NPAC course on Astroparticles

VI - PARTICLE ACCELERATION: DIFFUSIVE SHOCK ACCELERATION

Particle acceleration

Ohm's law

Static electric field -> $$\vec{E} = \frac{\vec{j}}{\sigma} \approx 0$$

electric conductivity -> infinity
in astrophysical plasmas!

Lorentz's force

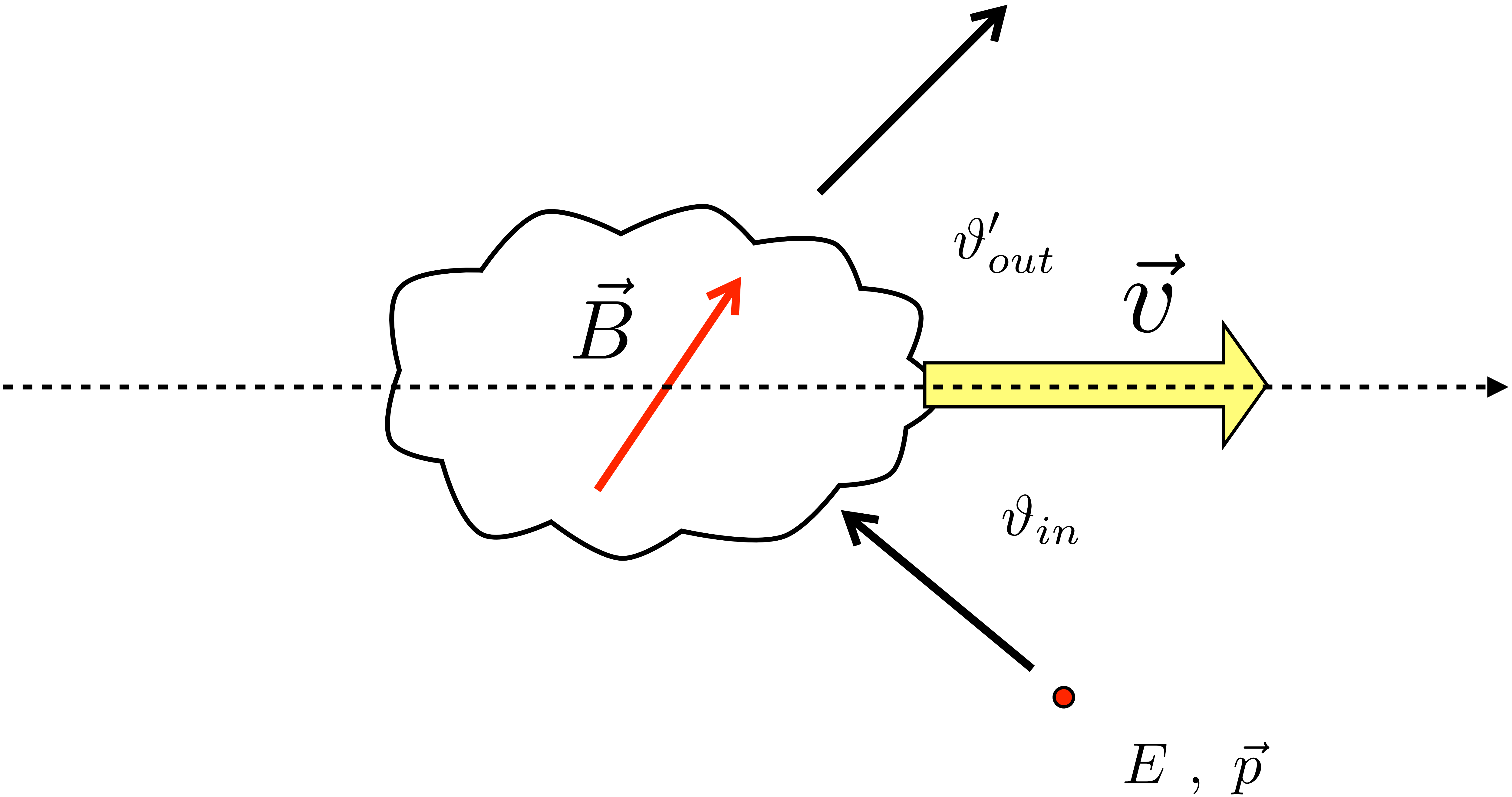
Static magnetic field -> $$\vec{F} = q\,\vec{v} \times \vec{B}$$

no work done on the particle

Induced E-field -> $$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

Moving B-field -> $$\vec{E} = -\frac{1}{c}\,\vec{v} \times \vec{B}$$

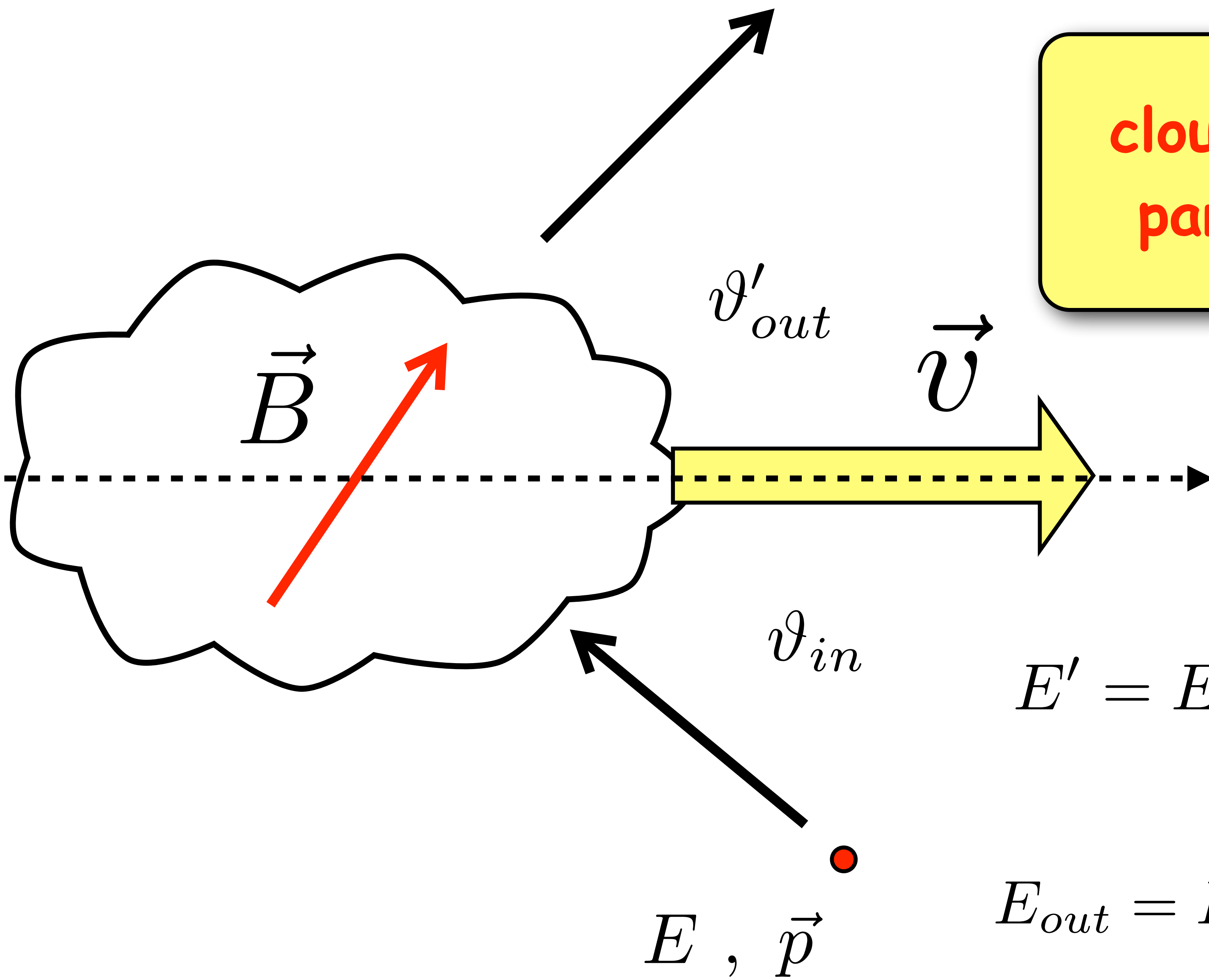
Fermi's idea (1949, 1954)



