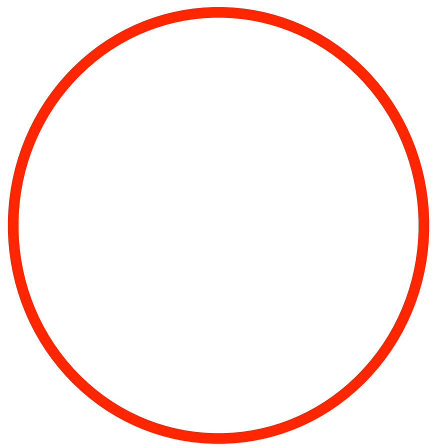
# NPAC course on Astroparticles

## Exercise #7 — Solution

# Gamma rays from SNRs

Supernova remnant



Energy in form of cosmic rays: $W_{CR} = 10^{50} \mathrm{erg}$

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p)E_p^2$$

How do we convert $Q_p$ (a rate) in $W_{CR}$ (an energy)?

Let's start by computing $\eta_\pi$

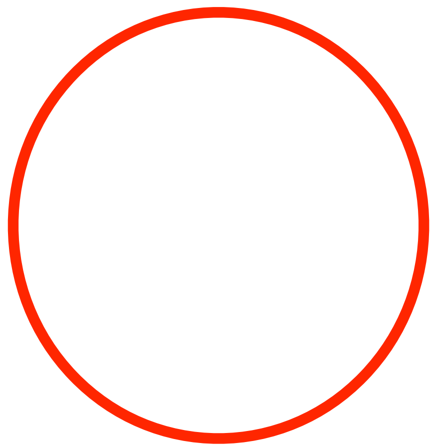
$$\eta_\pi = 1 - \exp\left(-\frac{\tau_{res}}{\tau_{pp}}\right)$$

$$\tau_{pp} \sim (n_{gas}\sigma_{pp}ck)^{-1} \sim 60 \left(\frac{n_{gas}}{\mathrm{cm}^{-3}}\right)^{-1} \mathrm{Myr} \rightarrow \tau_{pp} \sim 15 \mathrm{~Myr}$$

In the interstellar medium $n_{gas} \sim 1 \mathrm{~cm}^{-3}$ In the SNR shell (shock compression) $n_{gas} \sim 4 \mathrm{~cm}^{-3}$

# Gamma rays from SNRs

Supernova remnant



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$$\eta_\pi = 1 - \exp\left(-\frac{\tau_{res}}{\tau_{pp}}\right) \longrightarrow \frac{\tau_{res}}{\tau_{pp}}$$

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$$\tau_{res} \leq \tau_{age} \lesssim 10^4\mathrm{yr} \ll \tau_{\mathrm{pp}}$$

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\tau_{res}}{3\tau_{pp}} Q_p(E_p) E_p^2$$

$N_p(E_p)$ —> # of CR of energy $E_p$ in the SNR

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{N_p(E_p)}{3\tau_{pp}} E_p^2$$

$$N_p(E_p) = N_* \left(\frac{E_p}{E_*}\right)^{-2.4} \qquad E_* = 1 \text{ TeV}$$

$$W_{CR} = \int_{1 \text{ GeV}} \mathrm{d}E_p N_p(E_p) E_p = N_* E_*^2 \int_{10^{-3}} \mathrm{d}x x^{-1.4} = N_* E_*^2 \frac{x^{-0.4}}{0.4}|_{10^{-3}} = 10^{1.2} \frac{N_* E_*^2}{0.4}$$

why 1 GeV? (see Exercise 1)

$$N_* = \frac{0.4}{10^{1.2}} \frac{W_{CR}}{E_*^2}$$

$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{N_p(E_p)}{3\tau_{pp}}E_p^2$$

$$N_* = \frac{0.4}{10^{1.2}}\frac{W_{CR}}{E_*^2}$$

$$Q_\gamma(E_\gamma) = \frac{N_p(E_p)}{3\tau_{pp}}\frac{E_p^2}{E_\gamma^2} = \frac{100}{3}\frac{N_p(E_p)}{\tau_{pp}}$$

