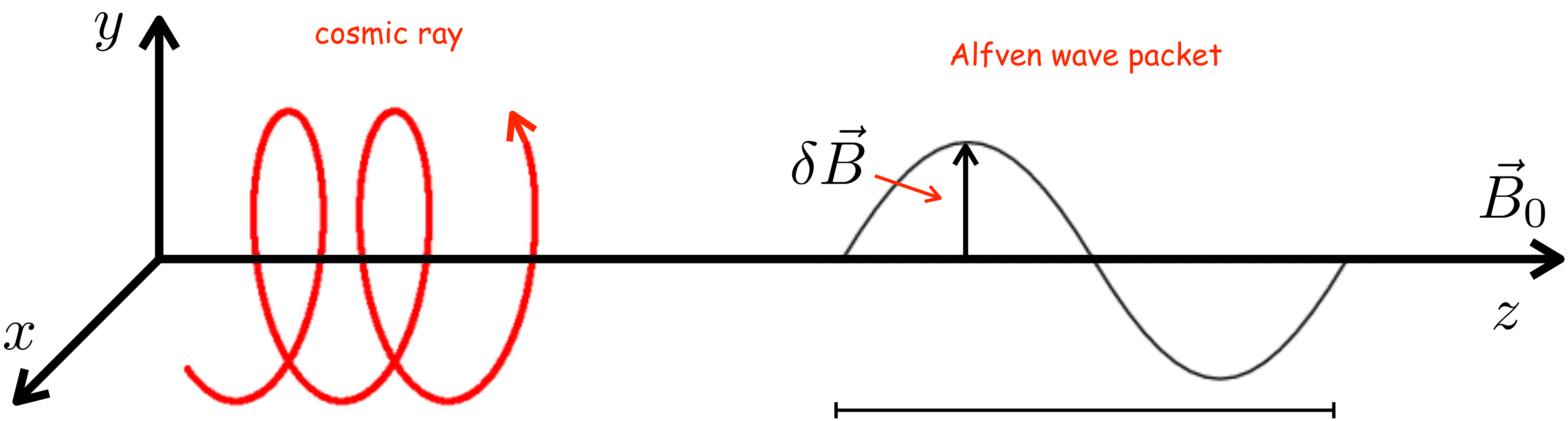
NPAC course on Astroparticles

V - PARTICLE TRANSPORT

Cosmic ray scattering off Alfven waves

$v \sim c$ $\quad$ $v_A \ll c$ —> wave at rest



angle between particle velocity and B0

pitch angle -> $\mu = \cos\vartheta$

$$L \approx \lambda = \frac{2\pi}{k}$$

helical motion

$$\begin{cases} v_\parallel = \mu\, v \\ v_\perp = (1-\mu^2)^{1/2} v \end{cases}$$

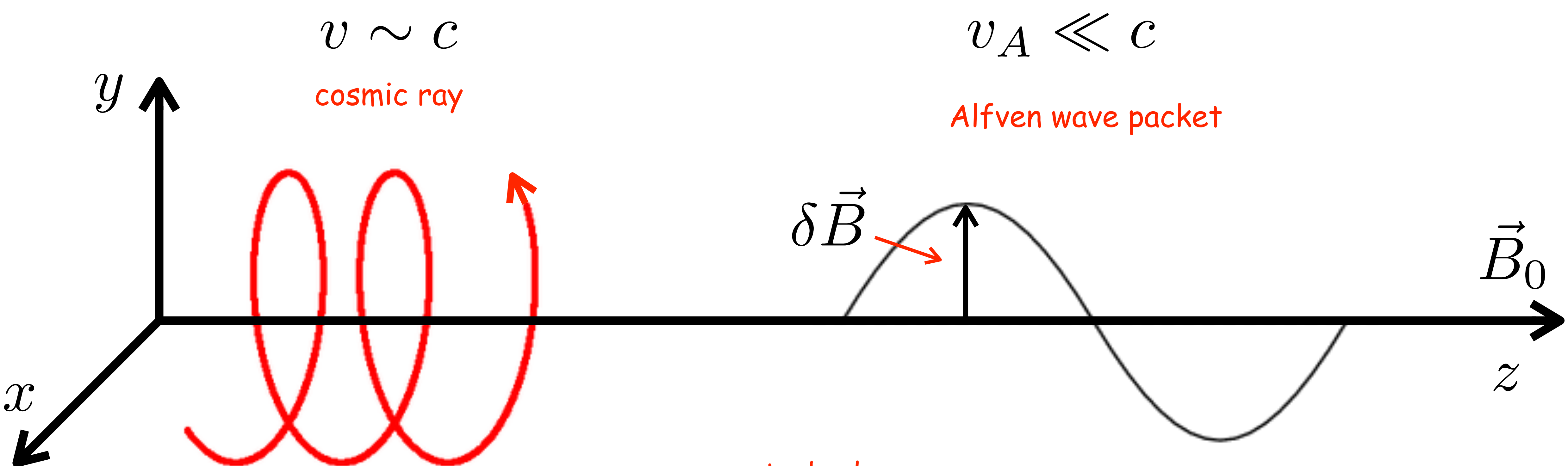
$$\delta\vec{B}_y \sim \delta B \sin{(kz)}\,\vec{y}$$

arbitrary phase

$$v_x = v_\perp \sin(\Omega\; t + \phi)$$

gyration frequency

Cosmic ray scattering off Alfven waves



unperturbed trajectory

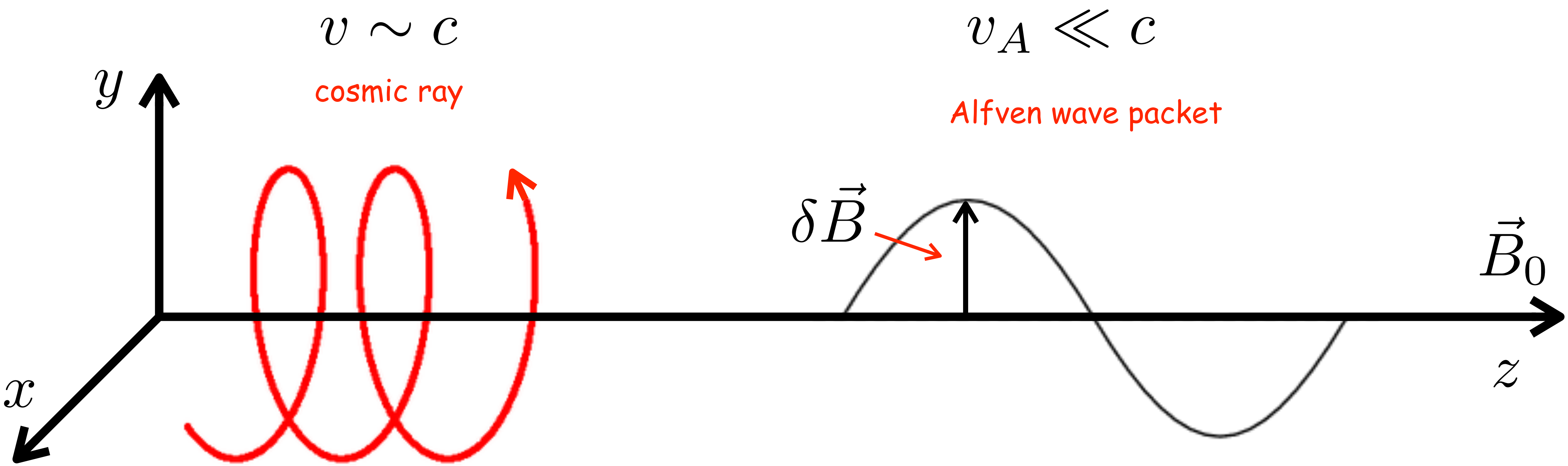
perturbed field

quasi-linear theory

$$F_L = \frac{q}{c}\, \vec{v} \times \vec{B} \qquad \vec{B} = B_0 \vec{z} + \delta \vec{B}_y$$

$$(\vec{v} \times \vec{B})_z = v_x \, \delta B_y$$

Cosmic ray scattering off Alfven waves

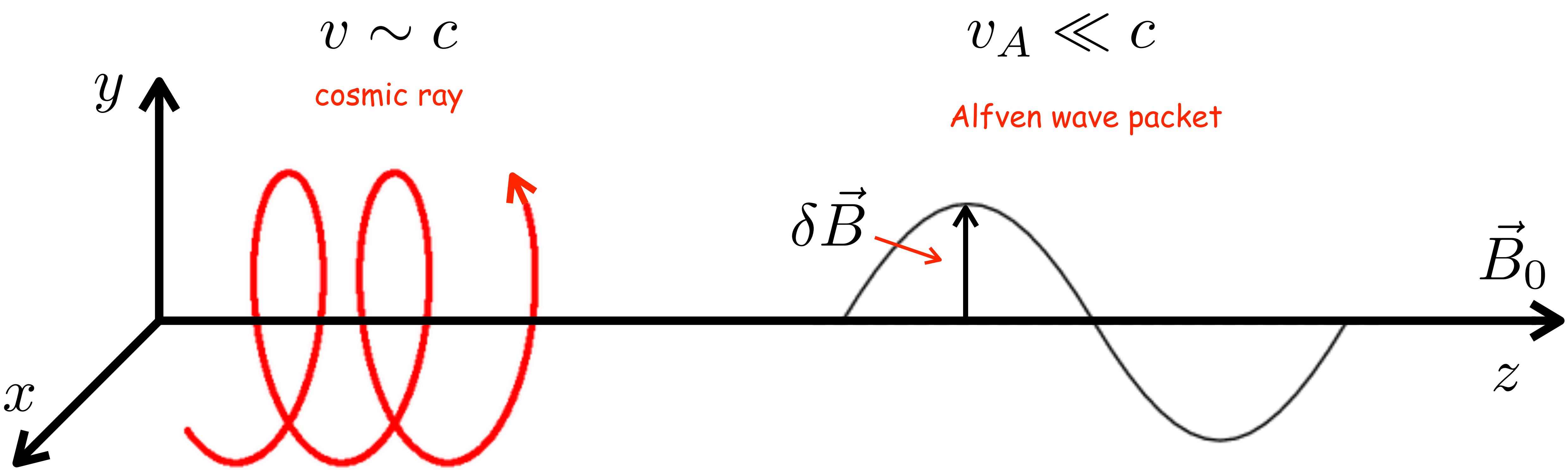


$$\delta \vec{B}_y \sim \delta B \sin(kz)\, \vec{y} \qquad v_x = v_\perp \sin(\Omega\, t + \phi)$$

$$(\vec{v} \times \vec{B})_z = v_x\ \delta B_y \ = v_\perp \delta B \sin(kz) \sin(\Omega\, t + \phi)$$