* primed quantities are measured in the rest frame of the cloud

Energy gain (loss) per interaction

$$E' = \gamma_v \left(E - v\, p\, \cos(\vartheta_{in})\right)$$



cloud -> non-relativistic
particle -> relativistic

cloud speed

$$E' = E\left(1 - \beta\, \cos(\vartheta_{in}) + \frac{\beta^2}{2}\right)$$

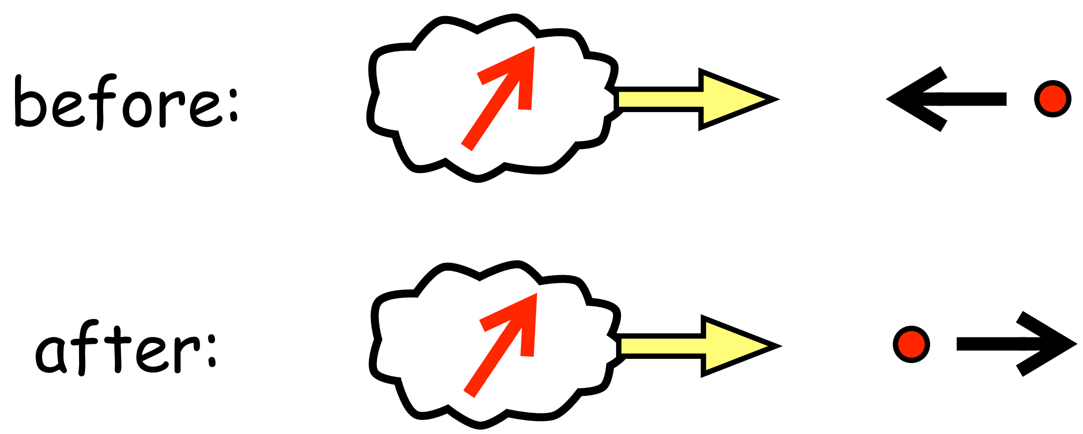
$$E_{out} = E'\left(1 + \beta\, \cos(\vartheta'_{out}) + \frac{\beta^2}{2}\right)$$

* primed quantities are measured in the rest frame of the cloud

Energy gain (loss) per interaction

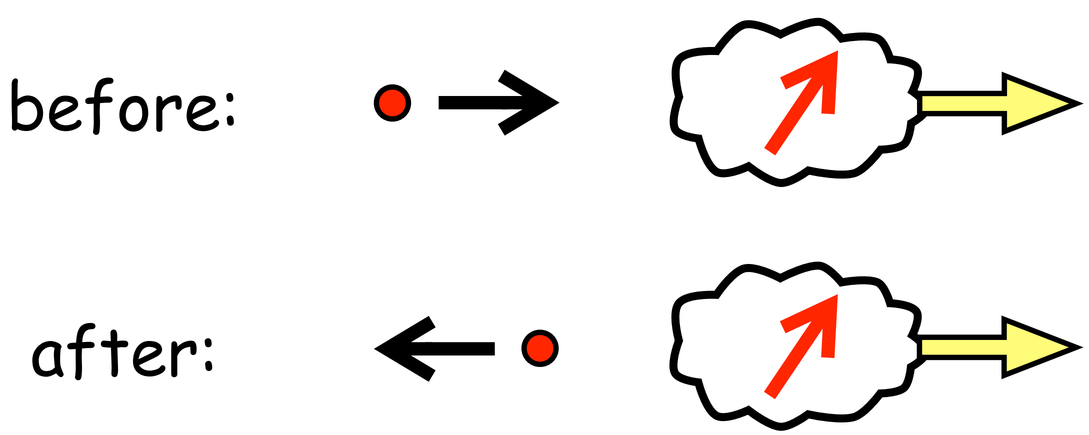
$$\frac{\Delta E}{E} = \beta\left[\cos(\vartheta'_{out}) - \cos(\vartheta_{in})\right] + \beta^2\left[1 - \cos(\vartheta_{in})\cos(\vartheta'_{out})\right]$$

head-on collision



$$\frac{\Delta E}{E} = 2\,\beta\,(1+\beta)$$

tail-on collision



$$\frac{\Delta E}{E} = -2\,\beta\,(1-\beta)$$

Average energy gain

$$\frac{\Delta E}{E} = \beta\left[\cos(\vartheta'_{out}) - \cos(\vartheta_{in})\right] + \beta^2\left[1 - \cos(\vartheta_{in})\cos(\vartheta'_{out})\right]$$

- isotropy of particles in the cloud frame ----> $\langle\cos(\vartheta'_{out})\rangle = 0$
- # particles between ϑ_{in} and $\vartheta_{in} + \mathrm{d}\vartheta_{in}$ ----> $\propto \sin(\vartheta_{in})\mathrm{d}\vartheta_{in}$
- rate at which particles enter the cloud prop. to ---> $\propto 1 - \beta\cos(\vartheta_{in})$

$$\langle\cos(\vartheta_{in})\rangle = -\frac{\beta}{3}$$

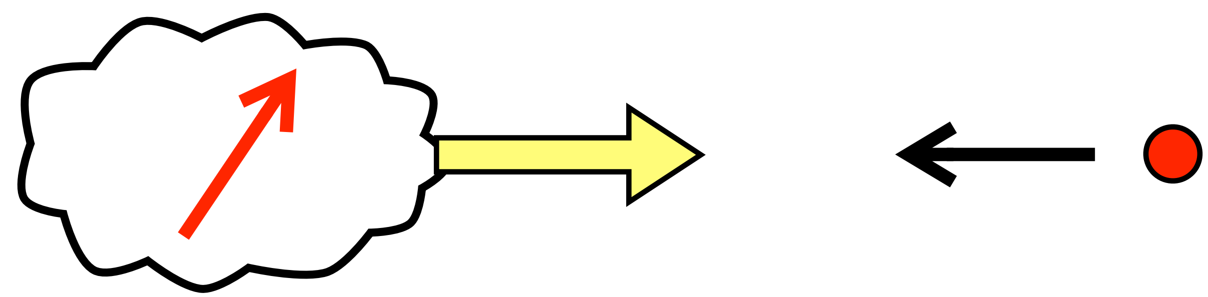
Second order Fermi mechanism

$$\left\langle \frac{\Delta E}{E} \right\rangle \approx \frac{4}{3} \beta^2$$

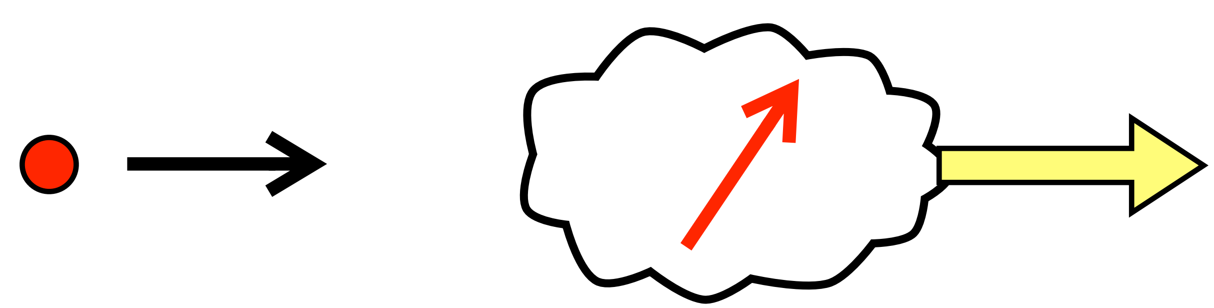
inefficient -> too slow...

