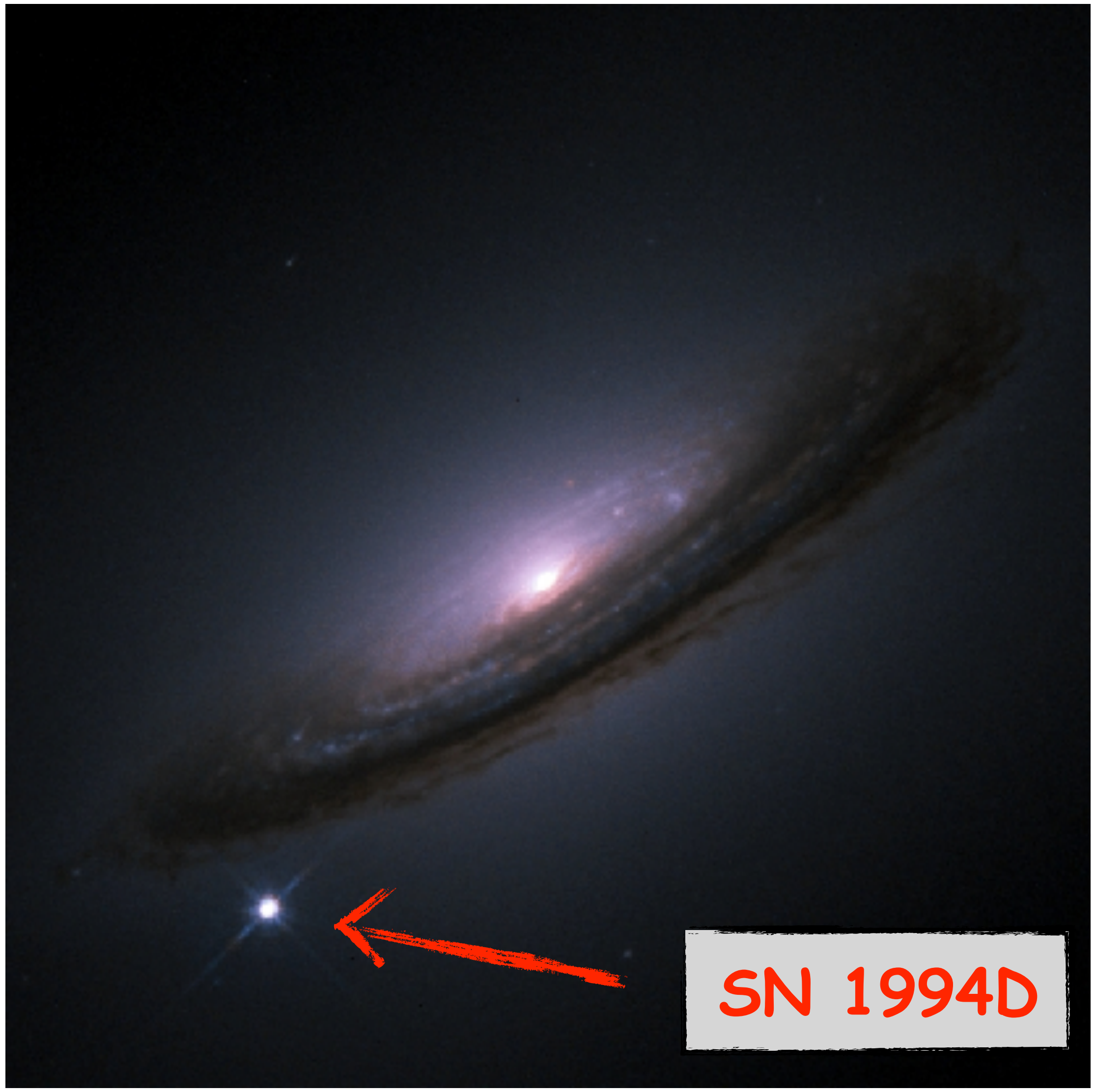
NPAC course on Astroparticles

III - ASTROPHYSICS: SUPERNOVA REMNANTS

Supernova explosions

last phase of the evolution of some stars->huge increase of the star's luminosity



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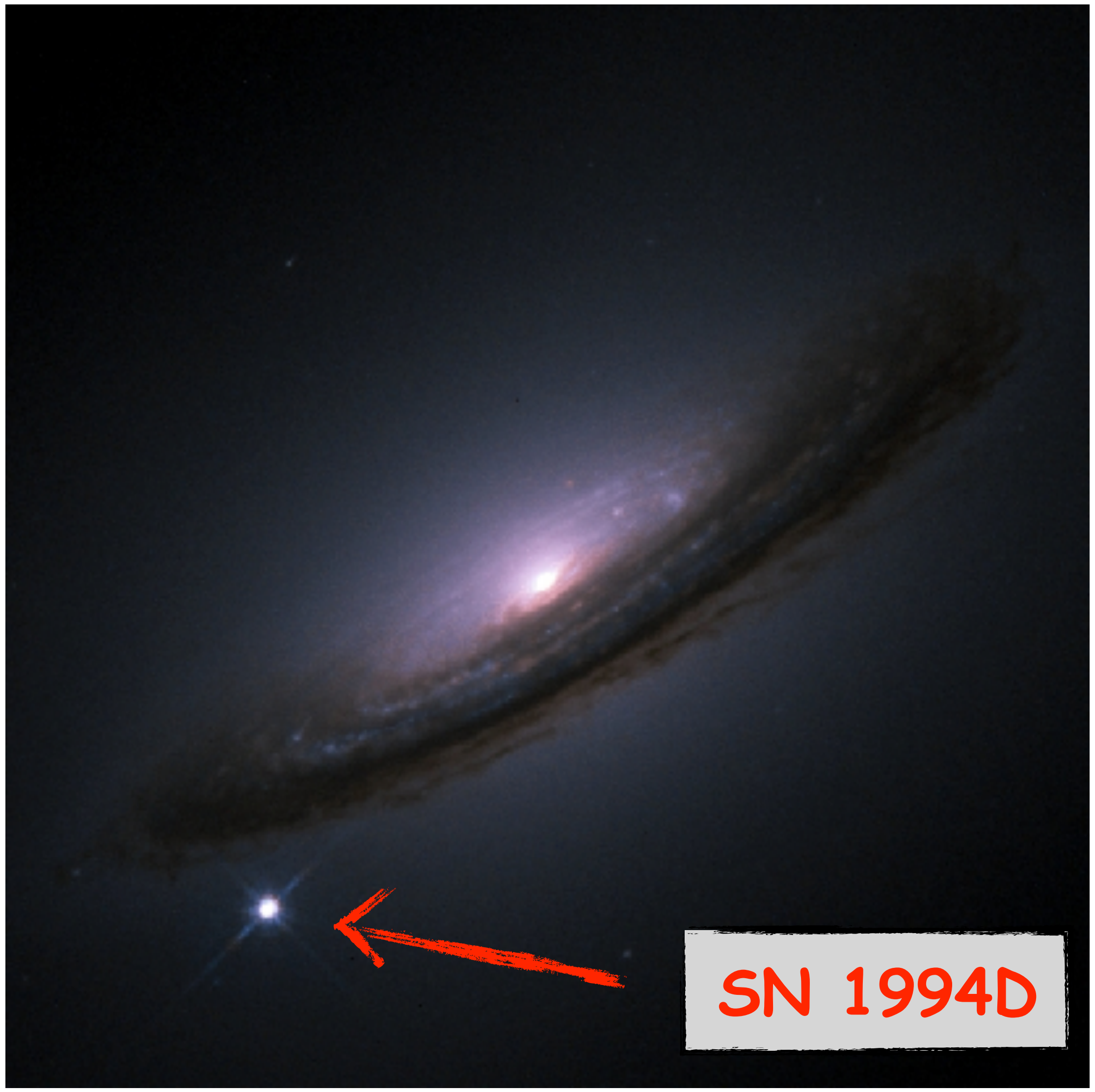

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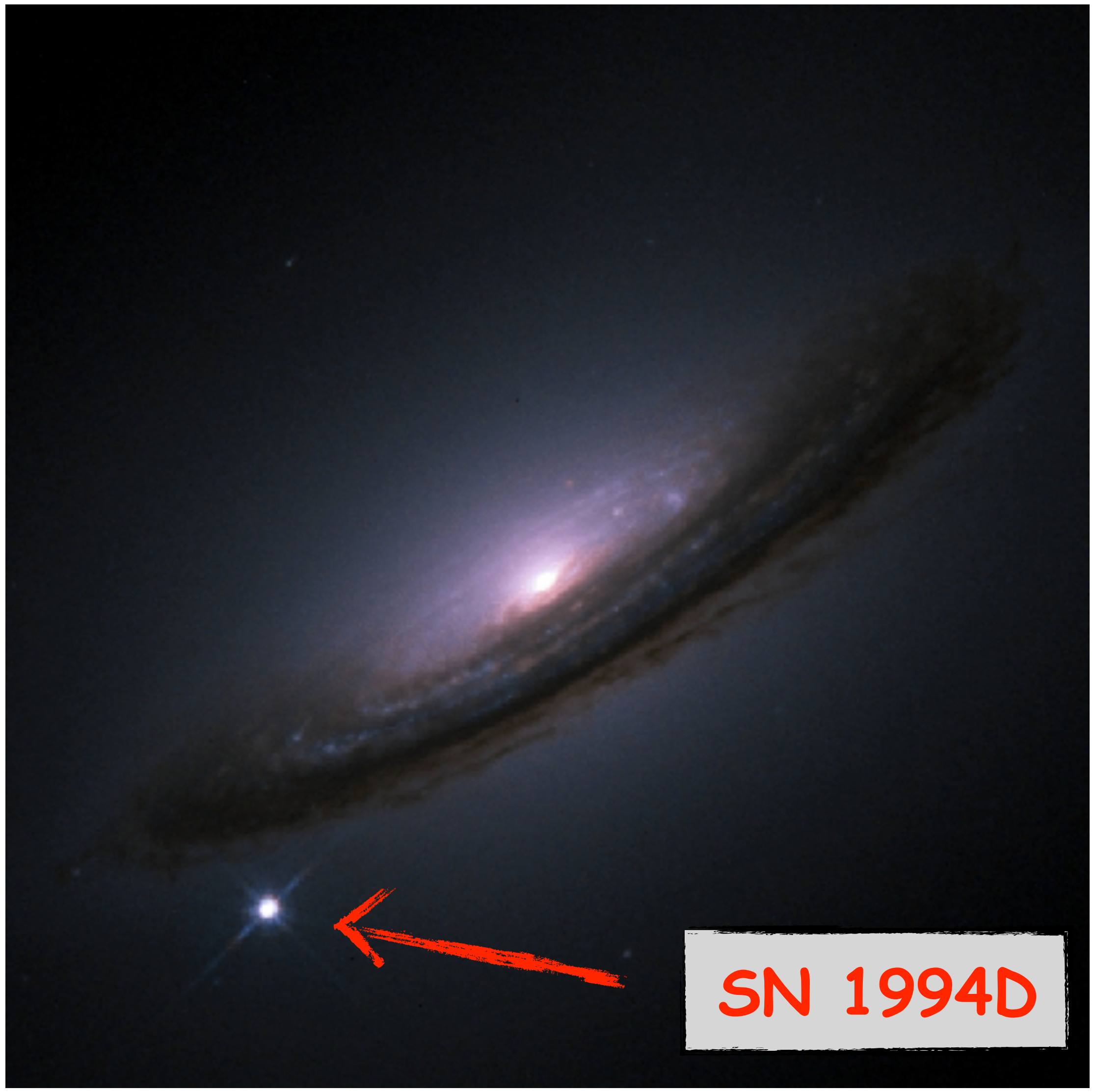

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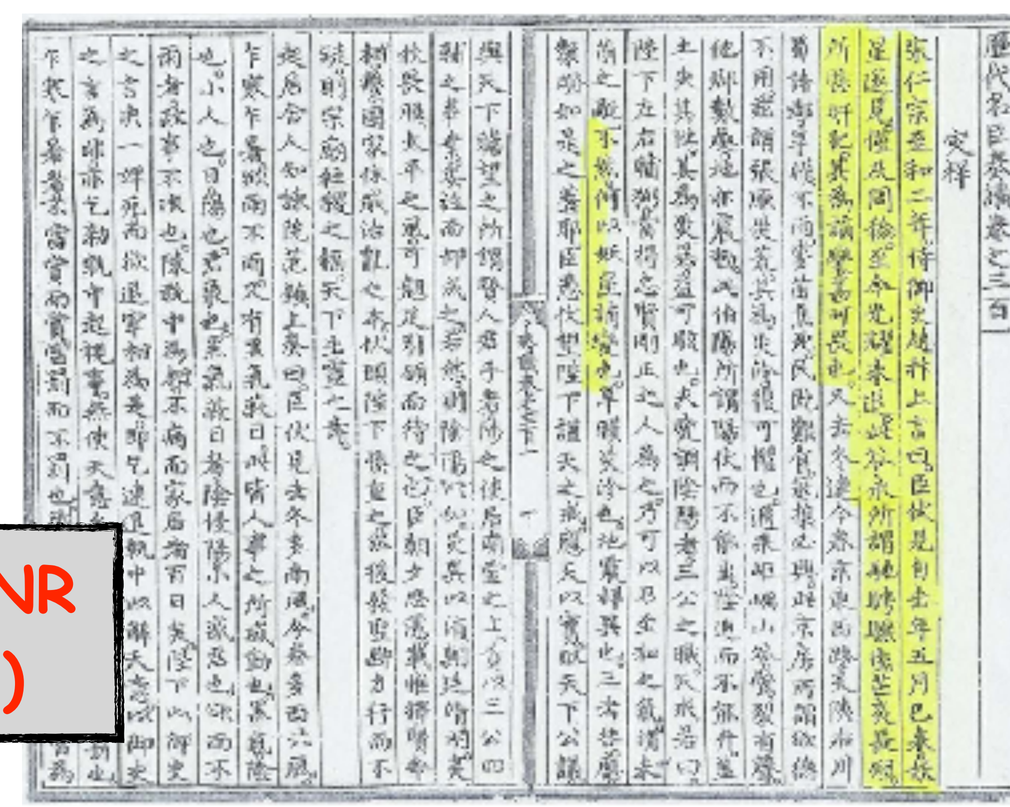

Crab SNR (1059)

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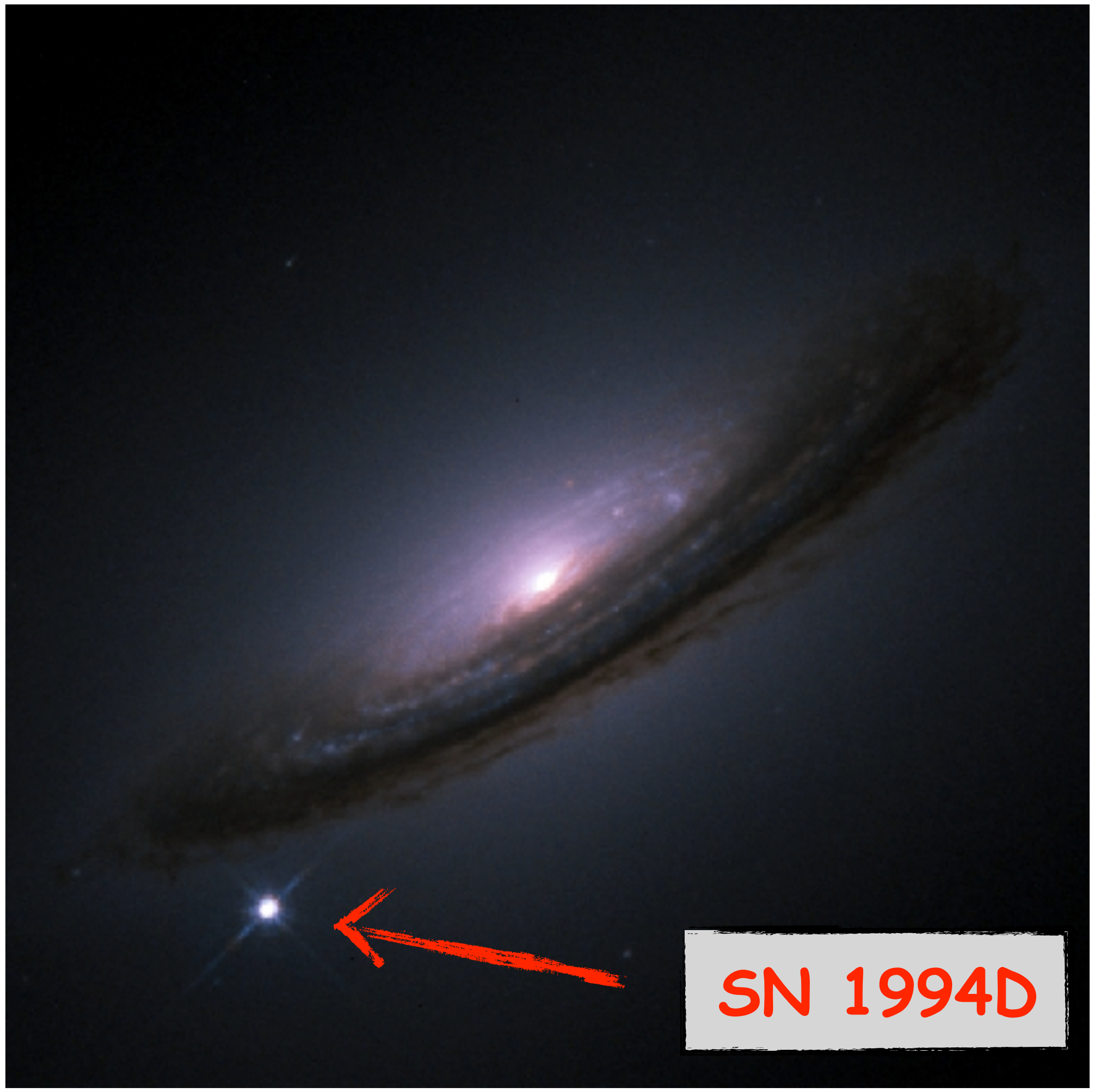

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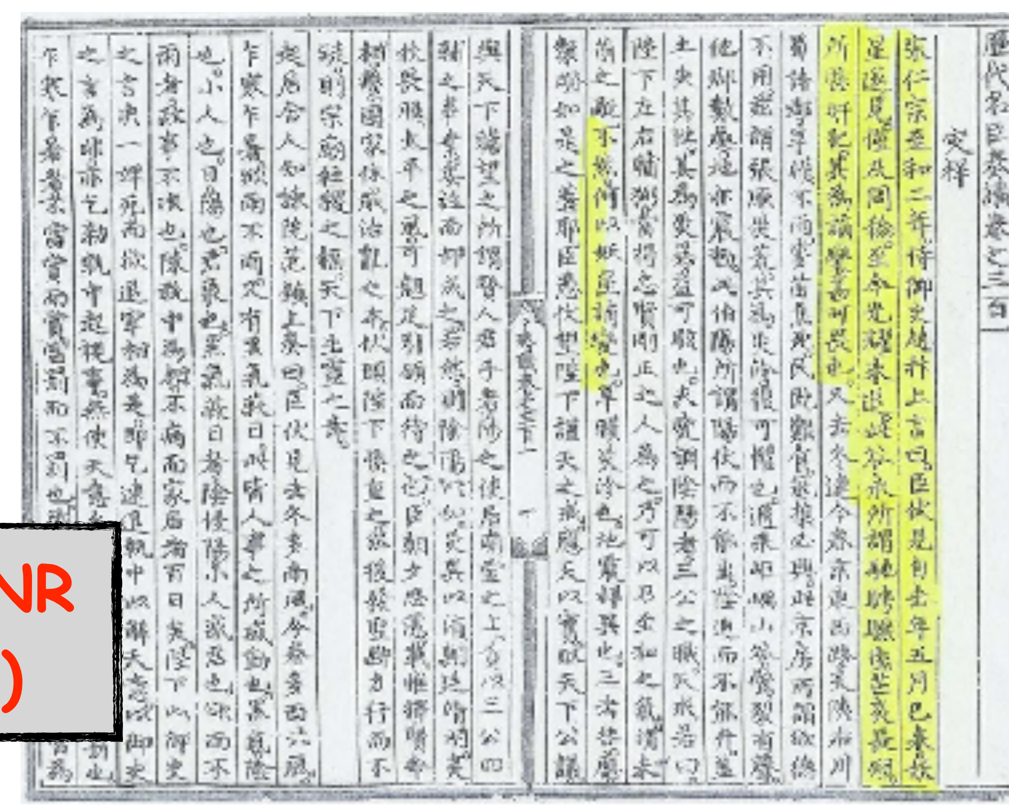
in 2016, ~6400 extragalactic SNe have been reported



Crab SNR (1059)

“Recent” supernova explosions

year	SNR	distance
185	RC W86 (?)	~3 kpc
393	RX J1713.7-3946 (?)	~1 kpc
1006	SN 1006	~2 kpc
1054	Crab	~2 kpc
1181	3C 58 (?)	~2 kpc
1572	Tycho	~2-3 kpc
1604	Kepler	~6 kpc

ALL recorded SNe associated with SNRs in the MW

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-> ~3 SN/century in the MW

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some other notable SNe

SNR	age	distance
Cas A	~350 yr	~3 kpc
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SN not recorded: youngest known galactic SNR → G1.9+0.3

