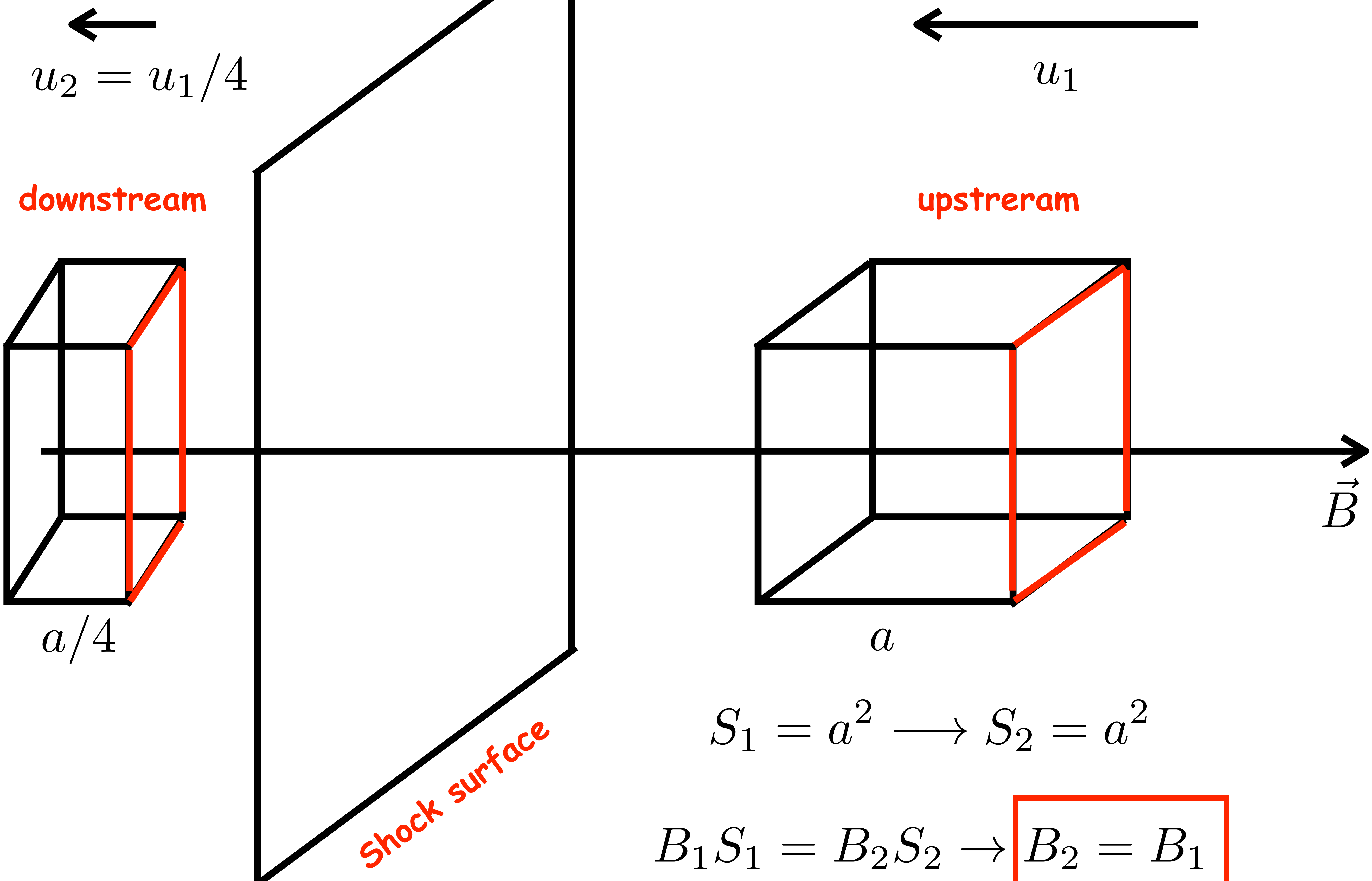
NPAC course on Astroparticles

Exercise #5 – Solution

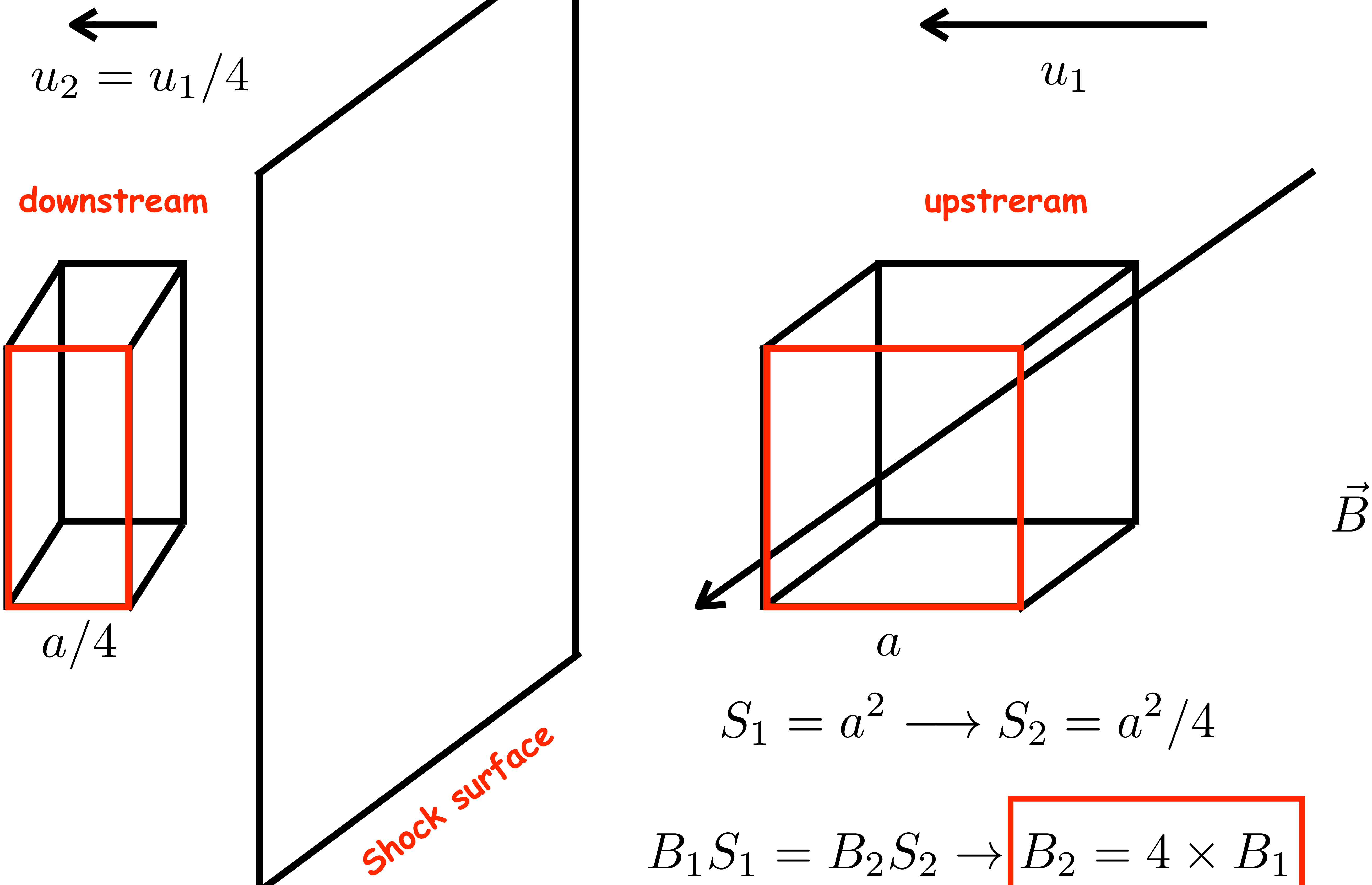
PARALLEL SHOCK



$$S_1 = a^2 \longrightarrow S_2 = a^2$$

$$B_1 S_1 = B_2 S_2 \rightarrow \boxed{B_2 = B_1}$$

PERPENDICULAR SHOCK



$$S_1 = a^2 \longrightarrow S_2 = a^2/4$$

$$B_1 S_1 = B_2 S_2 \rightarrow \boxed{B_2 = 4 \times B_1}$$

TURBULENT FIELD

Shock normal along z axis

Isotropic field —> $\langle B_{1,x}^2 \rangle = \langle B_{1,y}^2 \rangle = \langle B_{1,z}^2 \rangle$

$$\begin{aligned} B_2^x &= 4B_1^x \\ B_2^y &= 4B_1^y \\ B_2^z &= B_1^z \end{aligned}$$

$$\langle B_{2,tot}^2 \rangle = \langle B_{2,x}^2 \rangle + \langle B_{2,y}^2 \rangle + \langle B_{2,z}^2 \rangle$$

$$= 16\langle B_{1,x}^2 \rangle + 16\langle B_{1,y}^2 \rangle + \langle B_{1,z}^2 \rangle = 33\langle B_{1,z}^2 \rangle$$

$$\boxed{\sqrt{\langle B_{2,tot}^2 \rangle}} = \sqrt{33\langle B_{1,z}^2 \rangle} = \sqrt{33\frac{\langle B_{1,tot}^2 \rangle}{3}} = \boxed{\sqrt{11}\sqrt{\langle B_{1,tot}^2 \rangle}}$$

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Exercise #6 — Solution

(VERY ROUGH!)

