The Cosmic Microwave Background

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Outline

- Generalities
- Power spectrum and correlation function
- The sound horizon, r_d
- the ΛCDM parameters $(\Omega_\Lambda, \Omega_{cdm}, \Omega_b, H_0, A_s)$
- H_0 from local distance ladder
- Polarization and gravitational waves

Recombination: before and after

electrons and protons combine to form hydrogen atoms at $T \sim 3000K$ corresponding to $z \sim 1090$

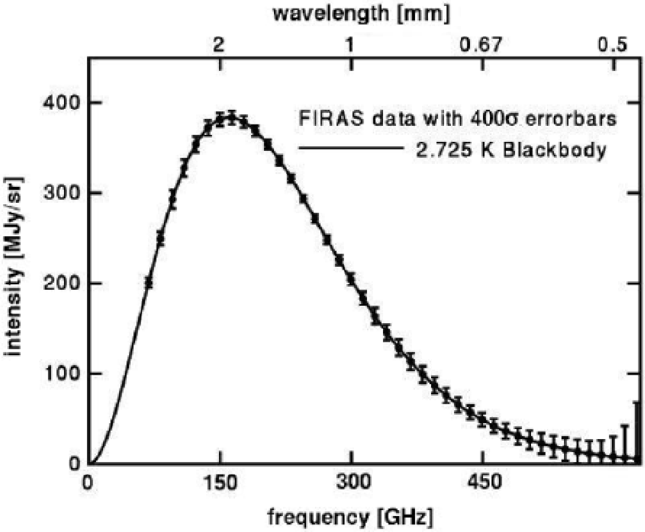
Before recombination:

- Compton scattering $\Rightarrow$ photon mean-free-path much less then c/H. $\Rightarrow$ universe is opaque
- (electron-proton-photon) system is a nearly perfect fluid supporting acoustic waves with $c_s \sim c/\sqrt{3}$

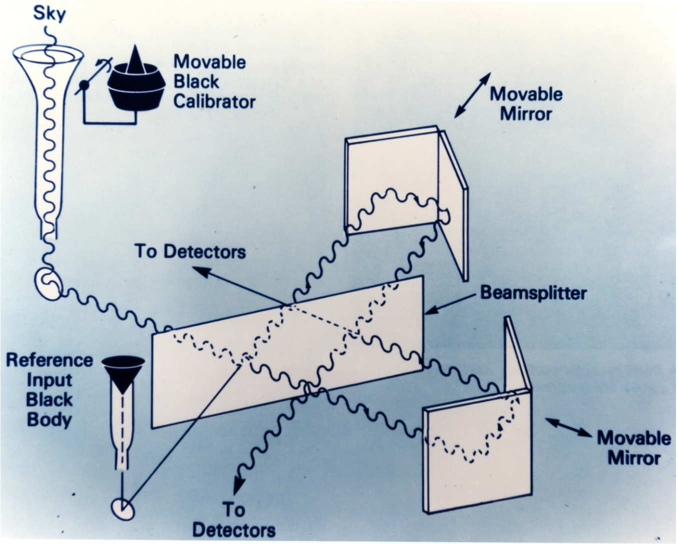
After recombination:

- Photons "free-stream":mean-free-path much greater than c/H $\Rightarrow$ universe is transparent.
- Acoustic waves stop

CMB spectrum from COBE



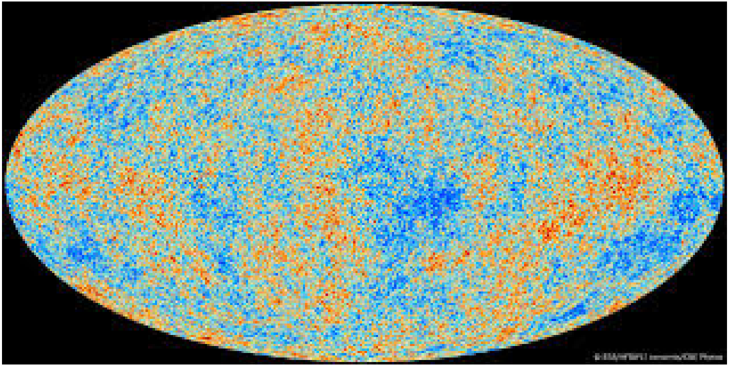
CMB spectrum from COBE-FIRAS



Fourier transform spectrometer

Planck all-sky CMB temperature map

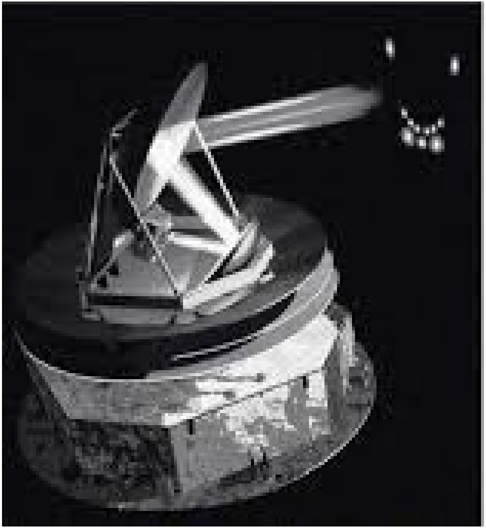
(after subtraction of solar-system-velocity effect, $v/c\ 2 \times 10^{-3}$)