A very simple idea

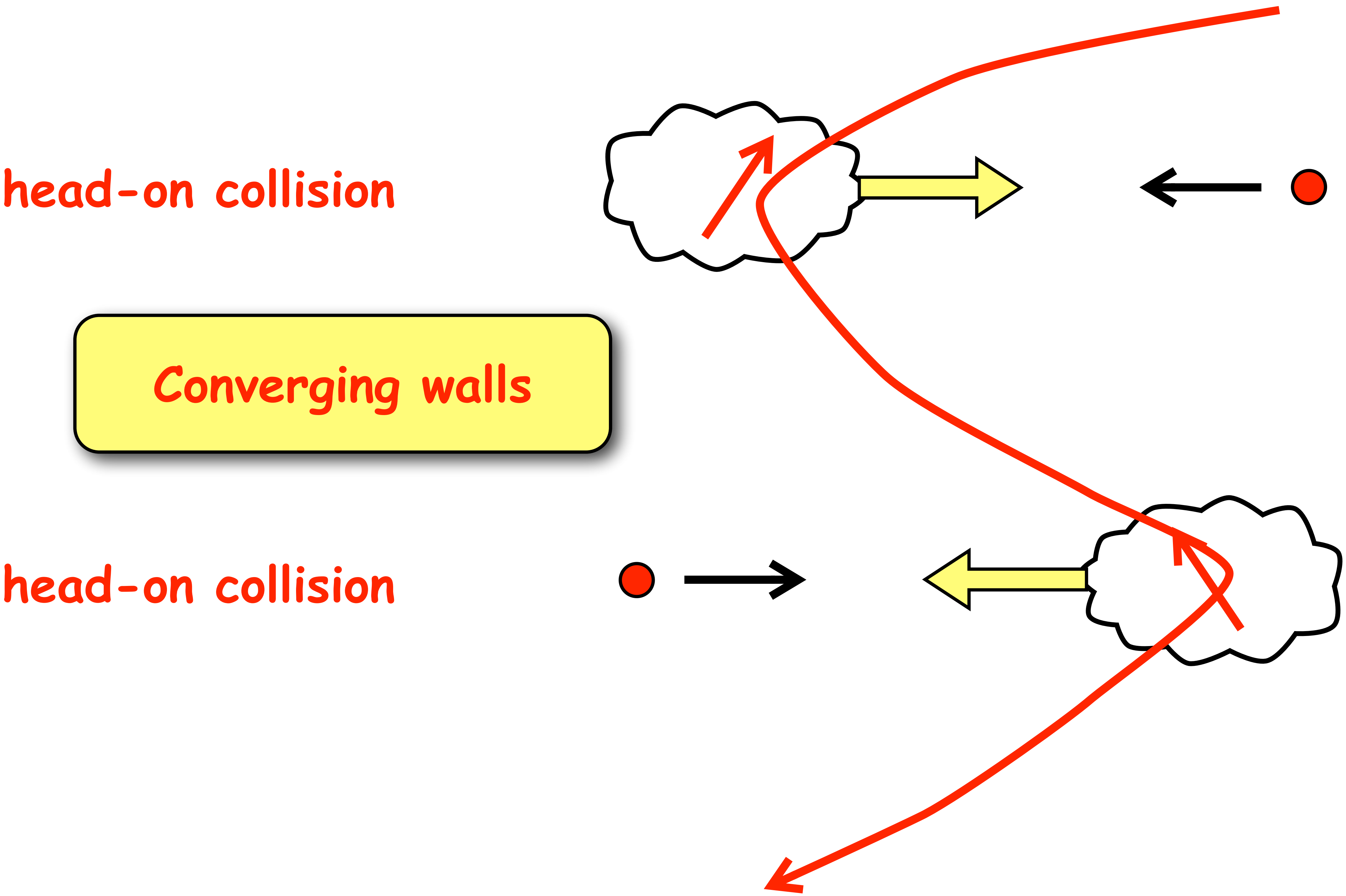
head-on collision



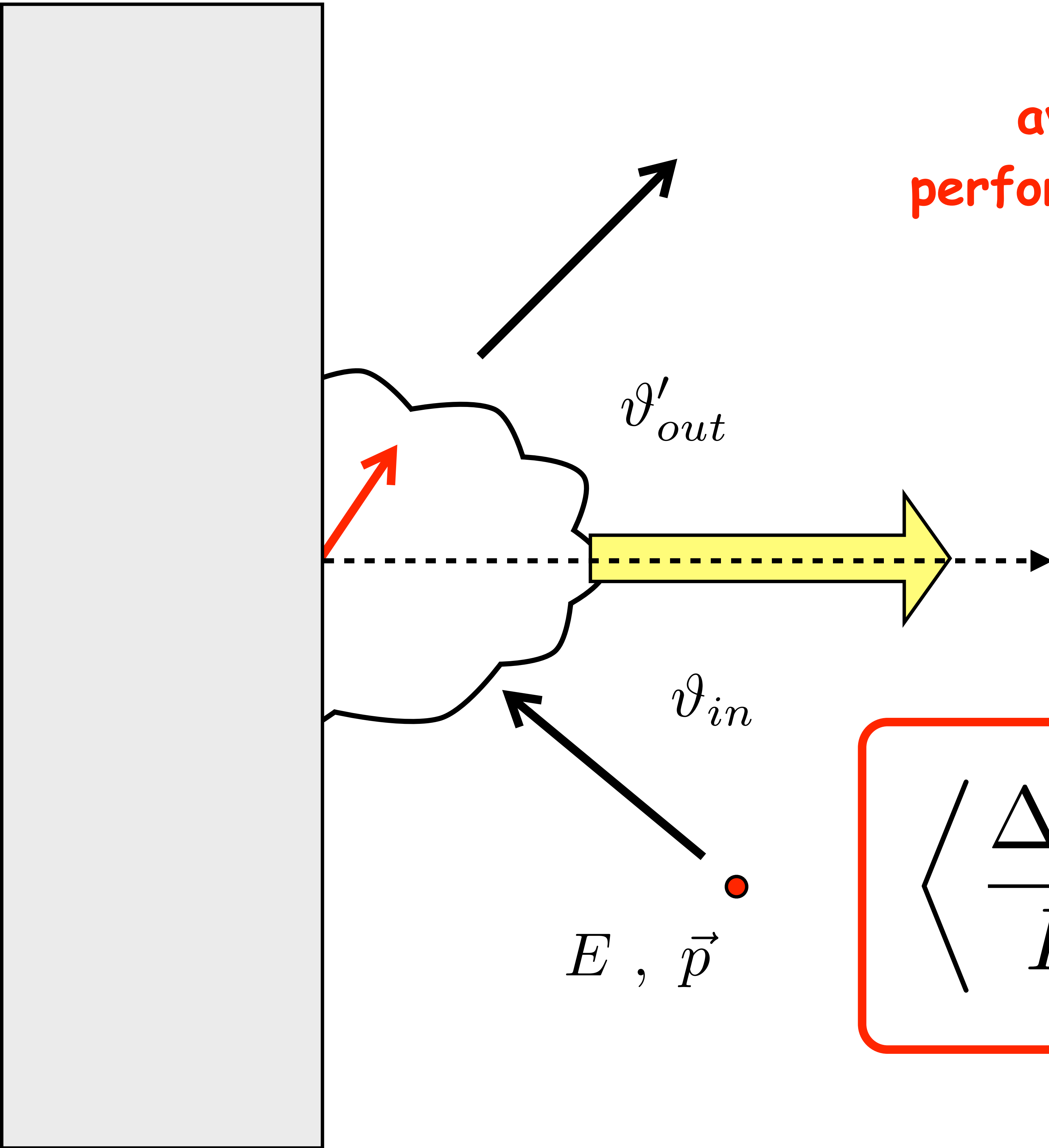
tail-on collision



A very simple idea



First order Fermi mechanism



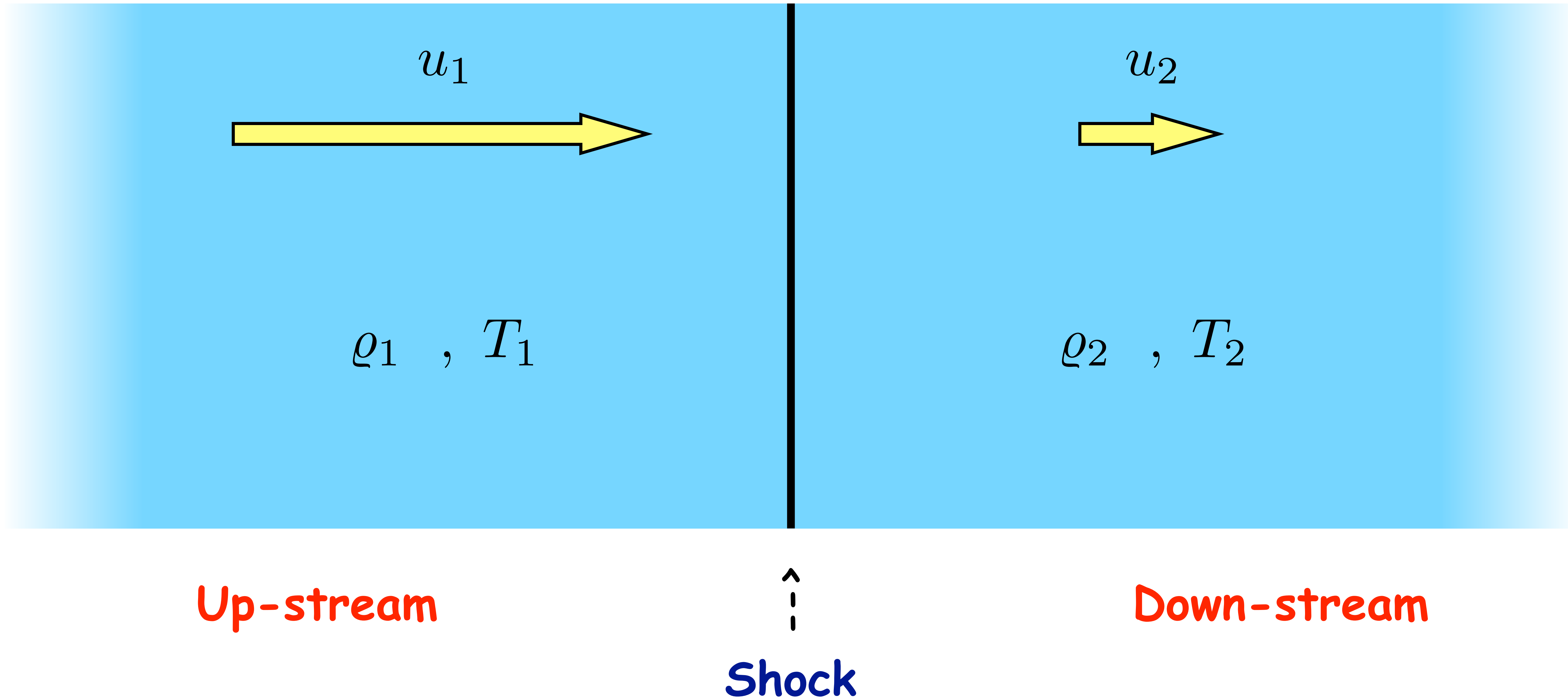
averages have to be performed over the interval:

$$0 < \vartheta < \frac{\pi}{2}$$

First order!