Background rate/point source —> $$R_B^P = \phi_B \pi \left(\frac{\vartheta_{PSF}}{2}\right)^2 A_{eff}$$

Background rate/extended source —> $$R_B^E = \phi_B \pi \left(\frac{\vartheta_s}{2}\right)^2 A_{eff}$$

$$R_B^E = R_B^P \left(\frac{\vartheta_s}{\vartheta_{PSF}}\right)^2$$

For extended sources more background is collected...

To simplify things let's consider the case: $2\eta_B R_B > R_s$

$$\sigma = \frac{R_s}{\sqrt{R_s + 2\eta_B R_B}} T^{1/2}$$

$$\rightarrow \sigma \approx \frac{R_s}{\sqrt{2\eta_B R_B^E}} T^{1/2} = \frac{R_s}{\sqrt{2\eta_B R_B^P}} T^{1/2} \left(\frac{\vartheta_{PSF}}{\vartheta_s}\right)$$

For a given value of σ we get:

$$(R_s^{min})^E \approx (R_s^{min})^P \left(\frac{\vartheta_s}{\vartheta_{PSF}} \right)$$

Very brutal result, but it is often used to make order of magnitude estimates…

MINIMUM DETECTABLE LUMINOSITY

Point source: $\Phi^P_{min}(>1\ \mathrm{TeV}) \approx 10^{-13}\mathrm{cm}^{-2}\mathrm{s}^{-1}$

$$\frac{\mathrm{d}\Phi^P_{min}}{\mathrm{d}E} = \Phi_0 \left(\frac{E}{E_0}\right)^{-2.4}$$

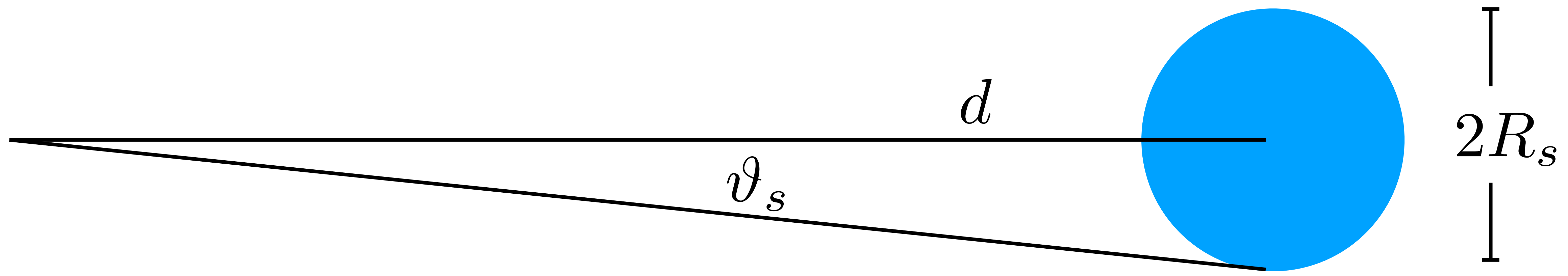
$$\Phi^P_{min}(>1\ \mathrm{TeV}) = \int_{1\ \mathrm{TeV}} \mathrm{d}E \frac{\mathrm{d}\Phi^P_{min}}{\mathrm{d}E} \longrightarrow \Phi_0$$

Flux

$$F^P_{min}(>1\ \mathrm{TeV}) = \int_{1\ \mathrm{TeV}} \mathrm{d}E \frac{\mathrm{d}\Phi^P_{min}}{\mathrm{d}E} E = 5.6 \times 10^{-13} \mathrm{erg/cm^2/s}$$

$$L^P_{min} = 4\pi d^2 F^P_{min} \rightarrow L^P_{min}(>1\ \mathrm{TeV}) = 6.7 \times 10^{31} \left(\frac{d}{\mathrm{kpc}}\right)^2 \mathrm{erg/s}$$