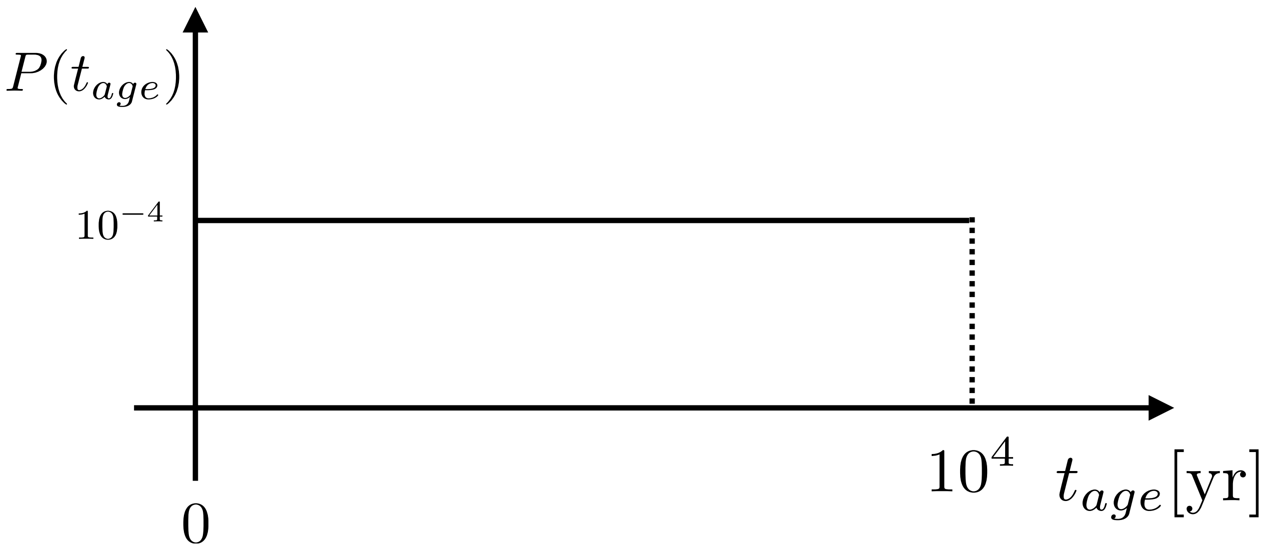
$$\int_{1\ \mathrm{TeV}} \mathrm{d}E_\gamma Q_\gamma(E_\gamma) = \frac{100}{3\tau_{pp}}\int_{1\ \mathrm{TeV}} \mathrm{d}E_\gamma N_p(E_p) = \frac{10}{3\tau_{pp}}\int_{10\ \mathrm{TeV}} \mathrm{d}E_p N_p(E_p) = \frac{10\ N_*}{3\tau_{pp}}\int_{10\ \mathrm{TeV}} \mathrm{d}E_p \left(\frac{E_p}{E_*}\right)^{-2.4}$$

$$= \frac{10\ N_* E_*}{3\tau_{pp}}\int_{10} \mathrm{d}x\ x^{-2.4} = \frac{10}{3\tau_{pp}}\frac{0.4}{10^{1.2}}\frac{W_{CR}}{E_*}\frac{10^{-1.4}}{1.4} \sim 3.2\times 10^{32}\mathrm{s}^{-1}$$

flux →

$$\phi = \frac{\int \mathrm{d}E_\gamma Q_\gamma}{4\pi d^2} = 2.6\times 10^{-12}\left(\frac{d}{\mathrm{kpc}}\right)^{-2}\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