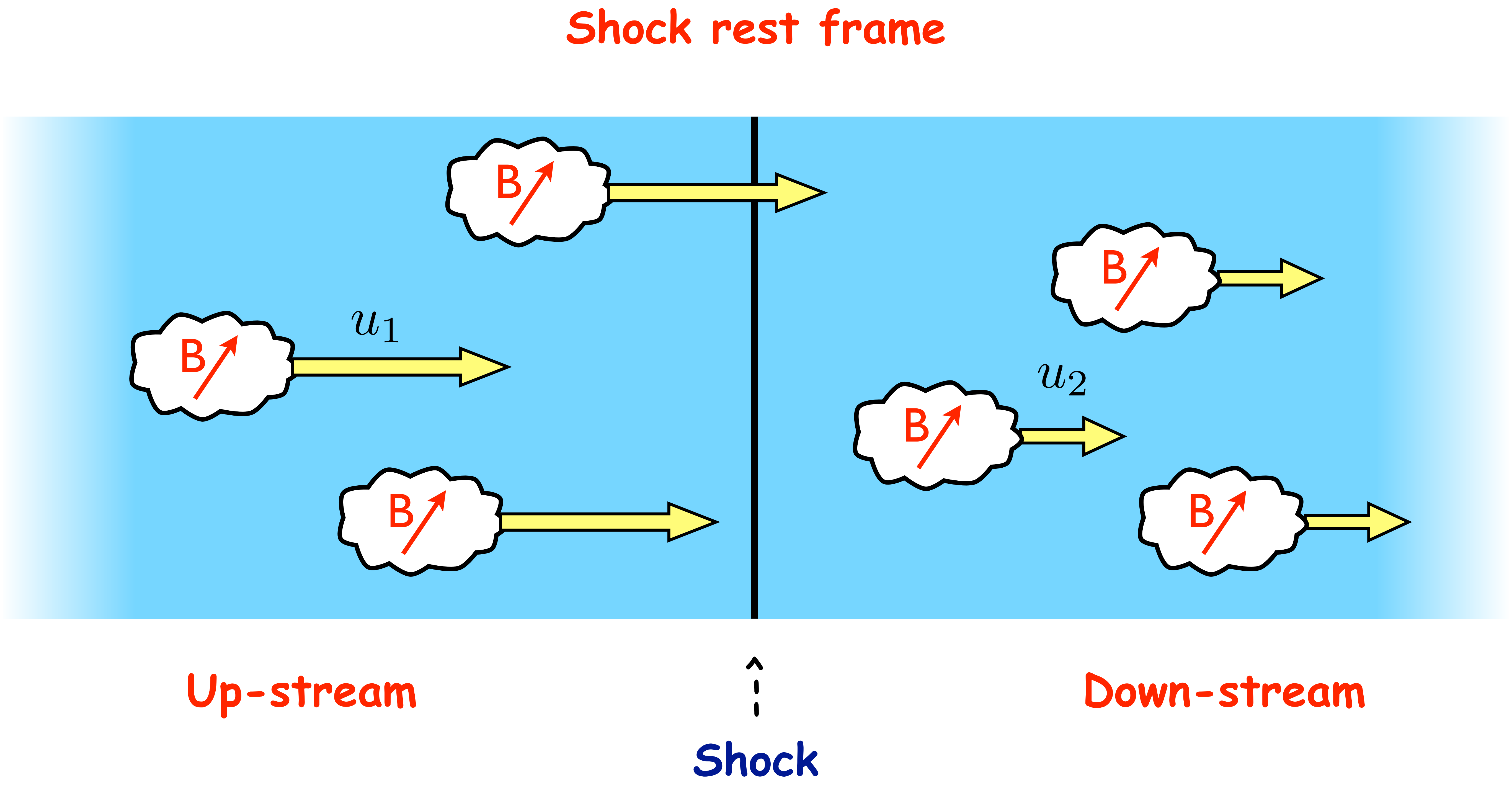
$$\left\langle \frac{\Delta E}{E} \right\rangle \approx \frac{4}{3}\,\beta$$

$$\frac{\varrho_2}{\varrho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1} = 4 \qquad\qquad p_2 = \frac{2}{\gamma+1}\,\varrho_1 u_1^2$$



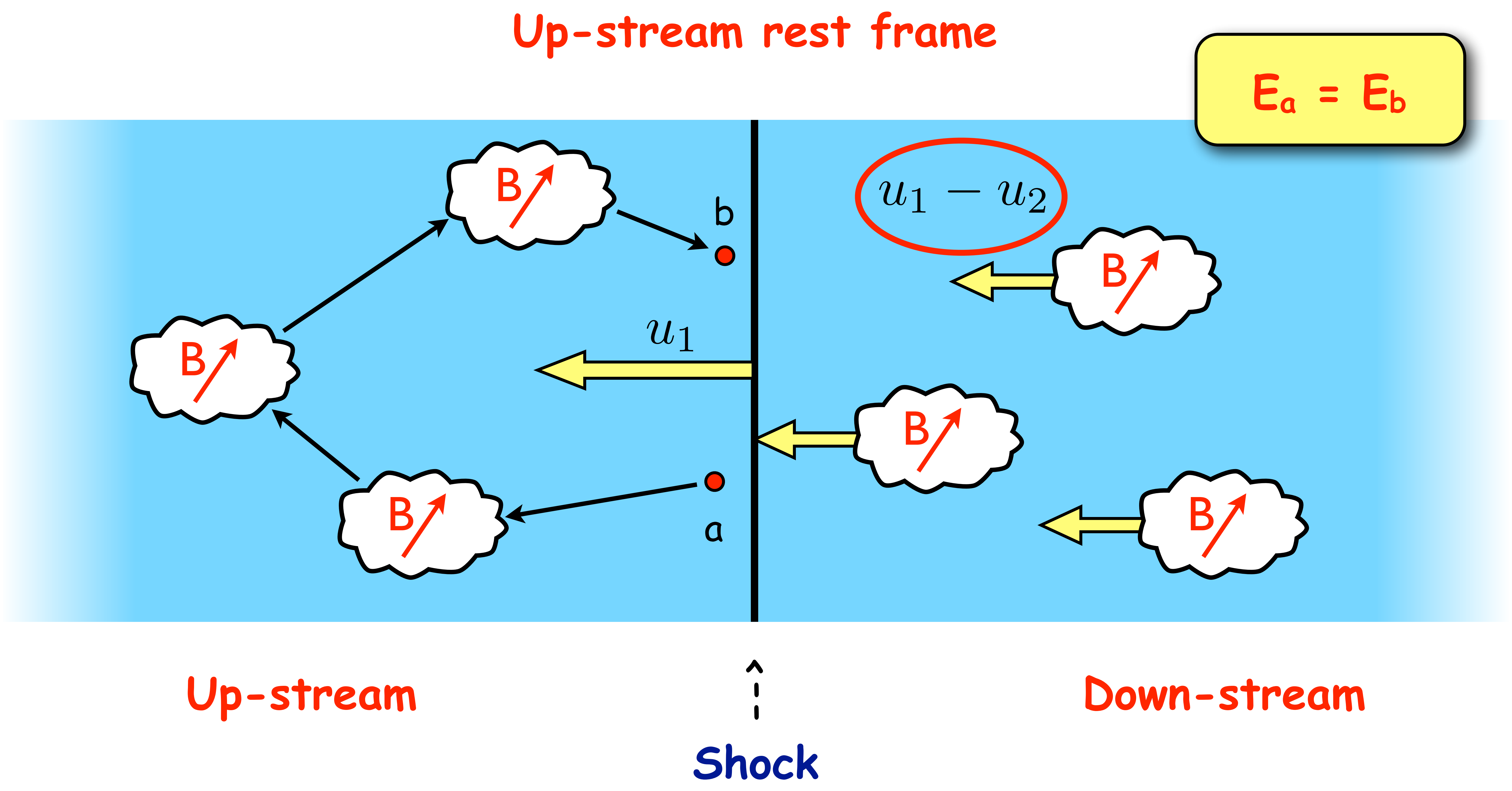
In 1960 (!) F. Hoyle (first?) suggests that shocks accelerate CRs