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closest extragalactic SN (Large Magellanic Cloud) → SN 1987A

Small perturbations in a fluid: sound waves

fixed fluid element $\longrightarrow$ $$\frac{\mathrm{d}}{\mathrm{d}t} \longrightarrow \frac{\partial}{\partial t} + \vec{u} \cdot \nabla$$ $\longleftarrow$ fixed position

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mass conservation

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \vec{v}) = 0$$

momentum conservation

$$\varrho \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\nabla P \longrightarrow \frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{\nabla P}{\varrho}$$

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equilibrium

$$\begin{aligned} \vec{v} &= 0 \\ \varrho &= \varrho_0 \\ P &= P_0 \end{aligned}$$

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equilibrium **perturbed**

$$\begin{array}{ccccc} \vec{v} & = & 0 & \longrightarrow & \vec{\delta v} \\ \varrho & = & \varrho_0 & \longrightarrow & \varrho_0 + \delta \varrho \\ P & = & P_0 & \longrightarrow & P_0 + \delta P \end{array}$$

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equilibrium				**perturbed**
$\vec{v}$	$=$	0	$\longrightarrow$	$\delta\vec{v}$
ϱ	$=$	ϱ_0	$\longrightarrow$	$\varrho_0 + \delta\varrho$
P	$=$	P_0	$\longrightarrow$	$P_0 + \delta P$

$$\frac{\partial}{\partial t}\delta\varrho + \varrho_0 \nabla \cdot \delta\vec{v} = -\nabla \cdot \left(\delta\varrho\delta\vec{v}\right)$$

$$\frac{\partial}{\partial t}\delta\vec{v} = -\delta\vec{v} \cdot \nabla\delta\vec{v} - \frac{1}{\varrho_0 + \delta\varrho}\nabla\delta P$$

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Small perturbations in a fluid: sound waves

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Small perturbations in a fluid: sound waves

$$\frac{\partial}{\partial t}\delta\varrho + \varrho_0 \nabla \cdot \vec{\delta v} = 0 \quad \rightarrow \frac{\partial}{\partial t} \rightarrow \quad \frac{\partial^2}{\partial t^2}\delta\varrho + \varrho_0 \frac{\partial}{\partial t}\nabla \cdot \vec{\delta v} = 0$$

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$\boxed{\text{adiabatic eq. of state}}$

$$P = K\varrho^\gamma$$

$$\gamma = \frac{5}{3}$$ monoatomic gas

$$\delta P = \left(\frac{\partial P}{\partial \varrho}\right)_s \delta\varrho = \frac{\gamma P}{\varrho}\delta\varrho \equiv c_s^2 \delta\varrho$$

constant entropy

sound speed

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$$\frac{\delta\varrho}{\varrho_0} = A\ e^{i\left(\vec{k}\cdot\vec{x}\pm\omega t\right)}$$

$$A \ll 1 \quad , \quad \omega^2 = k^2 c_{s,0}^2$$

The formation of shock waves

let's relax the assumption of small perturbations —> finite amplitude perturbations

$$c_s \sim c_{s,0}$$

The formation of shock waves

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$$c_s \quad c_{s,0}$$

supersonic

$$c_s > c_{s,0}$$

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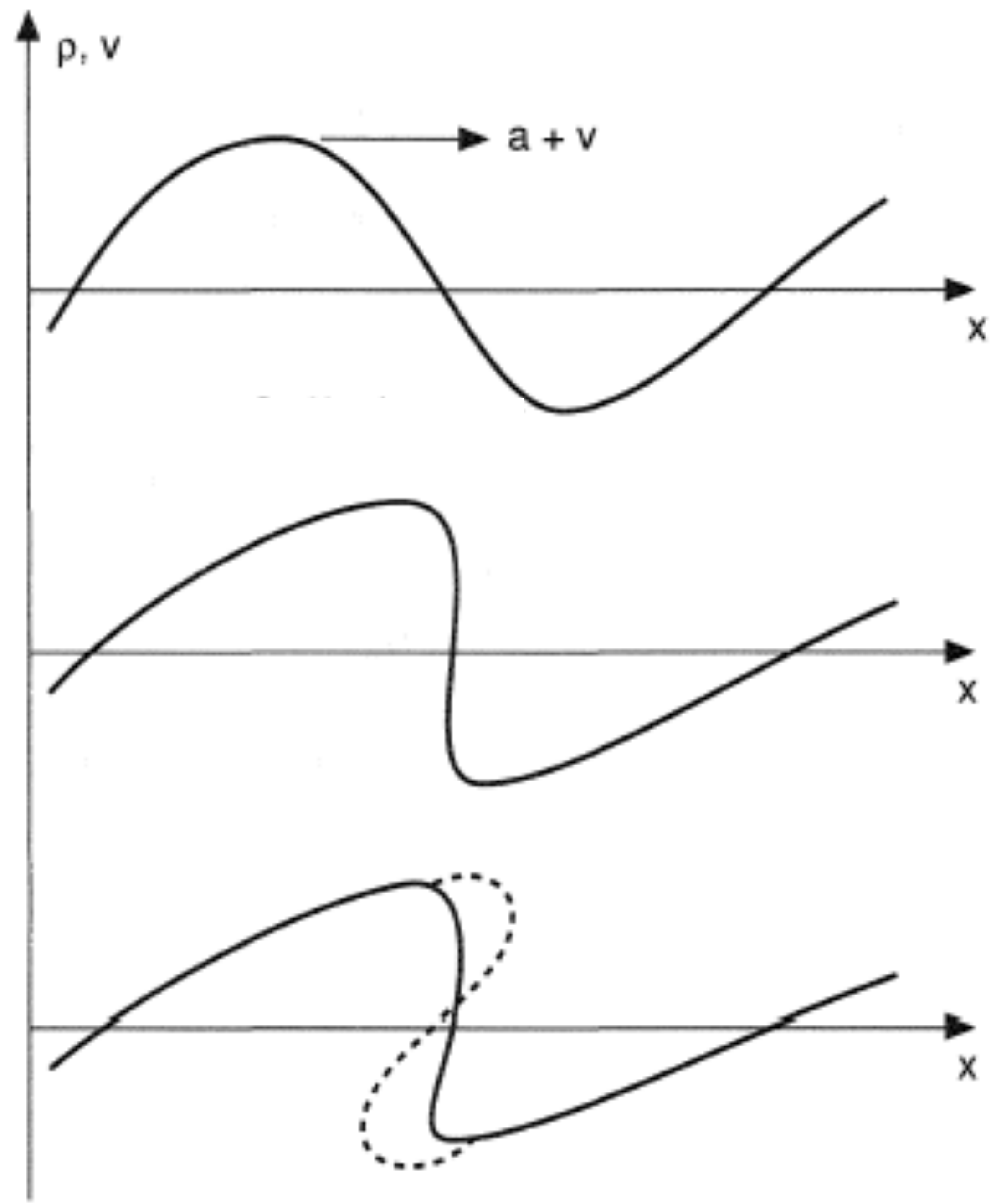

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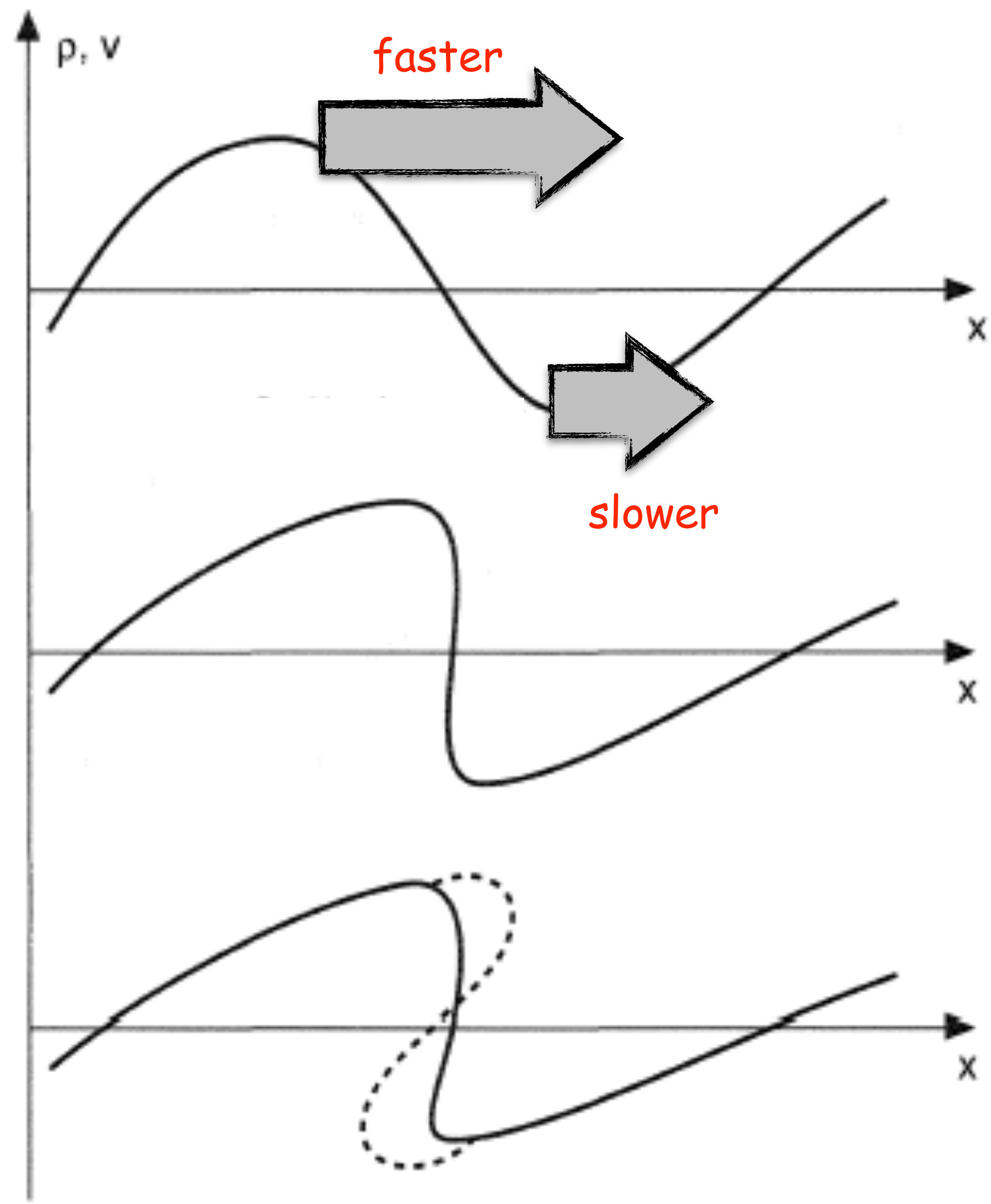
let's relax the assumption of small perturbations —> finite amplitude perturbations

$$c_s \cancel{=} c_{s,0}$$

supersonic

$$c_s > c_{s,0}$$

the propagation speed is faster when the density is larger



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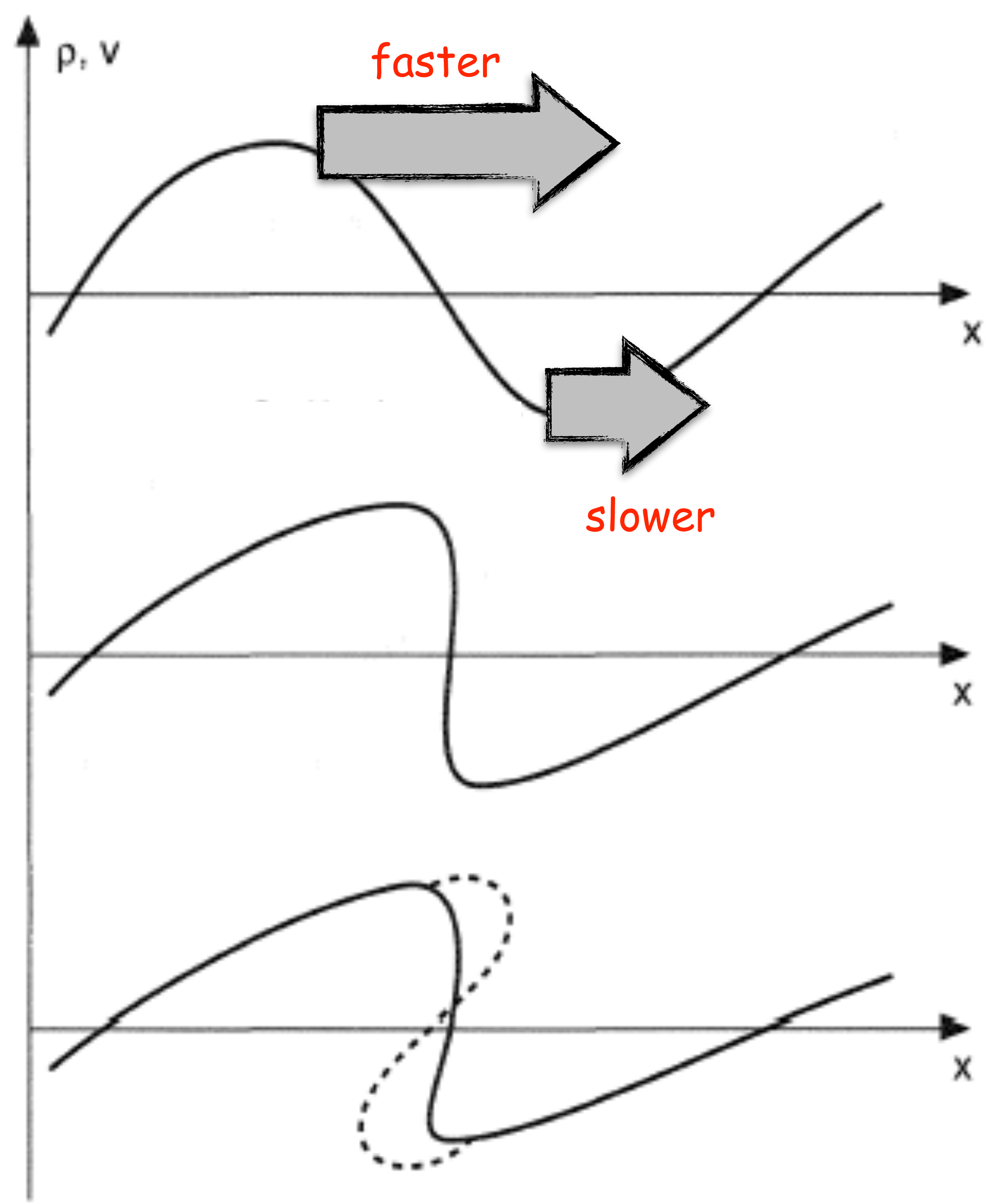
steepening of the wave



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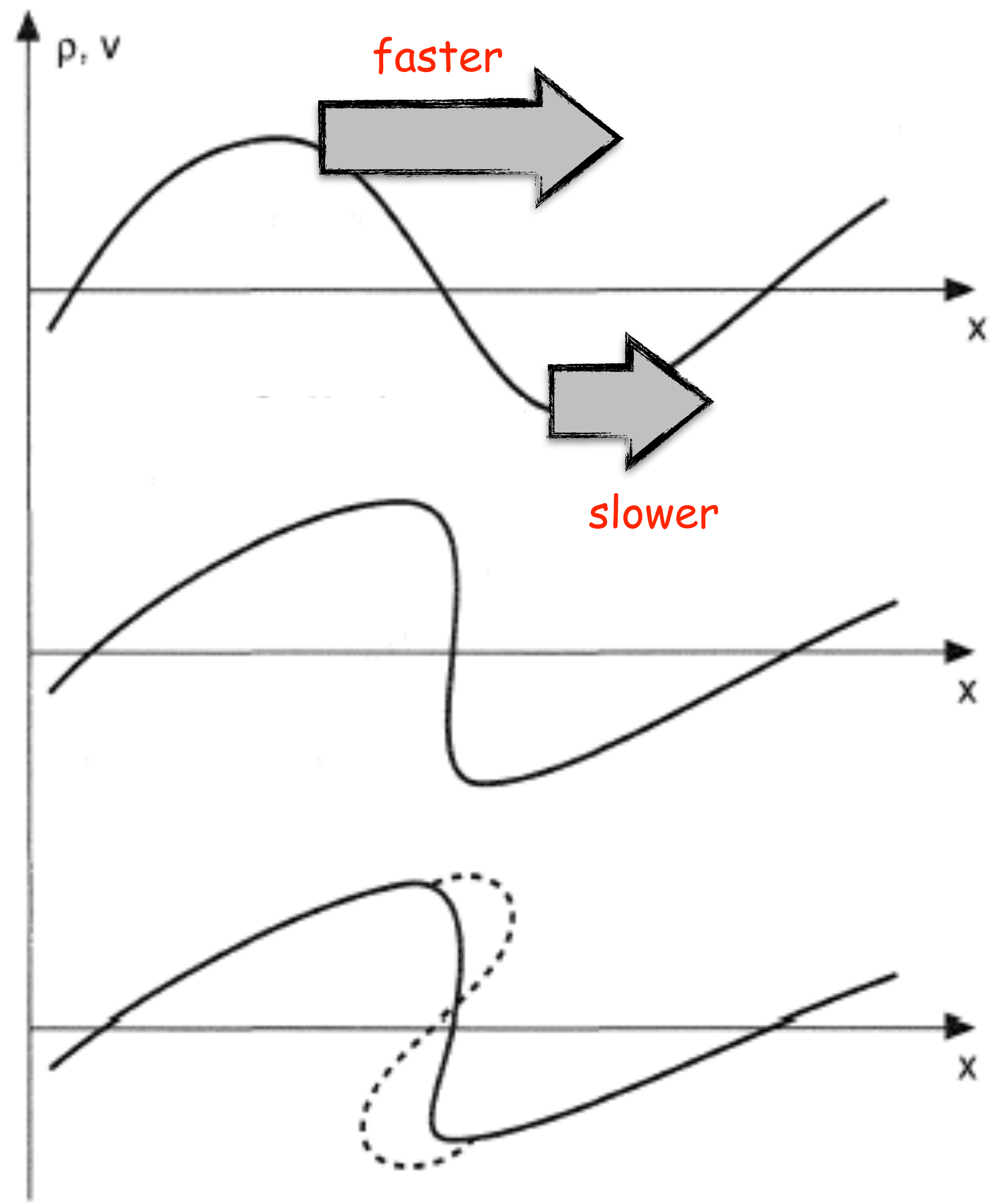
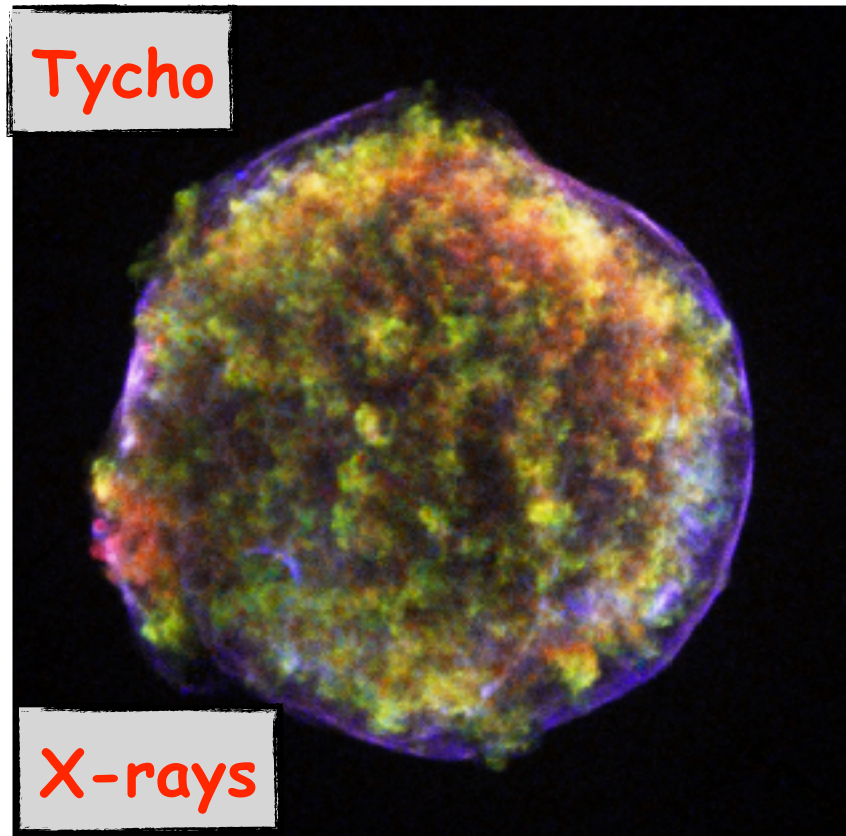
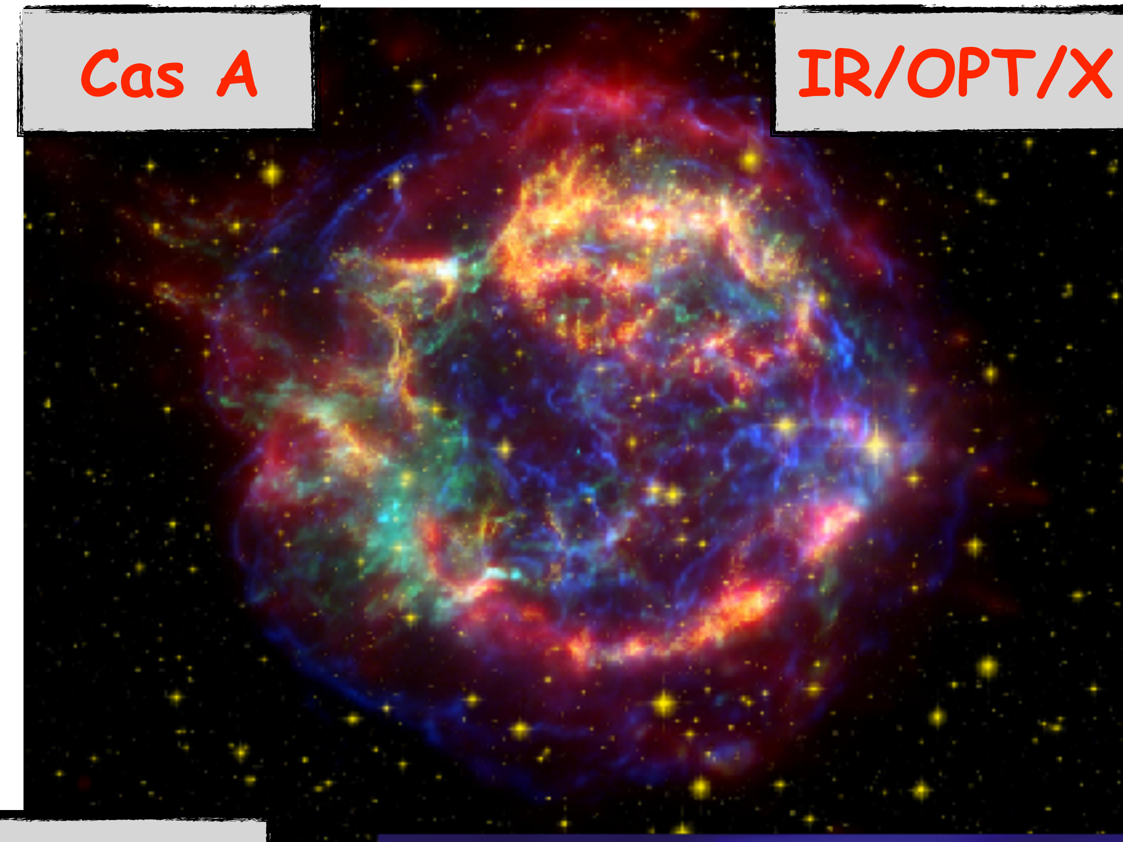
steepening of the wave