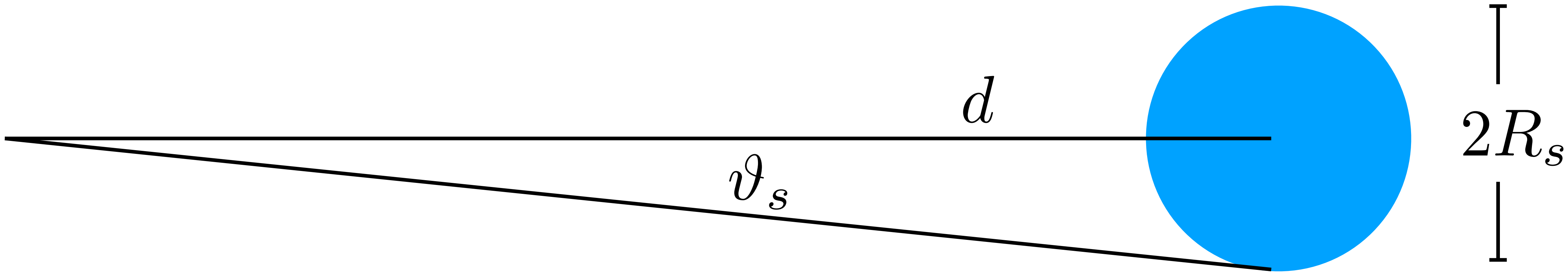
we should consider all SNR of age between 0 and 10000 years. SN explosions happen at a regular rate (3 per century). the probability distribution of SNR ages is flat. let's estimate the average age of SNRs.




$$\langle t_{age} \rangle = \int_0^{10^4 \ \mathrm{yr}} \mathrm{d}t_{age} P(t_{age}) t_{age} = 5 \times 10^3 \mathrm{yr}$$

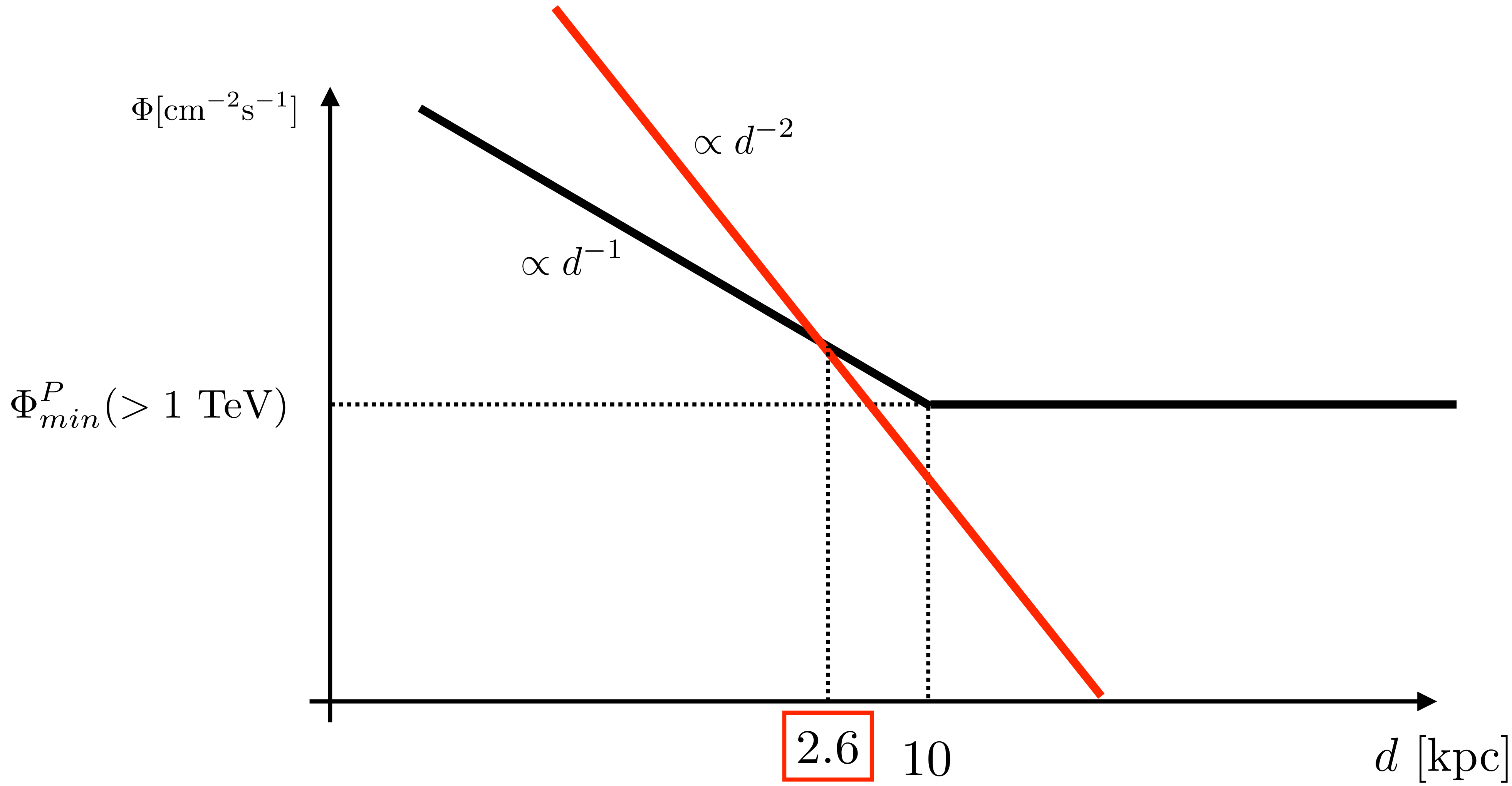
$$R_s(\langle t_{age} \rangle) \approx 10 \ \mathrm{pc}$$