Diffusive Shock Acceleration



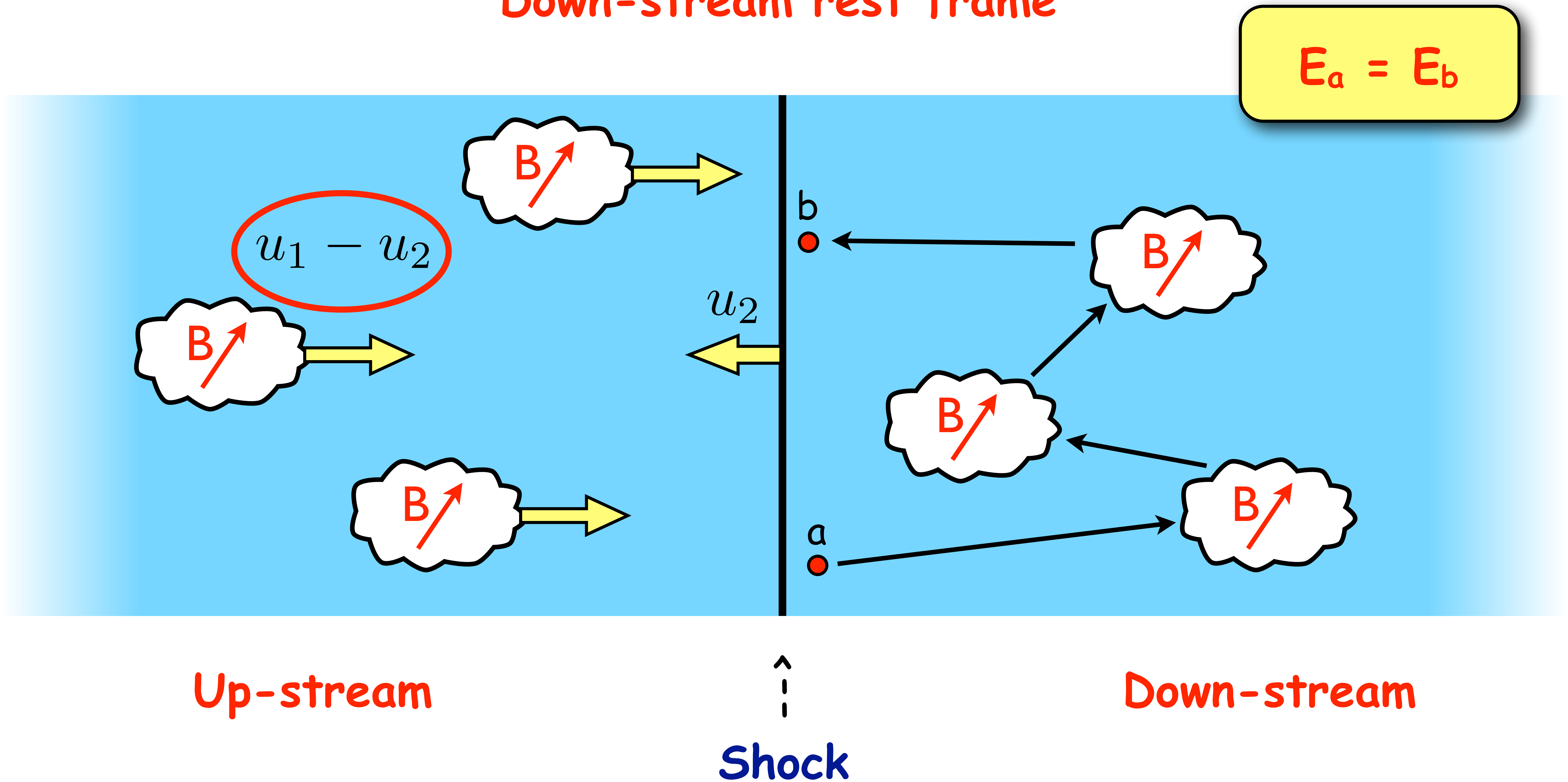
Krymskii 1977, Axford et al. 1977, Blandford & Ostriker 1978, Bell 1978

Diffusive Shock Acceleration



Diffusive Shock Acceleration

Down-stream rest frame



Diffusive Shock Acceleration

Symmetry

Every time the particle crosses the shock (up -> down or down -> up), it undergoes an head-on collision with a plasma moving with velocity u_1-u_2

Asymmetry

(Infinite and plane shock:) Upstream particles always return the shock, while downstream particles may be advected and never come back to the shock

Universality of diffusive shock acceleration

Let's search for a test-particle solution

Assumption: scattering is so effective at shocks that the distribution of particles is isotropic

-> an universal solution of the problem can be found

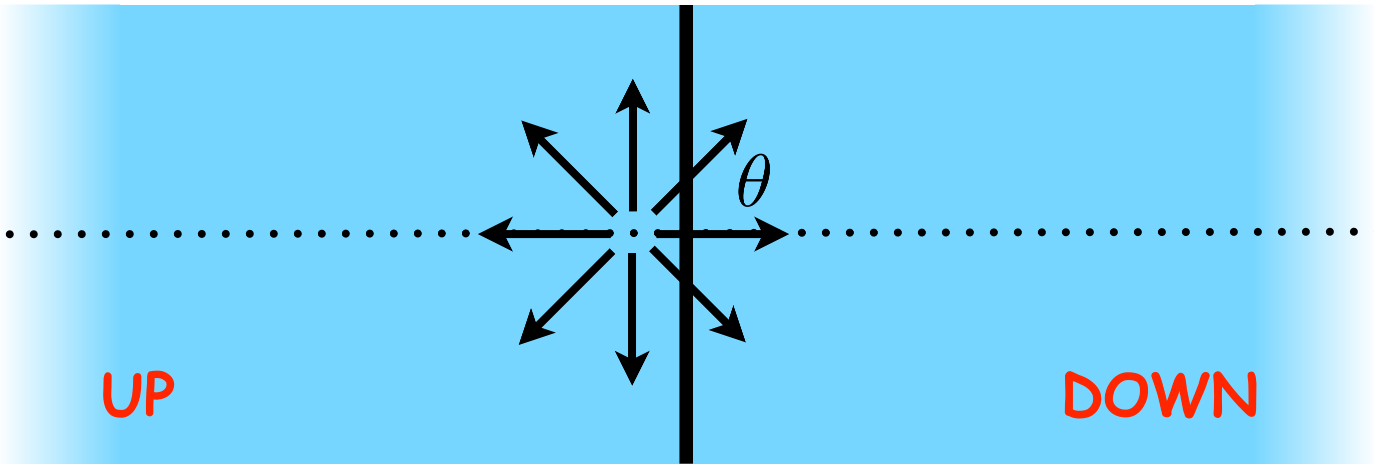
Rate at which particles cross the shock

Let's calculate R_{in}...

n -> density of accelerated particles close to the shock

n is isotropic: $$dn = \frac{n}{4\pi} d\Omega$$

velocity across the shock: $c \cos(\theta)$

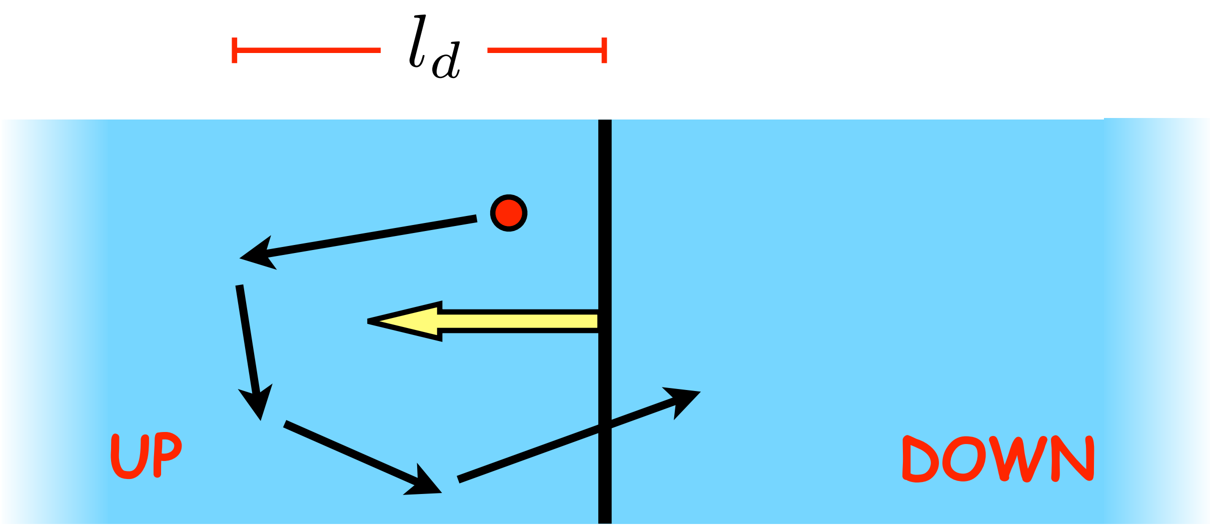


$$R_{in} = \int_{up \rightarrow down} dn \; c \; \cos(\theta) = \frac{n \; c}{4\pi} \int_0^{\frac{\pi}{2}} \cos(\theta) \sin(\theta) d\theta \int_0^{2\pi} d\psi = \frac{1}{4} n \; c$$

-> the same result is obtained for down -> up

Residence time upstream

-> let's find the STEADY STATE solution upstream of the shock



behavior of particles is diffusive
D(E) -> diffusion coefficient

very poorly constrained (from both observations and theory)

