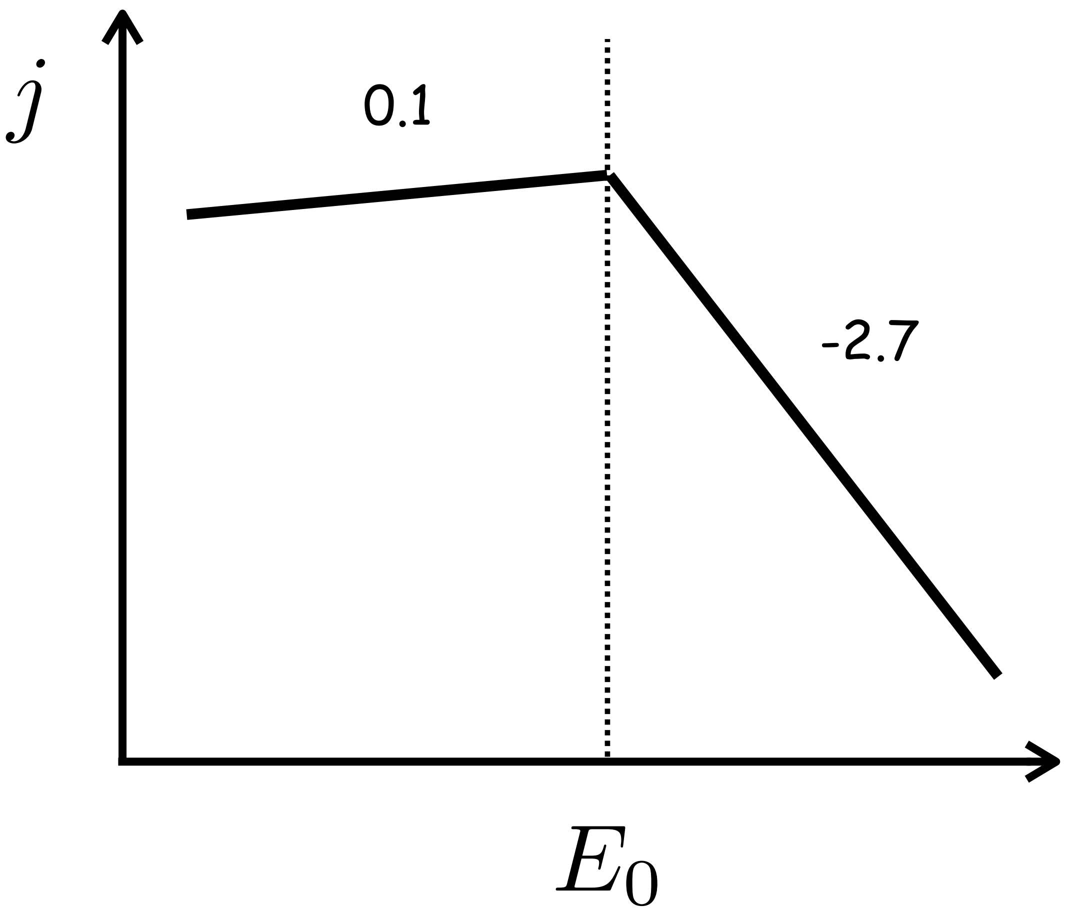
$$= \frac{4\pi A E_0}{c}\left[\frac{1}{\sqrt{2}}\frac{x^{1.6}}{1.6}\Big|_{x_{min}}^{1} - \frac{x^{-0.7}}{0.7}\Big|_{1}^{x_{max}}\right]$$