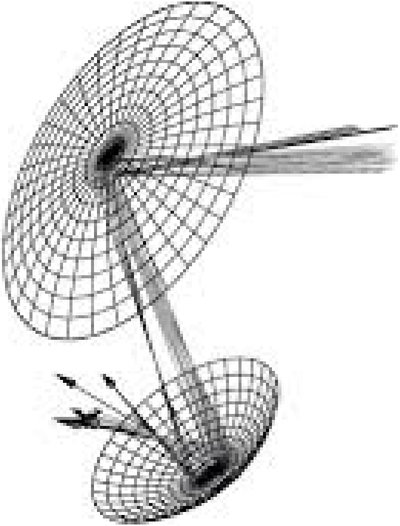


$T_0(1 - 10^{-5})$
$T_0(1 + 10^{-5})$

Thermal spectrum: $T_0 = 2.728K$, $kT = 2.348 \times 10^{-4} eV$
Photons last scattered (on electrons) when
redshift$=z \sim 1090$ $\quad t \sim 4 \times 10^5$yr, $\quad kT \sim 0.2$eV

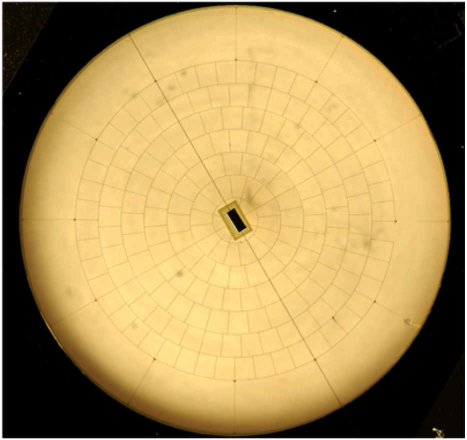
Planck Satellite





Planck focal plane

Planck Bolometers



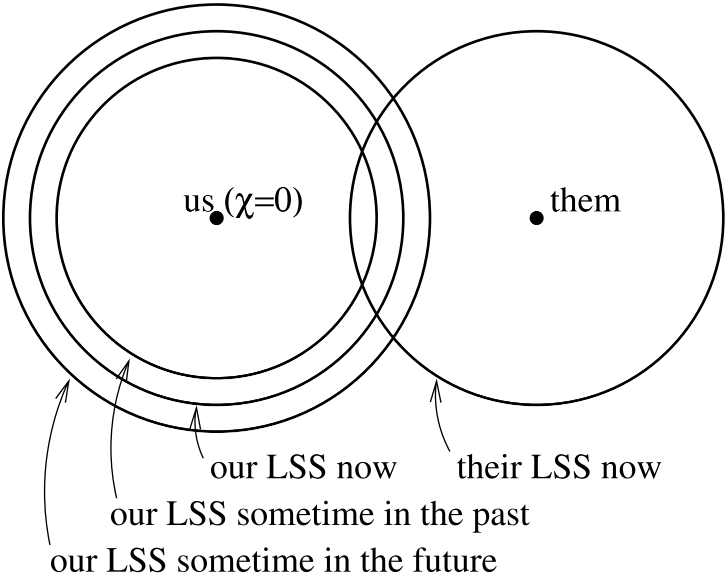
“spider web”

$T = 0.1K$

1 micron thick gold-plated silicon nitride wires.

Germanium thermister at center.
$30 \times 100 \times 300$ micron

Last scattering surface



Two distances and an angle

Sound horizon expanded to today:

$$r_d = \int_{z_{rec}}^{\infty} \frac{c(z)dz}{H(z)} \approx \frac{c}{H(z_{rec})}$$

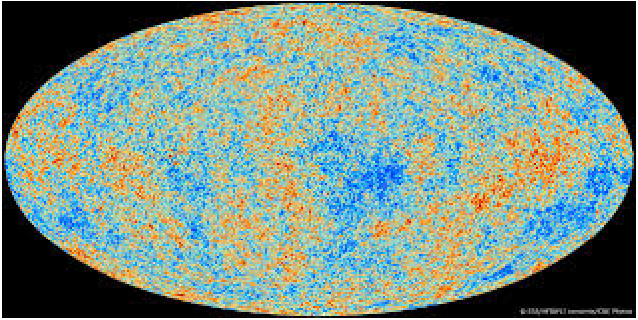
Present distance to LSS:

$$D(z = z_{rec}) = \int_0^{z_{rec}} \frac{dz}{H(z)}$$

Angle on the sky subtended by the sound horizon on the LLS:

$$\theta_d = \frac{r_d}{D_M(z = z_{rec})} \qquad D_M = D \text{ for a flat universe}$$

Three origins of temperature fluctuations



cold regions:

1. potential wells on LSS (concentrations of DM+baryons)
   $\Delta T/T \sim \Delta\Phi$
2. plasma is colder than average
   $\Delta T/T \sim \Delta T_{plasma}/T$
3. plasma's peculiar velocity points away from us.
   $\Delta T/T \sim \Delta v_p/c$

Potential fluctuations dominate for $\theta > \theta_d$

Sphere on the LSS of radius R and mass excess ΔM.
Potential fluctuation are related to density fluctuations:

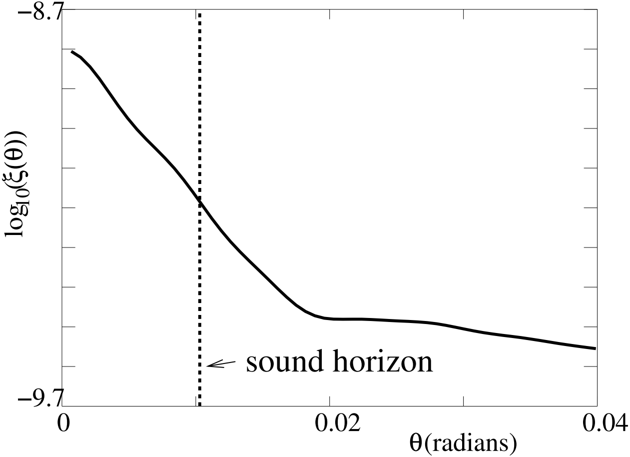
$$\Delta\Phi_g \approx \frac{G\Delta M}{R} \approx G\rho\frac{\Delta\rho}{\rho}R^2 \approx \frac{\Delta\rho}{\rho}\frac{R^2}{r_d^2}$$

