NPAC course on Astroparticles

II - Particle interactions: cosmic rays, gamma rays, and neutrinos

The electroscope

- simple device used to measure the
- electric charge of objects:
- it works because of the repulsion of objects of like charge



ELECTROSCOPE

How does it work



Charging by contact



Charging by induction

The problem...



in 1785 Coulomb noted that charged electroscopes discharge spontaneously; in 1835 Faraday confirmed Coulomb's results, using a better insulation system -> it is not an instrumental problem; in 1879 Crookes noted that the discharge time changes with the pressure of the air -> the discharge is induced by the ionisation of the air in 1896 Bequerel discovers radioactivity

Radioactivity from the Earth







hypothesis: the Earth's crust contains radioactive isotopes (natural radioactivity) -> this might be the source of the ionizing radiation needed to explain the spontaneous discharge of electroscopes.

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in 1910 spends his Easter holidays in Paris, where he brings his electroscopes to measure the discharge time at the top and at the bottom of the Eiffel tower, during the day and during the night (the sun?);



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| 31 | | | | | 14,4 |



though the effect was smaller than expected, Wulf concluded that Earth's radioactivity remained the most plausible hypothesis

Pacini's (forgotten) experiment

in 1911 Pacini performed measurements on a boat off the coast of Livorno (300 m from the coast). Measurements were performed on the sea surface (8 m from sea bottom) and at 3 m of depth.

~20% drop of the ionization rate underwater

-> the ionization radiation comes from the atmosphere and NOT from the Earth!

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Which is the nature of the ionizing radiation in the atmosphere?



















Between April and August **1912** Hess performed 7 balloon flights. During the 7th flight he reached an altitude of 5200 meters.

The ionizing radiation has an extra-terrestrial origin



What are Cosmic Rays?

Cosmic rays particles hit the Earth's atmosphere at the rate of about 1000 per square meter per second. They are ionized nuclei - about 90% protons, 9% alpha particles and the rest heavy nuclei - and they are distinguished by their high energies. Most cosmic rays are relativistic, having energies comparable or somewhat greater than their masses. A very few of them have ultrarelativistic energies extending up to 10²⁰ eV (about 20 Joules), eleven order of magnitudes greater than the equivalent rest mass energy of a proton. The fundamental question of cosmic ray physics is, "Where do they come from?" and in particular, "How are they accelerated to such high energies?".

T. Gaisser "Cosmic Rays and Particle Physics"

Also electrons are present in the cosmic radiation -> $\sim 1\%$













Energy density

Cosmic Ray energy density:

 $w_{CR} \sim 1 \text{ eV cm}^{-3}$

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Magnetic field energy density: $w_B = \frac{B^2}{8\pi} \sim 1 \text{ eV cm}^{-3}$
Thermal gas energy density: $w_{gas}^{turb} = \rho_{gas} v_{turb}^2 \sim 1 \text{ eV cm}^{-3}$

Variations in time and space

CR flux at Earth constant during the last 10° yr (from radiation damages in geological and biological samples, meteorites, and lunar rocks)
thus the CR flux must be constant along the orbit of the Sun around the galactic centre (many revolutions in a Gyr)

Stability in time and (hints for) spatial homogeneity

Cosmic Ray anisotropy: δ

$$= \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

(I -> CR intensity)



figure from Iyono et al, 2005

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Hayakawa's conjecture (1952)



Cosmic Rays undergo hadronic interactions in the InterStellar Medium
-> the Galaxy should shine in gamma rays!

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y-ray observations from ground...

Cherenkov telescopes/arrays


y-ray observations from ground...



y-ray observations from ground...



...and from space





...and from space





Schematic view of the Milky Way



Schematic view of the Milky Way



Schematic view of the Milky Way



Schematic view of the Milky Way



Schematic view of the Milky Way

Are CRs universal?



Cosmic rays are homogeneously distributed in the galactic disk. Hypothesis: are they homogeneously distributed in the whole Universe?

Are CRs universal?



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We play the same game with the Small Magellanic Cloud. Total gas mass -> expected gamma rays

We observe less gammas than expected!





The electromagnetic spectrum



Which CRs are confined in the Galaxy?



It depends on the values of the magnetic field and thickness of the halo (both poorly constrained...)

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Confinement condition:



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Cosmic Ray composition



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Cosmic Ray composition: spallation

Spallation: production of light elements as fragmentation products of the interaction of high energy particles with cold matter.

The anomaly is explained if (~ GeV) CRs transverse $~\lambda~pprox~5~{
m g/cm}^2$

Cosmic Ray composition: spallation

Spallation: production of light elements as fragmentation products of the interaction of high energy particles with cold matter.