position of the cosmic ray at time t $\quad z = v_z t$

Cosmic ray scattering off Alfven waves

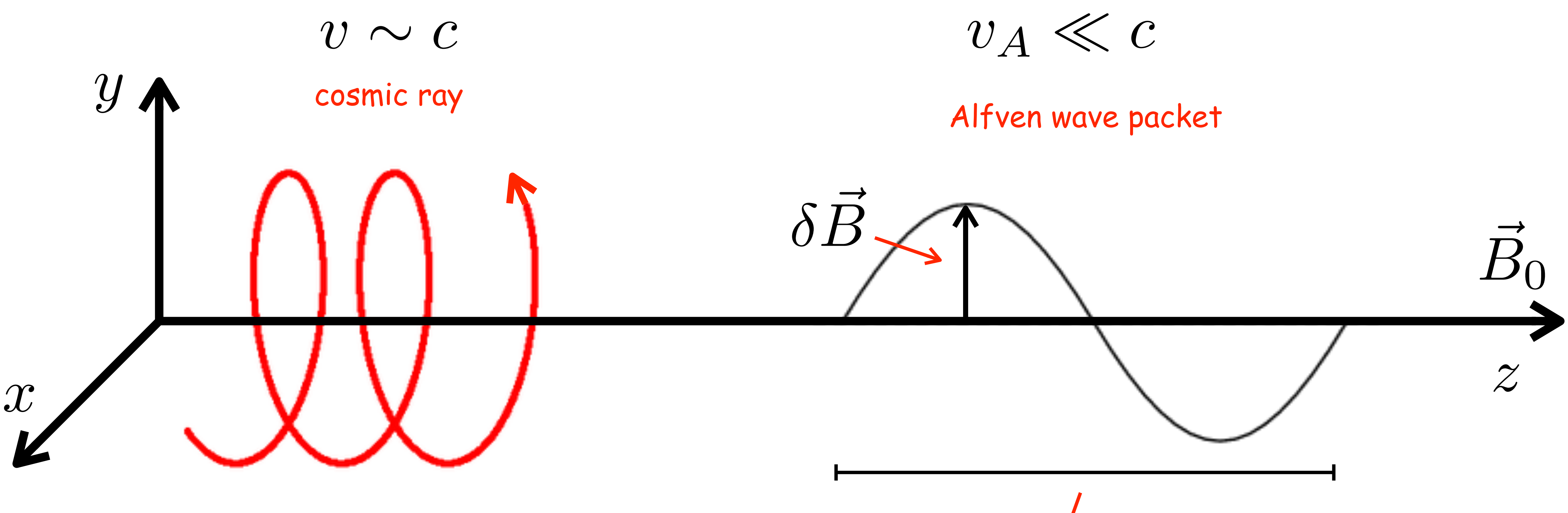


$$(\vec{v} \times \vec{B})_z = v_x \ \delta B_y \ = v_\perp \delta B \sin(k v_z t) \sin(\Omega \ t + \phi)$$

$$= \frac{v_\perp \delta B}{2} \left[\cos(k v_z t - \Omega \ t - \phi) - \cos(k v_z t + \Omega \ t + \phi)\right]$$

$$= \frac{v_\perp \delta B}{2} \left\{\cos\left[(k v_z - \Omega) \ t - \phi\right] - \cos\left[(k v_z + \Omega) \ t + \phi\right]\right\}$$

Cosmic ray scattering off Alfven waves



crossing time —> $$\tau_c = \frac{\lambda}{v_z} = \frac{2\pi}{k v_z}$$

this averages out

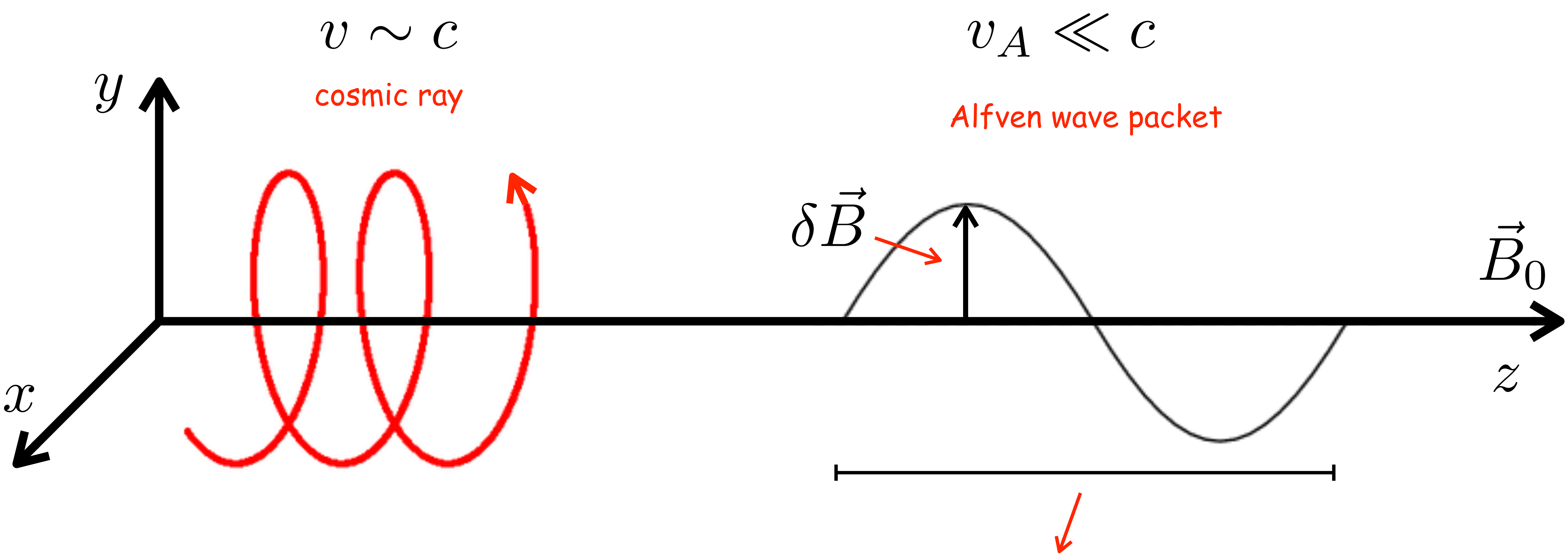
frequency

$$(\vec{v} \times \vec{B})_z = \frac{v_\perp \delta B}{2} \left\{ \cos\left[(k v_z - \Omega)\ t - \phi\right] - \cos\left[(k v_z + \Omega)\ t + \phi\right] \right\}$$

# of oscillations of the second cos in a crossing time

$$(k v_z + \Omega)\tau_c = (k v_z + \Omega)\frac{2\pi}{k v_z} = 2\pi\left(1 + \frac{\Omega}{k v_z}\right) \gg 1$$

Cosmic ray scattering off Alfven waves



crossing time –> $\tau_c = \dfrac{\lambda}{v_z} = \dfrac{2\pi}{k v_z}$

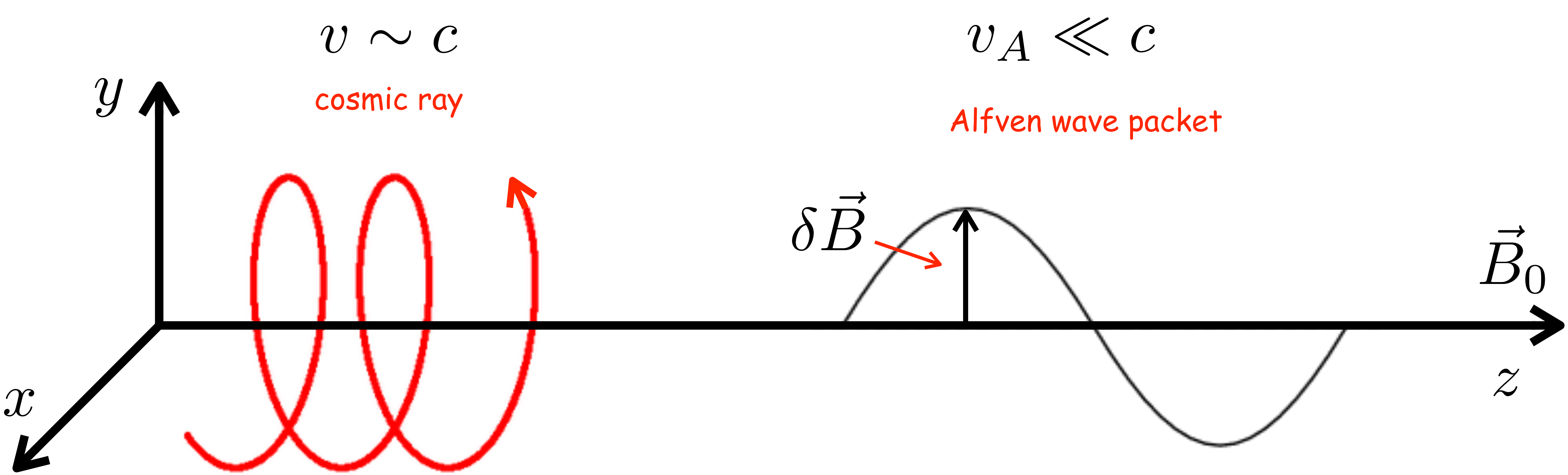
frequency

$$(\vec{v} \times \vec{B})_z = \frac{v_\perp \delta B}{2} \left\{ \cos\left[(k v_z - \Omega)\, t - \phi \right] - \cos\left[(k v_z + \Omega)\, t + \phi \right] \right\}$$

Φ

the first cos does NOT averages out when: $k v_z \approx \Omega$ –> does NOT depend on time

Resonant scattering



$$kv_z \approx \Omega = \frac{v_\perp}{R_L}$$

$$\tau_c = \frac{\lambda}{v_z} = \frac{2\pi}{kv_z} \approx \frac{2\pi}{\Omega} = \tau_g$$