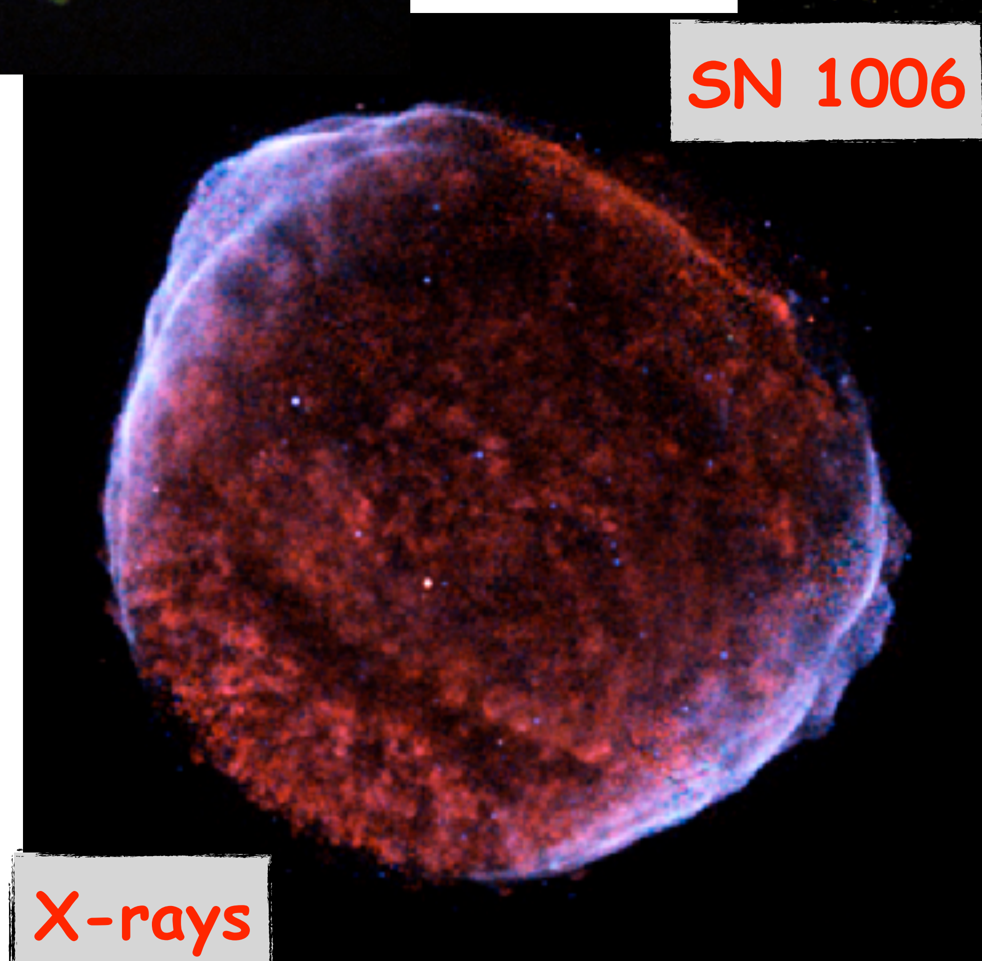
formation of a discontinuity

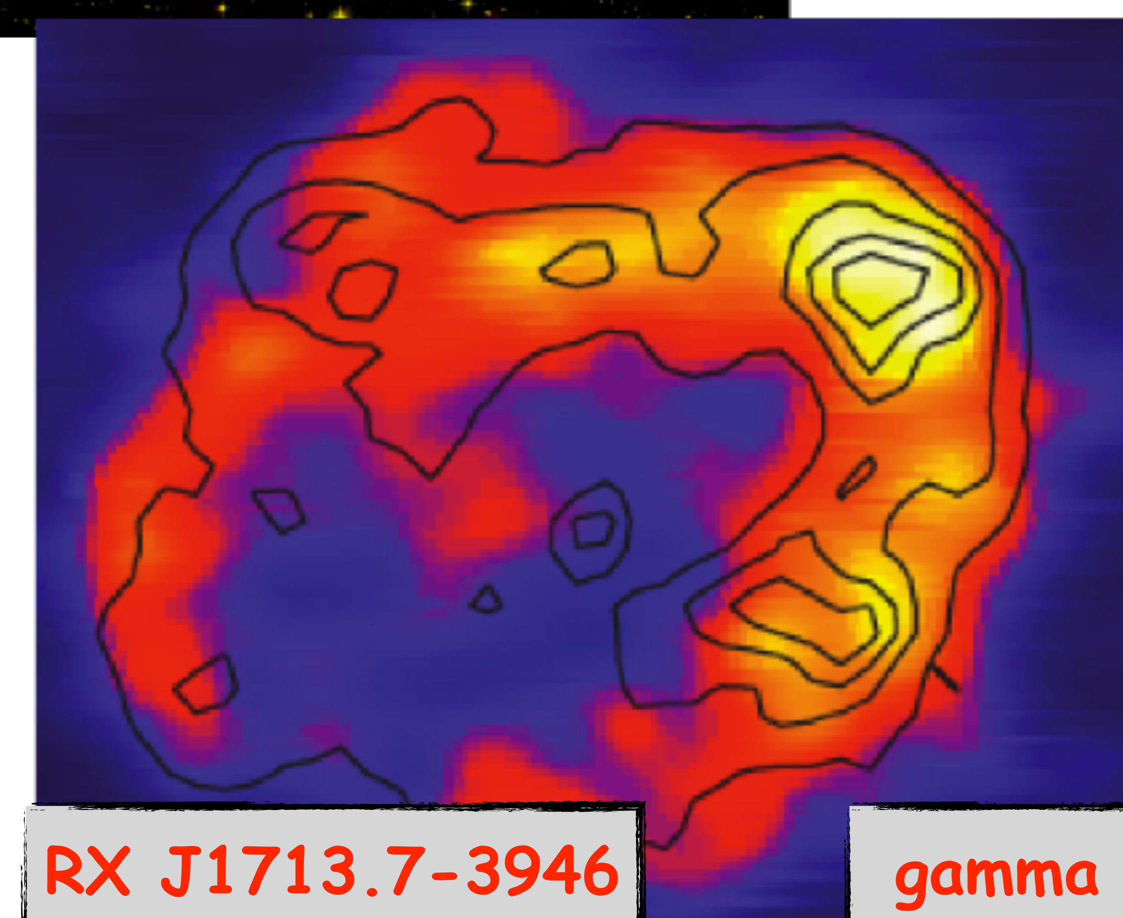


Supernova remnant shocks

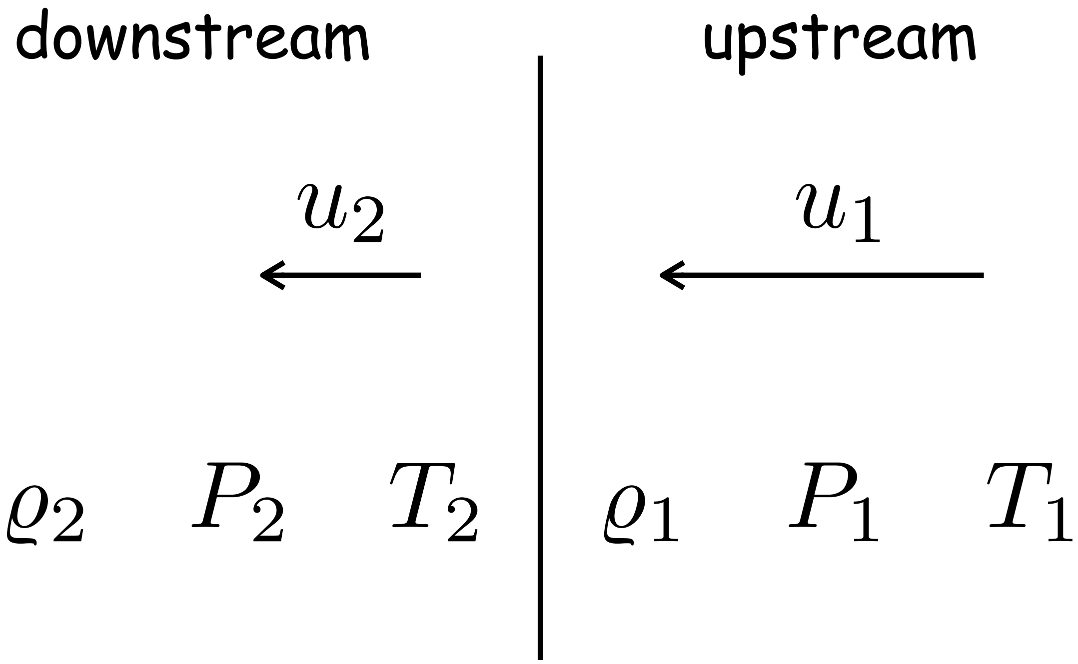








Shock waves: conservation laws

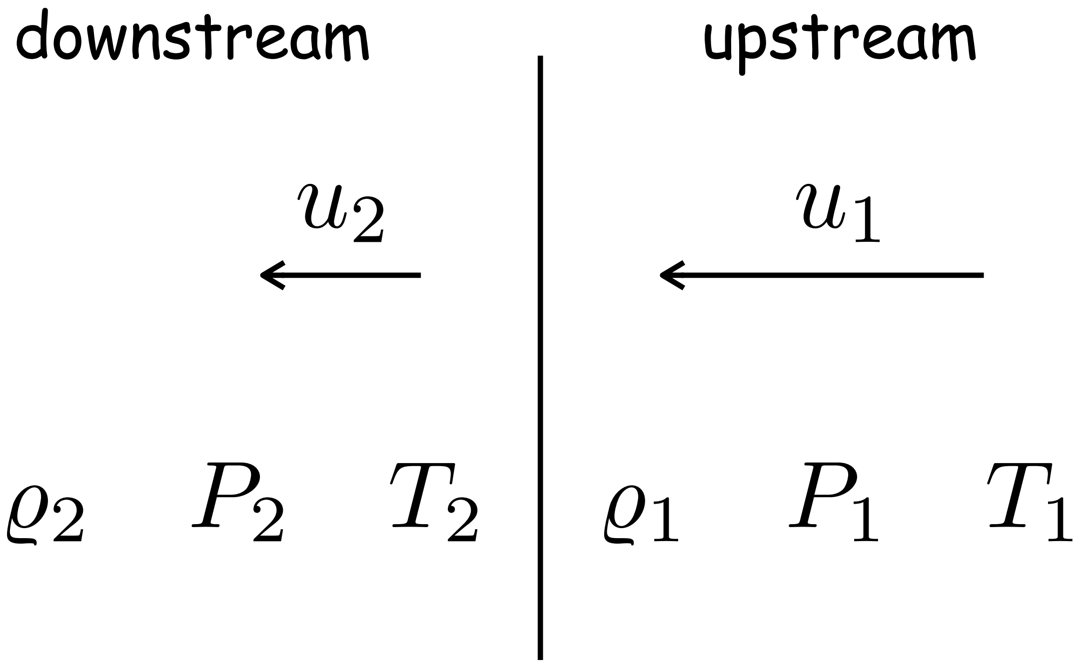


shock rest frame

Mach number

$$\mathcal{M} = \frac{u_1}{c_s}$$

Shock waves: conservation laws



shock rest frame

Mach number

$$\mathcal{M} = \frac{u_1}{c_s}$$

mass

$$\varrho_1 u_1 = \varrho_2 u_2$$

momentum

$$\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2$$

energy

$$\frac{1}{2}\varrho_1 u_1^3 + \varrho_1 u_1 \left(\epsilon_1 + \frac{P_1}{\varrho_1}\right) = \frac{1}{2}\varrho_2 u_2^3 + \varrho_2 u_2 \underbrace{\left(\epsilon_2 + \frac{P_2}{\varrho_2}\right)}_{\text{specific enthalpy}}$$

internal energy per unit mass

Shock waves: conservation laws

downstream | upstream

$u_2 \longleftarrow$ $\quad$ $u_1 \longleftarrow$

$\varrho_2 \quad P_2 \quad T_2 \quad | \quad \varrho_1 \quad P_1 \quad T_1$

shock rest frame

Mach number

$$\mathcal{M} = \frac{u_1}{c_s}$$

mass

$$\varrho_1 u_1 = \varrho_2 u_2$$

momentum

$$\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2$$

$$1 + \frac{P_1}{\varrho_1 u_1^2} = \frac{u_2}{u_1} + \frac{P_2}{\varrho_1 u_1^2}$$

energy

$$\frac{1}{2}\varrho_1 u_1^3 + \varrho_1 u_1 \left(\epsilon_1 + \frac{P_1}{\varrho_1}\right) = \frac{1}{2}\varrho_2 u_2^3 + \varrho_2 u_2 \underbrace{\left(\epsilon_2 + \frac{P_2}{\varrho_2}\right)}_{\text{specific enthalpy}}$$

internal energy per unit mass

Shock waves: conservation laws

downstream | upstream

$u_2 \leftarrow$ | $u_1 \longleftarrow$

$\varrho_2 \quad P_2 \quad T_2$ | $\varrho_1 \quad P_1 \quad T_1$

shock rest frame

Mach number

$$\mathcal{M} = \frac{u_1}{c_s}$$

strong shock

$$\mathcal{M}^2 \gg 1 \longrightarrow \frac{\varrho_1 u_1^2}{\gamma P_1} \gg 1$$

mass

$$\varrho_1 u_1 = \varrho_2 u_2$$

momentum

$$\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2$$

$$\left.\begin{array}{l} \varrho_1 u_1 = \varrho_2 u_2 \\ \varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2 \end{array}\right\} \quad 1 + \frac{P_1}{\varrho_1 u_1^2} = \frac{u_2}{u_1} + \frac{P_2}{\varrho_1 u_1^2}$$

energy

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internal energy per unit mass

specific enthalpy

Shock waves: conservation laws

downstream		upstream
$\xleftarrow{u_2}$		$\xleftarrow{\quad u_1 \quad}$
		cold medium
$\varrho_2 \quad P_2 \quad T_2$		$\varrho_1 \quad \cancel{P_1} \quad \cancel{T_1}$

shock rest frame

Mach number

$$\mathcal{M} = \frac{u_1}{c_s}$$

strong shock

$$\mathcal{M}^2 \gg 1 \longrightarrow \frac{\varrho_1 u_1^2}{\gamma P_1} \gg 1$$

mass

$$\varrho_1 u_1 = \varrho_2 u_2$$

momentum

$$\varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2$$

$$\left.\begin{array}{l} \varrho_1 u_1 = \varrho_2 u_2 \\ \varrho_1 u_1^2 + P_1 = \varrho_2 u_2^2 + P_2 \end{array}\right\} \; 1 + \cancel{\frac{P_1}{\varrho_1 u_1^2}} = \frac{u_2}{u_1} + \frac{P_2}{\varrho_1 u_1^2}$$

energy

$$\frac{1}{2}\varrho_1 u_1^3 + \cancel{\varrho_1 u_1 \left(\epsilon_1 + \frac{P_1}{\varrho_1}\right)} = \frac{1}{2}\varrho_2 u_2^3 + \varrho_2 u_2 \underbrace{\left(\epsilon_2 + \frac{P_2}{\varrho_2}\right)}_{\text{specific enthalpy}}$$

internal energy per unit mass

specific enthalpy

Jump (Rankine-Hugoniot) conditions

compression factor

$$r = \frac{\varrho_2}{\varrho_1} = \frac{u_1}{u_2}$$

ideal gas

$$P_2 = (\gamma - 1)\epsilon_2 \varrho_2$$

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$$\rightarrow r = \frac{2\gamma}{\gamma-1} - 1 = \frac{\gamma+1}{\gamma-1} \xrightarrow[\gamma = 5/3]{} 4$$

gas compression

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ram-pressure —> thermal pressure

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gas compression

Strong shocks

- compress moderately the gas $\quad r = \frac{\varrho_2}{\varrho_1} = \frac{\gamma+1}{\gamma-1} \xrightarrow[\gamma\,=\,5/3]{} 4$

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typical SNR shock speed

$$kT_2 = \frac{2(\gamma-1)}{(\gamma+1)^2} m_p u_1^2 \xrightarrow[\gamma=5/3]{} \frac{3}{16} m_p u_1^2 \sim 2\left(\frac{u_1}{1000\ \mathrm{km/s}}\right)^2 \ \mathrm{keV}$$

thermal energy

kinetic energy

hot plasma!!!