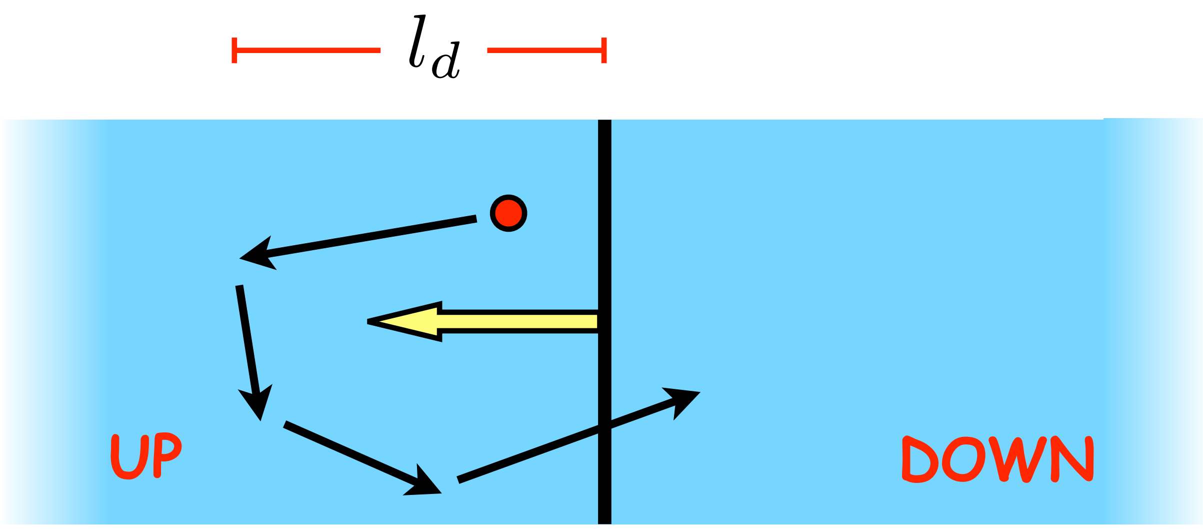
-> due to diffusion particles spread over $l \approx \sqrt{D\ t}$

-> at the same time the shock moves $l = u_1\ t$

$$l_d \approx \frac{D}{u_1}$$

Residence time upstream

-> let's find the STEADY STATE solution upstream of the shock



behavior of particles is diffusive
D(E) -> diffusion coefficient

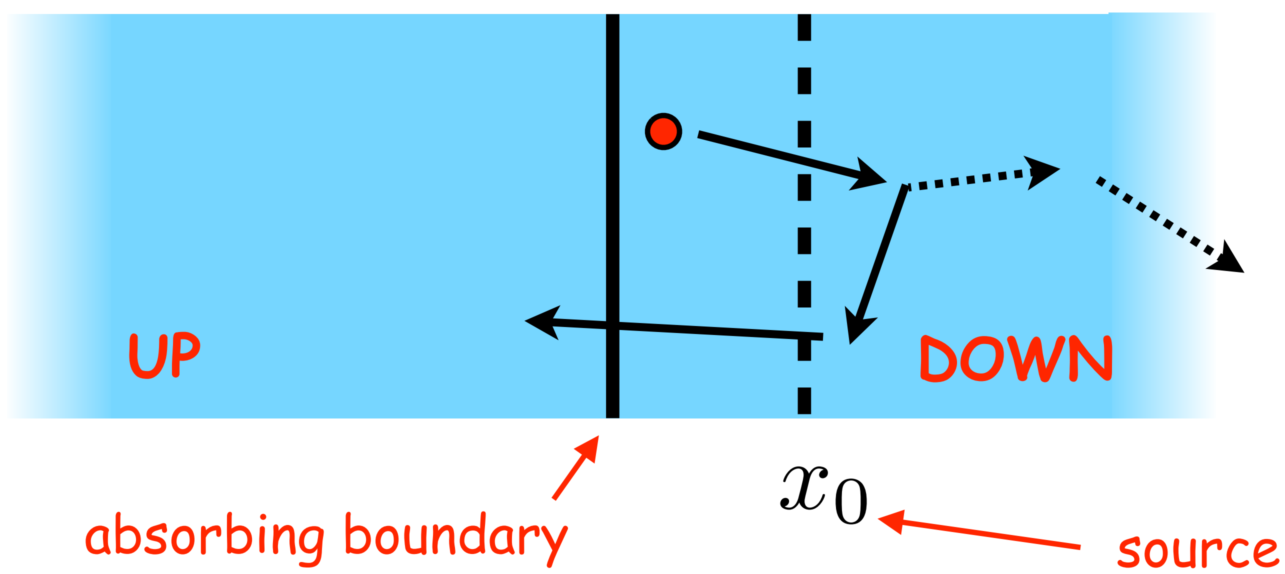
very poorly constrained (from both observations and theory)

cosmic ray precursor -> n ~constant up to $l_d \approx \frac{D}{u_1}$

residence time upstream -> $\tau_{up} = \frac{N_{up}}{R_{in}} = \frac{n\ l_d}{\frac{1}{4}\ n\ c} = \frac{4\ D}{u_1\ c}$

Residence time downstream

-> a bit more subtle...



n is constant downstream of the shock

$$u_2 \frac{\partial n}{\partial x} = D \frac{\partial^2 n}{\partial x^2} + Q\, \delta(x - x_0) \qquad n(0) = 0$$

we need to know the returning flux $\left. D \frac{\partial n}{\partial x} \right|_{x=0} \longrightarrow P_{ret} = \frac{\left. D \frac{\partial n}{\partial x} \right|_{x=0}}{Q}$

$$P_{ret} = \exp\left(-\frac{x_0 u_2}{D} \right)$$

Residence time downstream

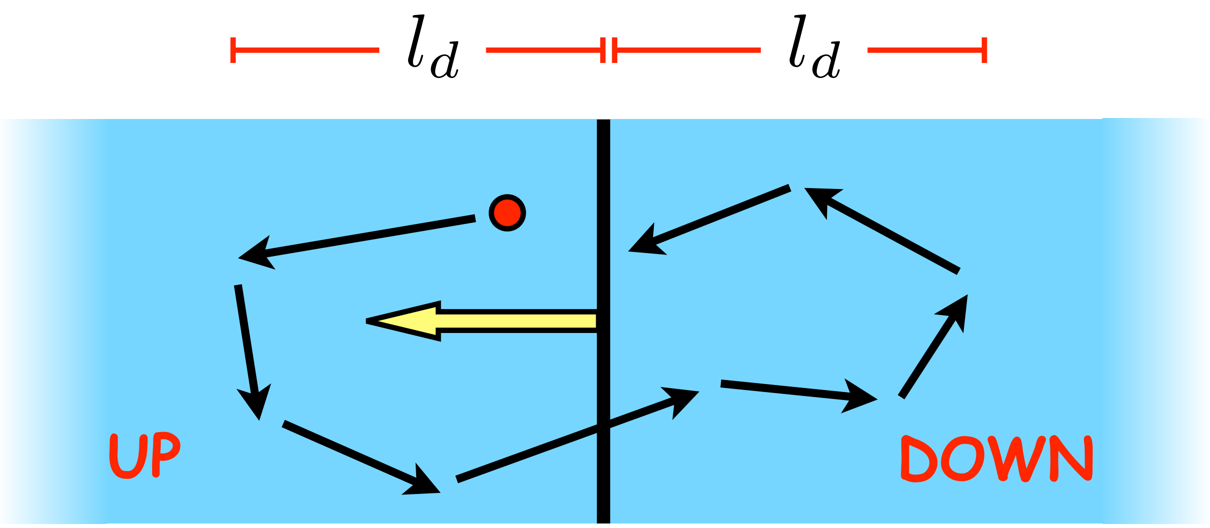
number of downstream particles that will return to the shock:

$$\int_0^\infty \mathrm{d}x \; P_{ret}(x) \; n \; = \; \frac{D \; n}{u_2}$$

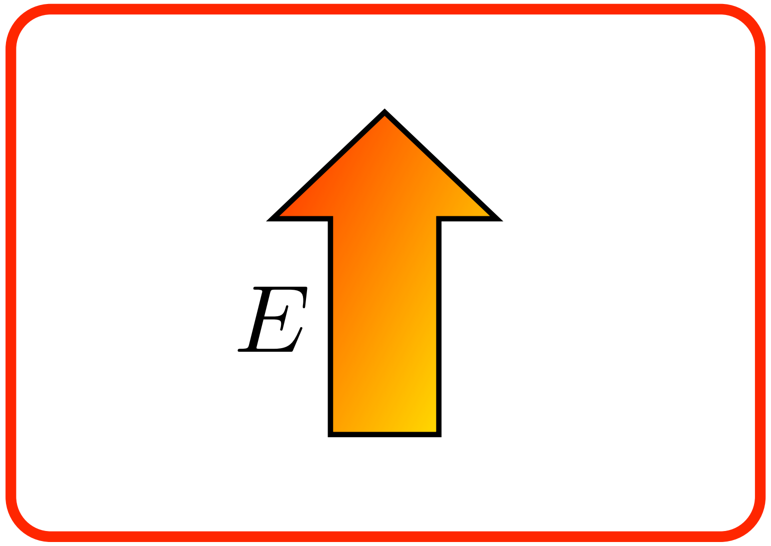
same expression upstream!