The anomaly is explained if (~ GeV) CRs transverse $\lambda \approx 5 \text{ g/cm}^2$ Assuming propagation in the galactic disk: $l_s = \frac{\lambda}{\varrho_{ISM}} \approx 1 \text{ Mpc}$ $\int_{\text{much larger than the size of the disk!!!}}$

> CRs don't go straight but are confined in the disk -> diffusive behavior -> isotropy!

CRs don't go straight: consequences

(1) We cannot doCR astronomy

-> difficult to identify sources















Is this correct?

CRs interact with the gas -> $\ p+p \rightarrow p+p+\pi^0$

Should we use this equation instead?

$$\frac{\mathrm{d}\mathcal{E}_{CR}}{\mathrm{d}t} = P_{CR} - \frac{\mathcal{E}_{CR}}{t_{disk}} - \dot{\mathcal{E}}_{pp} \underbrace{_{\text{energy loss term}}}_{\text{due to p-p}}_{\text{interactions}}$$

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Energy loss rate:

$$t_{pp} = (n_{gas} \sigma_{pp} c k)^{-1}$$

$$\int \int (1 - 26 cm^2) c k^{-1}$$

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We can safely neglect CR energy losses

Spallation measurements tell us that cosmic rays follow tortuous paths before escaping the Galaxy. Why?

The galactic magnetic field or, better, **irregularities in the Galactic magnetic field** are responsible for the diffusive propagation of cosmic rays.

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(Oversimplified picture)

magnetized cloudlets in an unmagnetized background



the particle energy is unchanged (Lorentz force)





 λ -> mean free path



 λ -> mean free path

 $\tau_c = \frac{\lambda}{c} \quad \text{-> collision time}$



 $\lambda \rightarrow$ mean free path λ

 $\tau_c = \frac{\lambda}{c} \quad \text{-> collision time}$

 $N = \frac{t}{\tau_c} \; \xrightarrow{} \; \text{\# collisions} \\ \text{after time t}$


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diffusion length ->
$$l_d = \lambda \sqrt{N}$$

random walk



diffusion length -> $l_d = \lambda \sqrt{N} = \lambda \sqrt{\frac{t}{\tau_c}} = \lambda \sqrt{\frac{t c}{\lambda}} = \sqrt{\lambda c t}$ this product determines the

diffusion properties of the particle

It is convenient to define the quantity $D = \lambda \ c$ called diffusion coefficient

diffusive propagation -> $l_d = \sqrt{D t} \propto \sqrt{t}$ straight line propagation -> $l_{sl} = c t \propto t$

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Spallation measurements allow us to measure the average diffusion coefficient in the Galaxy

$$l_{disk} = \sqrt{D \ t_{disk}} \longrightarrow D = \frac{l_{disk}^2}{t_{disk}} = 10^{28} \ \mathrm{cm}^2/\mathrm{s}$$

$$\int_{3 \text{ Myr (from spallation)}} \mathbb{O}(10 \ \text{GeV})$$

Spallation measurements at different energies -> $t_{disk} \propto E^{-0.3}$

which corresponds to -> $\,D \propto E^{0.3}$

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We can now constrain the CR injection spectrum in the Galaxy

$$0 = \underbrace{dN}_{E} (E) = Q_{CR}(E) - \frac{N_{CR}(E)}{t_{disk}}$$
escape rate
from the disk
$$Q_{CR}(E) = \frac{N_{CR}(E)}{t_{disk}} \propto N_{CR}(E)D(E) \propto E^{-2.4}$$

Spallation measurements at different energies -> $t_{disk} \propto E^{-0.3}$

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We can now constrain the CR injection spectrum in the Galaxy

$$0 = \underbrace{dN}_{E} = Q_{CR}(E) - \underbrace{N_{CR}(E)}_{t_{disk}} = \operatorname{escape rate}_{from the disk}$$
stability in time
$$CRs \text{ injected from sources in the disk}$$

$$Q_{CR}(E) = \frac{N_{CR}(E)}{t_{disk}} \propto N_{CR}(E)D(E) \propto E^{-2.4}$$

$$\operatorname{measured} \rightarrow E^{-2.7} \operatorname{which sources???}$$

A remarkable coincidence

Total CR power in the Galaxy ->

$$P_{CR} = 10^{41} \mathrm{erg/s}$$

A SuperNova is the explosion of a massive star that releases ~ 10^{51} ergs in form of kinetic energy. In the Galaxy the observed supernova rate is of the order of 1/30 yr⁻¹.