Emission from hot plasmas: thermal Bremsstrahlung

Radiation from a thermal electron-proton plasma

Emission from hot plasmas: thermal Bremsstrahlung

Radiation from a thermal electron-proton plasma

$$v = \sqrt{\frac{3kT}{m}} \longrightarrow v_e = \left(\frac{m_p}{m_e}\right)^{1/2} v_p \gg v_p$$

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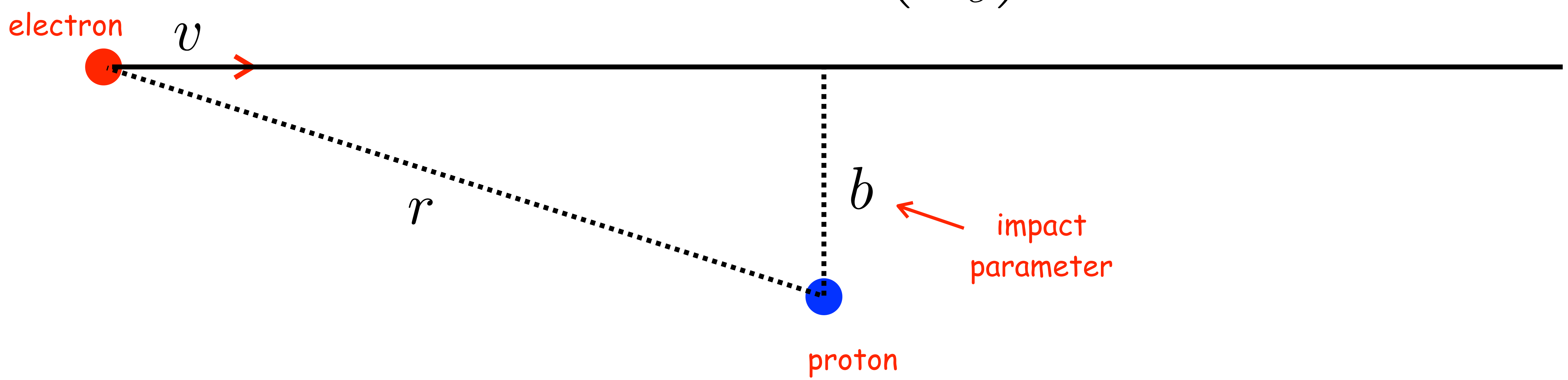

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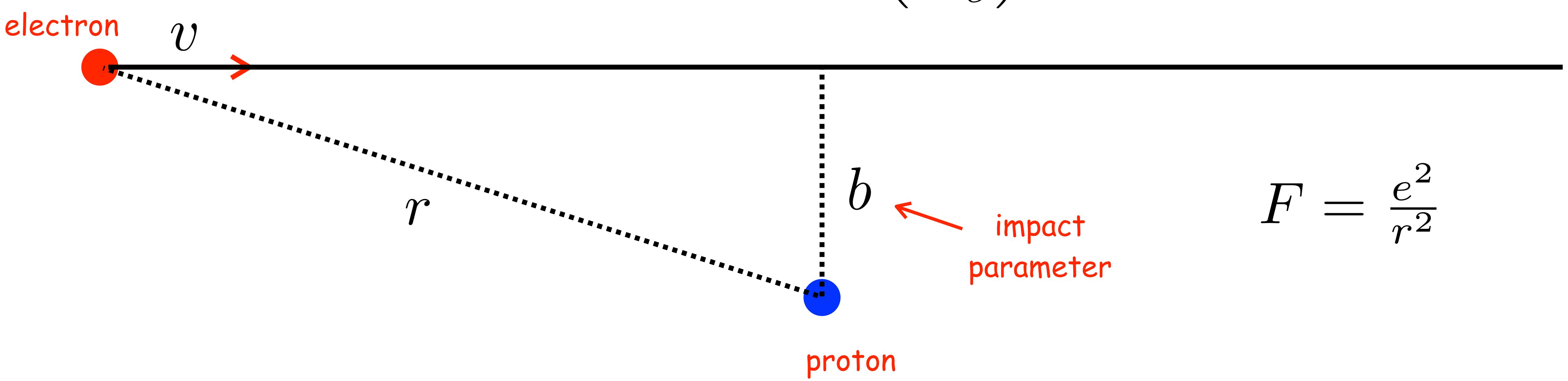

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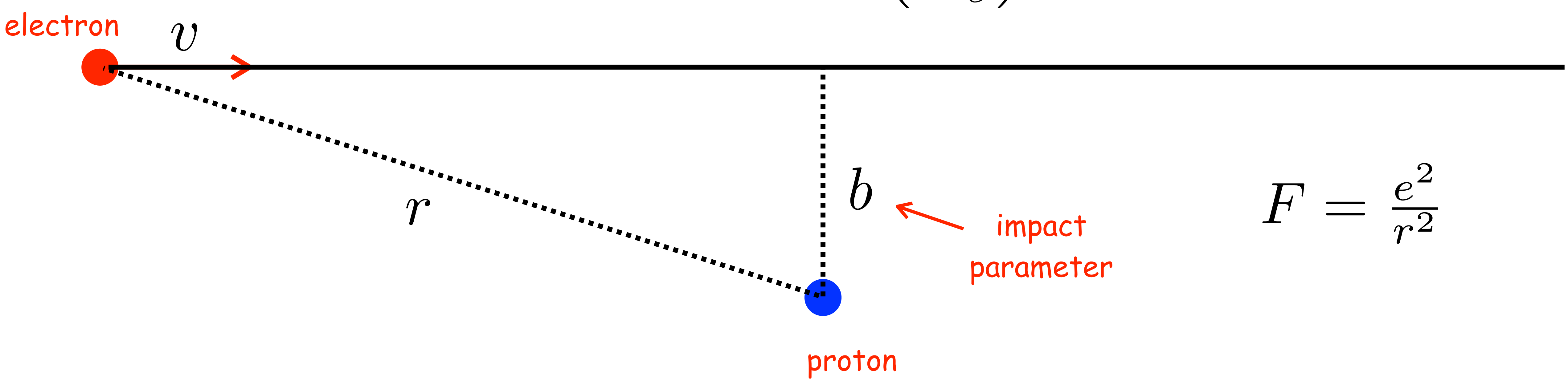

$$F = \frac{e^2}{r^2}$$

power emitted by an accelerated charge

$$P = \frac{2e^2}{3c^3} a^2$$

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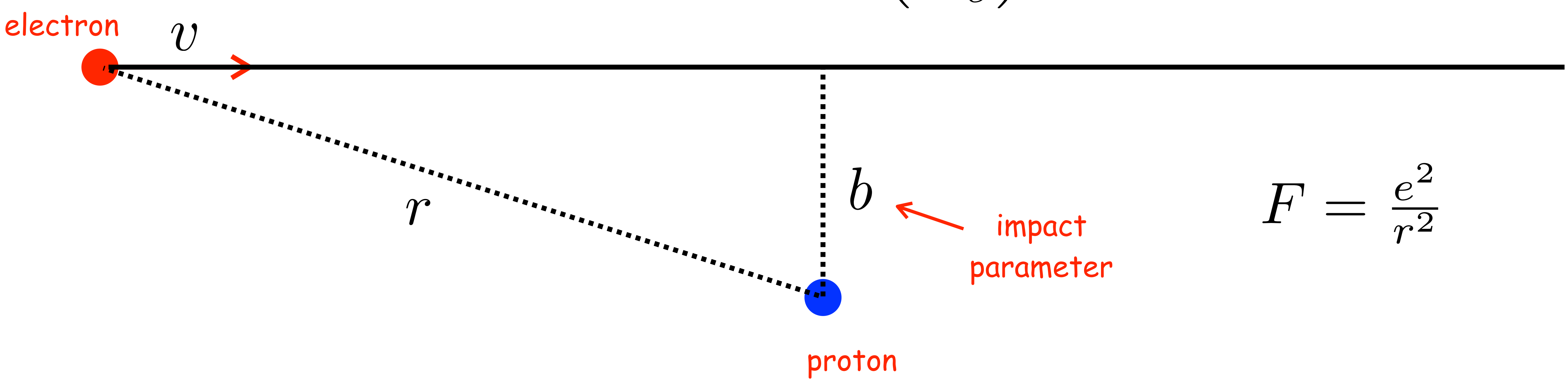

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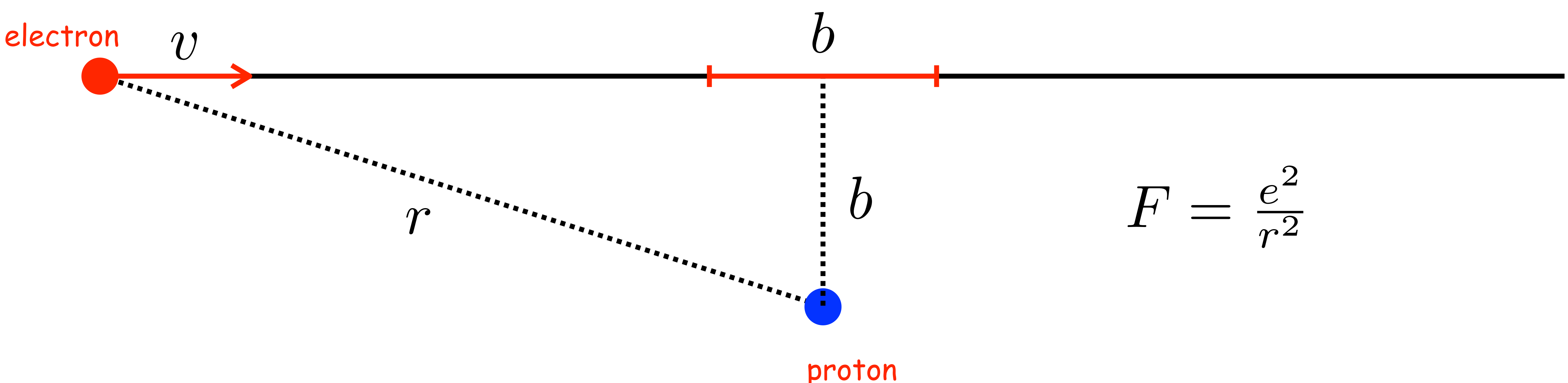
$$\begin{cases} r \approx b \rightarrow a \approx \dfrac{e^2}{m_e b^2} \\ r \gg b \rightarrow a \approx 0 \end{cases}$$

Thermal Bremsstrahlung

very brutal approximations...

characteristic time for the interaction $\tau \approx \frac{b}{v} \longrightarrow \omega \approx \frac{1}{\tau} = \frac{v}{b}$ characteristic frequency of the emitted radiation



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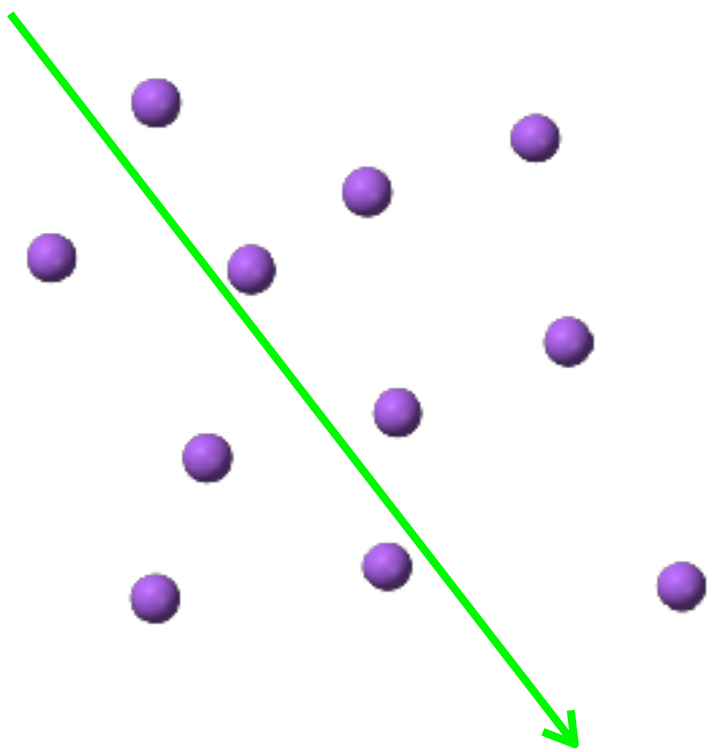
Thermal Bremsstrahlung

rough estimate of the impact parameter b

plasma proton density -> n_p

mean distance between protons -> $l_p \sim n_p^{-1/3} \approx b$



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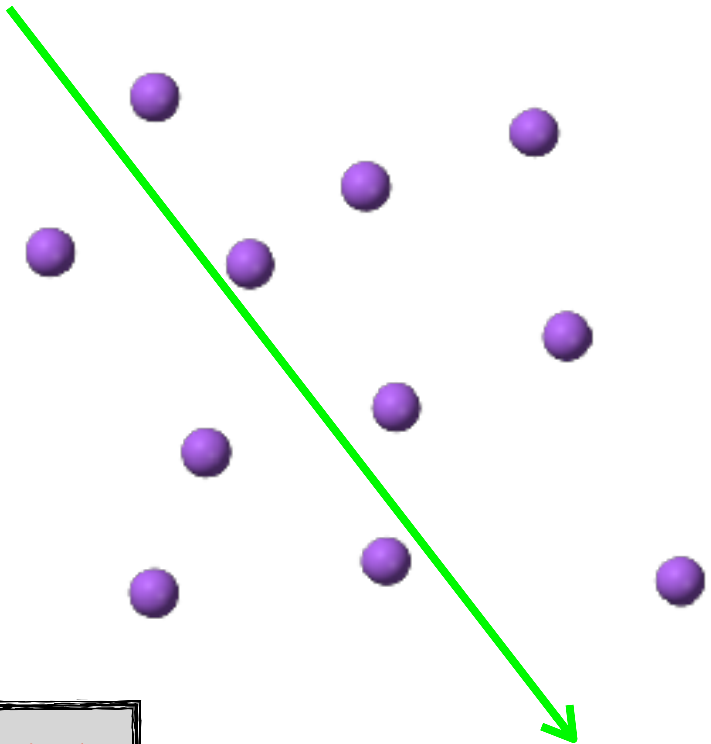

emissivity (power per unit frequency per unit solid angle)

ω -> ν = ω/2π

$$j(\nu) \approx \frac{n_e P}{(4\pi)} \frac{2\pi}{\omega} = \frac{n_e n_p e^6}{3c^3 m_e^2} \left(\frac{m_e}{kT}\right)^{1/2}$$

Thermal Bremsstrahlung

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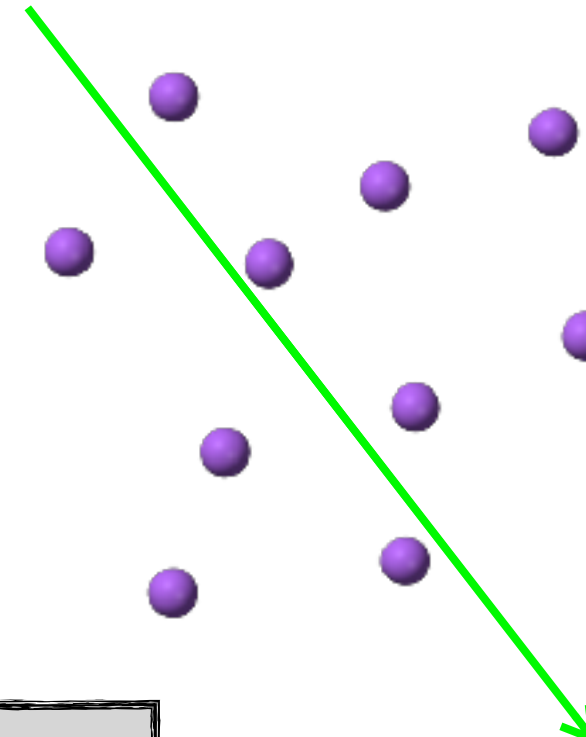

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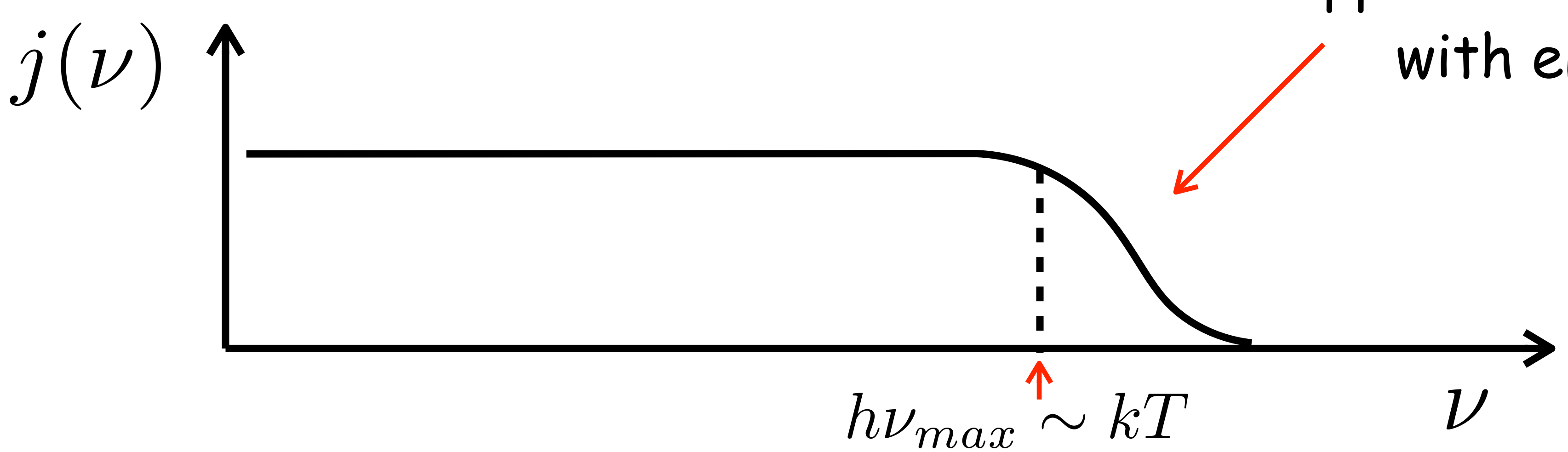
suppression -> no electrons with energy >> kT



Thermal Bremsstrahlung

exact solution

$$j(\nu) = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{n_e n_p e^6}{m_e^2 c^3} \left(\frac{m_e}{kT}\right)^{1/2} e^{-\frac{h\nu}{kT}} \bar{g}_{ff}$$

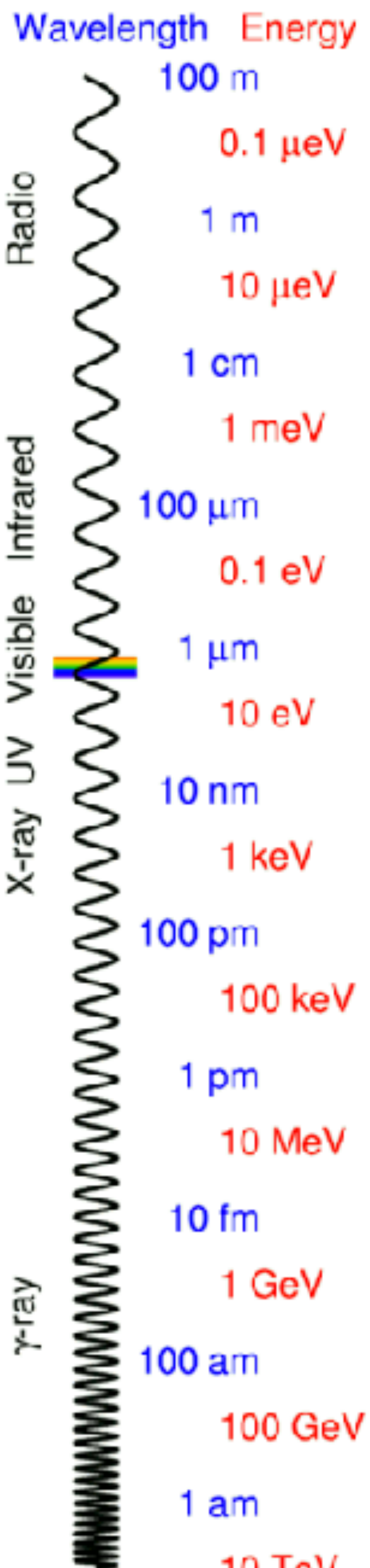
$$j = \int \mathrm{d}\nu j_\nu = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{n_e n_p e^6}{m_e^2 c^3} \frac{(m_e kT)^{1/2}}{h} \bar{g}_{ff}$$

in numbers (cgs units)

$$j(\nu) = 5.4 \times 10^{-39} Z^2 n_e n_i T^{-1/2} e^{-\frac{h\nu}{kT}} \bar{g}_{ff} \ \mathrm{erg/s/cm^3/Hz/sr}$$

$$j = 1.1 \times 10^{-28} Z^2 n_e n_i T^{1/2} \bar{g}_{ff} \ \mathrm{erg/s/cm^3/sr}$$