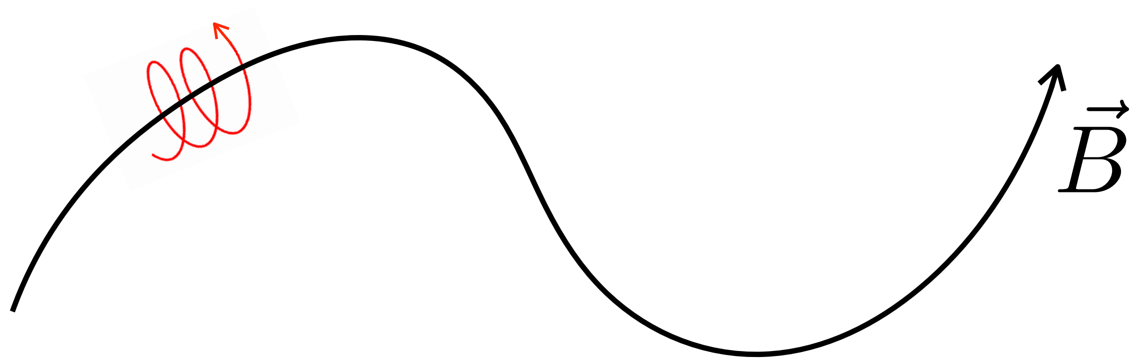
crossing time = gyration time

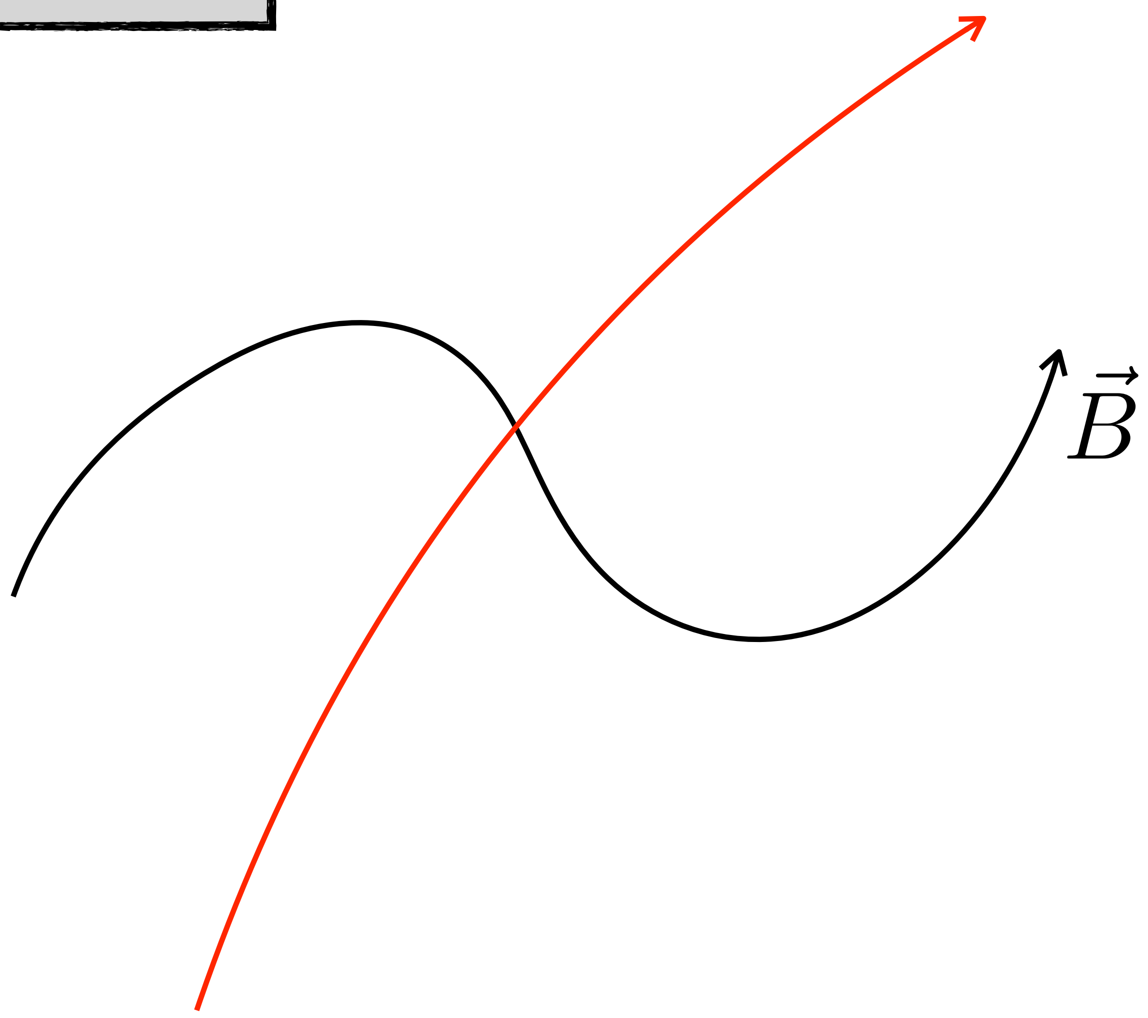
neglecting factors of order unity:

$$R_L \approx \frac{1}{k}$$

Resonant scattering

no scattering when:





$$R_L \ll \frac{1}{k}$$

$$R_L \gg \frac{1}{k}$$

Pitch angle scattering

$$\Delta p_z = \int_0^{\tau_c} \mathrm{d}t\ F_{L,z} = \frac{q}{c}\int_0^{\tau_c} \mathrm{d}t\ (\vec{v}\times\vec{B})_z = \frac{q}{c}\,\tau_c\,\frac{v_\perp \delta B}{2}\Phi$$

$$\left.\begin{array}{r} kv_z \approx \Omega = \dfrac{v_\perp}{R_L} \\ \tau_c = \dfrac{\lambda}{v_z} = \dfrac{2\pi}{kv_z} \approx \dfrac{2\pi}{\Omega} = \tau_g \end{array}\right\} \Delta p_z = \frac{\pi q \Phi}{c} R_L \delta B = \pi\Phi\left(\frac{\delta B}{B_0}\right) p_\perp$$

$$R_L = \frac{p_\perp c}{q B_0}$$

amplitude of the perturbation

v_A << v -> Alfven wave virtually at rest -> static B field -> CR particle energy is conserved!

$$\Delta p_z \sim \delta(p \cos\vartheta) \sim -p\sin\vartheta\,\delta\vartheta = -p_\perp \delta\vartheta$$