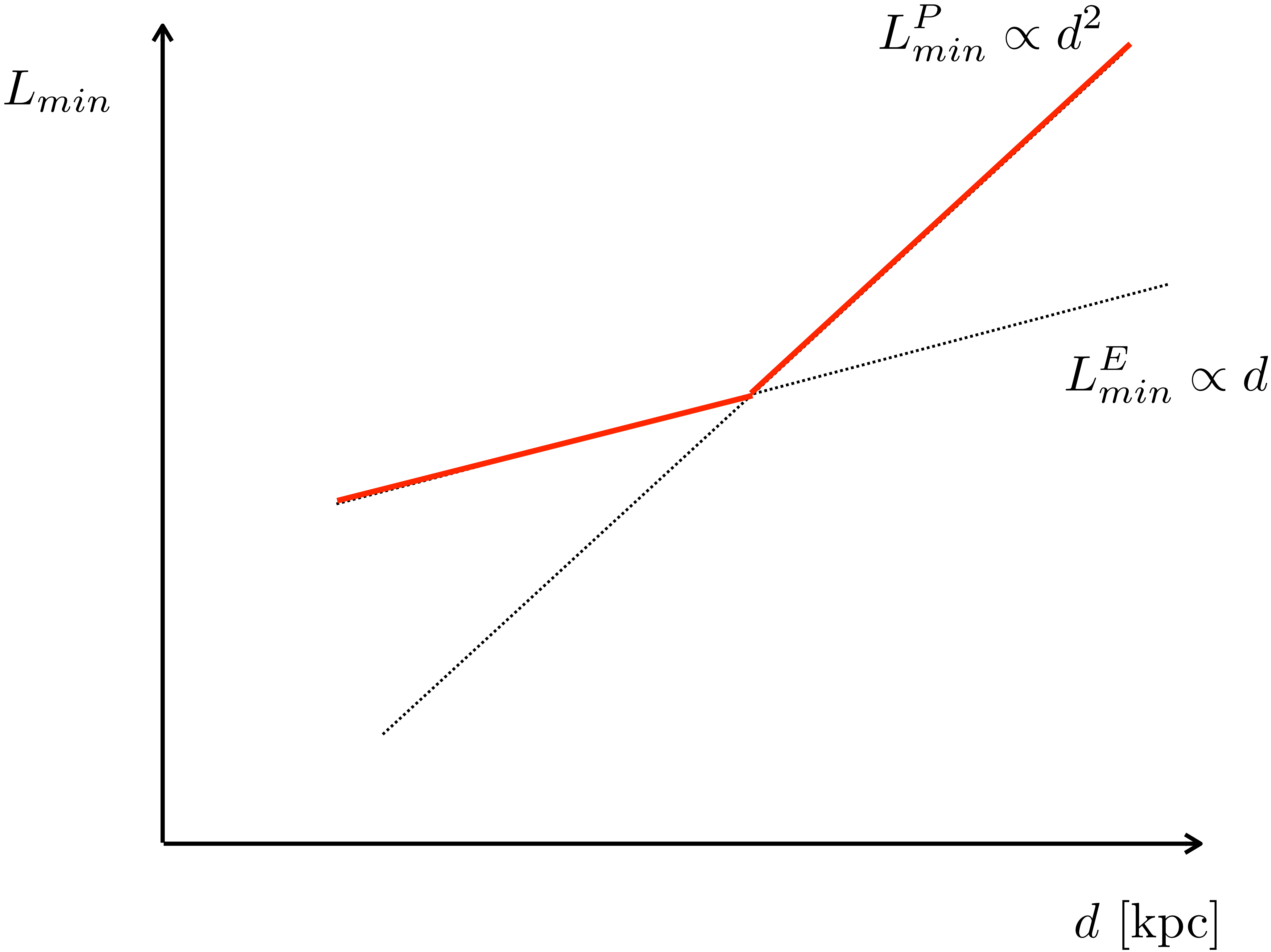
Extended source: $L^E_{min}(>1\ \mathrm{TeV}) = 6.7\times 10^{31}\left(\frac{d}{\mathrm{kpc}}\right)^2\left(\frac{\vartheta_s}{\vartheta_{PSF}}\right)\mathrm{erg/s}$



$$\vartheta_s \sim \frac{2R_s}{d} \quad > \vartheta_{PSF} \quad \rightarrow R_s > \frac{\vartheta_{PSF}}{2} d \approx 1\left(\frac{d}{\mathrm{kpc}}\right)\ \mathrm{pc}$$

$$L^E_{min}(>1\ \mathrm{TeV}) = 6.7\times 10^{31}\left(\frac{d}{\mathrm{kpc}}\right)^2 \vartheta_{PSF}^{-1}\frac{2R_s}{d}\mathrm{erg/s}$$

$$= 7.7\times 10^{31}\left(\frac{d}{\mathrm{kpc}}\right)\frac{R_s}{\mathrm{pc}}\mathrm{erg/s}$$


L_{min}
$L^{P}_{min} \propto d^2$
$L^{E}_{min} \propto d$
d [kpc]