Plasma tempertaure fluctuations:

$$\frac{\Delta T_{plasma}}{T} = (1/3)\frac{\Delta n_\gamma}{n_\gamma} = (1/3)\frac{\Delta n_b}{n_b} = (1/3)\frac{\Delta\rho}{\rho}$$

("Adiabatic" density fluctuations)

Temperature correlation function



$$\frac{\langle T(\vec{\theta}_1)\, T(\vec{\theta}_2)\rangle - \bar{T}^2}{\bar{T}^2}$$

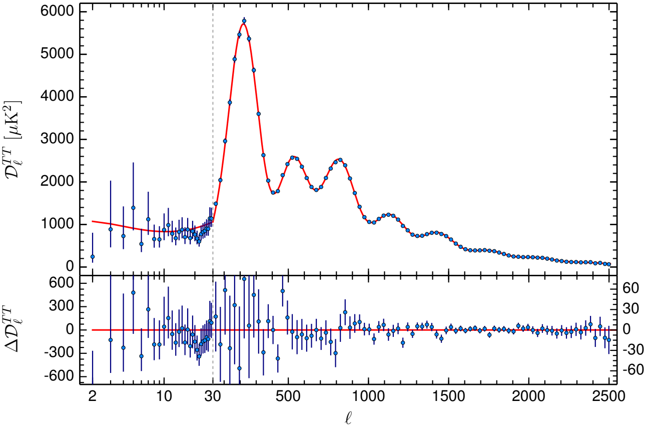
"Fourier transform" of power spectrum

Large correlations for $\theta < 2r_d/D(z_{rec})$
Small (primordial) correlations for $\theta > 2r_d/D(z_{rec})$

Planck CMB power spectrum

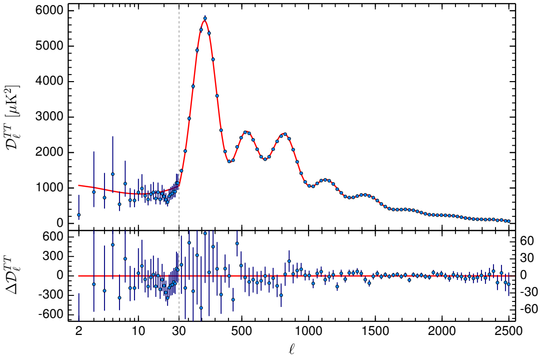
$$T(\theta,\phi) = \bar{T} + \sum_{\ell,m} a_{\ell m} Y^{\ell}_{m}(\theta,\phi) \qquad \bar{T}^2 - \bar{T}^2 = (2\pi)^{-1}\sum_{\ell,m} |a_{\ell,m}|^2$$

$$\frac{\ell(\ell+1)}{2\pi}\langle|a_{\ell m}|^2\rangle$$



$$\text{“}\theta_\ell\text{”} \sim \pi/\ell \qquad \text{“}\lambda_\ell\text{”} \sim \pi D_M(z_{rec})/\ell$$

Cosmological information in power spectrum



Peak positions
$\Rightarrow D_m(z_{lss})/r_d$

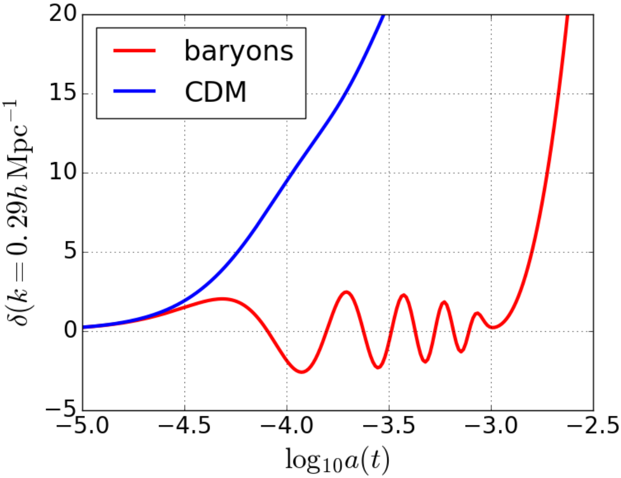
Spectrum shape (relative peak heights) gives $\rho_{\rm M}/\rho_\gamma$, $\rho_{\rm B}/\rho_\gamma$

$\Rightarrow \Omega_{\rm M} H_0^2, \Omega_{\rm B} H_0^2$

The (simplified) primary effects [Hu et al. 2001 ApJ 549, 669]:

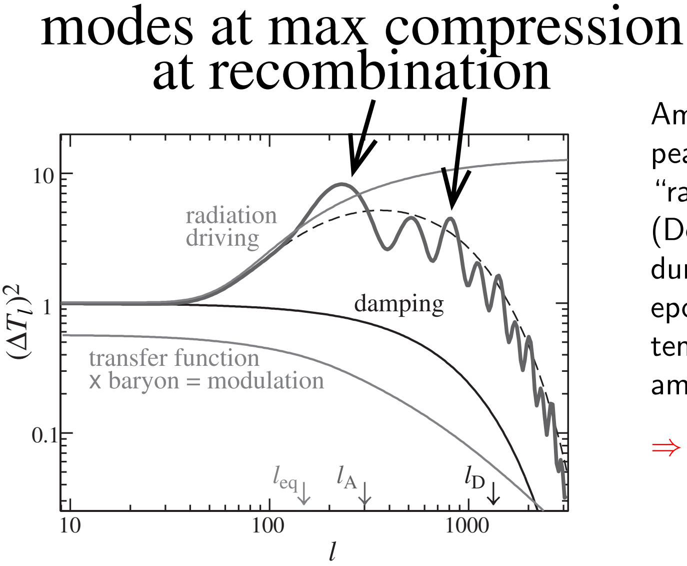
- Power($\ell < 30$) $\Rightarrow A_s$ (primordial fluctuations)
- Power(peak 1)/Power($\ell < 30$) $\Rightarrow \Omega_{\rm M} h^2/\Omega_{\rm R} h^2$
- Power(even peaks)/Power(odd peaks)$\Rightarrow \Omega_{\rm B} h^2/\Omega_{\rm M} h^2$
- Power($\ell > 1000$)/Power(peak 1)$\Rightarrow N_\nu$

$\delta_{\vec{k}}$ for CDM and baryons



baryons are alternately compressed and decompressed in CDM potential wells.