conserved

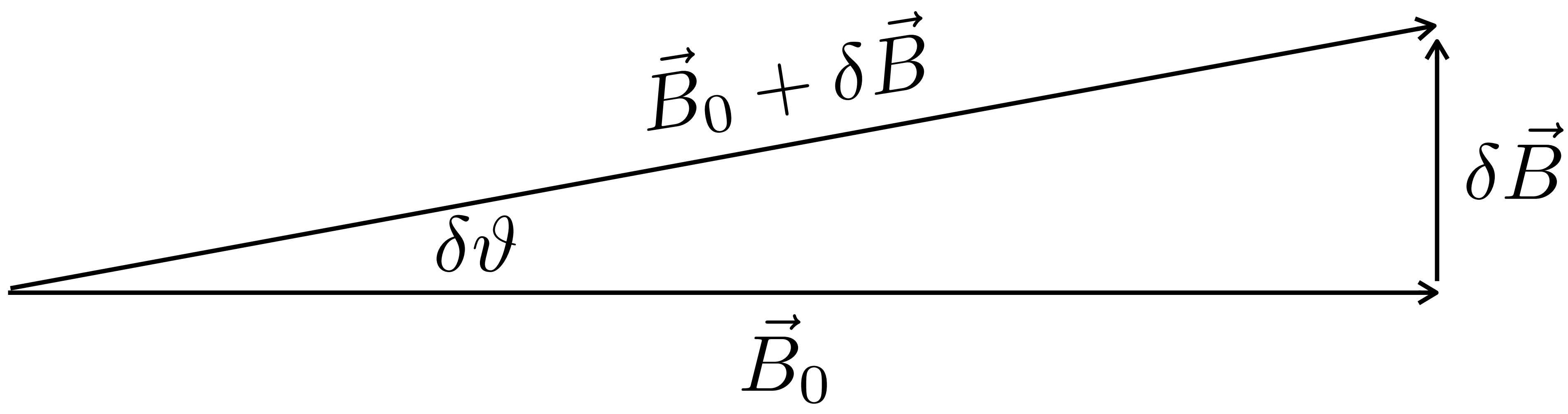
Pitch angle scattering

variation of the pitch angle after a scattering

$$\delta\vartheta \sim -\pi \left(\frac{\delta B}{B_0} \right) \Phi$$

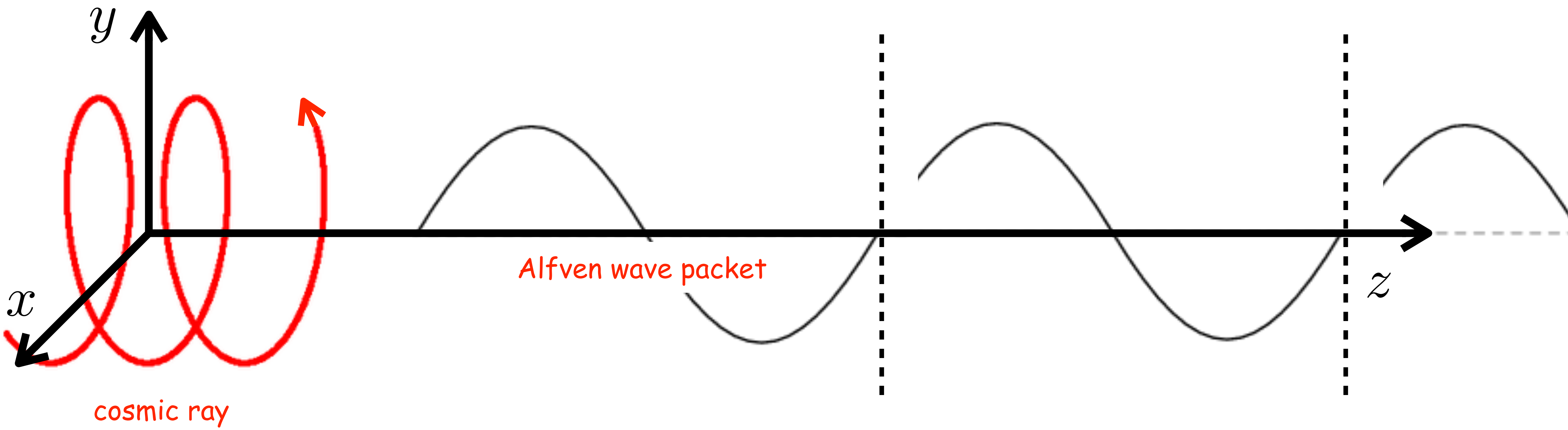
neglecting factors of order unity —>

$$\delta\vartheta \sim \pm \frac{\delta B}{B_0}$$



variation of the pitch angle = deflection of the field line due to the wave

Wave train



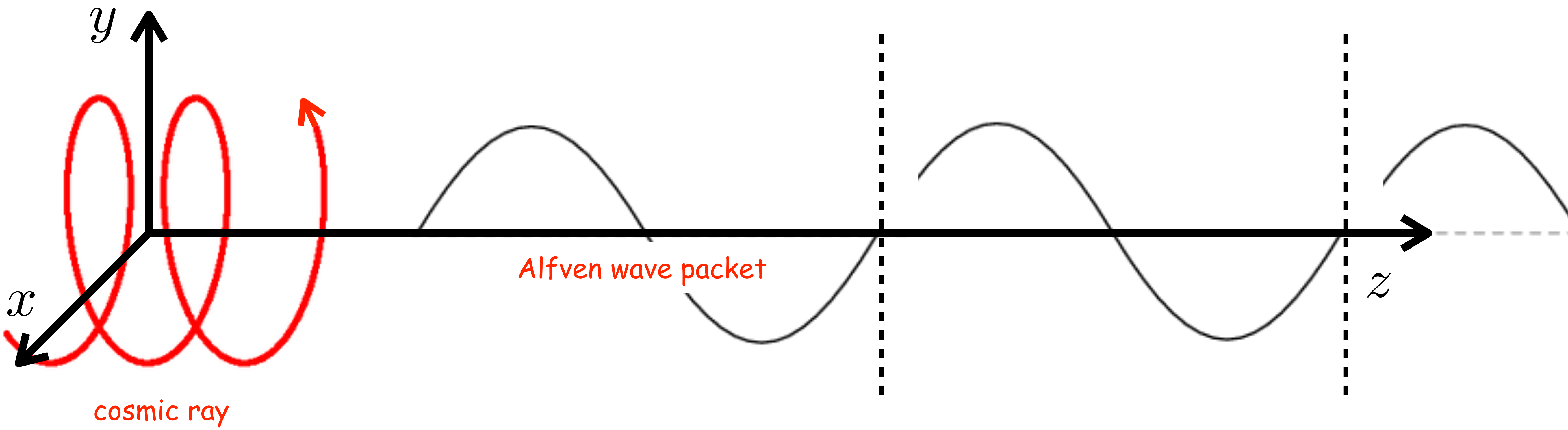
random walk

$$\langle (\Delta\vartheta)^2 \rangle = n \, \langle (\delta\vartheta)^2 \rangle$$

total mean squared displacement

mean squared displacement for a single interaction

Wave train



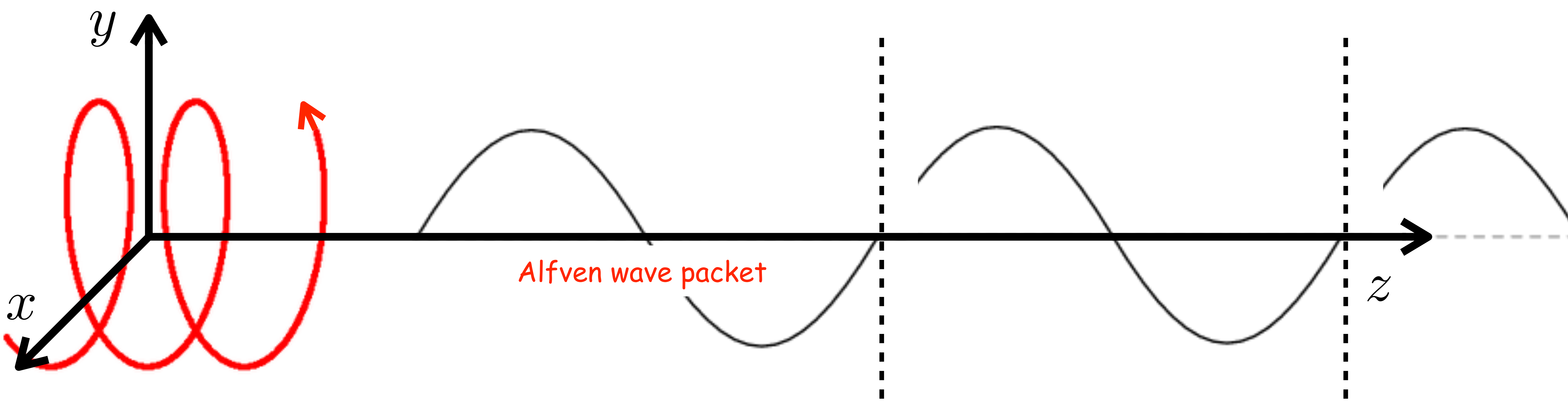
$$\langle (\Delta\vartheta)^2 \rangle = n \, \langle (\delta\vartheta)^2 \rangle = \frac{\Omega \, t}{2\pi} \langle (\delta\vartheta)^2 \rangle$$

$$n = \frac{t}{\tau_c}$$

resonance condition

$$\tau_c = \frac{\lambda}{v_z} = \frac{2\pi}{k v_z} \approx \frac{2\pi}{\Omega} = \tau_g$$

Wave train



n identical and densely packed waves, with random phases Φ

linear in t, as expected (random walk)

$$\langle (\Delta\vartheta)^2 \rangle = n \, \langle (\delta\vartheta)^2 \rangle = \frac{\Omega \, t}{2\pi} \langle (\delta\vartheta)^2 \rangle = \frac{\pi}{4} \, \Omega \, \langle \left(\frac{\delta B}{B_0} \right)^2 \rangle \, t$$

single scattering

$$\delta\vartheta^2 \sim \pi^2 \left(\frac{\delta B}{B_0} \right)^2 \Phi^2 \rightarrow \frac{\pi^2}{2} \langle \left(\frac{\delta B}{B_0} \right)^2 \rangle$$

average of Φ^2 = $\cos^2$(phase)

Diffusion coefficient

$$D_\vartheta = \frac{\langle (\Delta\vartheta)^2 \rangle}{2\ t} = \frac{\pi}{8}\ \Omega\ \left\langle \left(\frac{\delta B}{B_0} \right)^2 \right\rangle$$

$$\frac{1}{D_\vartheta} \rightarrow$$ characteristic time to diffuse over 1 radian

transport equation

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} \left[(1 - \mu^2) D_\vartheta \frac{\partial f}{\partial \mu} \right]$$

particles move along B with velocity μv

no flux beyond the boundaries +1 and -1

and diffuse in pitch angle

The link with the turbulent spectrum

for a spectrum of waves

$$k_{min} < k < k_{max}$$

resonance condition

$$D_\vartheta = \frac{\langle (\Delta\vartheta)^2 \rangle}{2\, t} = \frac{\pi}{8}\, \Omega \, \langle \left(\frac{\delta B_k}{B_0} \right)^2 \rangle$$

wave spectrum

$$\langle \left(\frac{\delta B_k}{B_0} \right)^2 \rangle \equiv k \, W(k)$$

normalised energy per unit wave number

$$\langle \left(\frac{\delta B_{TOT}}{B_0} \right)^2 \rangle = \int \frac{\mathrm{d}k}{k} \langle \left(\frac{\delta B_k}{B_0} \right)^2 \rangle = \int \mathrm{d}k \; W(k)$$

resonance condition

$$R_L(E) \approx \frac{1}{k} \longrightarrow D_\vartheta(E)$$

energy dependent diffusion coefficient

Spatial diffusion coefficient

iotropisation time

$$\tau_s \sim \frac{1}{D_\vartheta}$$

particles lose memory of the initial pitch angle

spatial diffusion coefficient

Bohm

mean free path

particle velocity

$$D \sim \lambda v \sim (\tau_s v) v \sim \frac{v^2}{D_\vartheta} \sim \frac{v^2}{\Omega\ kW(k)} \sim \frac{R_L v}{kW(k)}$$

relativistic CRs

$R_L \sim E$

resonance condition

$$D \sim \frac{R_L c}{kW(k)} \propto \frac{E}{k^{1-\alpha}} \longrightarrow E^{2-\alpha}$$

$W(k) \sim k^{-\alpha}$

energy dependent diffusion!

from observations:

$$2 - \alpha \sim 0.3 \ldots 0.5$$

The diffusion equation

stationary

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left[(1-\mu^2) D_\vartheta \frac{\partial f}{\partial \mu}\right]$$

assumption: quasi isotropic particle distribution function

isotropic — anisotropic (and small)

$$f = f^{(0)} + f_\mu^{(1)} \qquad \int_{-1}^{1} \mathrm{d}\mu \; f_\mu^{(1)} = 0$$

$$\mu v \left(\frac{\partial f^{(0)}}{\partial z} + \frac{\partial f_\mu^{(1)}}{\partial z}\right) = \frac{\partial}{\partial \mu}\left[(1-\mu^2) D_\vartheta \frac{\partial f_\mu^{(1)}}{\partial \mu}\right]$$

The diffusion equation

$$\mu v \left(\frac{\partial f^{(0)}}{\partial z} + \frac{\partial f_\mu^{(1)}}{\partial z} \right) = \frac{\partial}{\partial \mu} \left[(1-\mu^2) D_\vartheta \frac{\partial f_\mu^{(1)}}{\partial \mu} \right]$$

let's integrate from -1 and μ

$$\frac{\mu^2 - 1}{2} v \frac{\partial f^{(0)}}{\partial z} + v \int_{-1}^{\mu} \mathrm{d}\mu \frac{\partial f_\mu^{(1)}}{\partial z} = (1-\mu^2) D_\vartheta \frac{\partial f_\mu^{(1)}}{\partial \mu}$$

$$\frac{v f^{(0)}}{D_\vartheta L} \qquad \frac{v f_\mu^{(1)}}{D_\vartheta L} \qquad f_\mu^{(1)}$$

strong scattering regime

$$\tau_s = \frac{1}{D_\vartheta} \ll \frac{L}{v} = \tau_c \longrightarrow \frac{v}{D_\vartheta L} \ll 1$$

The diffusion equation

$$\frac{\mu^2 - 1}{2} \; v \; \frac{\partial f^{(0)}}{\partial z} = (1 - \mu^2) D_\vartheta \frac{\partial f_\mu^{(1)}}{\partial \mu}$$

let's integrate again from -1 and μ

$$f_\mu^{(1)} = C - \frac{v}{2} \frac{\partial f^{(0)}}{\partial z} \int_{-1}^{\mu} \frac{\mathrm{d}\mu}{D_\vartheta}$$

integration constant

The diffusion equation

$$\mu v \left(\frac{\partial f^{(0)}}{\partial z} + \frac{\partial f_\mu^{(1)}}{\partial z} \right) = \frac{\partial}{\partial \mu} \left[(1-\mu^2) D_\vartheta \frac{\partial f_\mu^{(1)}}{\partial \mu} \right]$$

let's integrate this time from -1 to 1

$$\int_{-1}^{1} \mathrm{d}\mu \; \mu v \, \frac{\partial f_\mu^{(1)}}{\partial z} = 0$$

$$\frac{\partial}{\partial z} \left[\frac{v^2}{2} \int_{-1}^{1} \mathrm{d}\mu' \mu' \int_{-1}^{\mu'} \frac{\mathrm{d}\mu}{D_\vartheta} \frac{\partial f^{(0)}}{\partial z} \right] = 0$$

$$f_\mu^{(1)} = C - \frac{v}{2} \frac{\partial f^{(0)}}{\partial z} \int_{-1}^{\mu} \frac{\mathrm{d}\mu}{D_\vartheta}$$

diffusion equation!!!