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Total SN power in the Galaxy ->

$$P_{SN} = 10^{42} \mathrm{erg/s}$$

SuperNovae alone could maintain the CR population provided that about 10% of their kinetic energy is somehow converted into CRs

Energy threshold for neutral pion production:

$$p + p \rightarrow p + p + \pi^0$$



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$$E^2 - p^2 c^2 = \left(2m_p c^2 + m_{\pi^0} c^2\right)^2$$

Energy threshold for neutral pion production:

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 $\left(E_p + m_p c^2\right)^2 - p_p^2 c^2 = E^2 - p^2 c^2 = \left(2m_p c^2 + m_{\pi^0} c^2\right)^2$

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$$E_p - m_p c^2 > 2m_{\pi^0} c^2 + \left(\frac{m_{\pi^0}}{2m_p}\right) m_{\pi^0} c^2 \approx 280 \text{ MeV}$$

energy threshold

Let's calculate the spectrum of neutral pions:

We assume a power law spectrum for CRs: $N_p(E_p) \propto E_p^{-\delta}$ Fraction of proton kinetic energy transferred to pion (from data): $f_{\pi^0} \approx 0.17$ production total cross rate section $q_{\pi^0} = \int dE_p \ N_p(E_p) \ \delta(E_{\pi^0} - f_{\pi^0}E_{p,kin}) \ \sigma_{pp}(E_p) \ n_{gas} \ c$

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Let's now calculate the spectrum of photons from pion decay - I

The photon spectrum is the result of a "one-bodydecay" (neutral pion)



The photon spectrum MUST exhibit a feature at an energy relate to the pion mass

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Pion rest frame:



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The photon spectrum MUST exhibit a feature at an energy relate to the pion mass



Lab frame:

$$E_{\gamma} = \gamma \left(E_{\gamma}^* + v p_{\gamma}^* \cos \theta^* \right)$$

max and min energies -> $\cos \theta^* = \pm 1$

$$\frac{m_{\pi^0}}{2}\sqrt{\frac{1-\beta}{1+\beta}} \le E_{\gamma} \le \frac{m_{\pi^0}}{2}\sqrt{\frac{1+\beta}{1-\beta}}$$

Let's now calculate the spectrum of photons from pion decay - II

$$E_{\gamma}^{min} = \frac{m_{\pi^0}}{2} \sqrt{\frac{1-\beta}{1+\beta}} \le E_{\gamma} \le \frac{m_{\pi^0}}{2} \sqrt{\frac{1+\beta}{1-\beta}} = E_{\gamma}^{max}$$

(1)
$$\frac{\log E_{\gamma}^{max} + \log E_{\gamma}^{min}}{2} = \log\left(\frac{m_{\pi^0}}{2}\right)$$

in log-scale, the centre of the interval is half the pion mass

$$E_{\gamma}^{min} = \frac{m_{\pi^0}}{2} \sqrt{\frac{1-\beta}{1+\beta}} \le E_{\gamma} \le \frac{m_{\pi^0}}{2} \sqrt{\frac{1+\beta}{1-\beta}} = E_{\gamma}^{max}$$

(1)
$$\frac{\log E_{\gamma}^{max} + \log E_{\gamma}^{min}}{2} = \log\left(\frac{m_{\pi^0}}{2}\right) \qquad \begin{array}{l} \text{in log-scale, the centre} \\ \text{of the interval is half} \\ \text{the pion mass} \end{array}$$

(2) in the pion rest frame the photon distribution is isotropic
$$\frac{\mathrm{d}n_{\gamma}}{\mathrm{d}\Omega^{*}} = \frac{1}{4\pi}$$
$$\mathrm{d}\Omega^{*} \propto \mathrm{d}(\cos\theta^{*})$$
$$E_{\gamma} = \gamma \left(E_{\gamma}^{*} + vp_{\gamma}^{*}\cos\theta^{*}\right) \to \mathrm{d}E_{\gamma} \propto \mathrm{d}(\cos\theta^{*})$$
$$\frac{\mathrm{d}n_{\gamma}}{\mathrm{d}E_{\gamma}} = const$$
The spectrum is flat!