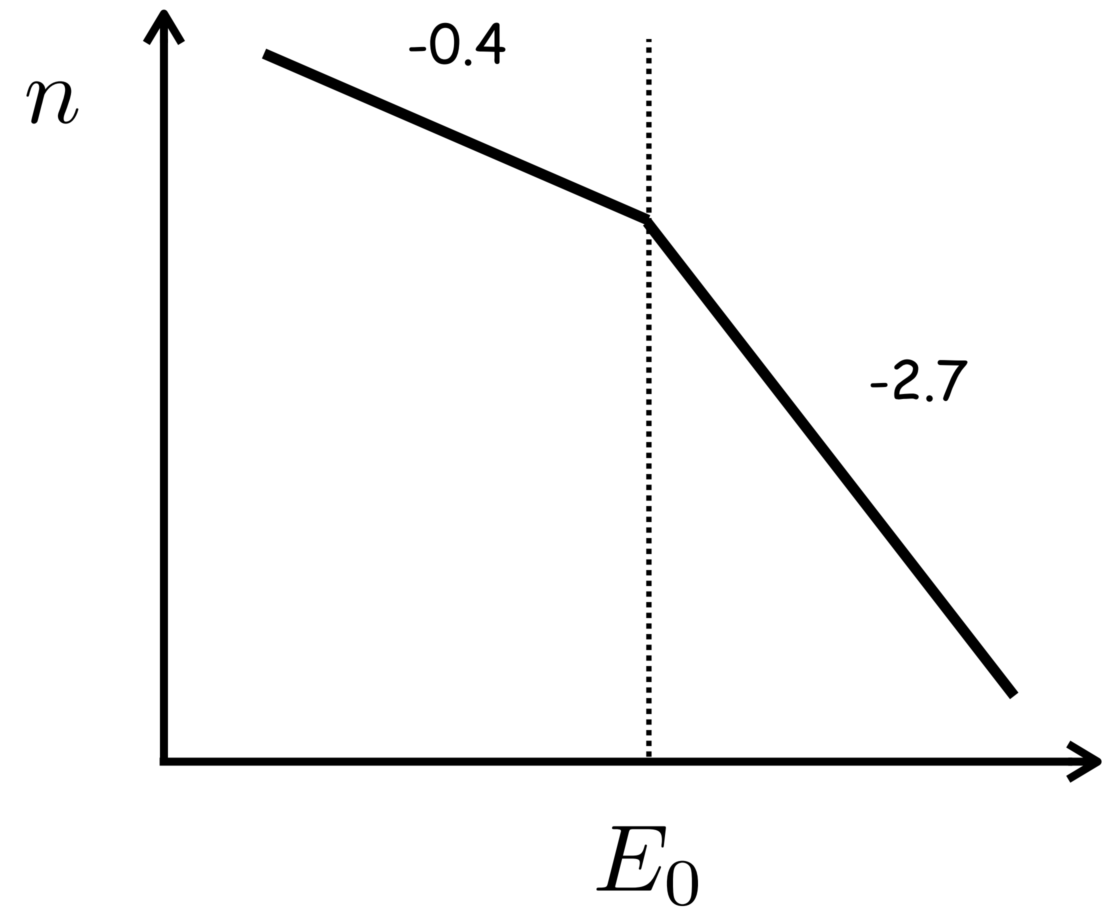
$$x_{min} \ll 1 \qquad x_{max} \gg 1$$

$$= \frac{4\pi A E_0}{c}\left[\boxed{\frac{1}{\sqrt{2}}\frac{1}{1.6}} + \boxed{\frac{1}{0.7}}\right] = 1.87\,\frac{4\pi A E_0}{c}$$