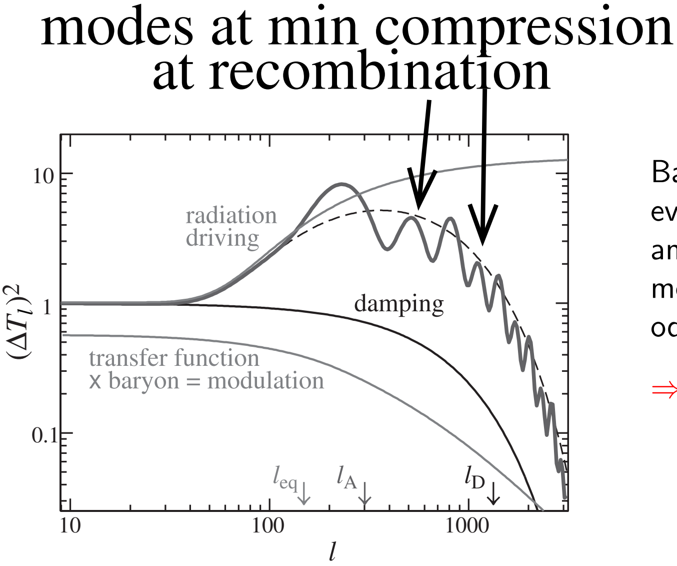
Modes at extrema at recombination



Amplitude of first peak determined by "radiation driving" (Decay of potential during radiation epoch increases temperature amplitude)

$\Rightarrow \rho_M/\rho_R$

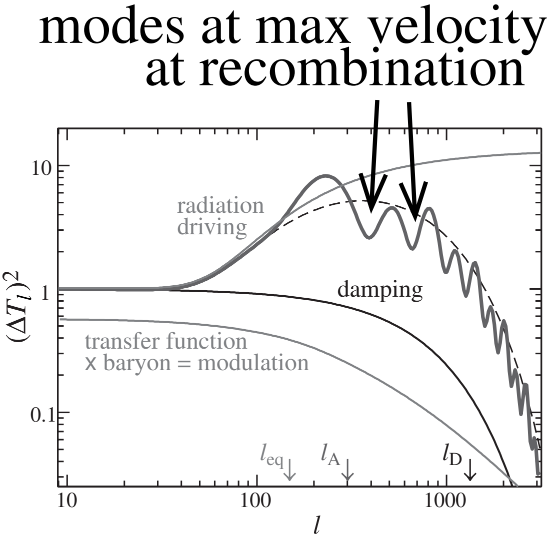
Modes at extrema at recombination



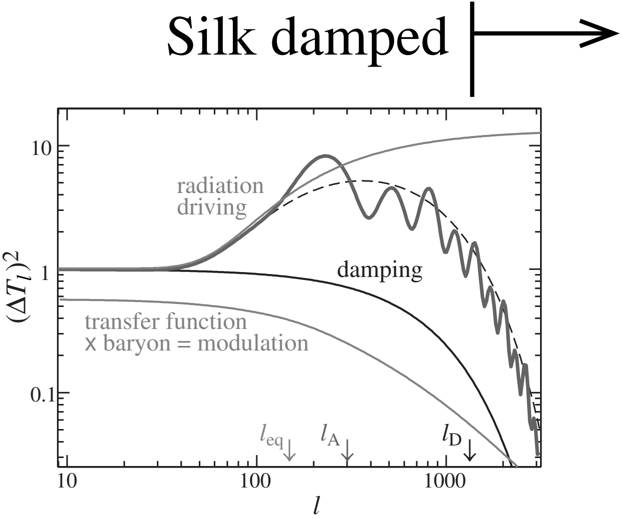
Baryon inertia makes even-peak amplitudes less than mean of neighboring odd-peak amplitudes

$\Rightarrow \rho_b/\rho_M$

Doppler effect suppressed by baryon mass

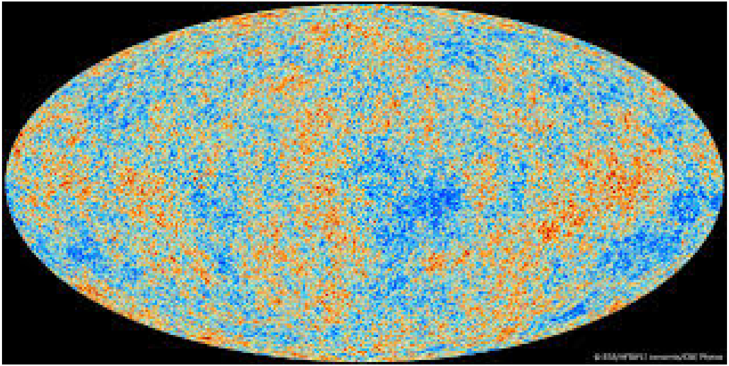


High-ℓ modes damped by photon random walk during one Hubble time at recombination



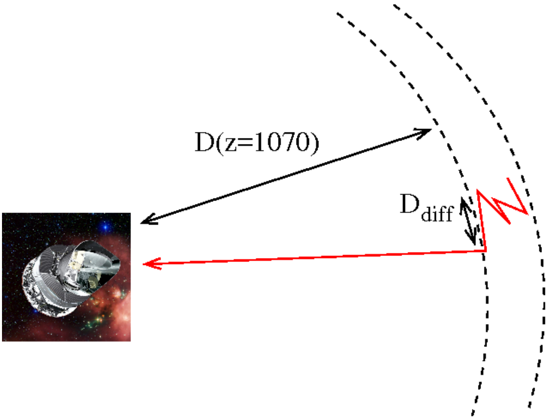
CMB image is "blurred"

Planck image of our "last-scattering surface":



The image is blurred because photons random-walk in the 100,000yr before recombination. Increasing the expansion rate, reduces the time for random walking, and makes the image sharper.