The electromagnetic spectrum



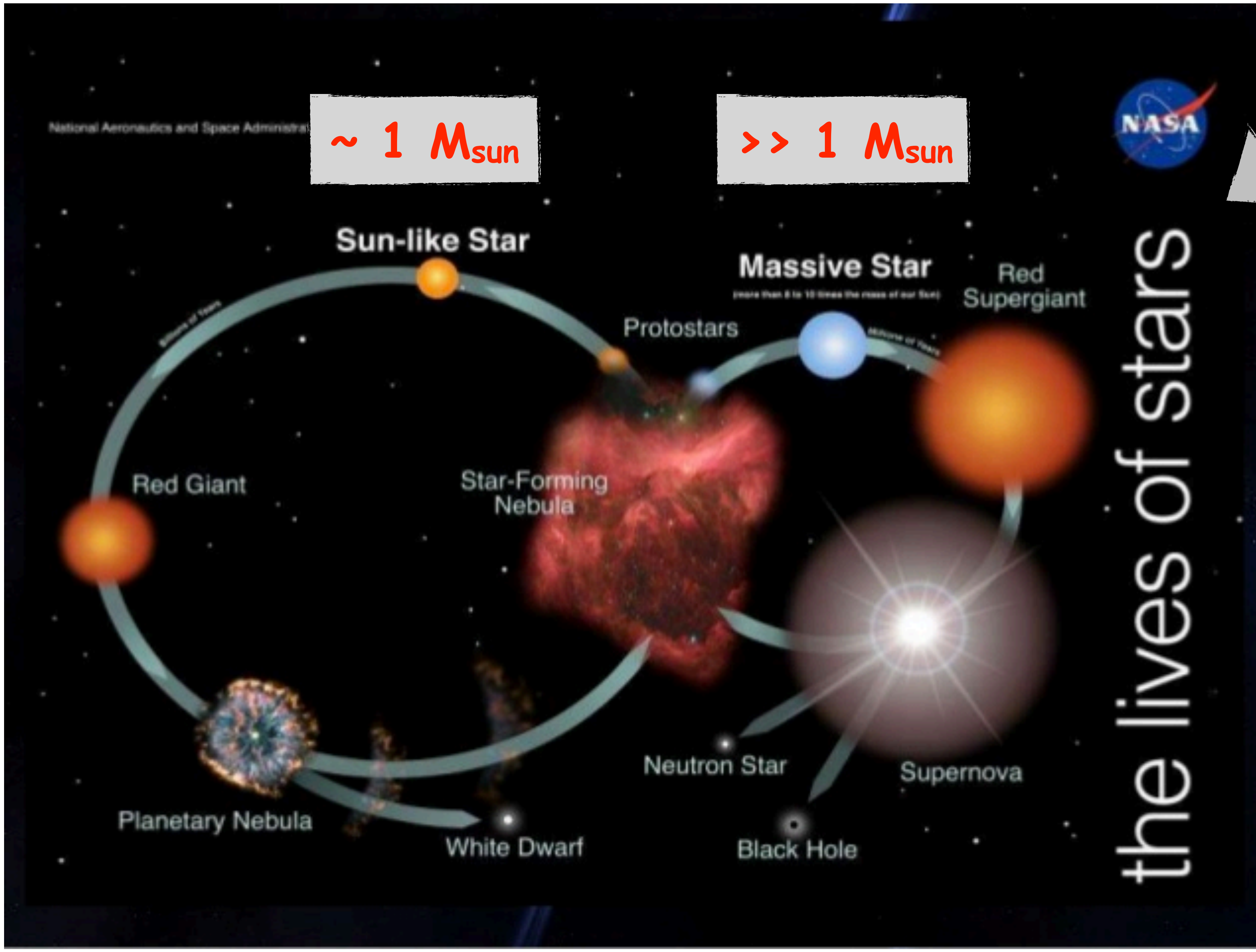
21 cm line -> atomic neutral Hydrogen (HI)

visible light -> stars

thermal emission from hot plasmas (millions of K)

interactions of cosmic ray protons with matter

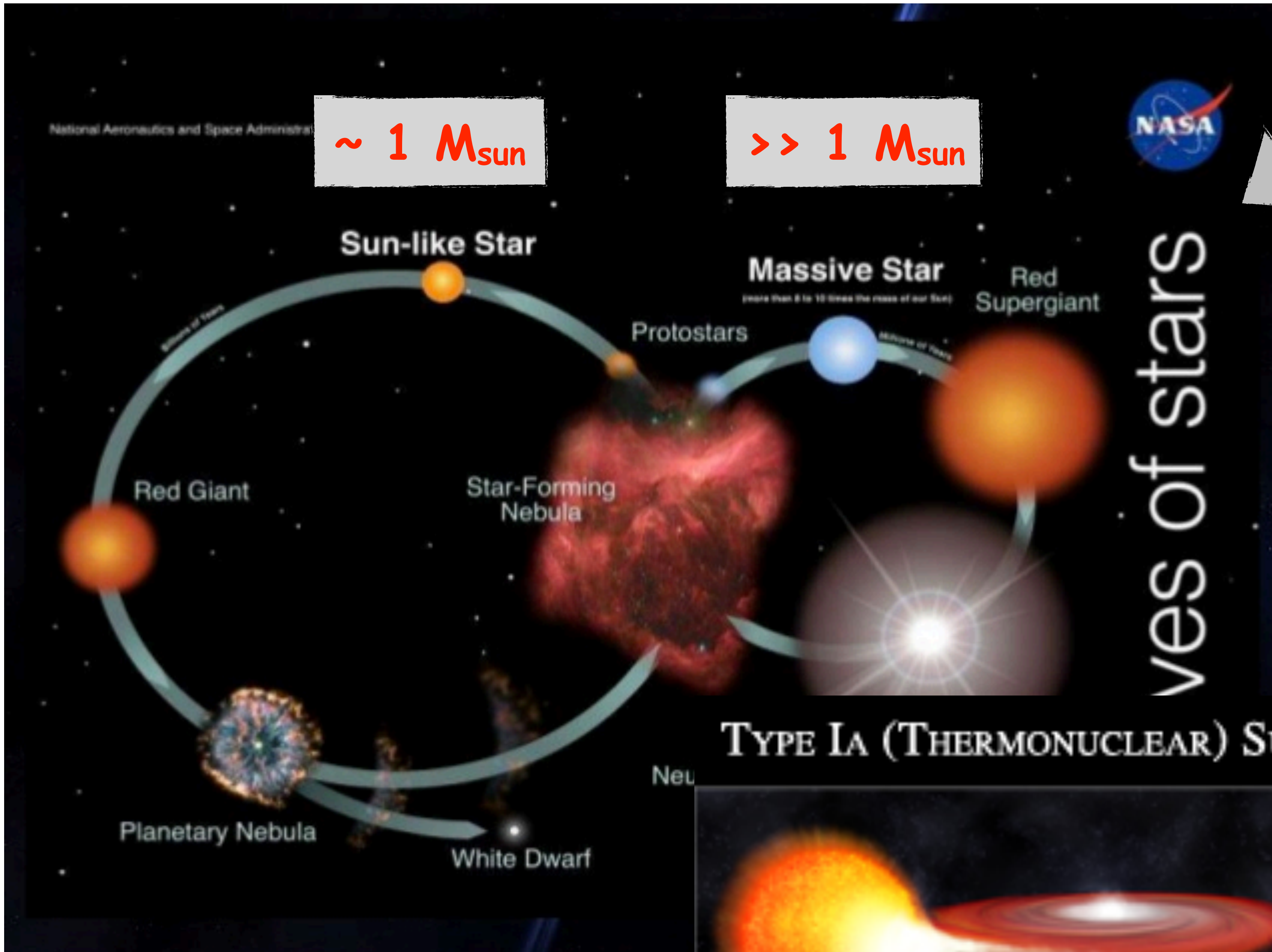
Thermonuclear & core-collapse supernovae



core-collapse

$M_{ej} \sim \text{several } M_\odot$

Thermonuclear & core-collapse supernovae



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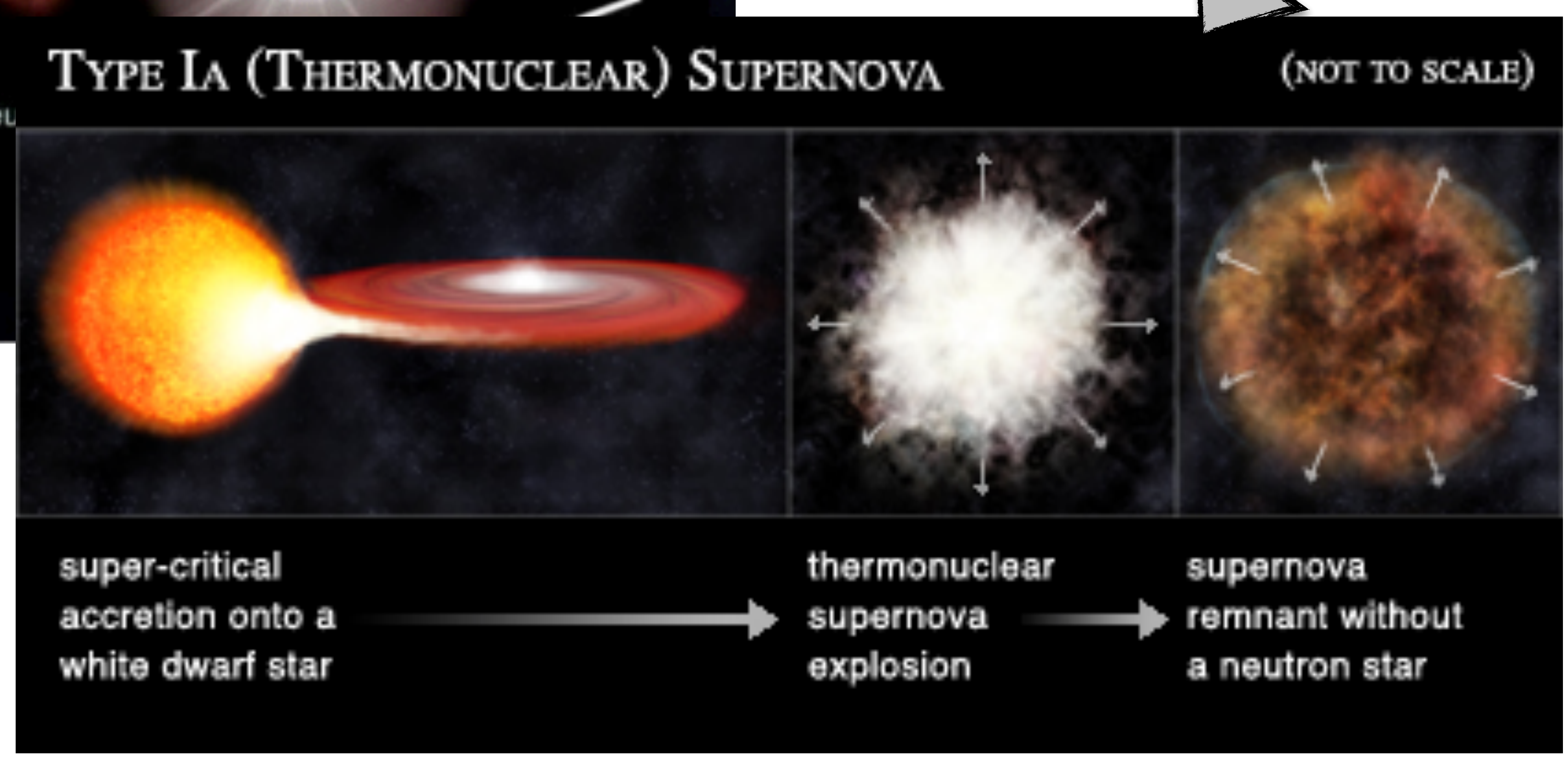
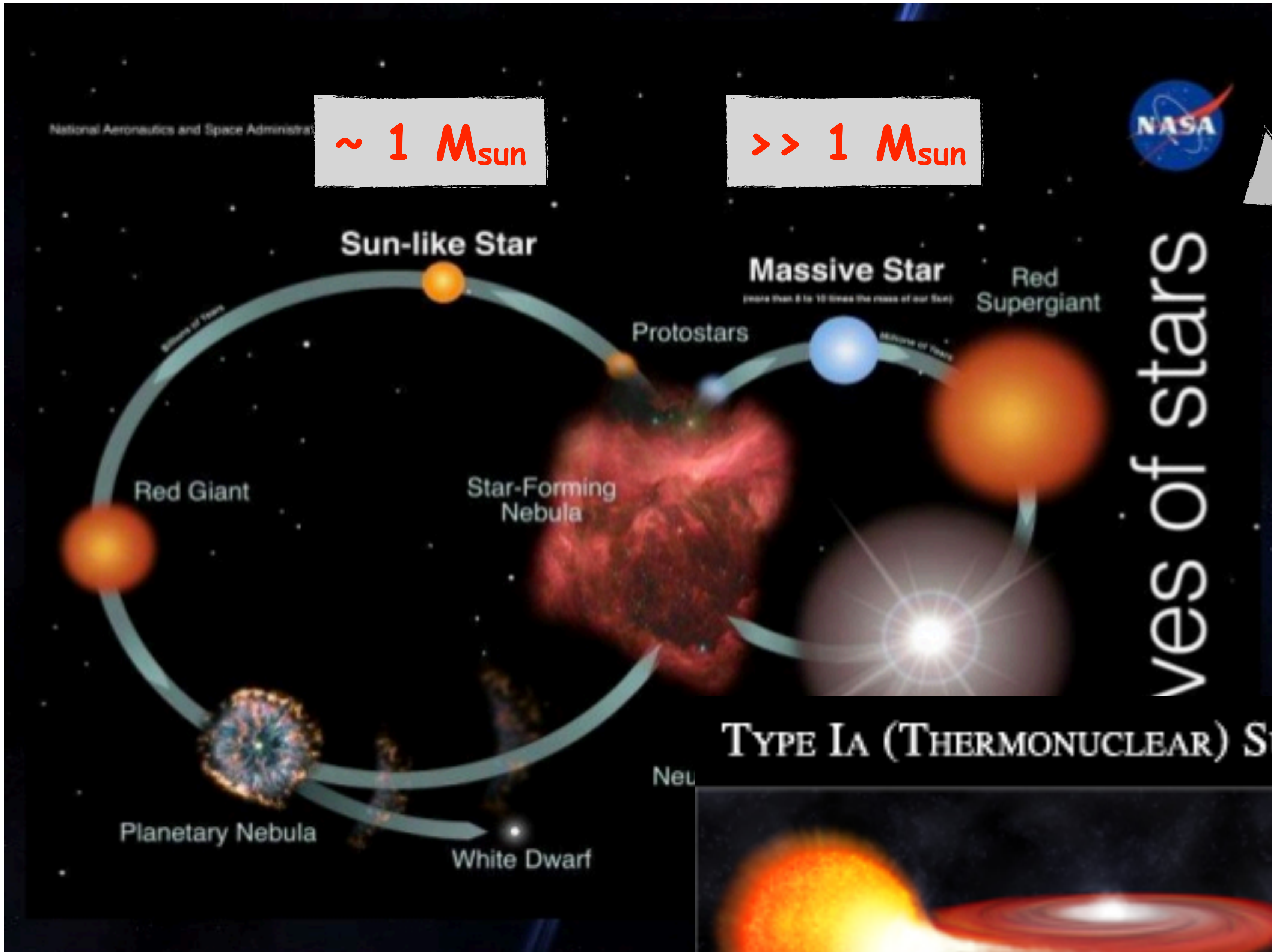
$$M_{ej} \sim \text{several } M_\odot$$

$$M_{ej} \sim 1.4 M_\odot$$

thermonuclear



Thermonuclear & core-collapse supernovae



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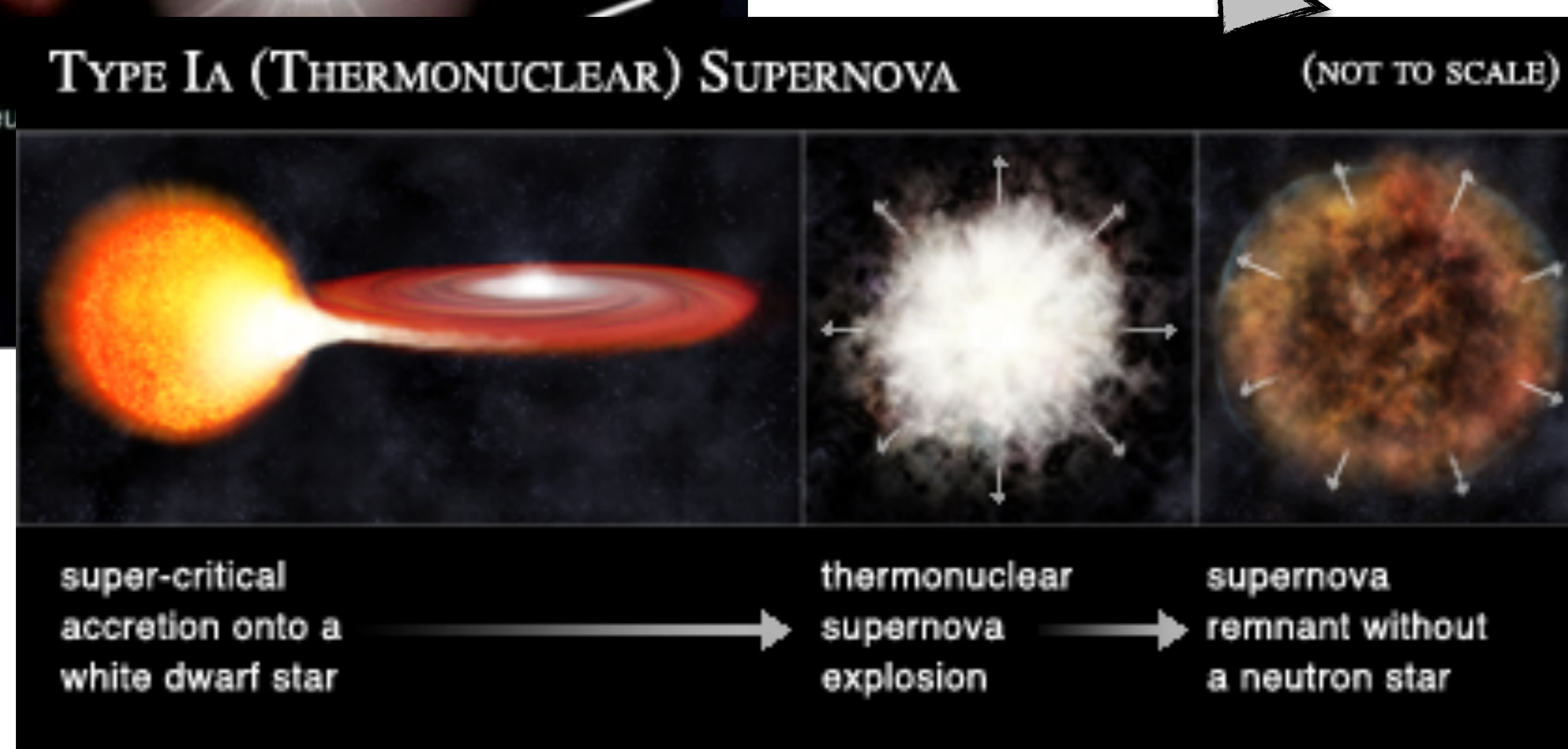

explosion energy

$E_{SN} \sim 10^{51} \text{erg}$

Dynamical evolution of supernova remnants
Free expansion phase

an amount of matter M_{ej} is ejected with velocity v_0 and kinetic energy E_{SN}

Dynamical evolution of supernova remnants
Free expansion phase

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$$E_{SN} = \frac{1}{2} M_{ej} v_0^2 \longrightarrow v_0 = 10000 \left(\frac{E_{SN}}{10^{51}\ \mathrm{erg}} \right)^{1/2} \left(\frac{M_{ej}}{M_\odot} \right)^{-1/2}\ \mathrm{km/s}$$

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sound speed in the ISM $\quad c_s = \left(\gamma \frac{kT}{m_p} \right)^{1/2} \approx 10 \left(\frac{T}{10^4 \ \mathrm{K}} \right)^{1/2} \ \mathrm{km/s}$

$$\mathcal{M} \approx 100 \Longrightarrow \boxed{\text{shock wave}}$$

Dynamical evolution of supernova remnants
Free expansion phase

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$$\mathcal{M} \approx 100$$ $\Rightarrow$ **shock wave**

density of the interstellar medium

$$\begin{cases} v_s & = & v_0 \\ R_s & = & v_0 t \end{cases}$$ as long as $$M_{ej} \gg M_{sw} = \frac{4\pi}{3} R_s^3 \varrho_0$$

mass of swept up medium

Dynamical evolution of supernova remnants
Free expansion phase

density of the interstellar medium

$$\begin{cases} v_s &= v_0 \\ R_s &= v_0 t \end{cases}$$

as long as

$$M_{ej} \ll M_{sw} = \frac{4\pi}{3} R_s^3 \varrho_0$$

mass of swept up medium

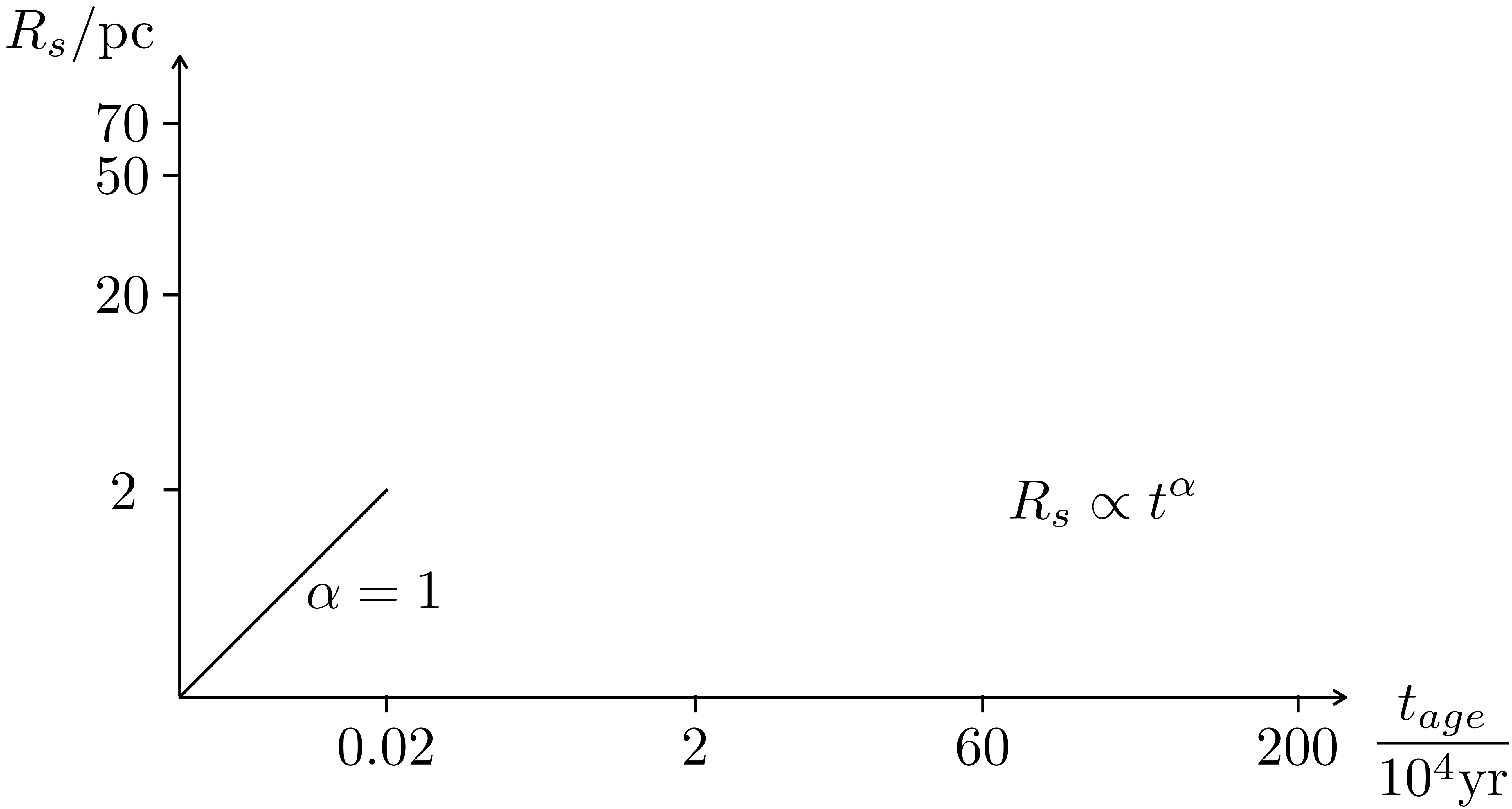
$$M_{ej} \approx M_{sw} \longrightarrow R_{ej} \sim 2 \left(\frac{M_{ej}}{M_\odot}\right)^{1/3} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}}\right)^{-1/3} \mathrm{pc}$$

$$v_{ej} \approx \frac{R_{ej}}{v_0} \sim 2 \times 10^2 \left(\frac{M_{ej}}{M_\odot}\right)^{5/6} \left(\frac{E_{SN}}{10^{51}\mathrm{erg}}\right)^{-1/2} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}}\right)^{-1/3} \mathrm{yr}$$