mean residence time upstream <-> mean residence time downstream

$$\frac{4D}{u_1 c} \qquad\qquad \frac{4D}{u_2 c}$$



Acceleration rate



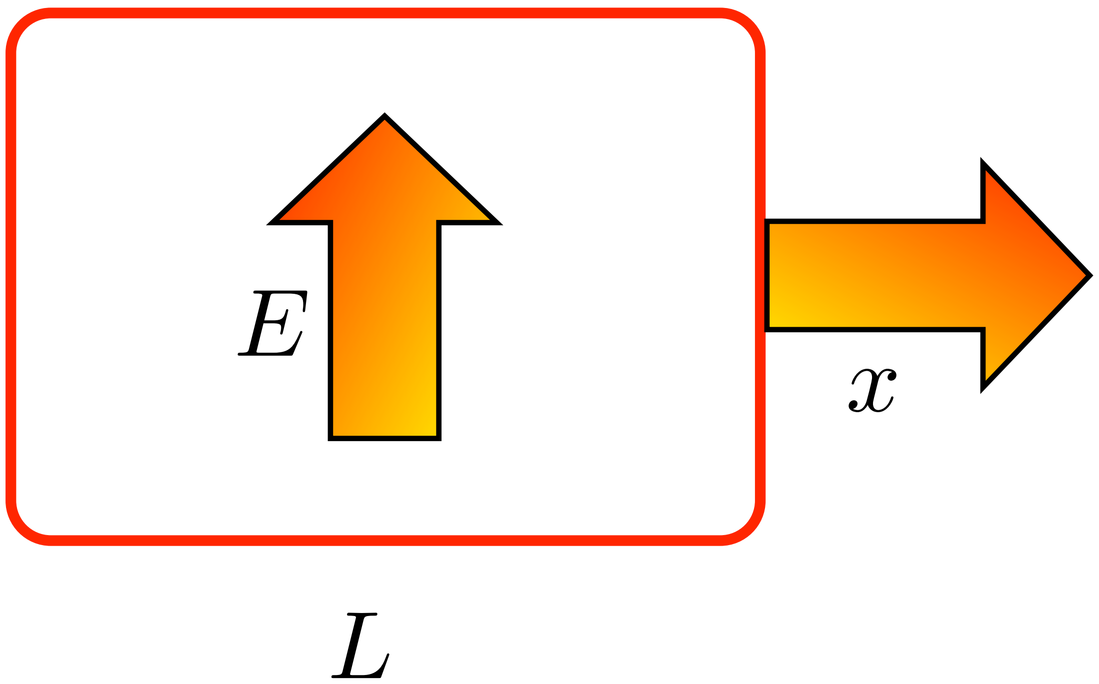
L

everything happens within a "box" of size:

$$L = \frac{D}{u_1} + \frac{D}{u_2} \quad = \quad c\,\frac{\tau_{up} + \tau_{down}}{4}$$

$$r_{acc} = \frac{\langle \frac{\Delta E}{E} \rangle}{\tau_{up} + \tau_{down}} \approx \frac{\frac{4}{3}\beta\ c}{4\ L} = \boxed{\frac{u_1 - u_2}{3\ L}}$$

Box model for shock acceleration



particles move up in energy

$$r_{acc} = \frac{u_1 - u_2}{3\ L}$$

particles exit the box downstream

$$r_{esc} = \frac{u_2}{L}$$

up-ward flux in E

down(stream)-ward flux in x

$$\frac{\partial}{\partial E}\Big(r_{acc} E N(E)\Big) \;=\; - r_{esc} N(E)$$

n(E)

= -1

$$\frac{L}{N(E)} \frac{\partial}{\partial E}\Big(E \boxed{\frac{N(E)}{L}}\Big) = -\frac{3\ u_2}{u_1 - u_2}$$

Universality
of diffusive shock acceleration

Let's search for a test-particle solution

Assumption: scattering is so effective at shocks that the distribution of particles is isotropic

-> an universal solution of the problem can be found

$$\frac{1}{n} \; \partial \; \ldots \; -1 \longrightarrow \boxed{n(E) \propto E^{-2}}$$

Independent on D !!!

Bell's approach

Let's start with N_0 particles of energy E_0...

-> # of particles starting a cycle per second: $nc/4$

-> # of particles leaving the system per second: $nu_2 = nu_1/4$

— divide —>

-> Probability to leave the system per cycle: u_1/c

-> Return probability to the shock per cycle: $P_R = 1 - \frac{u_1}{c}$

-> # of particles performing at least k cycles: $N_k = N_0 \left(1 - \frac{u_1}{c}\right)^k$

-> have an energy larger than: $E_k = E_0 \left(1 + \langle \frac{\Delta E}{E} \rangle\right)^k = E_0 \left(1 + \frac{u_1}{c}\right)^k$

Universal solution: Bell's approach

$$\log\left(\frac{N}{N_0}\right) = k\log\left(1-\frac{u_1}{c}\right)$$

– divide –>

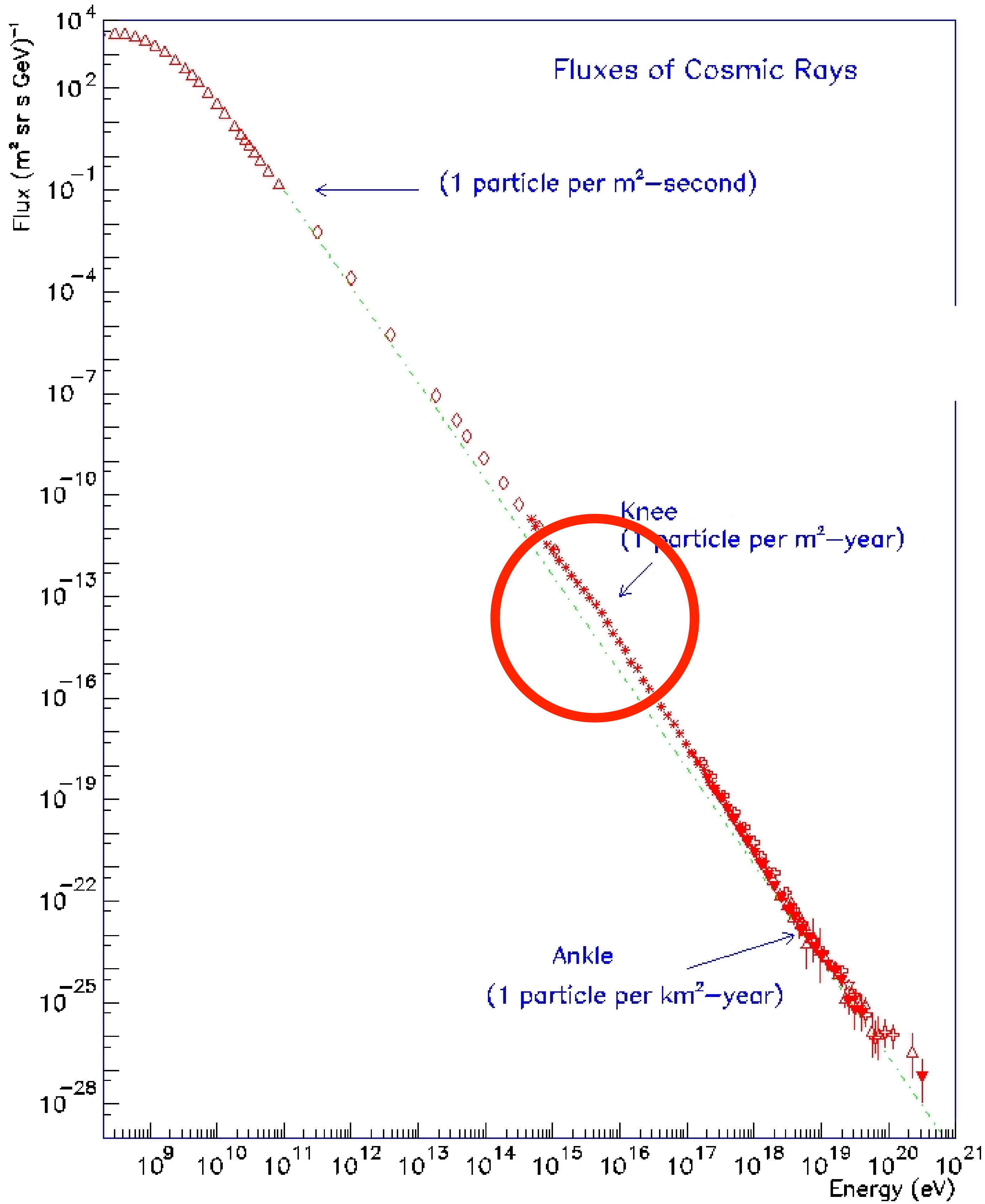
$$\log\left(\frac{E}{E_0}\right) = k\log\left(1+\frac{u_1}{c}\right)$$

$$N(>E) = N_0\left(\frac{E}{E_0}\right)^{\frac{\log\left(1-\frac{u_1}{c}\right)}{\log\left(1+\frac{u_1}{c}\right)}} \longrightarrow -1$$

Independent on D !!!

$$\boxed{\frac{\mathrm{d}N(E)}{\mathrm{d}E} \propto E^{-2}}$$

Getting to the knee



$$\tau_{acc} = \frac{1}{r_{acc}} \approx \frac{D(E)}{u^2}$$

maximum energy is given by:

$$\tau_{acc}(E) = \tau_{age}$$

this depends on D(E)

which age?

details matter

Getting to the knee

Lagage & Cesarsky 1983

$$\tau_{acc}(E) = \tau_{age}$$

-> SNR shocks do not decelerate until $\lesssim 1000 \text{ yr} \longrightarrow \tau_{age} \approx 1000\text{yr}$

wavelength

-> CRs are scattered by resonant MHD waves $\lambda \approx R_L$ ← Larmor radius

$$D \approx l_{mfp}\, c \approx R_L\, c \propto \frac{E}{B}$$

$$E_{max} \approx B\, u^2\, \tau_{age} = B\, u\, R \approx \boxed{10^{14}\text{eV}}$$