The spatial diffusion coefficient

integration by parts

$$D = \frac{v^2}{2} \int_{-1}^{1} \mathrm{d}\mu' \mu' \int_{-1}^{\mu'} \frac{\mathrm{d}\mu}{D_\vartheta} = \frac{v^2}{4} \int_{-1}^{1} \mathrm{d}\mu \, \frac{1-\mu^2}{D_\vartheta}$$

for isotropic scattering D_ϑ is independent on μ

$$\Rightarrow \quad D = \frac{v^2}{3 \, D_\vartheta}$$

(our rough estimate was v^2/D_ϑ)

diffusive flux

diffusion equation!!!

$$\frac{\partial}{\partial z}\left(D \frac{\partial f^{(0)}}{\partial z} \right) = 0$$

The diffusive flux

why do particles diffuse?

isotropic part

$$\int_{-1}^{1} \mathrm{d}\mu \ \mu v \ f^{(0)} = 0$$

anisotropic part

$$\int_{-1}^{1} \mathrm{d}\mu \ \mu v \ f_{\mu}^{(1)} = D \frac{\partial f^{(0)}}{\partial z}$$

Time dependent diffusion equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial f}{\partial z}\right) + Q$$

injection term (CR sources)

solution for an impulsive source and homogeneous diffusion

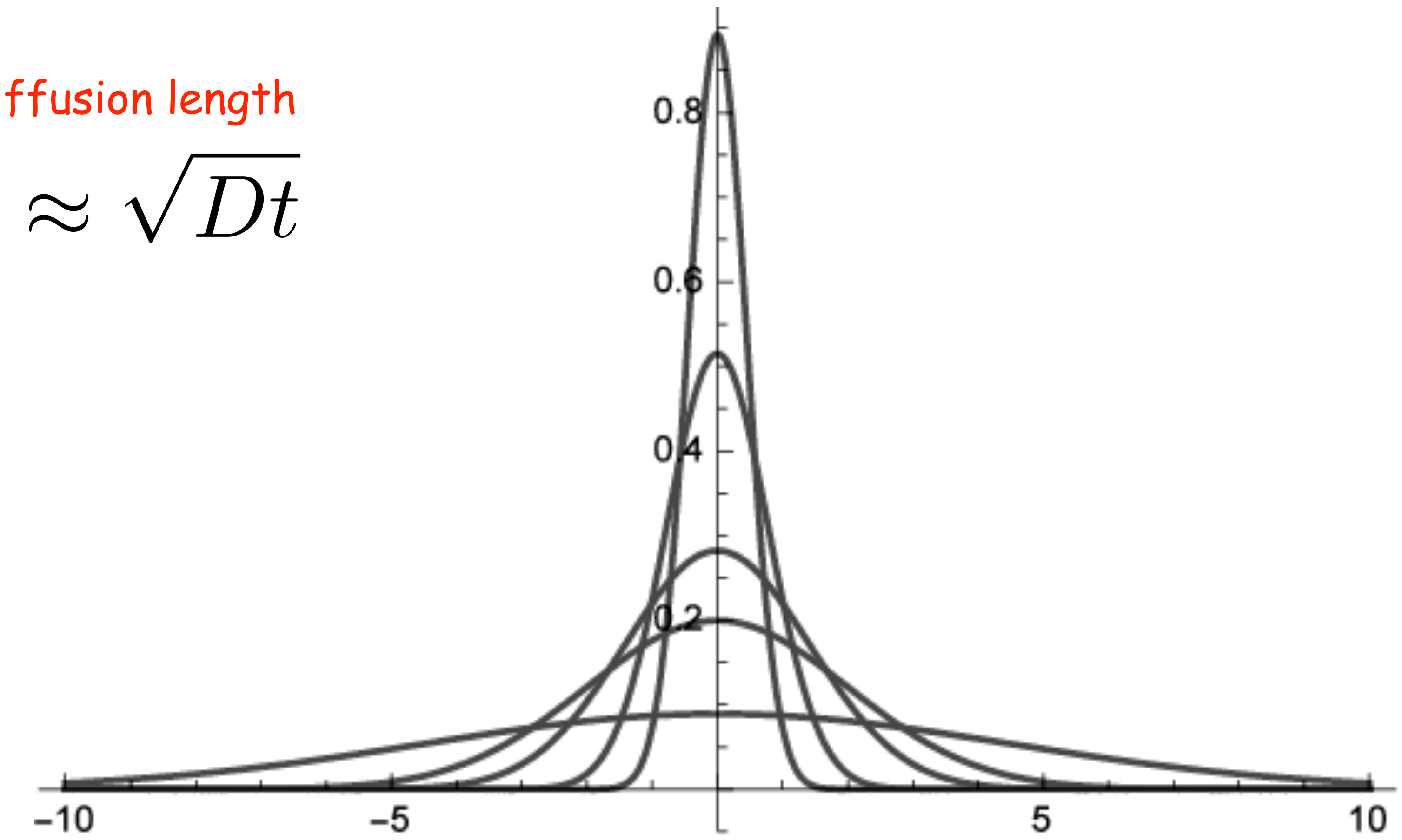
$$Q = Q_0\ \delta(t)\ \delta(z)$$

$$D(z) = D$$

diffusion length

$$z \approx \sqrt{Dt}$$

$$f(z,t) = \frac{Q_0}{\sqrt{4\pi D t}} e^{-\frac{z^2}{4Dt}}$$



Bohm diffusion

minimum plausible value for the diffusion coefficient

typically invoked in highly turbulent media

$$D_\vartheta \approx \Omega \approx \frac{v}{R_L}$$

particles are isotropised in one gyration

Bohm diffusion

$$D = \frac{v^2}{3D_\vartheta} \longrightarrow \frac{1}{3} R_L v$$

quasi-linear theory

$$D = \frac{D_B}{kW(k)} = D_B / \langle \left(\frac{\delta B_k}{B_0} \right)^2 \rangle$$

$$D_B \sim 10^{23} \left(\frac{E}{10 \text{ GeV}} \right) \left(\frac{B}{3\ \mu\text{G}} \right)^{-1} \text{cm}^2/\text{s}$$

Perpendicular diffusion

$$D_{\parallel} = \frac{v^2}{3\ D_{\vartheta}} \sim \frac{D_B}{kW(k)}$$

$$\frac{D_{\perp}}{D_{\parallel}} = k^2 W(k)^2 = \left(\frac{\delta B_k}{B_0}\right)^4 \ll 1$$

$$D_{\parallel} D_{\perp} = D_B$$

$$\sim 1 \text{ for } t \sim 1/D_{\vartheta}$$

$$\lambda_{\perp} \sim \sqrt{N}\,(\delta\vartheta\, R_L) \sim R_L$$ perpendicular displacement after $t \sim \frac{1}{D_{\vartheta}}$

after N scatterings (random walk)

displacement perpendicular to the field after one scattering

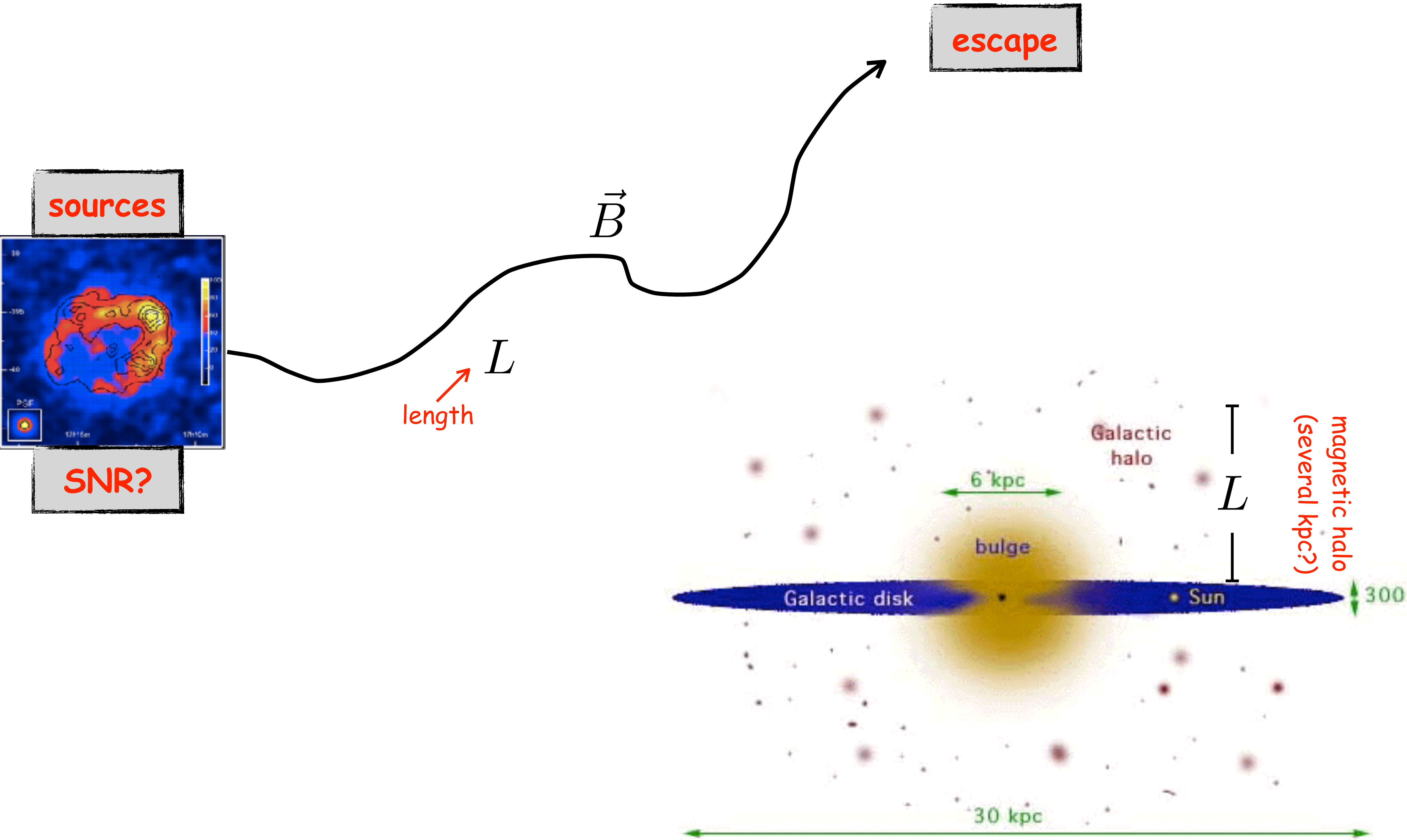
$$D_{\perp} \sim \lambda_{\perp}\,(\lambda_{\perp} D_{\vartheta}) \sim R_L^2 D_{\vartheta} \sim R_L^2\ \Omega\ kW(k) \sim D_B\ kW(k)$$