Time evolution of a SNR

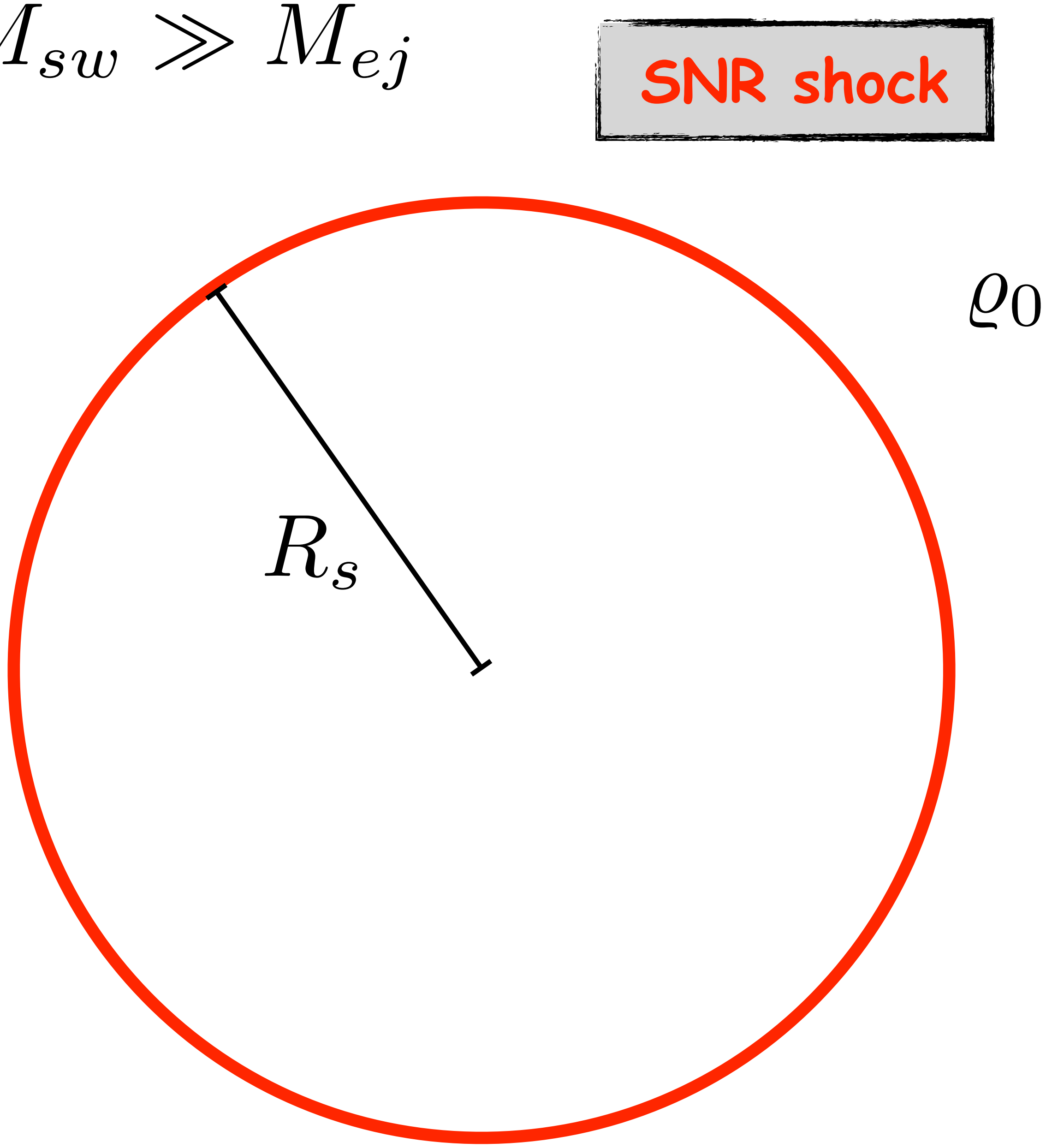
$$M_{sh}/M_\odot \approx \quad 1$$

$$\frac{v_s}{10^3 \text{ km/s}} \approx \quad 10$$



The thin shell approximation

$M_{sw} \gg M_{ej}$



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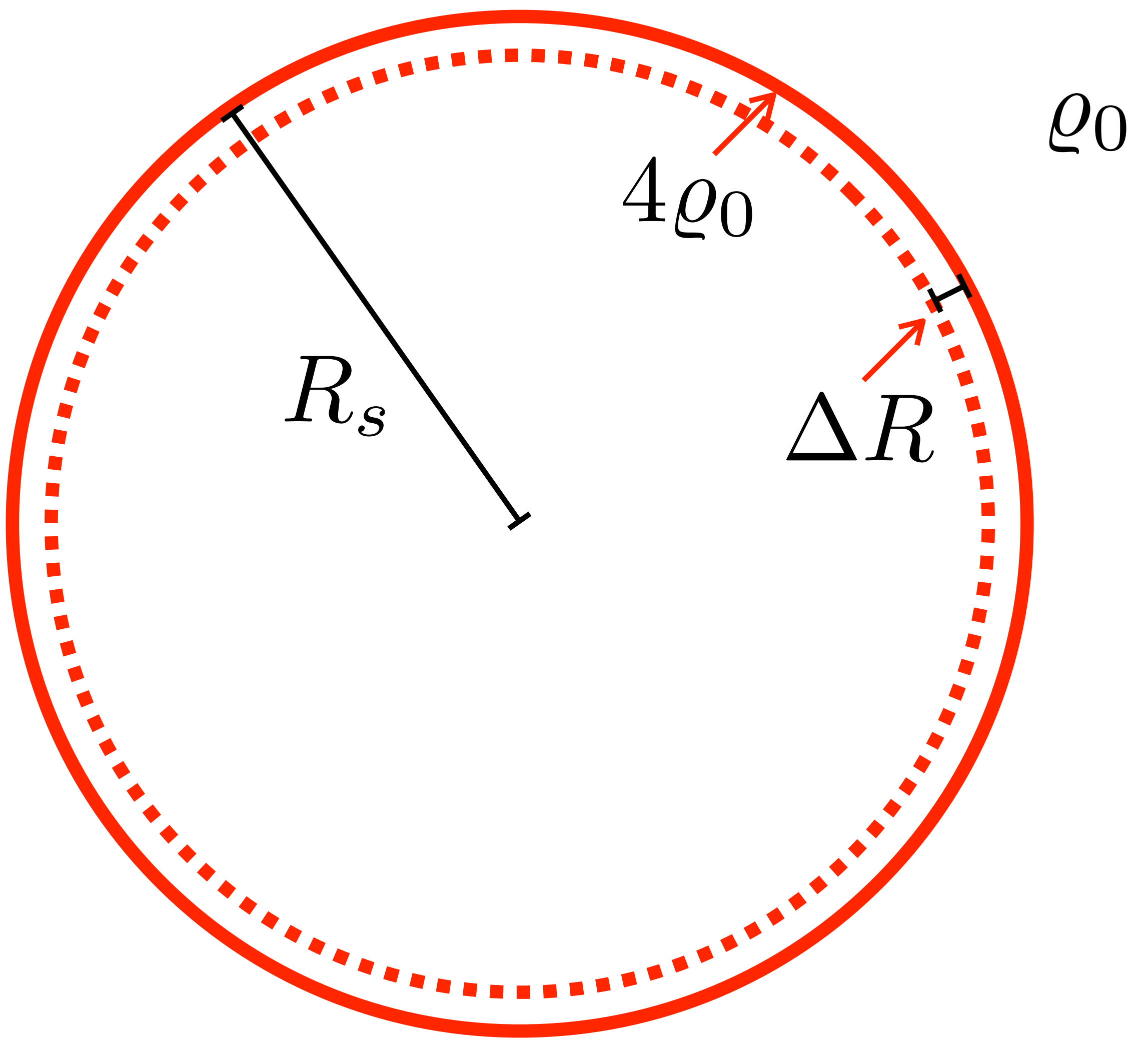
SNR shock



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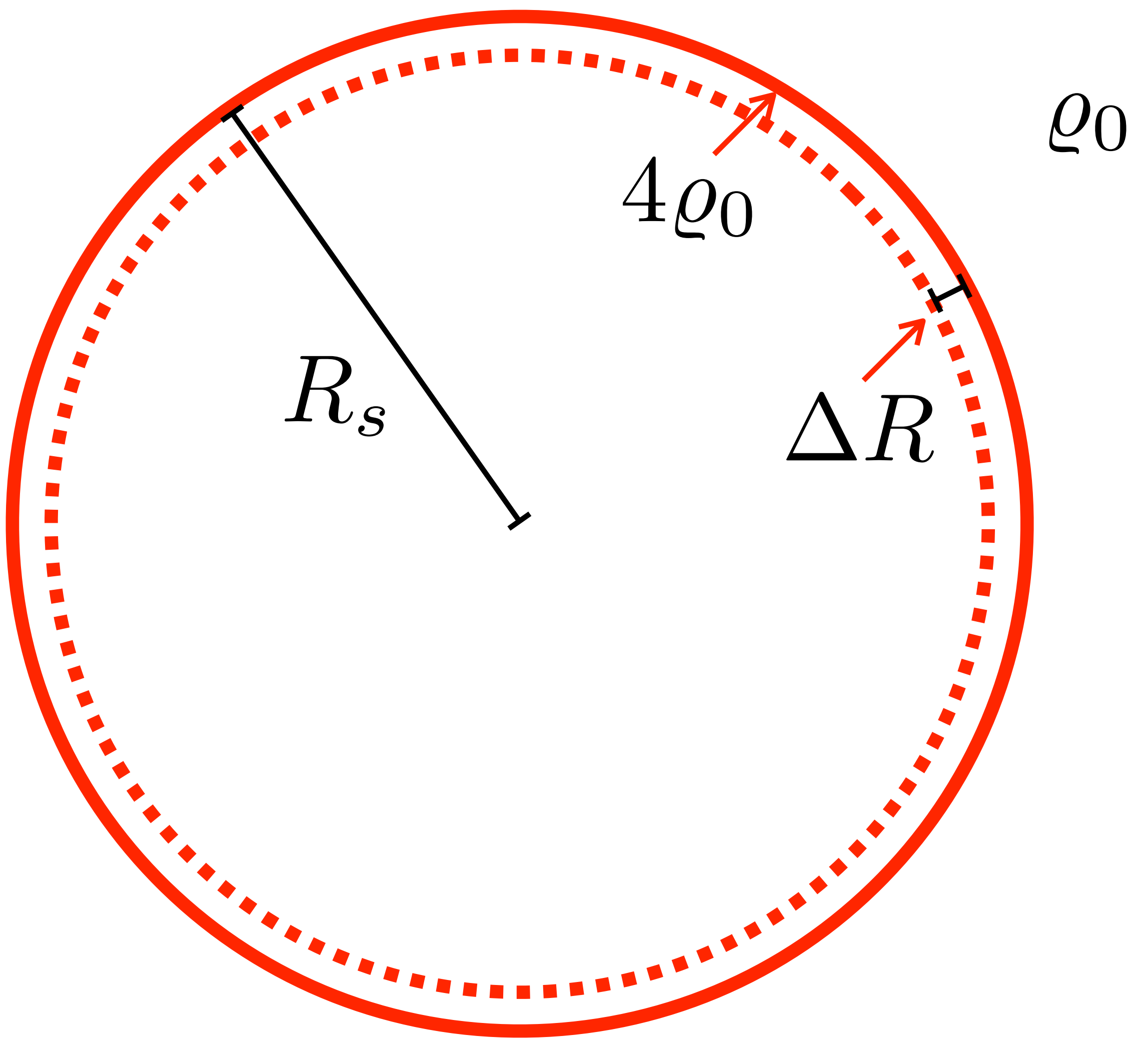


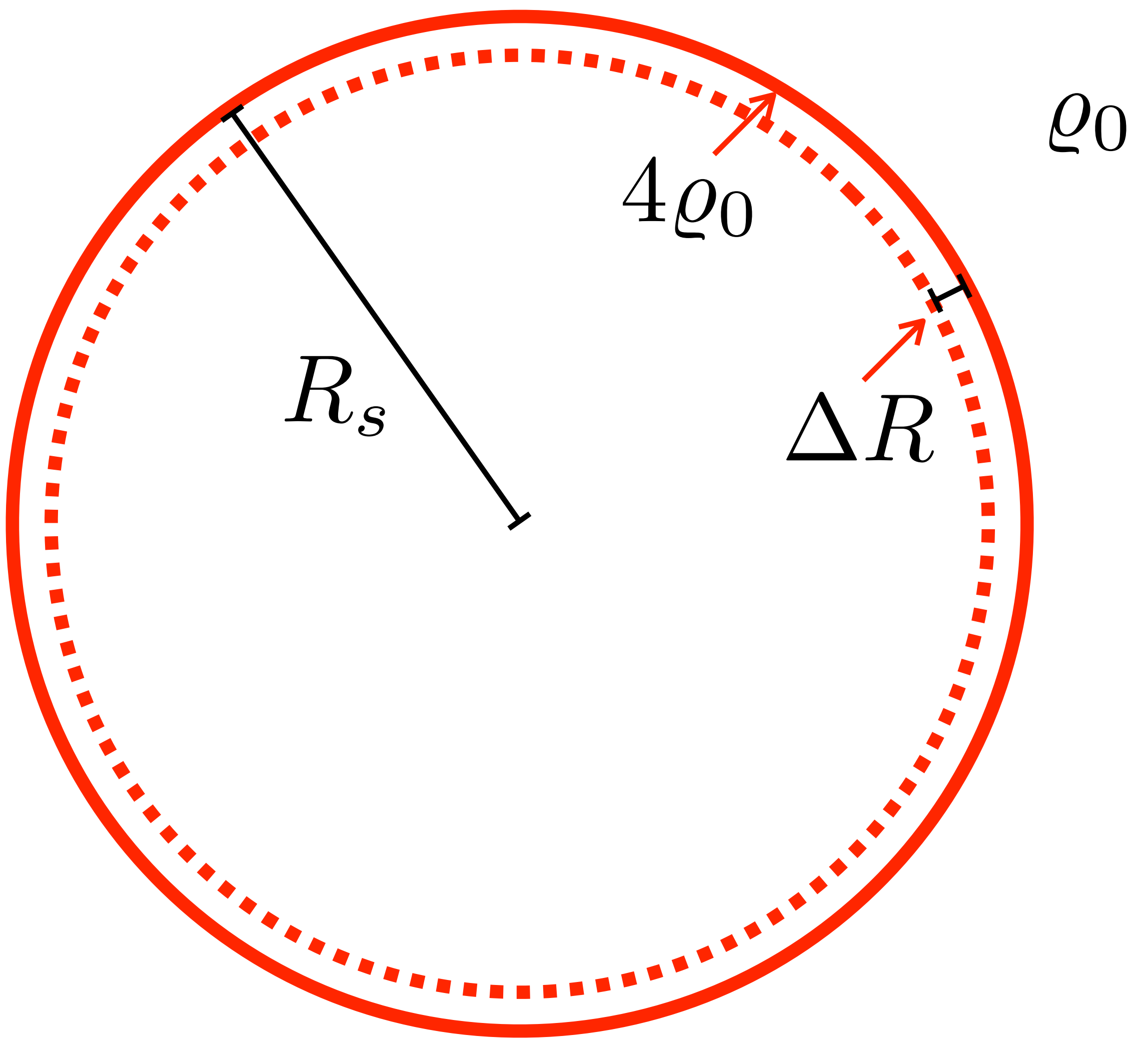
$$\frac{4\pi}{3} R_s^3 \varrho_0 = 4\pi R_s^2 \Delta R (4\varrho_0) \longrightarrow \frac{\Delta R}{R_s} = \frac{1}{12} \approx 0.08$$

thin shell

The thin shell approximation

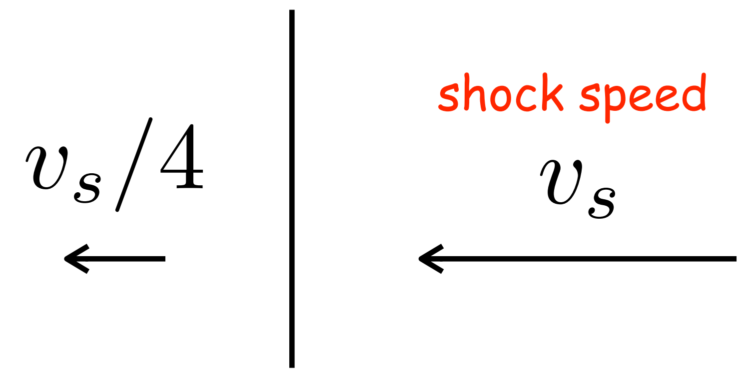
$M_{sw} \gg M_{ej}$

SNR shock



velocity of the shell

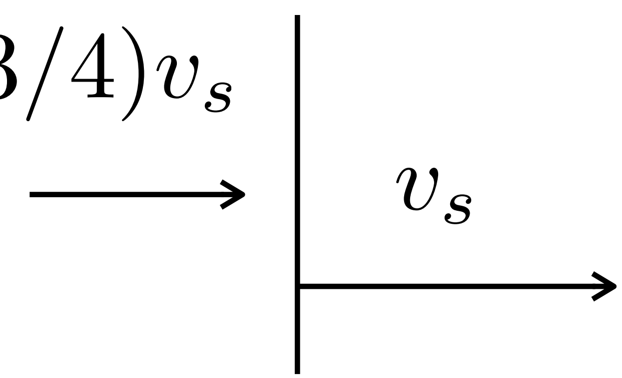
shock rest frame



lab rest frame

shell speed

$v_{sh} = (3/4)v_s$

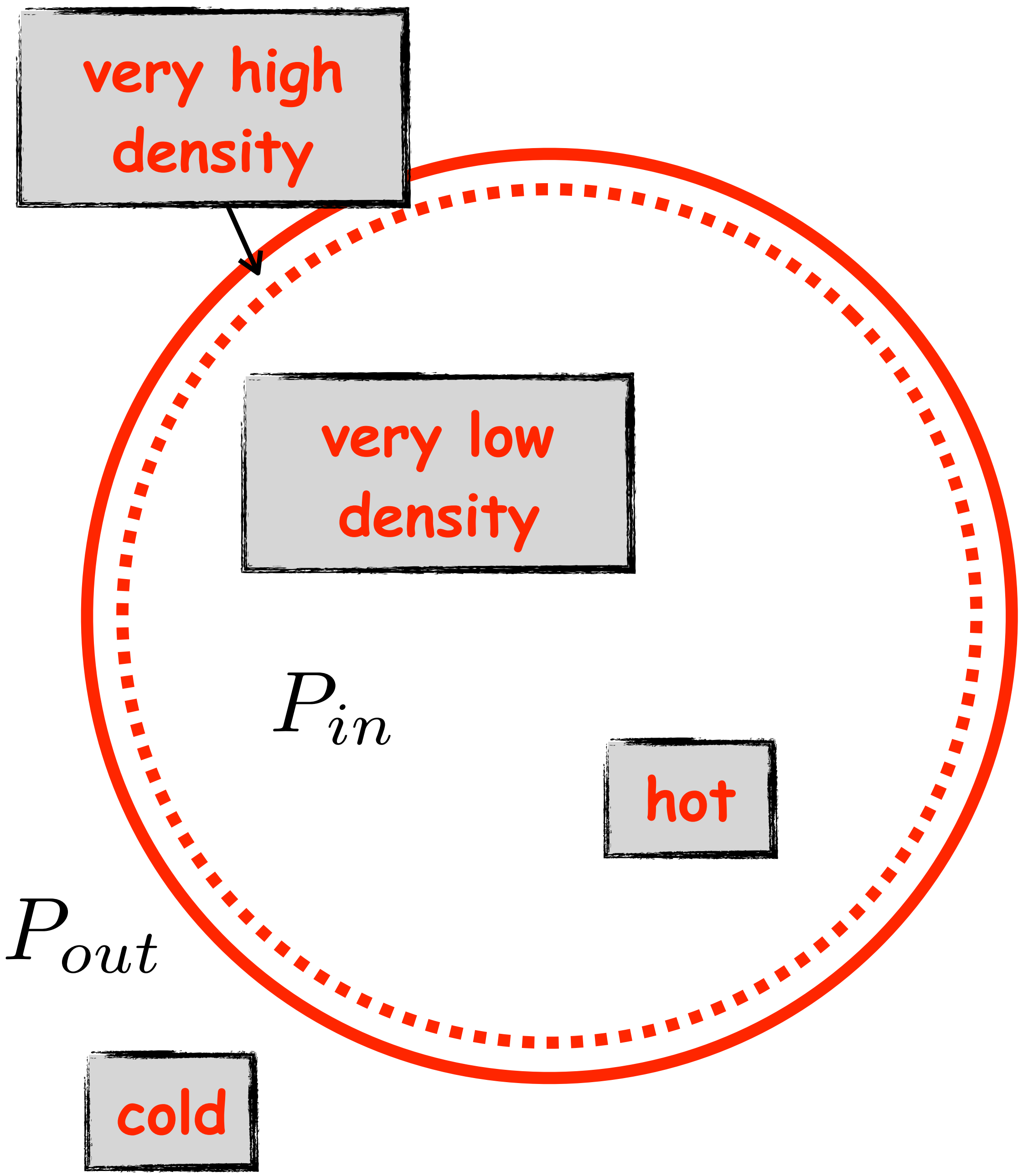


$$\frac{4\pi}{3} R_s^3 \varrho_0 = 4\pi R_s^2 \Delta R (4\varrho_0) \longrightarrow \frac{\Delta R}{R_s} = \frac{1}{12} \approx 0.08$$