Let's now calculate the spectrum of photons from pion decay - III



m extstyle the gamma ray spectrum is symmetric (in log-log) with respect to: $rac{m_{\pi^0}}{2} \sim 70 \,\, {
m MeV}$

> at high energy the spectrum mimic the CR spectrum, with (roughly): $E_\gamma pprox rac{E_{CR}}{10}$

Let's now calculate the spectrum of photons from pion decay - III



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m o}$ the gamma ray spectrum is symmetric (in log-log) with respect to: $rac{m_{\pi^0}}{2}\sim 70\,\,{
m MeV}$

 $^{(o)}$ at high energy the spectrum mimic the CR spectrum, with (roughly): $E_\gamma pprox rac{E_{CR}}{10}$

Why the same power law?

$$E_{\gamma}^{max} = \frac{m_{\pi^{0}}}{2} \sqrt{\frac{1+\beta}{1-\beta}} = \frac{m_{\pi^{0}}}{2} (1+\beta)\gamma \longrightarrow m_{\pi^{0}}\gamma$$

$$E_{\gamma}^{min} = \frac{m_{\pi^{0}}}{2} \sqrt{\frac{1-\beta}{1+\beta}} = \frac{m_{\pi^{0}}}{2} [(1+\beta)\gamma]^{-1} \longrightarrow \frac{m_{\pi^{0}}}{4}\gamma^{-1}$$

$$\boxed{\text{very large Lorentz factor}} \qquad 0 \lesssim E_{\gamma} \lesssim m_{\pi^{0}}\gamma \propto E_{p}$$

$$\text{to produce a photon of energy } E_{\gamma} \text{ we need: } E_{p} \gtrsim \frac{m_{p}}{m_{\pi^{0}}} E_{\gamma}$$

$$N_{\gamma}(E_{\gamma}) \propto \int_{\frac{m_{p}}{m_{\pi^{0}}} E_{\gamma}}^{\infty} dE_{p} \frac{N_{p}(E_{p})}{E_{\gamma}^{max} - E_{\gamma}^{min}} \propto \int_{\frac{m_{p}}{m_{\pi^{0}}} E_{\gamma}}^{\infty} dE_{p} \frac{E_{p}^{\delta}}{E_{p}} \propto E_{p}$$

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions

$$p + p \to p + p + \pi^0 + \pi^+ + \pi^-$$

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 $\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ $\mu^{\pm} \to e^{\pm} + \bar{\nu}_{\mu}(\nu_{\mu}) + \nu_{e}(\bar{\nu}_{e})$

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions



Final products of proton-proton interactions are not only gamma ray photons but also neutrinos, anti-neutrinos, electrons and positrons

$$E_e \approx E_\nu \approx \frac{E_p}{20}$$
Not only gammas: neutrinos & electrons

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions

$$\begin{array}{c} p+p \rightarrow p+p+\pi^{0}+\pi^{+}+\pi^{-} & \mbox{neutral and charged} \\ \pi^{0} \rightarrow \gamma + \gamma & \mbox{same probability} \\ \pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu}) \\ \mu^{\pm} - e^{\pm} + \bar{\nu}_{\mu}(\nu_{\mu}) + \nu_{e}(\bar{\nu}_{e}) \end{array}$$

same probability

(1/3, 1/3, 1/3)

Final products of proton-proton interactions are not only gamma ray photons but also neutrinos, anti-neutrinos, electrons and positrons

$$E_e \approx E_\nu \approx \frac{E_p}{20}$$

valid at large energies

gamma rays

number of protons of energy $E_{\rm p}\,produced\,per\,s$

 $Q_n(E_n)$

valid at large energies

gamma rays

number of protons of energy $E_{\rm p}\,produced\,per\,s$

 $Q_p(E_p)E_p^2$

power (energy per unit time)

valid at large energies

fraction of the energy converted into pions...\

gamma rays

number of protons of energy $E_{\rm p}\,produced\,per\,s$

 $\eta_{\pi}Q_{p}(E_{p})E_{p}^{2}$

power (energy per unit time)

valid at large energies

fraction of the energy converted into pions... \

gamma rays

number of protons of energy $E_{\text{p}}\,\text{produced per s}$

 $\eta_{\pi}Q_{p}(E_{p})E_{p}^{2}$

power (energy per unit time)

$$\eta_{\pi} = 1 - e^{-\left(\frac{\tau_{res}}{\tau_{pp}}\right)} \longrightarrow \frac{\tau_{res}}{\tau_{pp}} \qquad \tau_{pp} \gg \tau_{res}$$
$$\longrightarrow 1 \qquad \tau_{pp} \ll \tau_{res}$$

valid at large energies







neutrinos

valid at large energies

The same as for gammas but:



$$\Box \hspace{-1.5cm} > \hspace{-1.5cm} E_{\gamma} \longrightarrow E_{\nu} \sim 0.05 \times E_{p}$$

valid at large energies



 $Q_{\gamma}(E_{\gamma})E_{\gamma}^2 = \frac{\eta_{\pi}}{3}Q_p(E_p)E_p^2$



 $Q_{\nu}(E_{\nu})E_{\nu}^2 = \frac{1}{2}Q_{\gamma}(E_{\gamma})E_{\gamma}^2$