Photon random walk on last-scattering surface



$$D_{diff} \sim \sqrt{\lambda_{mf} \times ct_{walk}}$$

$$t_{walk} \sim 1/H(t_{rec})$$

$$\ell_{damp}/\ell_{peak1} \sim r_d/D_{diff}$$

$$\Rightarrow H(t_{rec}) \qquad \Rightarrow \rho(t_{rec})$$

$$\rho_\nu = \rho_{tot} - \frac{\rho_m}{\rho_{rel}}(\rho_\gamma + \rho_\nu) - \rho_\gamma \qquad N_\nu = 2.99 \pm 0.20$$

Calculation of the sound horizon

Same as "particle horizon" except $c_s < c$
$c_s = (c/\sqrt{3})f(\rho_\mathrm{B}/\rho_\gamma)$ (baryon inertia slows sound)

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)dz}{H(z)} = \sqrt{\frac{3}{8\pi G}} \int_{z_d}^{\infty} \frac{c_s(z)dz}{\sqrt{\rho_\mathrm{M} + \rho_\gamma + \rho_\nu}}$$

We normalize to the present photon density

$$r_d = \sqrt{\frac{3}{8\pi G \rho_\gamma(0)}} \int_{z_d}^{\infty} \frac{c_s(\rho_\mathrm{B}/\rho_\gamma)dz}{(1+z)^2\sqrt{\rho_\mathrm{M}(z)/\rho_\gamma(z) + 1 + \rho_\nu(z)/\rho_\gamma(z)}}$$

COBE gives us $\rho_\gamma(0)$ and the CMB spectrum shape (Planck) gives us the density ratios $\rho_\mathrm{M}/\rho_\gamma$, $\rho_\mathrm{B}/\rho_\gamma$, ρ_ν/ρ_γ.

r_d from COBE-Planck

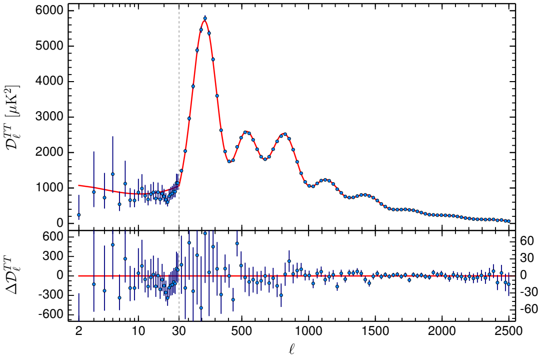
Imposing three neutrino families ($\rho_\nu = 0.23 N_\nu \rho_\gamma$) gives

$$r_d = (147.3 \pm 0.5)\mathrm{Mpc}$$

Fitting the CMB spectrum for N_ν gives $N_\nu = 2.99 \pm 0.2$ and

$$r_d = (147.4 \pm 1.5)\mathrm{Mpc}$$

Peak positions $D_m(z_{lss})/r_d \Rightarrow H_0$ (if $\Omega_k = 0$)



$$\theta_{BAO} = \frac{r_d}{D(z{=}z_{rec})}$$

$$\ell_{BAO} \approx \ell_1 \approx \frac{\pi}{\theta_{BAO}}$$

First peak: $\ell_1 \sim 200 \sim D(z = 1090)/r_d$:

$$D(z) = \int_0^z \frac{dz}{\left[H_0^2 + \Omega_{\mathrm{M}} H_0^2[(1+z)^3 - 1]\right]^{1/2}} \quad \Rightarrow H_0$$

($\sim 10\%$ of integral in H_0 dominated region)

Flat ΛCDM: CMB is enough

Planck 2015 (arXiv:1502.01589) (TT + LowP)

- $H_0 = 67.31 \pm 0.96$
- $\Omega_{\rm M} = 0.315 \pm 0.013$
- $\Omega_\Lambda = 1 - \Omega_{\rm M} = 0.685 \pm 0.013$
- $\Omega_{\rm B} h^2 = 0.02222 \pm 0.00023$

plus

- $A_s = (21.95 \pm 0.79) \times 10^{-10}$
  Amplitude of primordial scalar perturbations
- $n_s = 0.9655 \pm 0.0062$
  spectral index for scalar perturbations
- $\tau = 0.078 \pm 0.019$
  optical depth to last-scattering surface (reionization)

$D(z) \rightarrow D_M(z)$ with curvature

$$D(z) = a_0\chi(z) \;=\; \int_0^z D_H(z)dz \qquad D_H(z) = c/H(z)$$

$$D_M(z) = a_0 \begin{array}{c} \sin\chi \\ \chi \\ \sinh\chi \end{array} \;\mathrm{for}\; \begin{array}{c} \Omega_T > 1 \\ \Omega_T = 1 \\ \Omega_T < 1 \end{array}$$

We need a_0 from Friedman equation:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{k}{a^2} \quad \rightarrow \quad H_0^2 = H_0^2\Omega_T + \frac{k}{a_0^2} \quad \rightarrow \quad \frac{1}{a_0} = \frac{\sqrt{|1-\Omega_T|}}{c/H_0}$$