mean free path

velocity

v/R_L

CR escape from the Galaxy: a toy-model

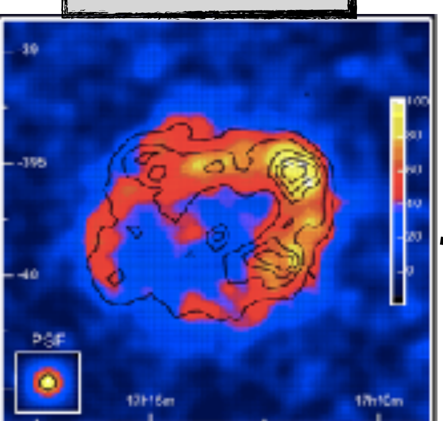


CR escape from the Galaxy: a toy-model

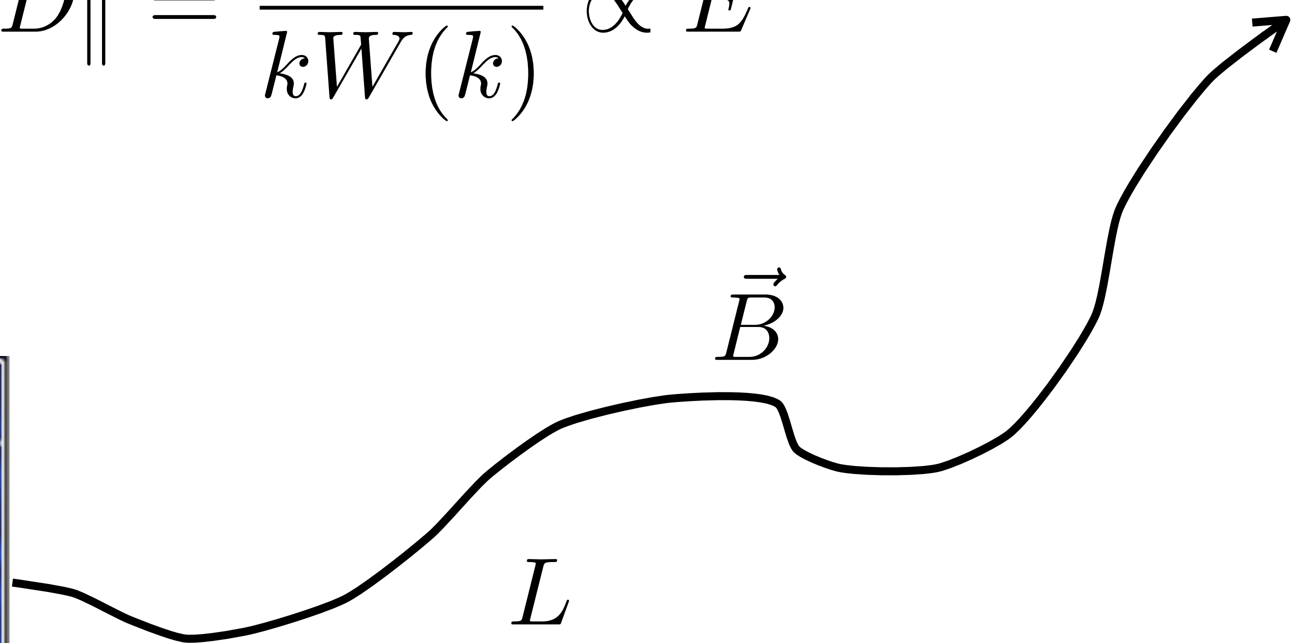
$$D_{\parallel} = \frac{D_B}{kW(k)} \propto E^{0.3\ldots0.5}$$

escape

sources



SNR?



3 kpc

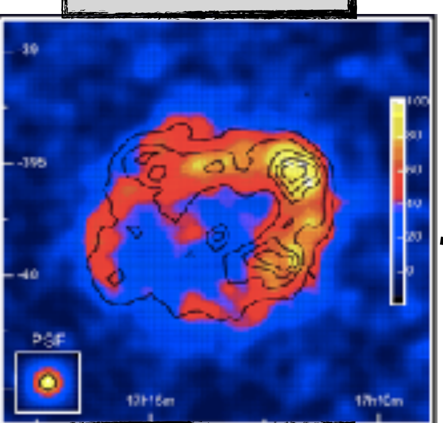
$$\tau_{esc} \sim \frac{L^2}{D_{\parallel}} \approx 10 \text{ Myr} \quad \longrightarrow \quad \frac{\delta B_k}{B_0} \sim 6 \times 10^{-4}$$

CR escape from the Galaxy: a toy-model

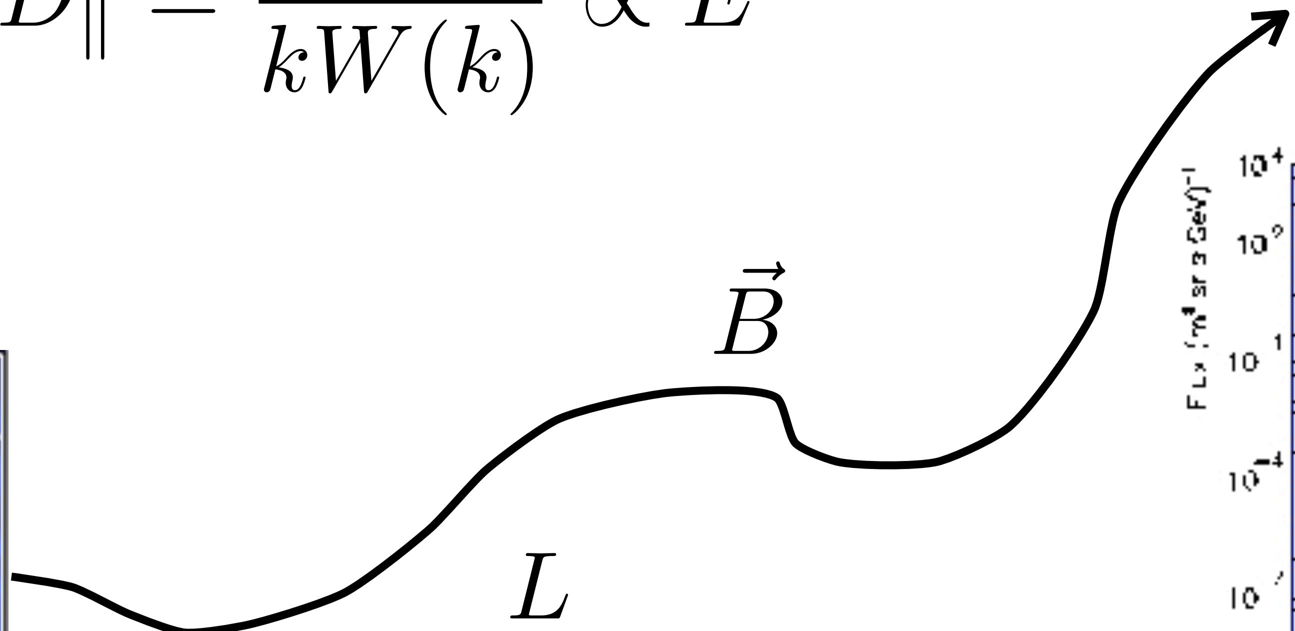
$$D_{\parallel} = \frac{D_B}{kW(k)} \propto E^{0.3\ldots0.5}$$

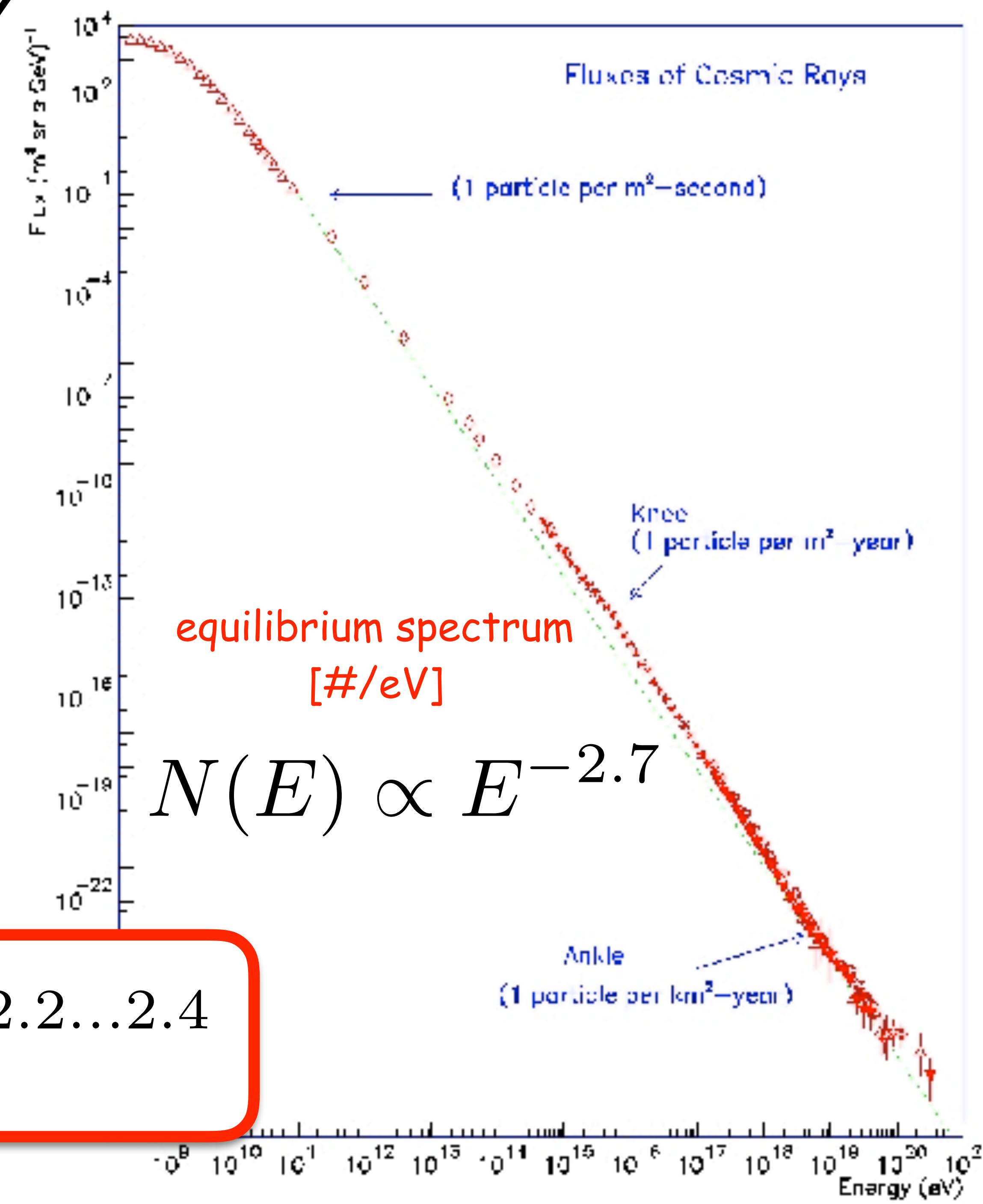
escape

sources



SNRs?