angular resolution —> $\vartheta_{PSF} \approx 0.1^\circ$ up to which distance we see SNRs as extended sources?




result from exercise #6: $\rightarrow R_s > \frac{\vartheta_{PSF}}{2} d \approx 1 \left( \frac{d}{\mathrm{kpc}} \right) \ \mathrm{pc} \longrightarrow d_{ext} \sim 10 \ \mathrm{kpc}$

minimum flux detectable by Cherenkov telescopes (point source) —> $\Phi^P_{min}(>1\ \mathrm{TeV}) \approx 10^{-13}\mathrm{cm}^{-2}\mathrm{s}^{-1}$




result from exercise #6: $\Phi^E_{min}(>1\ \mathrm{TeV}) \sim \Phi^{\mathrm{P}}_{\mathrm{min}}(>1\ \mathrm{TeV}) \left(\frac{\vartheta_{\mathrm{s}}}{\vartheta_{\mathrm{PSF}}}\right) \propto d^{-1}$

flux from a SNR: $\phi = \frac{\int \mathrm{d}E_\gamma Q_\gamma}{4\pi d^2} = 2.6 \times 10^{-12} \left(\frac{d}{\mathrm{kpc}}\right)^{-2} \mathrm{cm}^{-2}\mathrm{s}^{-1} \propto d^{-2}$