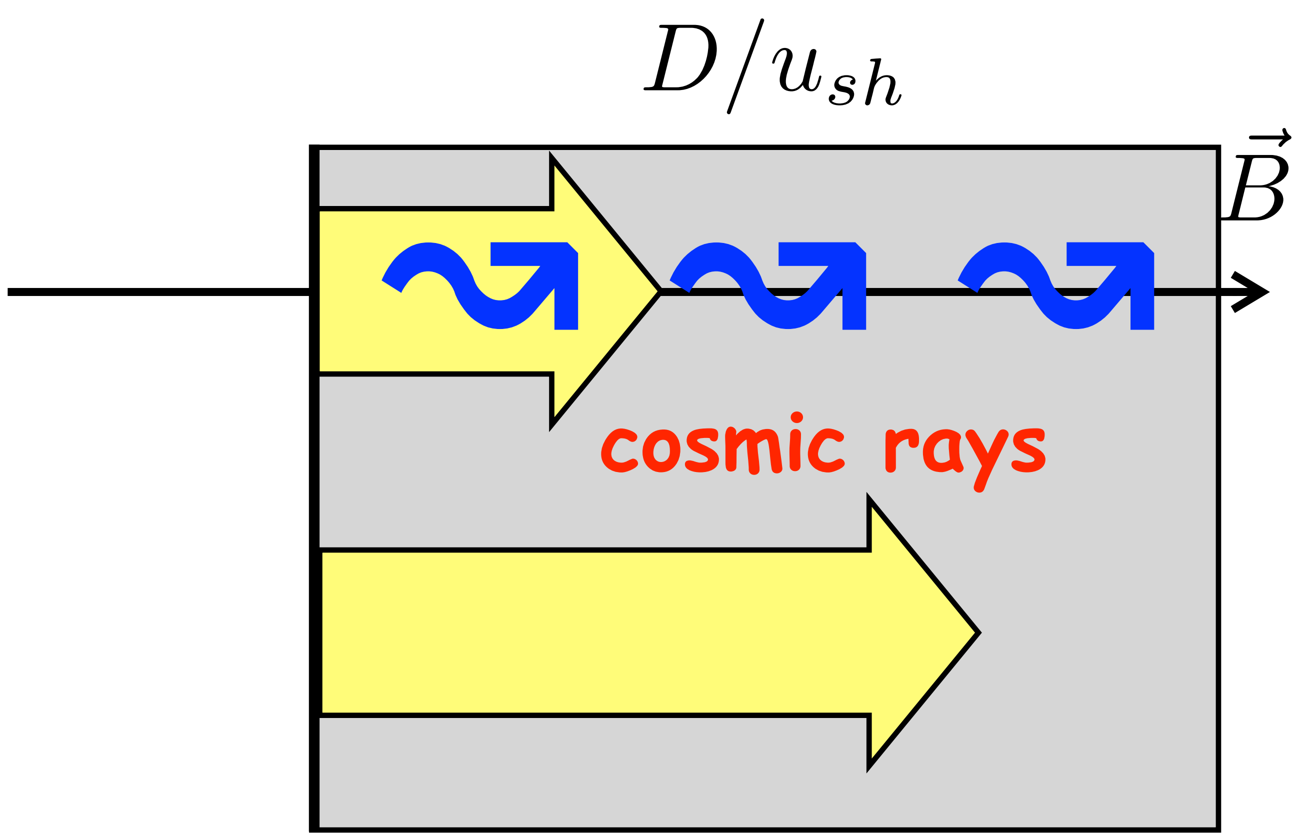
>10 times below the knee

How to solve the problem

horribly oversimplified, for a proper treatment see Bell 2004

the only way is to increase B

$$E_{max} \approx B\ u\ R$$



shock

Alfven speed

$$V_A = \frac{B}{\sqrt{4\ \pi\ \varrho}}$$

$$u_{sh} \gg V_A$$

-> CRs move with the shock -> faster than waves -> CR and waves strongly coupled -> V_A increases -> **B increases!**

Observational test: X-ray filaments

electrons

$$\tau_{acc}(E) = \tau_{age}$$

$$\tau_{acc} \sim \frac{D}{u_s^2} = \tau_{syn} \approx E^{-1}B^{-2} \longrightarrow E_{max} \sim u_s B^{-1/2}$$

max energy of synchrotron photons ->

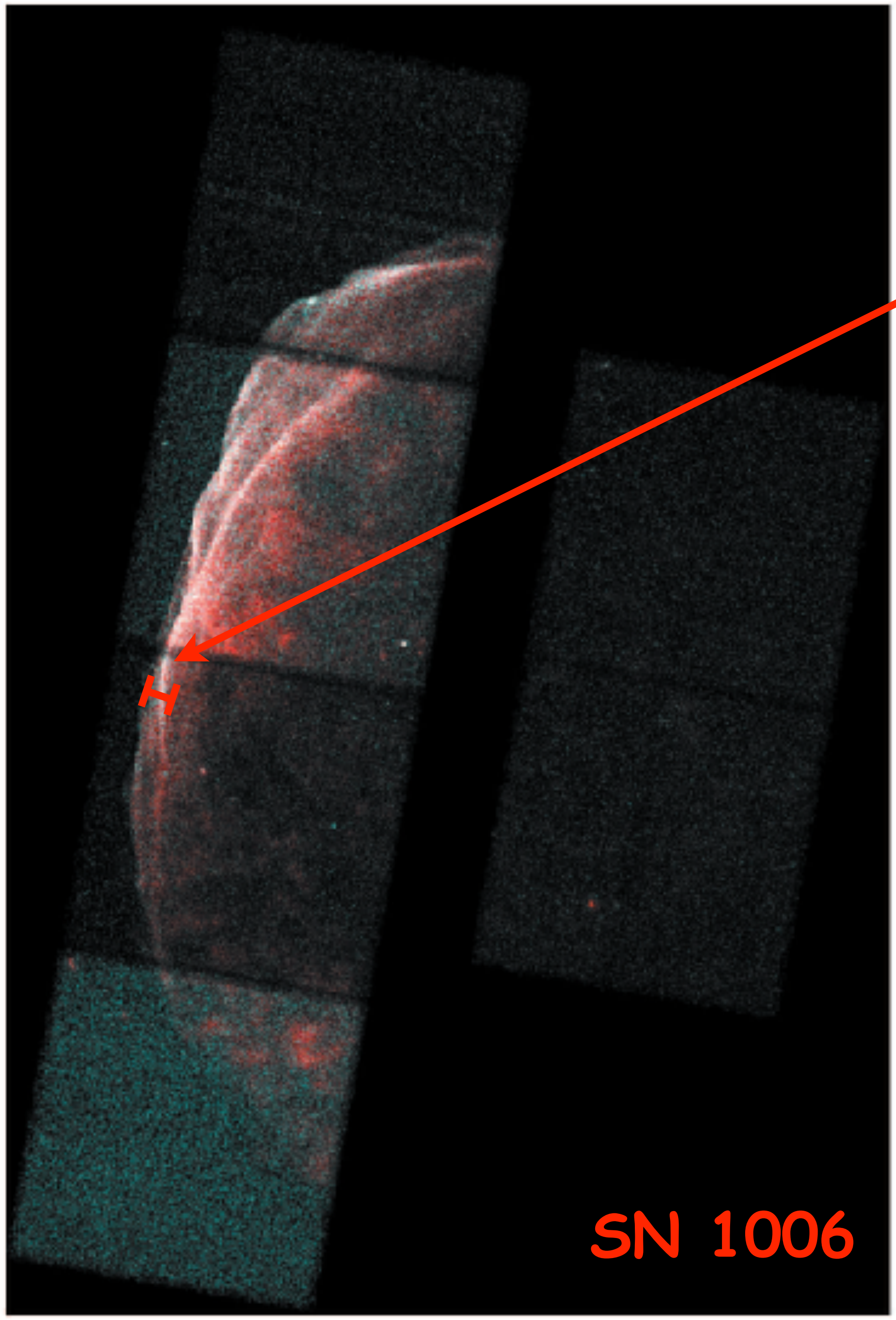
$$E_{syn} \sim E^2 B^2 \sim u_s^2$$

depends on velocity only!!!

$$u_s \approx 10^3 \mathrm{km/s} \longrightarrow E_{syn}^{max} \approx 1\mathrm{kev}$$

X-rays!

Observational test: X-ray filaments



$$\Delta l_X \sim \tau_{syn} u_2 \sim B^{-3/2}$$

B ~ hundreds of microGauss !

The supernova remnant paradigm: does it work?

- diffusive transport of cosmic rays in the galaxy —> **ISOTROPY**
- slope of the spectrum —> **E^{-2} is too hard!**
  - what we see from gamma ray **observations** of SNRs seems to suggest that shock accelerate **steeper spectra**
  - theoreticians proposed tricks (modification of the diffusive shock acceleration theory) to explain this
- if **magnetic field amplification** operates at shocks (???) —> protons can be accelerated **up to the knee (~10^{15} eV)**
- things we did not discuss: chemical composition, electrons, ...