Exercises (5)

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16 Isotropic compression of a magnetic field

Consider a cubic volume of size L containing a plasma of density ϱ and a magnetic field of strength B directed along the x axis. The edges of the cube are parallel to the x, y, and z axis. The volume of plasma is now compressed by the same amount along all of the three axis. After the compression the size of the cube is L/a, with $a > 1$, and its volume $(L/a)^3$. Compute the variation of B as a function of the density of the plasma

[**Solution:** the compression along x leaves the field strength unchanged, while the compressions along y and z increase the field by a factor of a each. So the final value of the magnetic field strength is $a^2 B$. The final value of the plasma density is $a^3 \varrho$. Therefore one gets $B \propto \varrho^{2/3}$.]

17 Cosmic rays electrons are most likely Galactic

Cosmic ray electrons are observed at Earth up to a particle energy of about 20 TeV. Can you show with a simple physical argument that cosmic ray electrons of energy larger than few hundreds GeV are most likely of Galactic origin?

[**Solution:** the Universe is filled with the cosmic microwave background (CMB) radiation, whose energy density is 0.25 eV/cm^3. Compute the energy loss time τ_{ICS} of electrons due to inverse Compton scattering off CMB photons. Then, the largest distance an electron of energy E can travel is $\tau_{ICS}(E) \times c$ (the maximum distance is obtained assuming that it moves along a straight line). Here, c is the speed of light. Show that for energies larger than few hundreds GeV such distance is shorter than ≈ 1 Mpc. The nearest galaxy to the Milky Way is Andromeda, which is at a distance of 1 Mpc. Therefore cosmic ray electrons in this energy domain cannot reach us from extragalactic objects.]

18 Adiabatic plane shock wave: scale free solution

We have seen that the evolution in time of a spherical shock generated by a point like explosion releasing an energy E in a cold gas (i.e. pressureless) of density ϱ is:

$$R_s \sim \left(\frac{E}{\varrho}\right)^{1/5} t^{2/5} \tag{1}$$

where the assumption of energy conservation was made (adiabatic or Sedov solution).

Imagine now that the energy E is not released in a point like explosion, but rather uniformly over a very large plane surface of area A. Let's take the surface to be perpendicular to the z axis and defined by z = 0. Such an explosion would generate a pair of plane shock waves moving away from z = 0 as $|z_s| \sim at^\alpha$. Use dimensional analysis to find the values of a and α under the assumption that the system is adiabatic.

[**Solution:** In this case, besides the density ϱ, the relevant quantity to be considered is the energy released per unit area $\sigma = E/A$. After expressing the solution as $|z_s| \sim \varrho^\beta \sigma^\gamma t^\alpha$ we can use dimensional analysis to find: $|z_s| \sim (\sigma/\varrho)^{1/3} t^{2/3}$.]