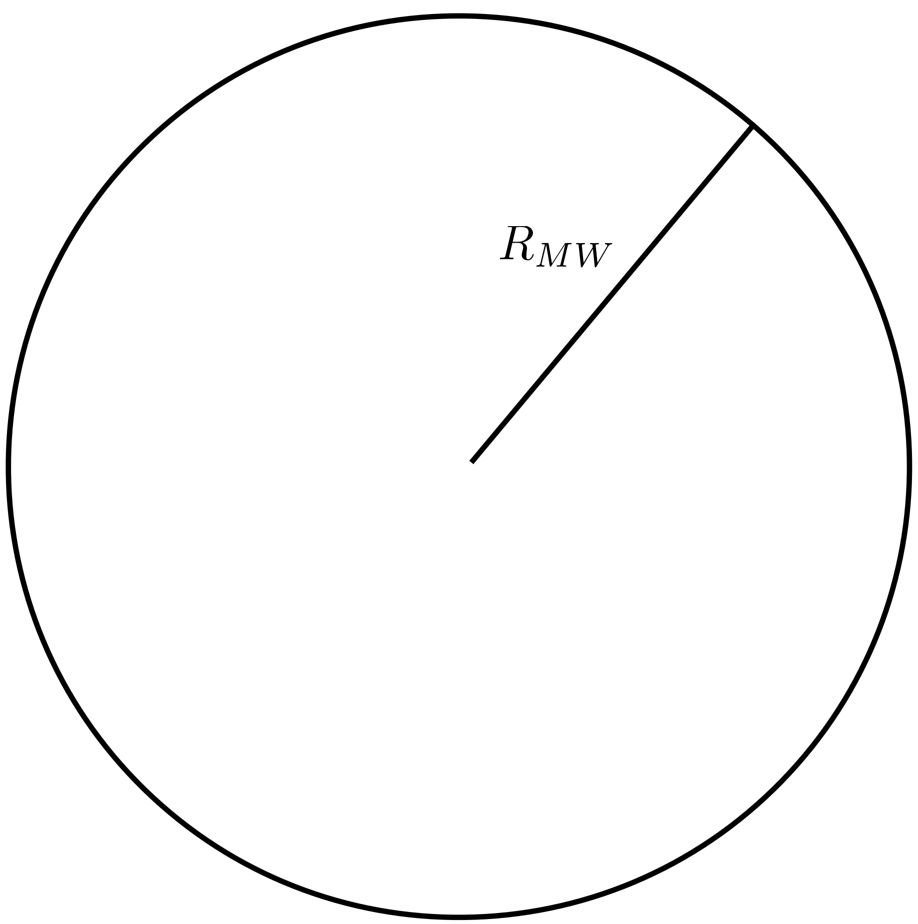
@ 10 kpc —> $\phi < \Phi^P_{min}$

how many SNR in the Galaxy?