equilibrium spectrum
[#/eV]

$$N(E) \propto E^{-2.7}$$

$$Q(E) \propto E^{-\alpha}$$

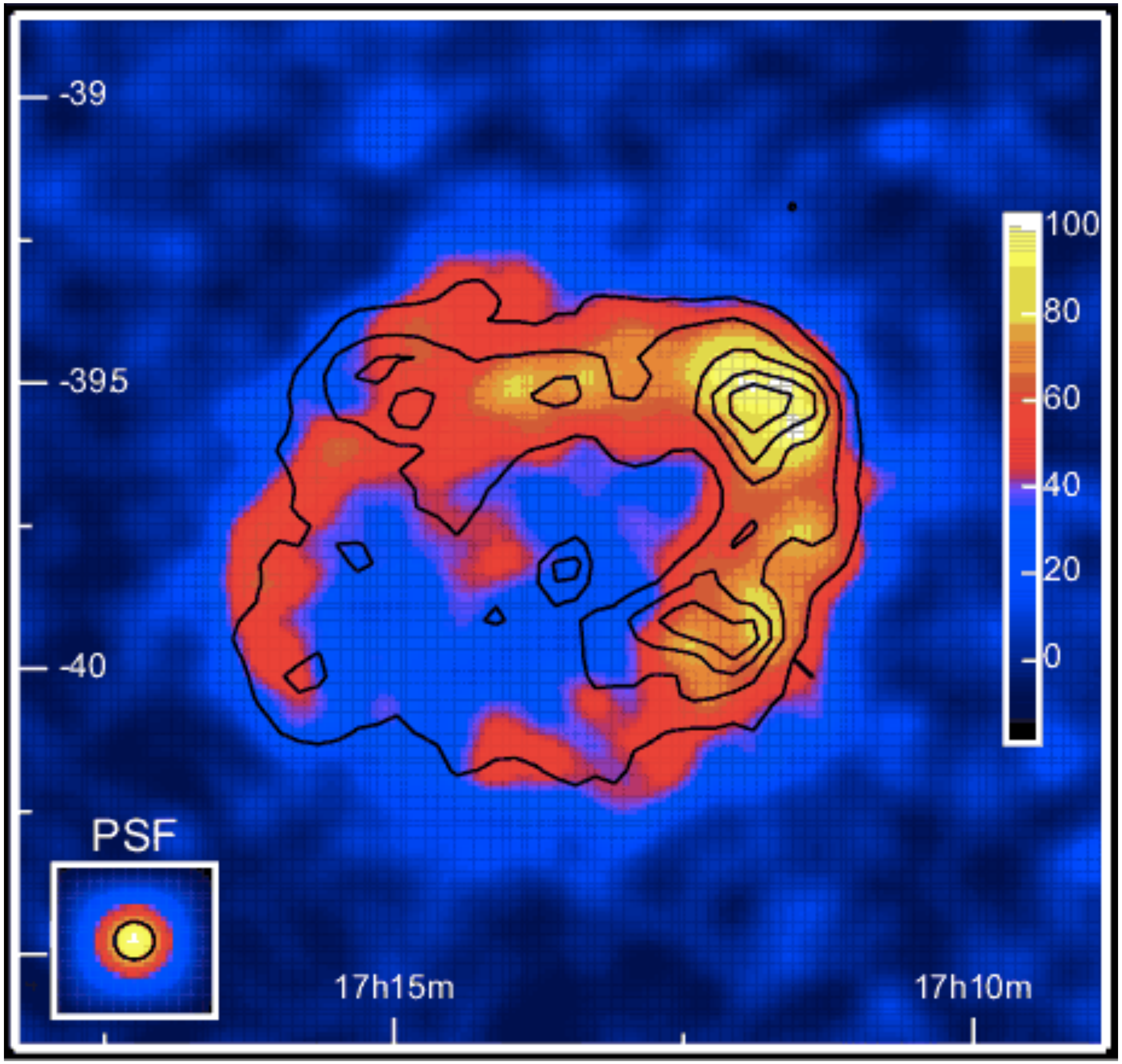
injection spectrum
[#/eV/s]

$$N(E) = Q(E)\tau_{esc} \rightarrow Q(E) \propto E^{-2.2\ldots2.4}$$

NPAC course on Astroparticles

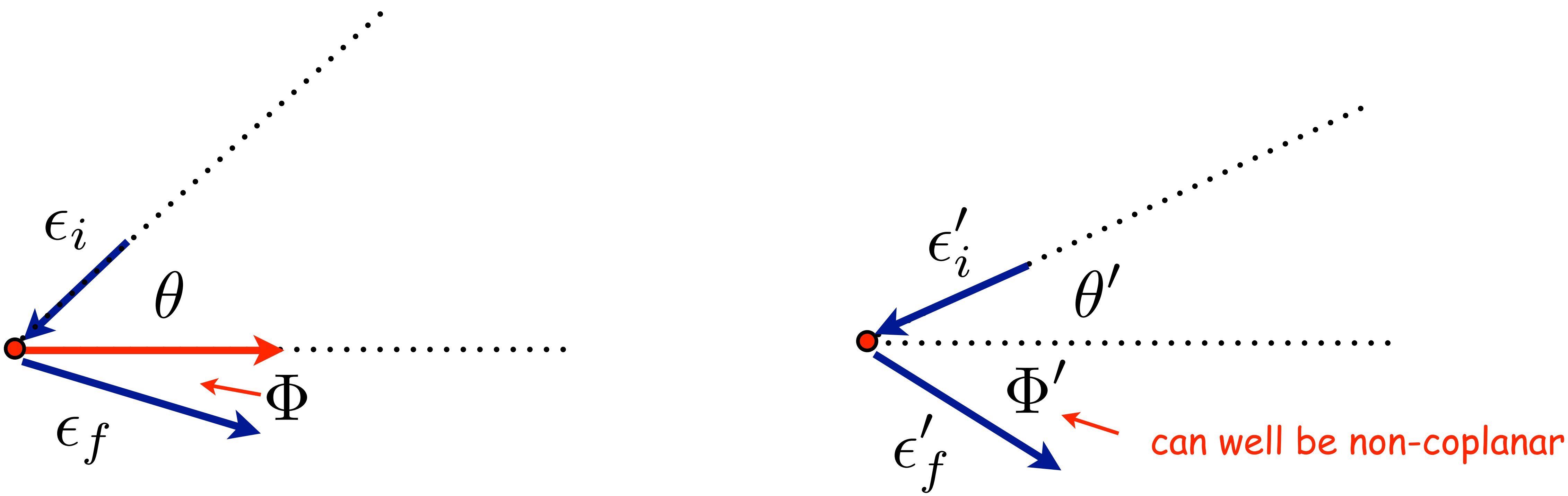
II ter - LEPTONIC GAMMA-RAYS: INVERSE COMPTON SCATTERING

Gamma-rays from supernova remnants: hadronic or leptonic?



Leptonic Gamma-Rays: Inverse Compton

Relativistic electrons can interact with soft background photons
(Cosmic Microwave Background, IR and Optical galactic background...)



In the electron rest frame (e.r.f.) the photon energy is: $\epsilon'_i = \epsilon_i \gamma (1 - \beta \cos\theta)$

Assumption: in the e.r.f the scattering is Thomson: $\epsilon'_f = \epsilon'_i$

In the lab rest frame the (final) photon energy is: $\epsilon_f = \epsilon'_f \gamma (1 + \beta \cos\Phi)$

Leptonic Gamma-Rays: Inverse Compton

$$\epsilon_f = \gamma^2 \epsilon_i \, G(\theta, \Phi)$$

After averaging over angles (tedious...):

$$\epsilon_f = \frac{4}{3} \gamma^2 \epsilon_i$$

Example:

Cosmic Microwave Background -> $T \sim 3 \text{ K}$ $\quad kT \approx 3 \times 10^{-4} \text{eV}$

- $E_e = 1 \text{ GeV} \rightarrow \epsilon_\gamma = 1,5 \text{ keV}$ X-rays
- $E_e = 1 \text{ TeV} \rightarrow \epsilon_\gamma = 1,5 \text{ GeV}$ gamma rays (FERMI)
- $E_e = 25 \text{ TeV} \rightarrow \epsilon_\gamma = 1 \text{ TeV}$ gamma rays (Cherenkov Telescopes)

Leptonic Gamma-Rays: Inverse Compton

is there a maximum energy for the up-scattered photons?

$$\epsilon_f = \frac{4}{3}\gamma^2 \epsilon_i \quad < \gamma \; m \; c^2$$

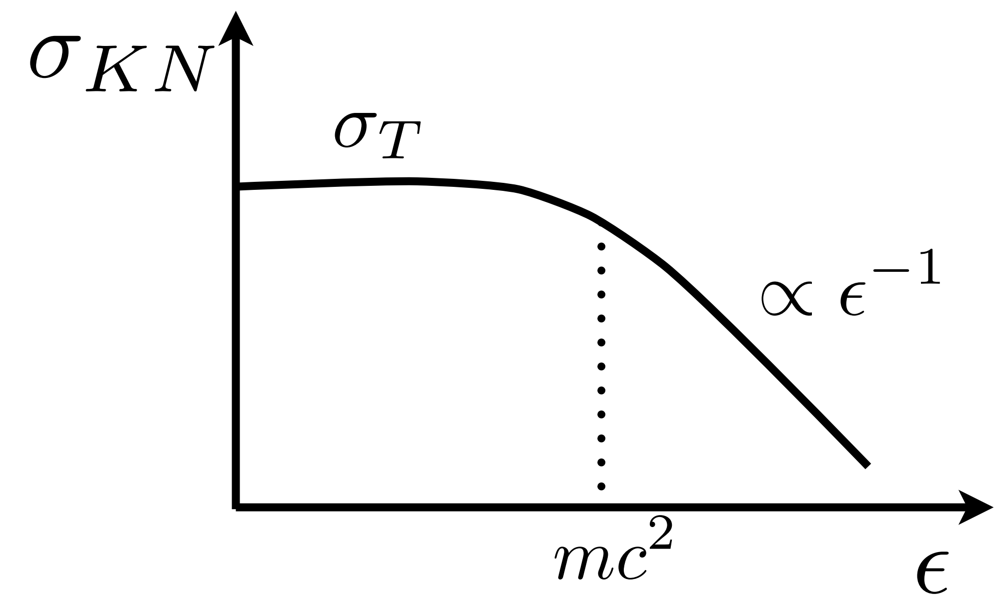
energy conservation...