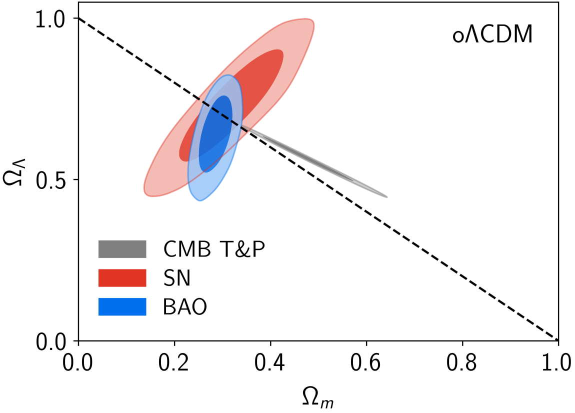
$$\chi(z) = \sqrt{|\Omega_k|}\frac{D(z)}{c/H_0} \quad \Rightarrow D_M(z) = D(z)\left[1 + \frac{\Omega_k}{6}\frac{D(z)^2}{(c/H_0)^2} + ...\right]$$

where $\Omega_k = 1 - \Omega_T$. For a given $D(z)$, $\Omega_T > 1 \Rightarrow$ objects look big.

Non-flat ΛCDM: CMB not enough

Distance to $z = 1090$ now depends also on Ω_k:

$D(H_0^2, \Omega_{\rm M} H_0^2) \;\rightarrow D_M(H_0^2, \Omega_{\rm M} H_0^2, \Omega_k H_0^2)$ or $D_M(H_0, \Omega_{\rm M}, \Omega_\Lambda)$



CMB (TT+EE)⇒

$\Omega_k \;= -0.04 \pm 0.04$

CMB + BAO ⇒

$\Omega_k \;= 0.0008 \pm 0.0040$

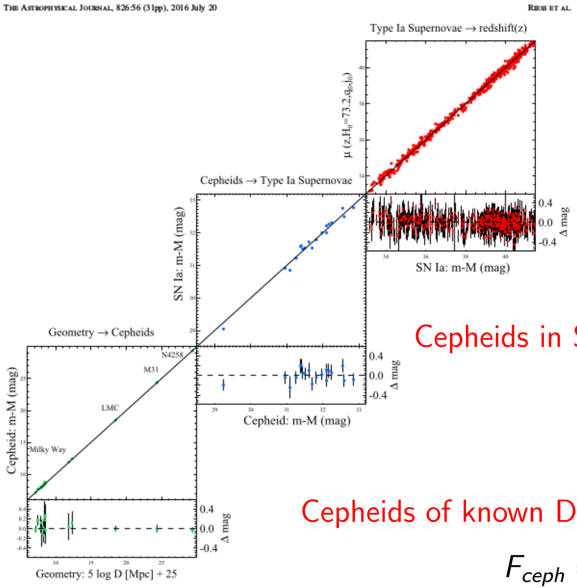
H_0 from distance ladder

$v = H_0 D$ (for $z \ll\sim 0.1$)
v = recession velocity from redshift
need small "peculiar" velocity $\Rightarrow v/c = z > 0.02$
D = distance from photon flux from objects of known luminosity
$F = L/4\pi D^2$

$\Rightarrow H_0 = 73.5 \pm 1.4 \mathrm{km\,s^{-1}Mpc^{-1}}$ (67.3 $\pm$ 1.0 from CMB-BAO)
A. Riess et al. arXiv:2001.03624

Objects of known luminosity:
Type Ia Supernovae (SNIa) (calibrated with cepheids)
Cepheid variable stars (calibrated with parallax/xallarap)

Distance Ladder: three steps



Hubble-flow SNI $\Rightarrow H_0 L_{SNIa}^2$:

$$F_{SNIa} = \frac{L_{SNIa}}{4\pi(zc/H_0)^2}$$

$$(0.01 < z < 0.05)$$

.

Cepheids in SNIa hosts $\Rightarrow L_{SNIa}/L_{ceph}$:

$$\frac{L_{SNIa}}{L_{ceph}} = \frac{F_{SNIa}}{F_{ceph}}$$

Cepheids of known D $\Rightarrow L_{ceph}$:

$$F_{ceph} = \frac{L_{ceph}}{4\pi D_{ceph}^2}$$

Distance Ladder: mostly small statistics

.

$$F_{SNIa} = \frac{L_{SNIa}}{4\pi(zc/H_0)^2}$$

$$\frac{L_{SNIa}}{L_{ceph}} = \frac{F_{SNIa}}{F_{ceph}}$$

$$F_{ceph} = \frac{L_{ceph}}{4\pi D_{ceph}^2}$$

Hundreds of SNIa in Hubble flow
$(0.01 < z < 0.05)$

18 SNIa in galaxies with cepheid distances

four galaxies of known distance and observed cepheids

- NGC4258: one maser-black hole binary
- M31: two detached stellar binaries
- LMC: eight detached stellar binaries
- Milky Way: 15 cepheid parallaxes

Systematics: do the cepheids in these four galaxies have the same luminosities and photometry as the cepheids in the 18 galaxies hosting SNIa?

Cepheid variable stars

Log(*flux*) vs. *Log*(*period*),
Cepheids in the Magellanic Clouds
Henrietta Leavitt, 1912