SN rate $\nu_{SN} \sim \frac{3}{100}\ \mathrm{yr}^{-1}$

$t_{age}^{max} = 10^4 \mathrm{yr}$

$$N_{SNR} = \nu_{SN}\ t_{age}^{max} \sim 300$$




Face-on view of the Milky Way

surface density of SNR

$$n_{SNR} = \frac{N_{SN}}{\pi R_{MW}^2} \sim 0.4\ \mathrm{kpc}^{-2}$$

Typical distance between SNR

$$l_{SNR} \sim n_{SNR}^{-1/2} \sim 1.6\ \mathrm{kpc} < 2.6\ \mathrm{kpc}$$

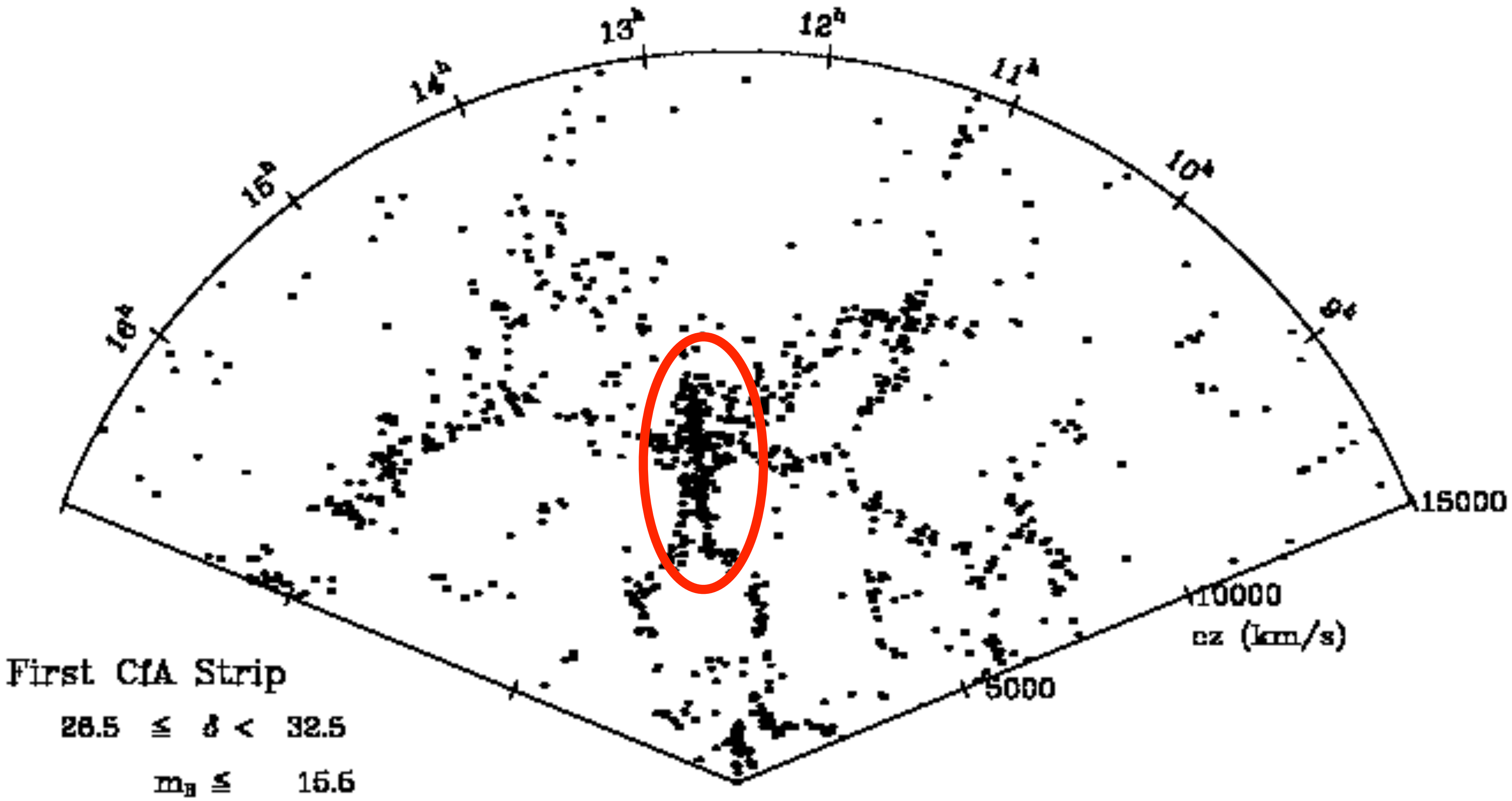
SNR are detectable! how many?

$$N_{SNR}^{\gamma} \sim n_{SNR} \pi d_{max}^2 \approx 10$$

# NPAC course on Astroparticles

# Exercise #10 — Solution

# Dark matter in the Coma cluster






$$\sigma_z \sim 1000 \text{ km/s}$$

Assuming isotropy in the velocity dispersion

$$\sigma_{tot}^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3\sigma_z^2$$

$$\sigma_{tot} = \sqrt{3}\sigma_z \sim 1700 \text{ km/s}$$

Stellar mass in Coma $M_* \sim 10^{13} M_\odot$

Escape velocity —> $$v_{esc} = \sqrt{\frac{2GM_*}{R}} \sim 300 \left(\frac{M}{10^{13} M_\odot}\right)^{1/2} \left(\frac{R}{\mathrm{Mpc}}\right)^{-1/2} \mathrm{km/s}$$

$$v_{esc} \ll \sigma_{tot}$$ <— we need more mass!

The cluster can exist only if $$v_{esc} > \sigma_{tot}$$

$$\sigma_{tot}^2 < \frac{2GM_{tot}}{R} \rightarrow M_{tot} > 3 \times 10^{14} M_\odot$$

Dark matter in galaxies? (Milky Way, dark matter = 10 times stellar mass) —> $10^{14}$ $M_{Sun}$ —> not enough!