above a given energy Inverse Compton scattering becomes ineffective

☑ let's check the assumption of Thomson scattering in the e.r.f.:

photon energy in the e.r.f: $\epsilon_i' \approx \gamma \; \epsilon_i$

Thomson scattering ONLY if: $\gamma \; \epsilon_i < mc^2$

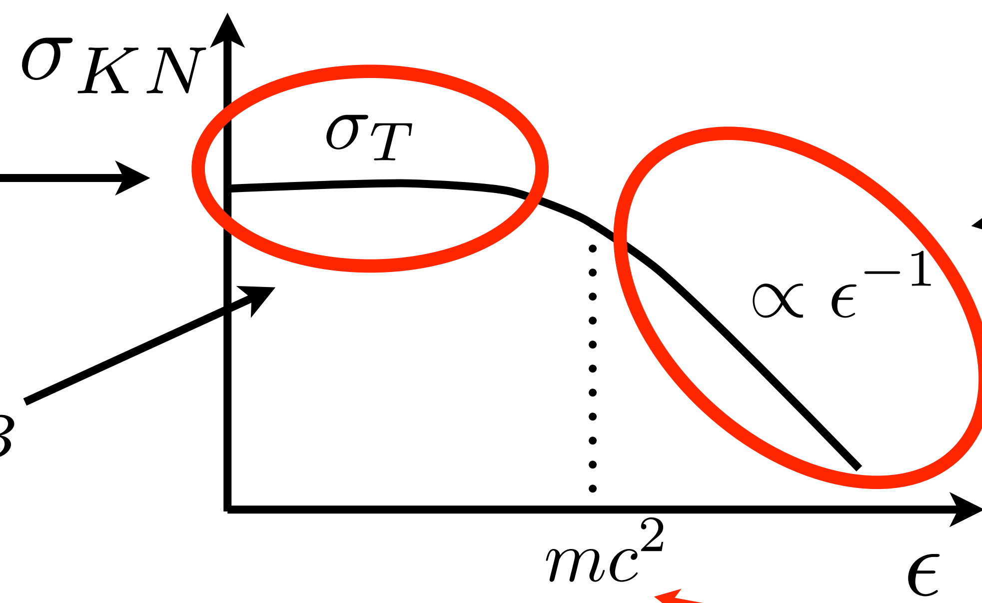


if $\gamma \; \epsilon_i \sim mc^2$ we must use the quantum relativistic (Klein-Nishina) cross section

Leptonic Gamma-Rays: Inverse Compton

$\gamma\ \epsilon_\gamma < mc^2$

$\epsilon_\gamma = \frac{4}{3}\ \gamma^2\ \epsilon_{CMB}$



$\gamma\ \epsilon_\gamma > mc^2$

$\epsilon_\gamma \approx E_e$

cutoff in the photon spectrum

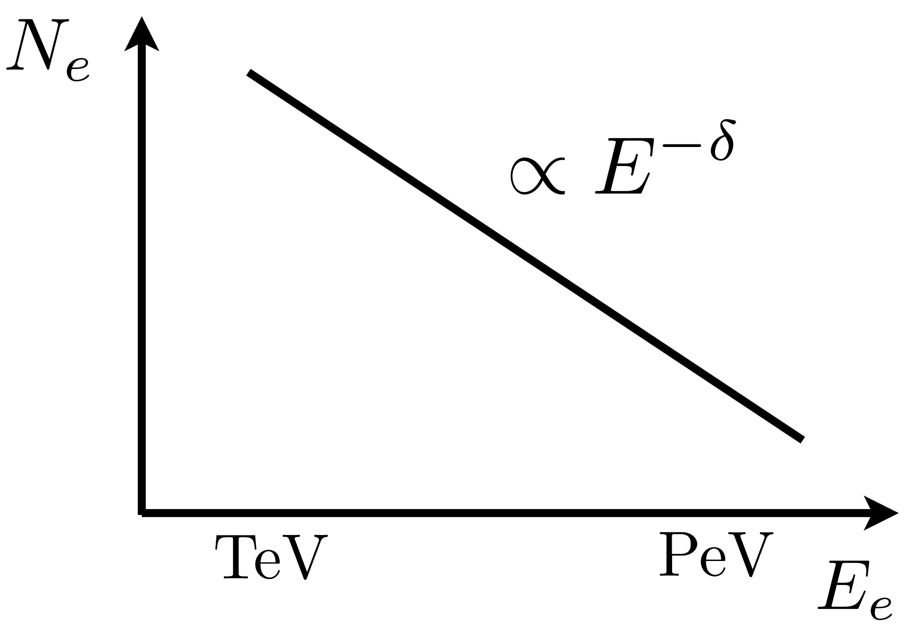
$E_e = \gamma\ m\ c^2$	$\frac{\epsilon_\gamma}{E_e}$	ϵ_γ
1 TeV	~0.2%	~1.5 GeV
25 TeV	~4%	~1 TeV
100 TeV	~15%	~15 TeV
200 TeV	~30%	~60 TeV
....	....	....
E_e >> 200 TeV	~100%	$E_e \sim E_\gamma$

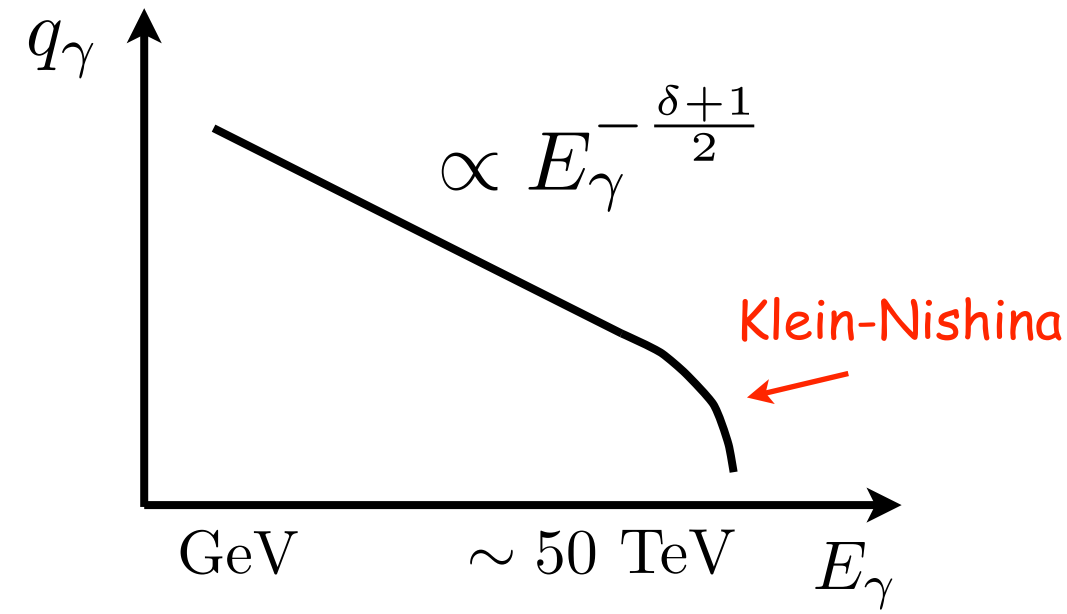
Thomson

Klein-Nishina

Leptonic Gamma-Rays: Inverse Compton

Photon spectrum:





$$q_\gamma(E_\gamma) = \int \mathrm{d}E_e N_e(E_e)\delta(E_\gamma - \frac{4}{3}\gamma^2\epsilon_{CMB})(n_{CMB}\sigma_T c) \propto E_\gamma^{-\frac{\delta+1}{2}}$$

$$\delta(g(x)) = \frac{\delta(x-x_0)}{|g'(x_0)|}$$

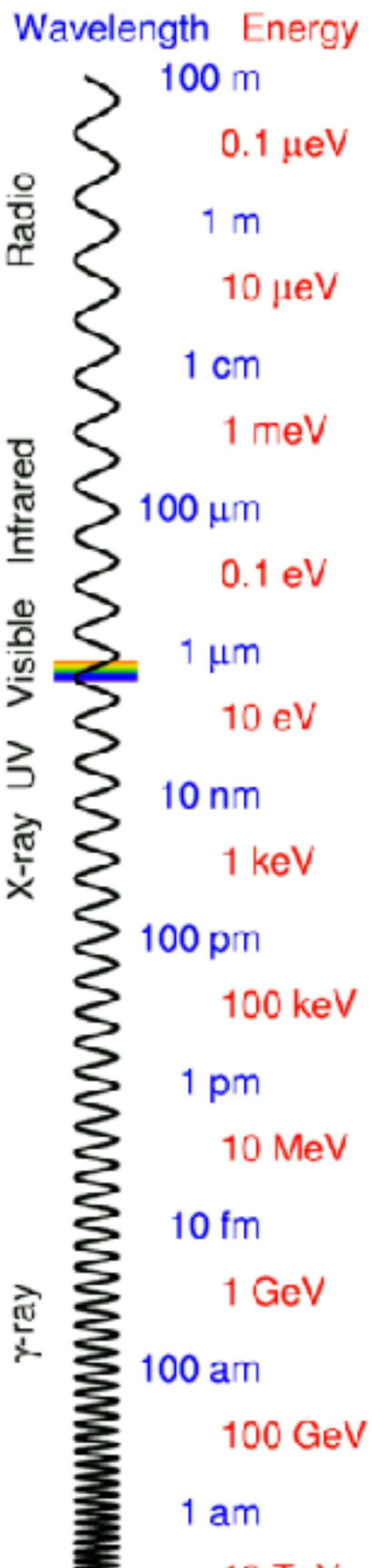
$$g(x_0) = 0$$

$g(x)$

$$\delta\left(E_\gamma - \frac{4}{3}\left(\frac{E_e}{mc^2}\right)^2 \epsilon_{CMB}\right) = \frac{\delta(E_e - aE_\gamma^{1/2})}{\frac{2}{a}E_\gamma^{1/2}}$$

$x_0 \propto E_\gamma^{1/2}$ $\quad g' \propto E_e$

The electromagnetic spectrum



21 cm line -> atomic neutral Hydrogen (HI)
synchrotron emission of multi-GeV electrons in a 3 μG field

visible light -> stars

thermal emission from hot plasmas (millions of K)
synchrotron emission of 100 TeV electrons in a 3 μG field
inverse Compton emission of ~GeV electrons in the CMB

interactions of cosmic ray protons with matter
inverse Compton emission of TeV/multi-TeV electrons in the CMB