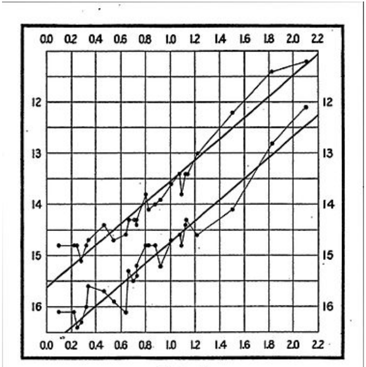
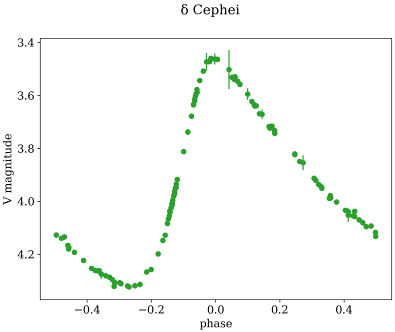
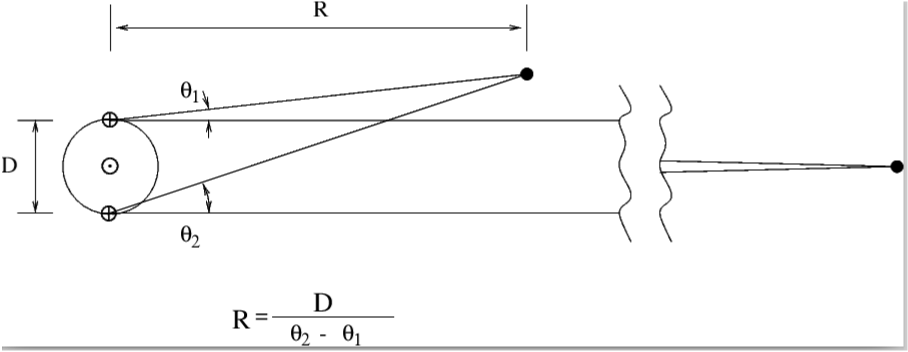


FIG. 2.

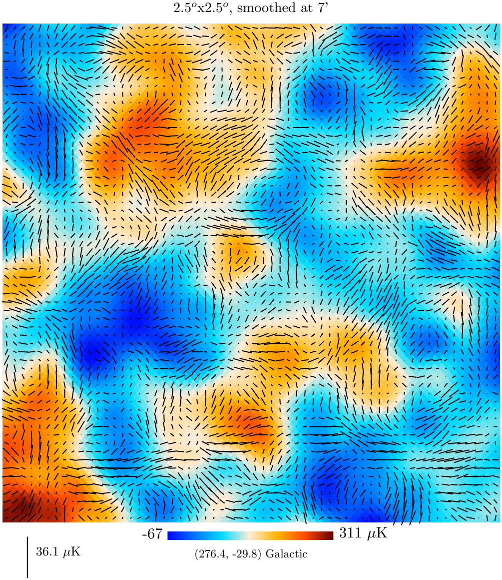


Parallax

The Earth's movement about the Sun changes the relative positions of nearby and distant objects:



The CMB is polarized

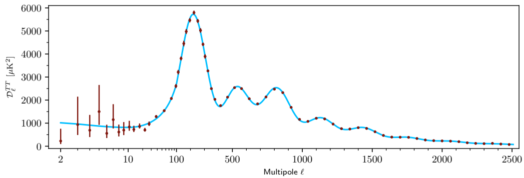


$2.5 \times 2.5\mathrm{deg}^2$ patch of the sky (Planck)
Colors give temperature
Black lines give polarization

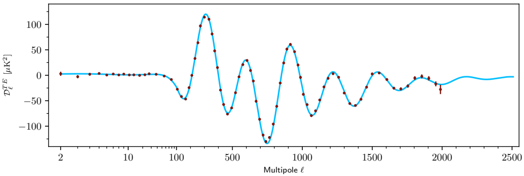
Correlations:
temperature-temperature
temperature-polarization
polarization-polarization

$\Rightarrow$ 3 power spectra

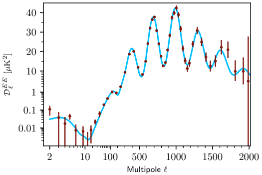
TT, TE, EE power spectra

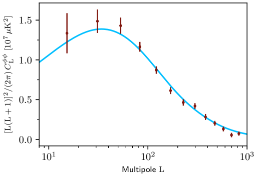


TT



TE

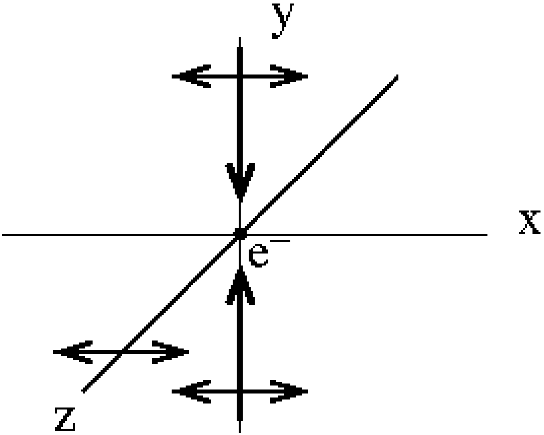




EE
Lensing

Thomson scattering on LSS polarizes photons

Compton scattering just before recombination;
LSS=xy-plane; Observer on z axis

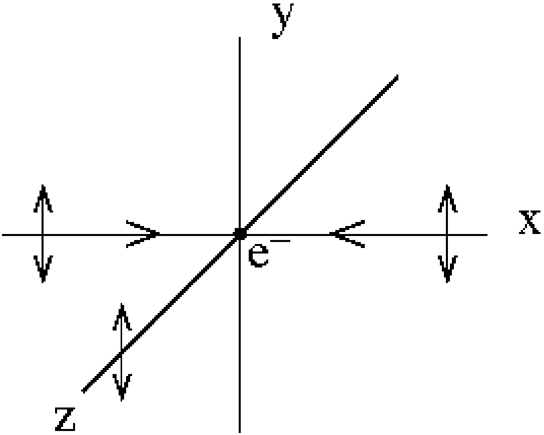


photon flux($\pm y$ directions)

$\Rightarrow$ photon polarization observed in x direction

Thomson scattering on LSS polarizes photons

Compton scattering just before recombination;
LSS=xy-plane; Observer on z axis

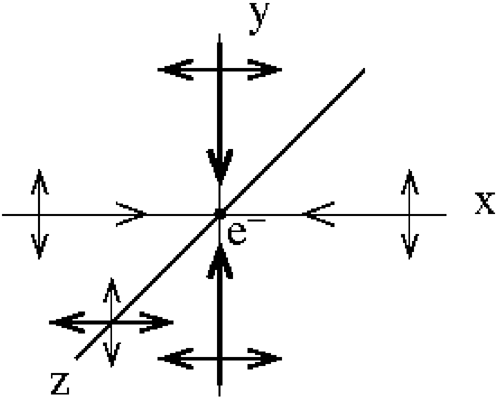


photon flux($\pm x$ directions)