X-ray emission from Coma $L_X \sim 10^{45} \mathrm{erg/s}$ $k_B T \sim 8\ \mathrm{keV}$

Bremsstrahlung –> $j_B \sim 1.1 \times 10^{-28} n_{gas}^2 T^{1/2} \mathrm{erg/cm^2/s/sr}$

$$L_X = 4\pi j_B V \sim 1.6 \times 10^{51} n_{gas}^2 \mathrm{erg/s} \quad \equiv 10^{45} \mathrm{erg/s}$$

$$\longrightarrow n_{gas} \sim 8 \times 10^{-4} \mathrm{cm^{-3}}$$

$$M_{gas} = n_{gas} m_p V \sim 8 \times 10^{13} M_\odot$$

Not enough!

Need for dark matter not associated to galaxies but diffuse in the entire cluster

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# Exercise #11 — Solution

# Inverse Compton scattering

Typical energy of CMB photons $\langle \epsilon \rangle = k_B T \sim 2.6 \times 10^{-4} \mathrm{eV}$

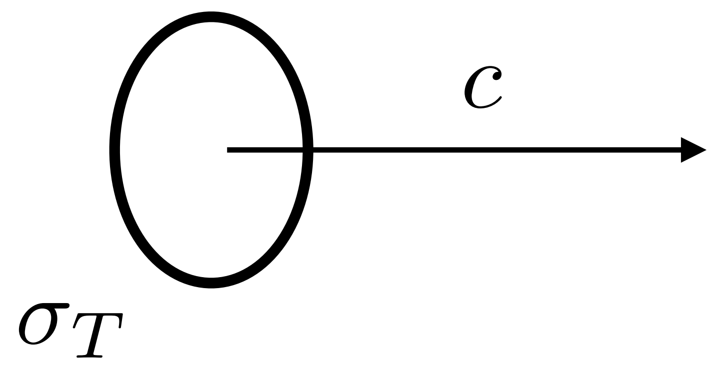
Energy of gamma ray photons $E_\gamma = \frac{4}{3} \gamma^2 \langle \epsilon \rangle = \frac{4}{3} \left( \frac{E_e}{m_e c^2} \right)^2 \langle \epsilon \rangle$

$$E_e = 1 \ \mathrm{TeV} \quad \longrightarrow \quad E_\gamma \sim 1 \ \mathrm{GeV}$$
$$E_e = 10 \ \mathrm{TeV} \quad \longrightarrow \quad E_\gamma \sim 100 \ \mathrm{GeV}$$
$$E_e = 100 \ \mathrm{TeV} \quad \longrightarrow \quad E_\gamma \sim 10 \ \mathrm{TeV}$$

Thomson regime $\gamma \langle \epsilon \rangle < m_e c^2$

$$E_e = \gamma m_e c^2 \ll \frac{(m_e c^2)^2}{\langle \epsilon \rangle} \sim 1 \ \mathrm{PeV}$$

Scattering rate




Volume swept per second $\sigma_T c$

Interaction rate $\sigma_T c n_{CMB}$ $n_{CMB} = \frac{\omega_{CMB}}{\langle \epsilon \rangle}$

Radiated power $$P_{IC} = \left( \sigma_T c \frac{\omega_{CMB}}{\langle \epsilon \rangle} \right) \left( \frac{4}{3} \gamma^2 \langle \epsilon \rangle \right) = \frac{4}{3} \sigma_T c \gamma^2 \omega_{CMB}$$

Same expression as synchrotron!