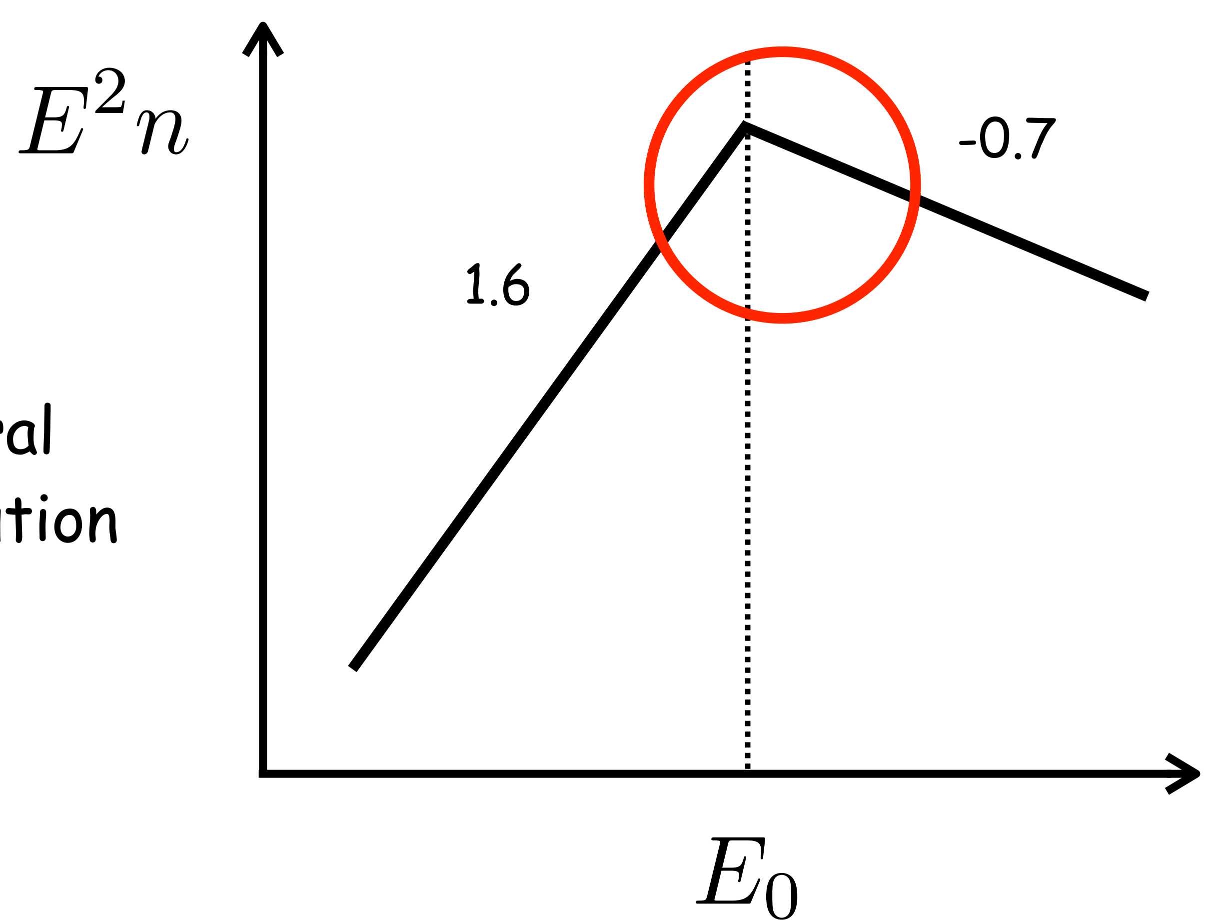
$0.44 \qquad 1.43$

$$\boxed{A = 1.3 \times 10^{-9}\mathrm{eV^{-1}cm^{-2}s^{-1}sr^{-1}}}$$





SED = Spectral
Energy Distribution



It tells you where
the energy is

The injection spectrum of CRs in the Galactic disk

Total spectrum of CRs $$[N(E)] = \mathrm{eV}^{-1}$$

$$N(E) \propto E^{-2.7}$$

Residence time in the disk $$\tau_c \propto E^{-0.3}$$

$$N(E) = Q(E)\tau_c \rightarrow Q(E) = \frac{N(E)}{\tau_c(E)} \propto E^{-2.7+0.3} = \boxed{E^{-2.4}}$$

$$[Q(E)] = \mathrm{eV}^{-1}\mathrm{s}^{-1}$$

CR power in the Galaxy $P_{CR} = 10^{41} \text{erg/s}$

$$Q(E) = A \left(\frac{E}{E_0} \right)^{-2.4}$$

$$P_{CR} = \int_{E_0} \mathrm{d}E Q(E) E = B \int_{E_0} \mathrm{d}E \left(\frac{E}{E_0} \right)^{-2.4} E$$

$$= B E_0 \int_1 \mathrm{d}x \; x^{-1.4} = \frac{B E_0^2}{0.4}$$

$$B = \frac{0.4 P_{CR}}{E_0^2} \sim 2.5 \times 10^{34} \text{eV}^{-1} \text{s}^{-1}$$