thin shell

The thin shell approximation

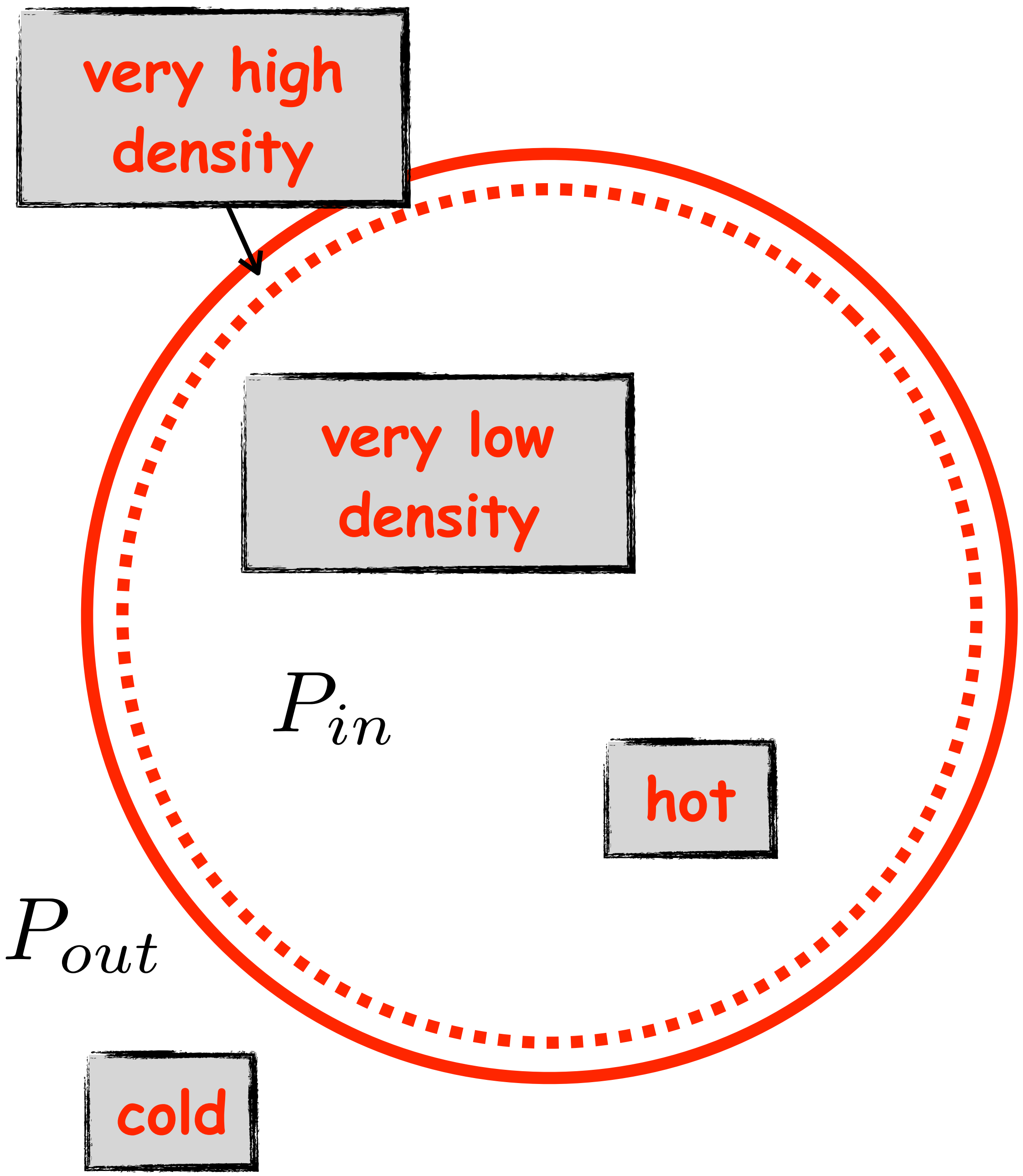
the mass is concentrated in an infinitesimally small shell behind the shock



The thin shell approximation

the mass is concentrated in an infinitesimally small shell behind the shock

mass inside the shell $$M = M_{ej} + 4\pi \int_0^{R_s} \mathrm{d}R\ R^2 \varrho_0 \approx \frac{4\pi}{3} R_s^3 \varrho_0$$



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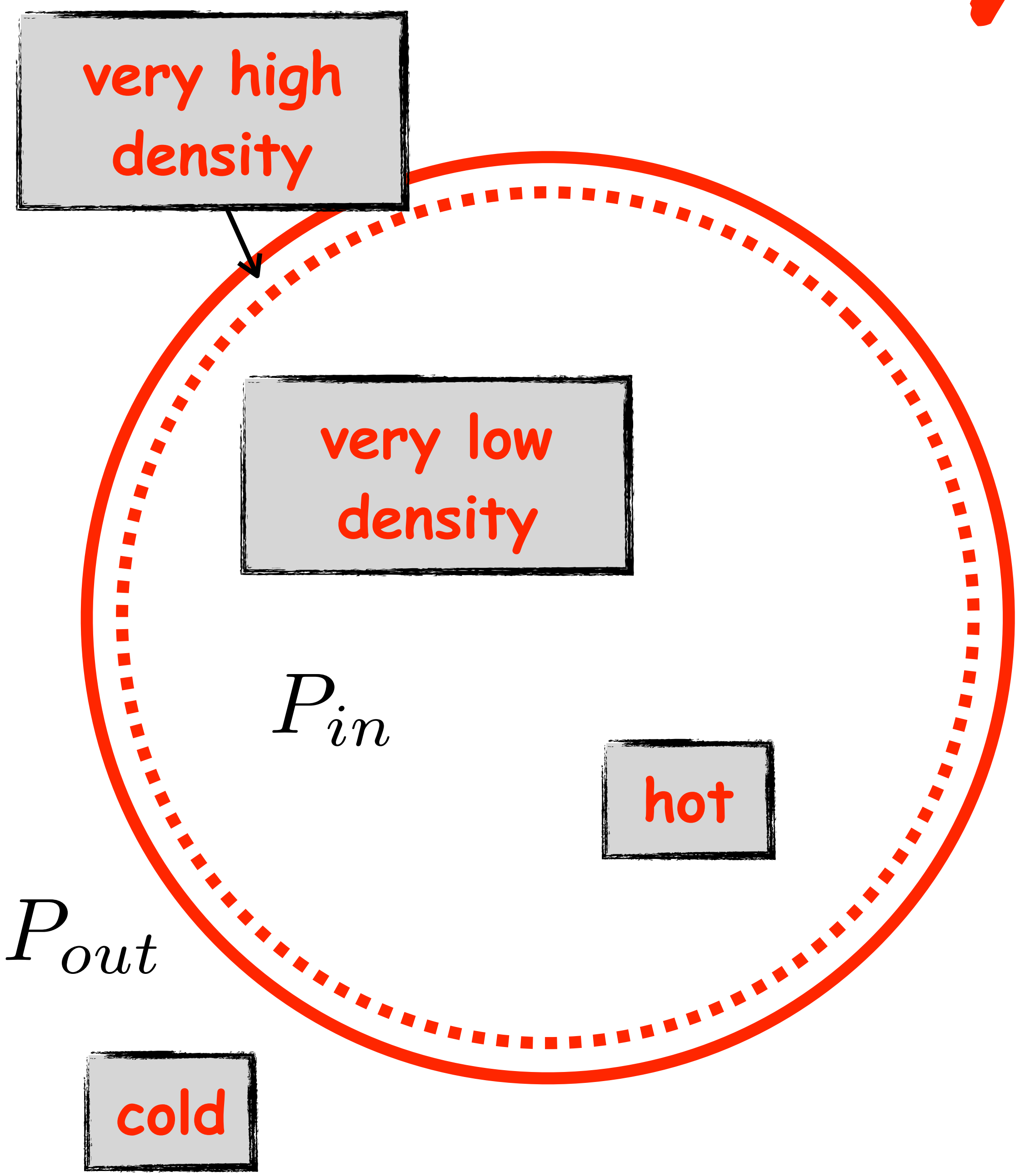

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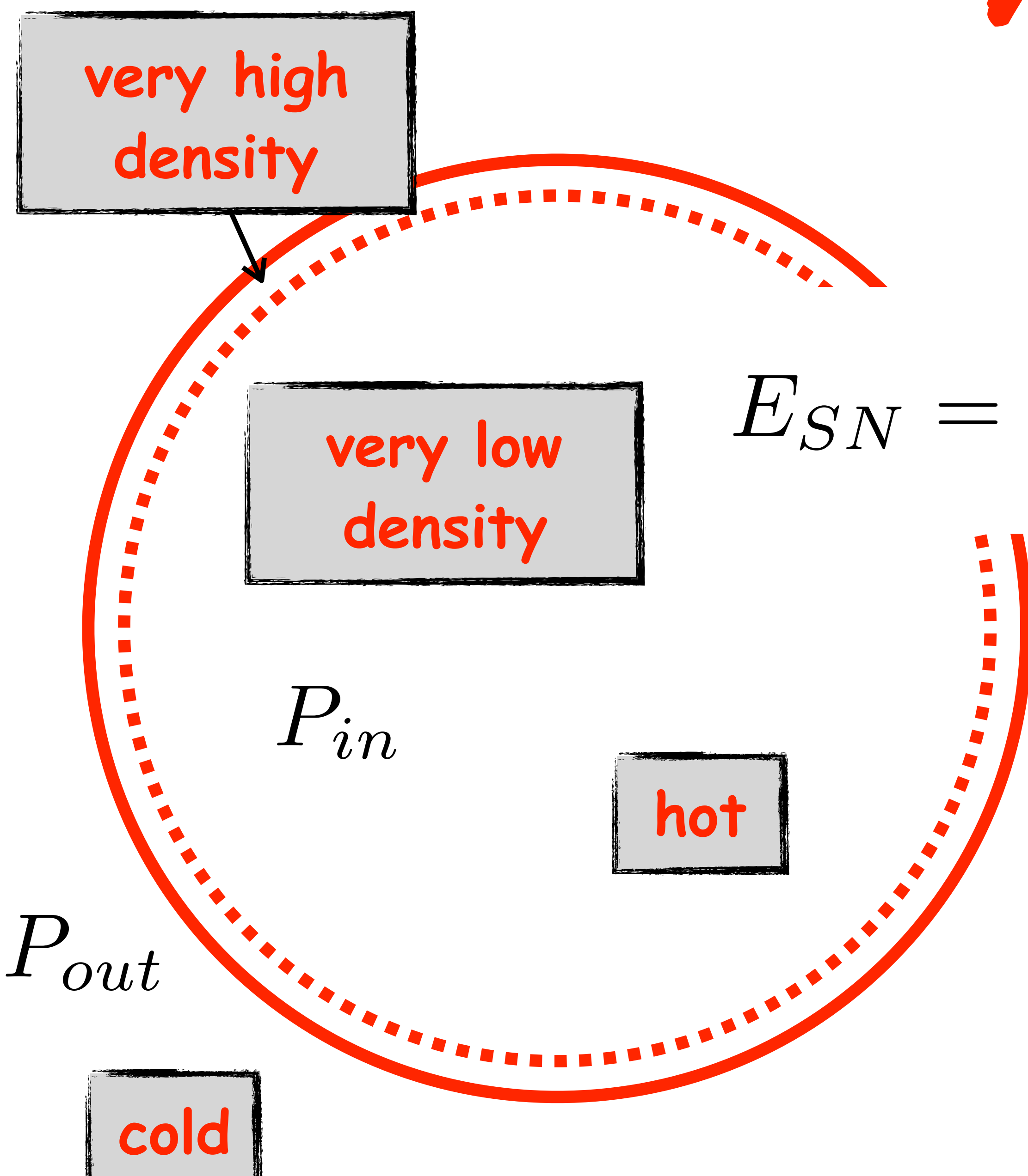

energy conservation

$$E_{SN} = E_k + E_{th} = \frac{1}{2} M v_{sh}^2 + \frac{P_{in}}{\gamma - 1} \frac{4\pi}{3} R_s^3$$

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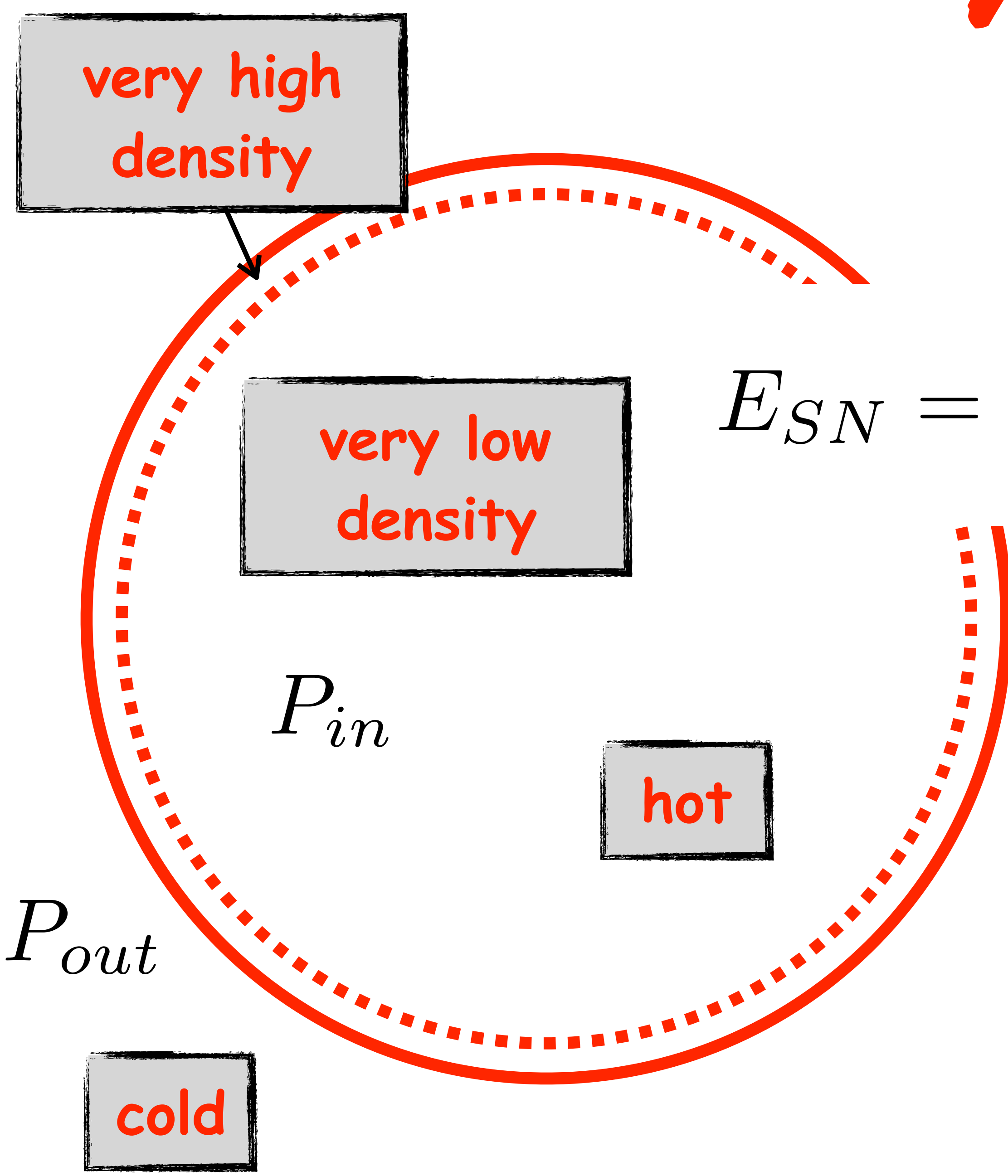

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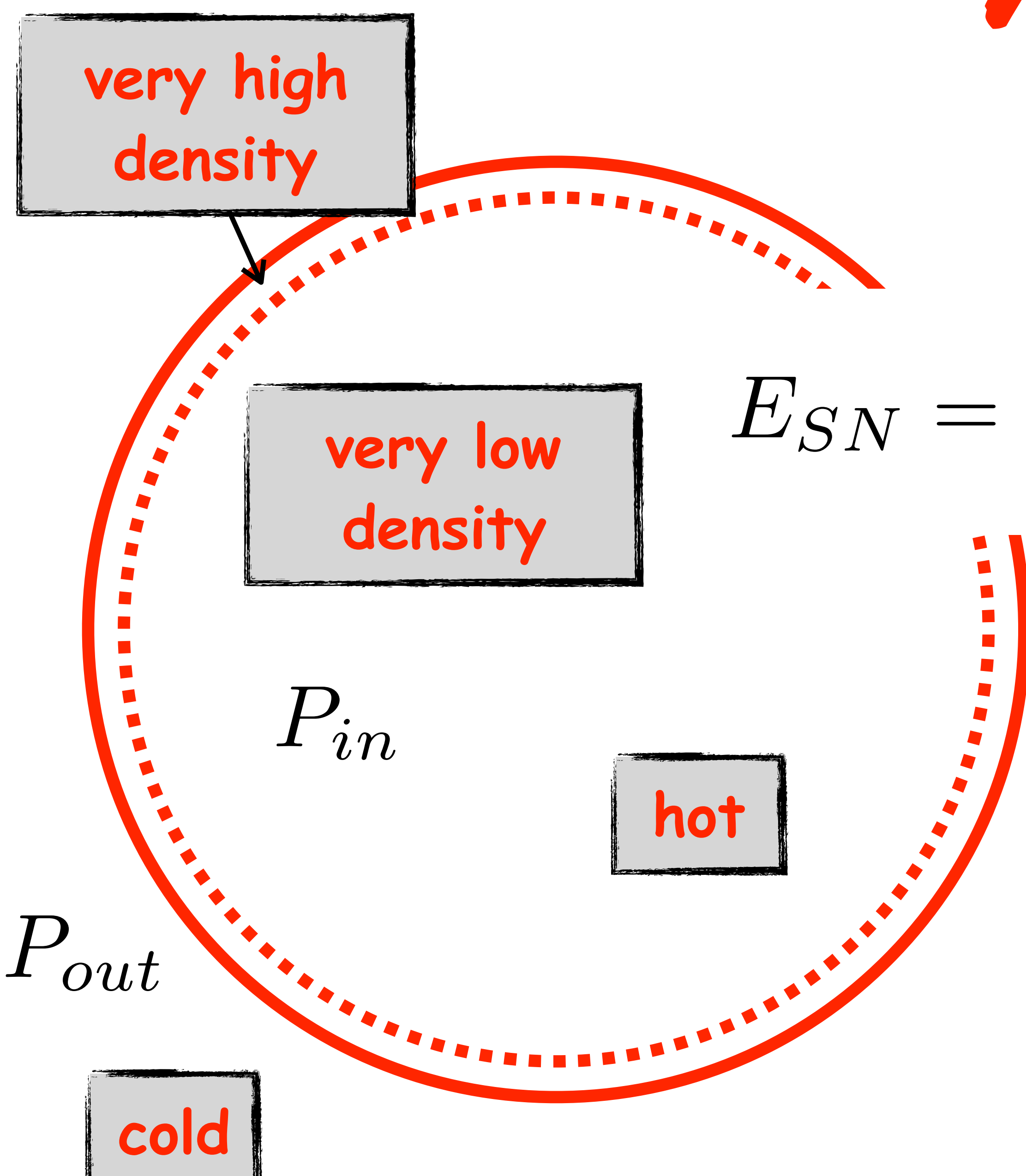