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# Exercise #12 — Solution

$$P_{IC} = \frac{4}{3}\sigma_T c\gamma^2 \omega_{CMB}$$

$$P_{syn} = \frac{4}{3}\sigma_T c\gamma^2 \omega_B \qquad \omega_B = \frac{B^2}{8\pi}$$

$$P_{IC} = P_{syn} \longrightarrow B_{CMB} = \sqrt{8\pi\omega_{CMB}} \sim 3\ \mu\mathrm{G}$$

This is the B field of the interstellar medium!
(synchrotron and inverse Compton contribute
equally to energy losses of CR electrons)

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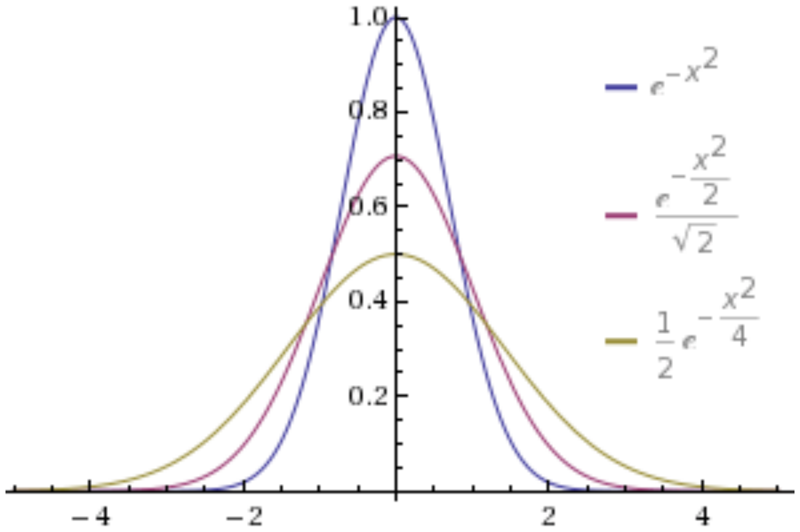
# Exercise #14 — Solution

The solution of the diffusion equation is:

$$N(t,z) = \frac{Q_0}{\sqrt{4\pi D t}} e^{-\frac{z^2}{4Dt}}$$




$$\langle z \rangle = \frac{\int_{-\infty}^{+\infty} \mathrm{d}z \; z N(t,z)}{\int_{-\infty}^{+\infty} \mathrm{d}z \; N(t,z)} = 0$$

—> symmetry

$$\langle z^2 \rangle = \frac{\int_{-\infty}^{+\infty} \mathrm{d}z \; z^2 N(t,z)}{\int_{-\infty}^{+\infty} \mathrm{d}z \; N(t,z)} = 2Dt$$

$$\sqrt{\langle z^2 \rangle} = \sqrt{2Dt}$$

that is why we use $(D\ T)^{1/2}$ as an estimate of the distance traveled by a CR in a time t!

# NPAC course on Astroparticles

# Exercise #15 — Solution

the goal of this exercise is to understand why we use the following expression to define the characteristic energy loss time of particles

$$\tau_{loss} \sim \frac{E}{P} = \frac{E}{-\frac{\mathrm{d}E}{\mathrm{d}t}}$$

power emitted by a particle due to (e.g.) synchrotron radiation

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -AE^2 \quad \longrightarrow \quad \tau_{loss} = \frac{E}{-\frac{\mathrm{d}E}{\mathrm{d}t}}\Big|_{E=E_0} = \frac{1}{AE_0}$$

$$\int_{E_0}^{E} \frac{\mathrm{d}E}{E^2} = -A \int_0^t \mathrm{d}t$$

$$\frac{1}{E_0} - \frac{1}{E} = E^{-1}\Big|_E^{E_0} = -At \quad \rightarrow \frac{E_0}{E} = 1 + AE_0 t$$

$$E = \frac{E_0}{1 + AE_0 t} = \frac{E_0}{1 + \frac{t}{\tau_{loss}}} \longrightarrow$$

$$t \ll \tau_{loss} \quad \rightarrow \quad E \sim E_0$$

$$t \gg \tau_{loss} \quad \rightarrow \quad E \propto t^{-1}$$

$$t = \tau_{loss} \quad \rightarrow \quad E = \frac{E_0}{2}$$