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strong shock

Solution of the equations

Let's search for power law solutions...

$$R_s = A\ t^{\alpha} \longrightarrow v_s = \frac{\mathrm{d}R_s}{\mathrm{d}t} = \frac{\alpha R_s}{t} \propto t^{\alpha-1}$$

$$M = \frac{4\pi}{3}\varrho_0 R_s^3 \qquad \gamma = \frac{5}{3} \qquad v_{sh} = \frac{3}{4}v_s$$

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$$\longrightarrow E_{SN} = \frac{\pi}{8}\varrho_0 A^5 \alpha(19\alpha-4)\ t^{5\alpha-2}$$

The Sedov (adiabatic) phase

adiabatic SNR

$$E_{SN} = \mathrm{const} \rightarrow E_{SN} \propto t^{5\alpha-2} \rightarrow \alpha = \frac{2}{5}$$

$$\rightarrow A = \left(\frac{50}{9\pi} \frac{E_{SN}}{\varrho_0} \right)^{1/5}$$

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$$\begin{cases} R_s \sim 5 \left(\frac{E_{SN}}{10^{51}\ \mathrm{erg}} \right)^{1/5} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-1/5} \left(\frac{t}{\mathrm{kyr}} \right)^{2/5} \quad \mathrm{pc} \\ u_s \sim 2 \times 10^3 \left(\frac{E_{SN}}{10^{51}\ \mathrm{erg}} \right)^{1/5} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-1/5} \left(\frac{t}{\mathrm{kyr}} \right)^{-3/5} \quad \mathrm{km/s} \end{cases}$$

Duration of the Sedov phase

Bremsstrahlung +
(mainly) lines emissions

cooling function

$$10^5 \lesssim T \lesssim 10^{7.5} \longrightarrow \Lambda \approx 2 \times 10^{-19} T^{-1/2} \mathrm{erg\ cm^3 s^{-1}}$$

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$$\tau_c \sim \frac{\epsilon_{th}}{n_i n_e \Lambda} = \frac{3nkT}{n^2 \Lambda}$$

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$$T \sim 2 \times 10^7 \left(\frac{u_s}{1000\ \mathrm{km/s}} \right)^2\ \mathrm{K}$$

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the shell becomes radiative at an age:

$$\tau_c \approx t_{age} \longrightarrow t_{age} \approx 2 \times 10^4 \left(\frac{E}{10^{51}\mathrm{erg}}\right)^{3/14} \left(\frac{n_{ISM}}{\mathrm{cm^{-3}}}\right)^{-4/7} \ \mathrm{yr}$$

Duration of the Sedov phase

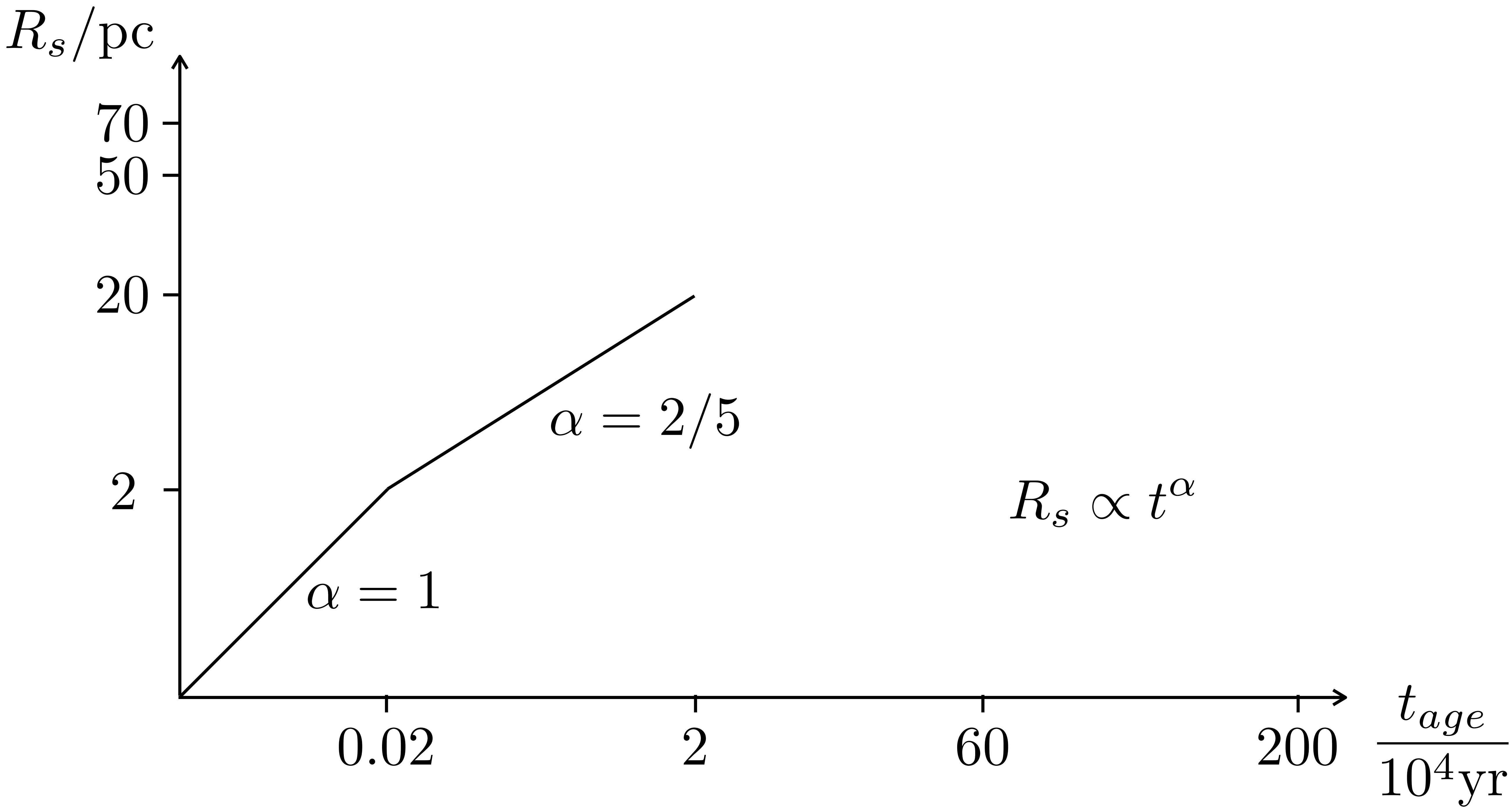
$$t_{ad} \approx 2 \times 10^4 \left(\frac{E_{SN}}{10^{51}\mathrm{erg}} \right)^{3/14} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-4/7} \mathrm{yr}$$

$$R_{ad} \approx 20 \left(\frac{E_{SN}}{10^{51}\mathrm{erg}} \right)^{2/7} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{-3/7} \mathrm{pc}$$

$$v_{ad} \approx 3 \times 10^2 \left(\frac{E_{SN}}{10^{51}\mathrm{erg}} \right)^{1/14} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}} \right)^{1/7} \mathrm{km/s}$$

Time evolution of a SNR

$M_{sh}/M_\odot \approx$	1	10^3
$\dfrac{v_s}{10^3\ \mathrm{km/s}} \approx$	10	0.3



Radiative (isothermal) shocks

as a particular case we consider a shock where radiative losses are so effective to have
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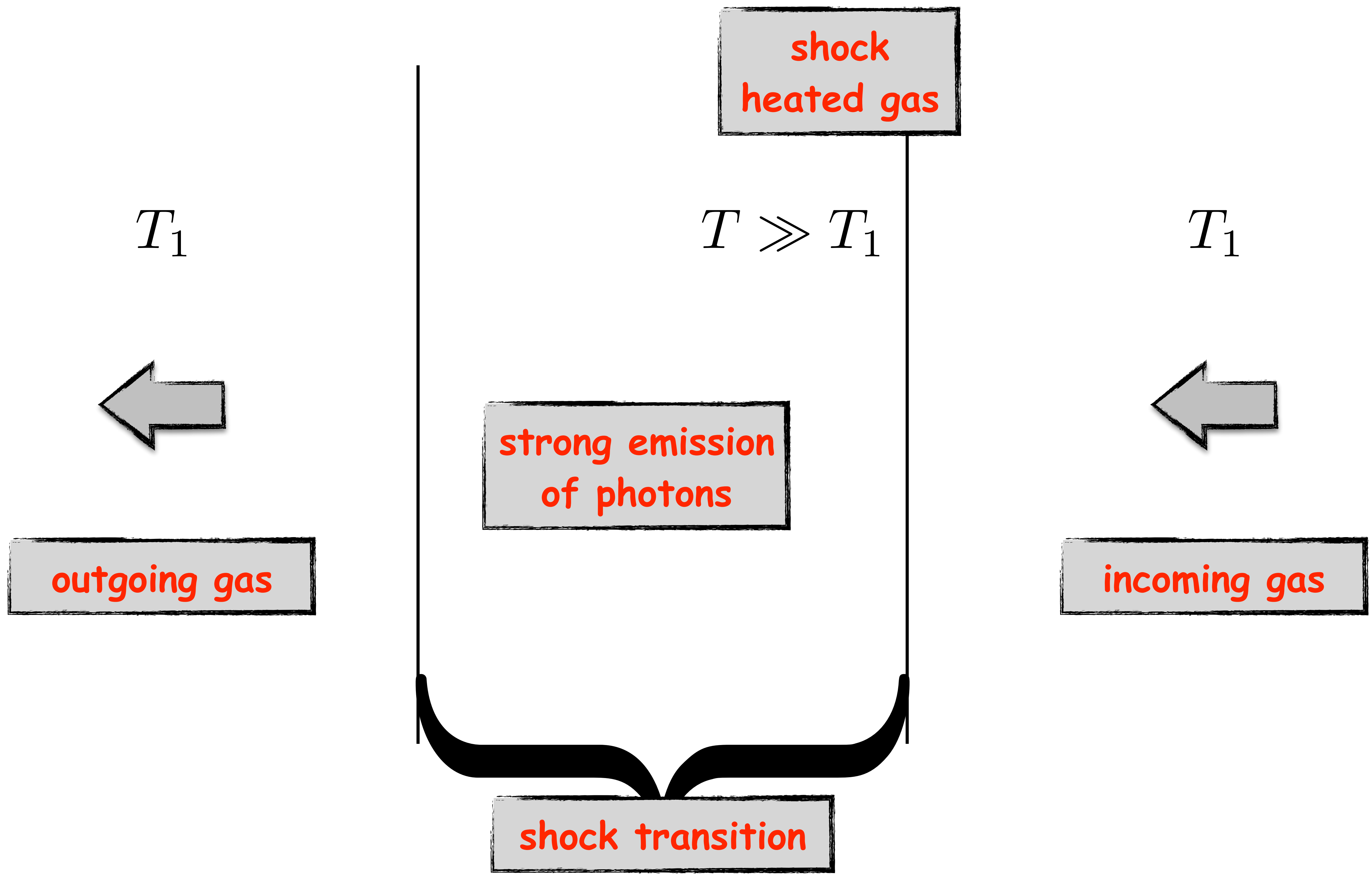

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strong shock

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unphysical

Radiative (isothermal) shocks

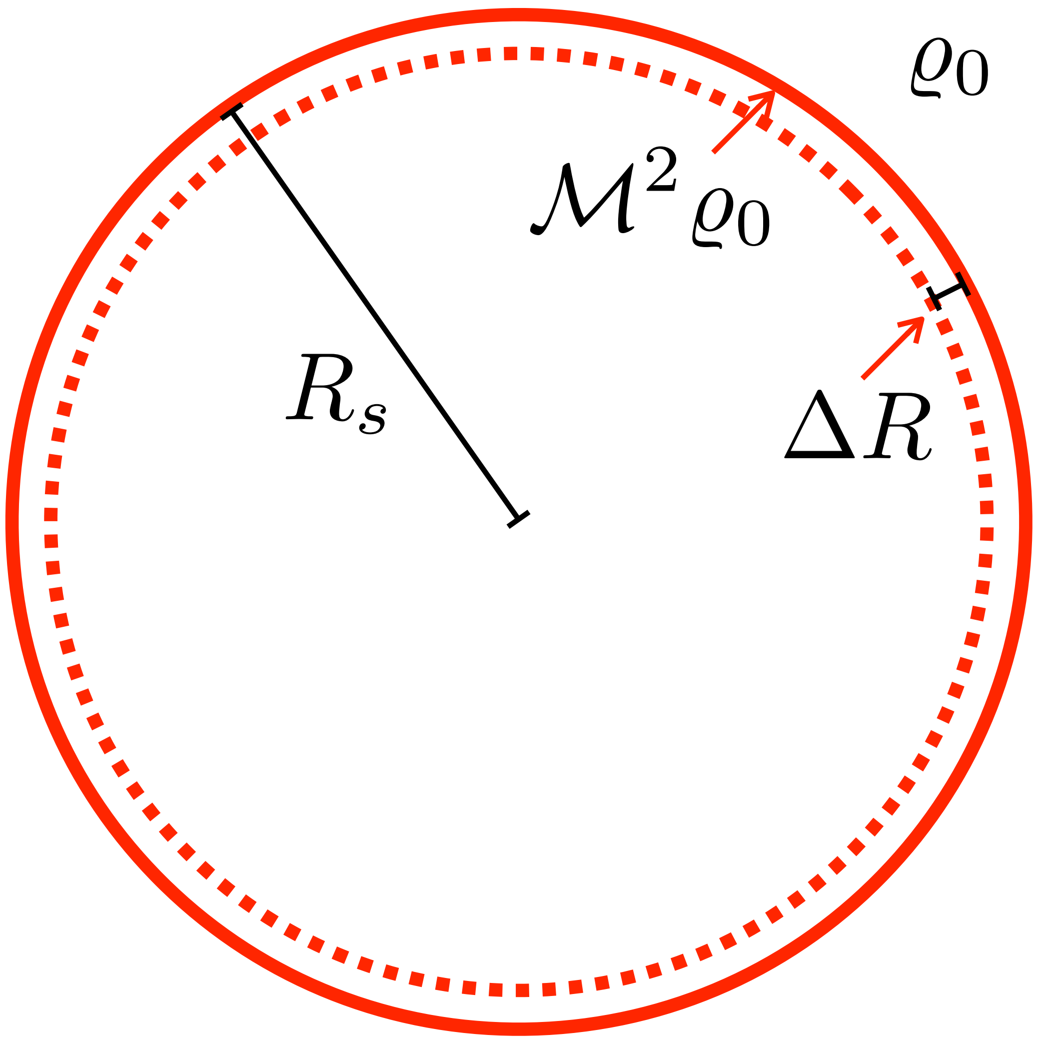
summary

$$u_2 = \frac{c_s^2}{u_1} = \frac{u_1}{\mathcal{M}^2} \xrightarrow{\mathcal{M}\to\infty} 0$$

$$r = \frac{\varrho_2}{\varrho_1} = \frac{u_1}{u_2} = \mathcal{M}^2 \xrightarrow[\mathcal{M}\to\infty]{} \infty$$

for strong shocks a very thin and dense shell forms, matter in the shell moves roughly at the same velocity of the shock -> the thin shell approximation is even more justified for radiative shocks!

The pressure driven snowplough phase



at first, the dense shell cools, while the interior does not (due to its very low density) -> all the energy dissipated at the shock is radiated away, and the SNR interior cools adiabatically

$$P_{in}V^{\gamma} = const \rightarrow P_{in} \propto R_s^{-3\gamma}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(Mv_s) = 4\pi R_s^2 P_{in}$$

$$R_s \propto t^{\alpha} \longrightarrow \alpha = \frac{2}{7}$$

Momentum conserving snowplough

$$n_{in} = \epsilon \; n_{sh}$$

density in the SNR interior

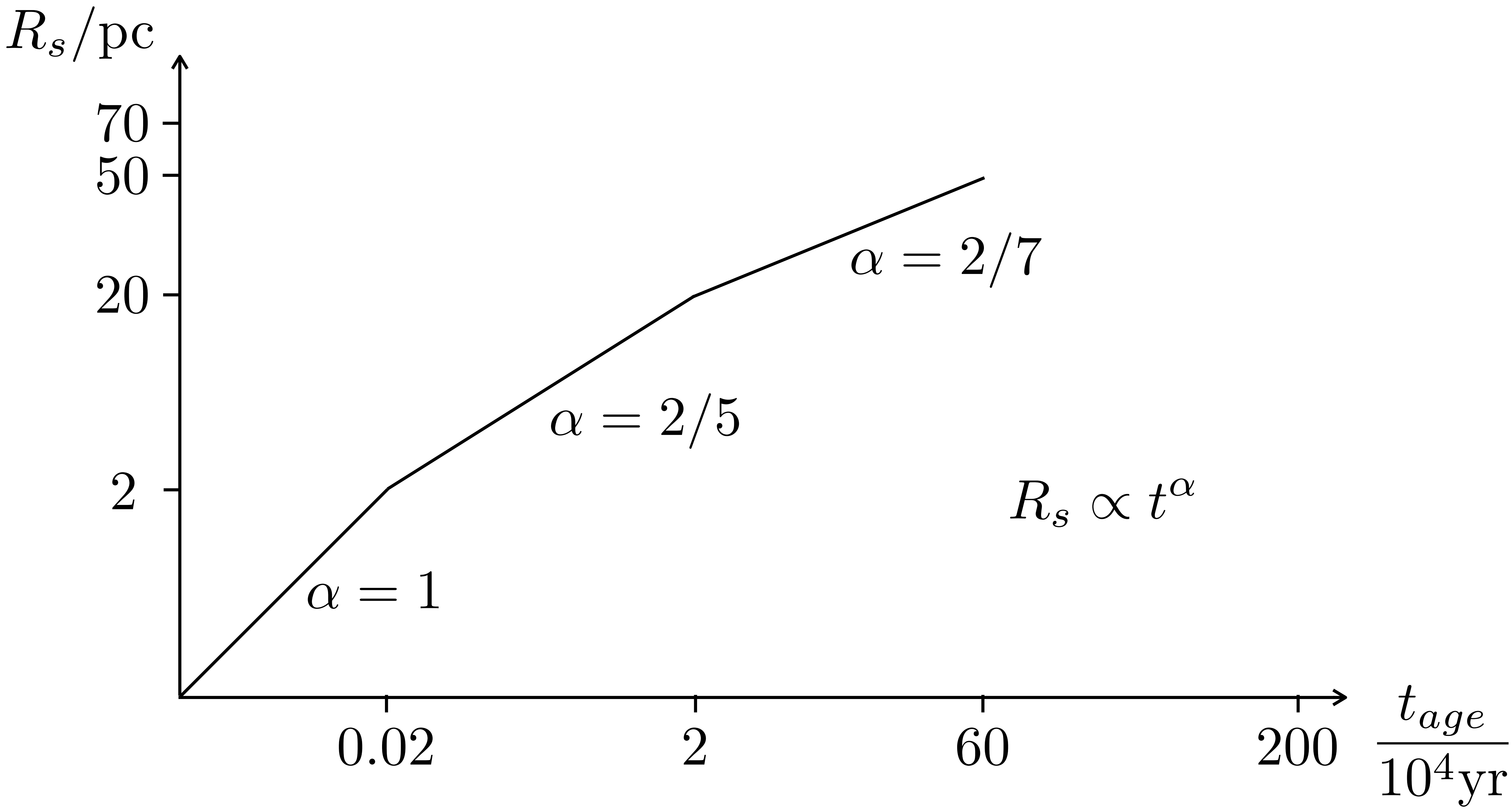
<< 1

density in the shell

the SNR interior cools much later than the SNR shell. according to the numerical simulations of Cioffi et al. 1988 the pressure driven snowplough lasts few tens of t_{ad}

Time evolution of a SNR

$M_{sh}/M_\odot \approx$	1	10^3	10^4
$\dfrac{v_s}{10^3\ \mathrm{km/s}} \approx$	10	0.3	2×10^{-2}



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density in the shell

the SNR interior cools much later than the SNR shell. according to the numerical simulations of Cioffi et al. 1988 the pressure driven snowplough lasts few tens of t_{ad}

once the interior cools, the motion of the shell is determined by momentum conservation

$$M v_s = const \;\Rightarrow\; M v_s = \frac{4\pi}{3} R_s^3 \varrho_0 v_s = \frac{4\pi}{3} \varrho_0 A^4 t^{4\alpha - 1} \longrightarrow \alpha = \frac{1}{4}$$

$$\begin{cases} R_s \propto t^{1/4} \\ v_s \propto t^{-3/4} \end{cases}$$

the SNR dissolves in the ISM when the shock Mach number becomes ~1 ($v_s \sim c_s \sim$ 10 km/s)

Time evolution of a SNR

$M_{sh}/M_\odot \approx$	1	10^3	10^4	3×10^4
$\dfrac{v_s}{10^3 \text{ km/s}} \approx$	10	0.3	2×10^{-2}	10^{-2}