$\Rightarrow$ photon polarization observed in y direction

Inhomogeneities on LSS ⇒ linear polarization

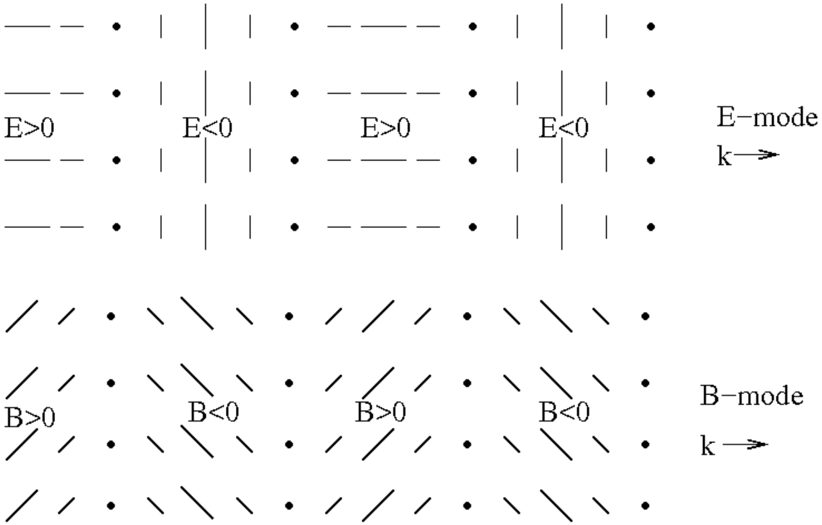
Compton scattering just before recombination;
LSS=xy-plane; Observer on z axis



photon flux($\pm y$ directions)
$>$ photon flux($\pm x$ directions)
⇒ photon polarization
observed in x direction

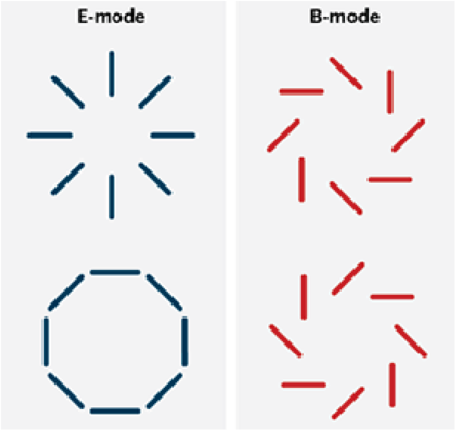
Primary source of photon flux differentials: plasma velocity gradients

Decompose polarization maps into E and B modes



E-modes: pol. alternatively parallel/perpendicular to $\vec{k}$
B-modes: pol. alternatively $\pm$45deg to $\vec{k}$

Superpositions of E and B modes



Polarization from inhomogeneities

Things that make the pre-recombination plasma move:

Movement parallel to $\vec{k} \Rightarrow$ E-modes only

- gravitational potential gradient
- pressure gradient

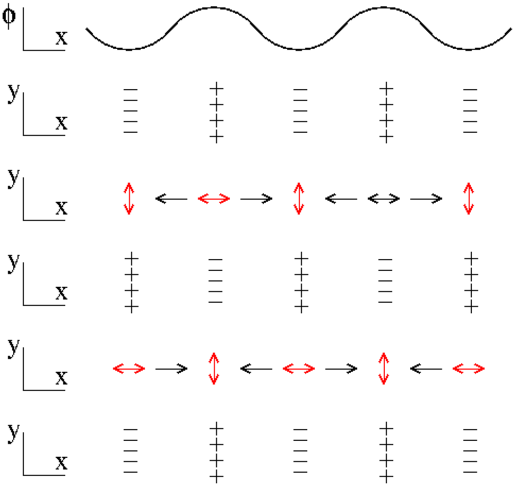
Movement perpendicular to $\vec{k} \Rightarrow$ E- and B-modes

- gravitational waves on LSS
  (predicted to exist by inflation)

Furthermore

- Lensing of the CMB on foreground density inhomogenites can cause "leakage" of E modes into B modes

E mode due to periodic potential



gravity potential ϕ

plasma at high ϕ

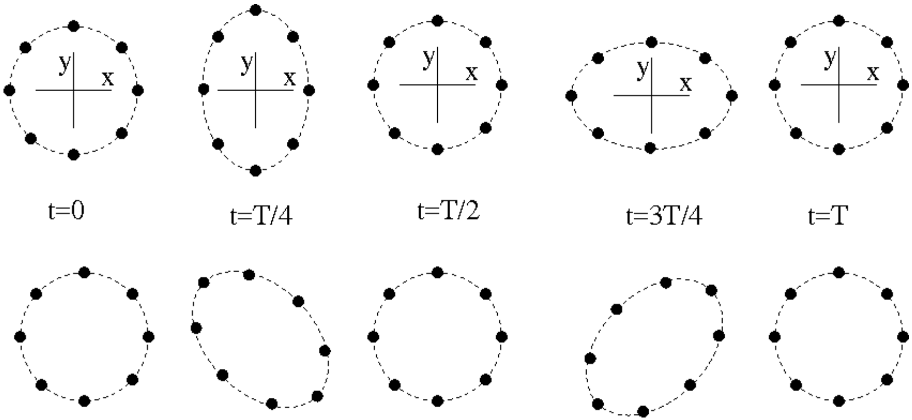
plasma falls down

plasma at low ϕ

plasma bounces up

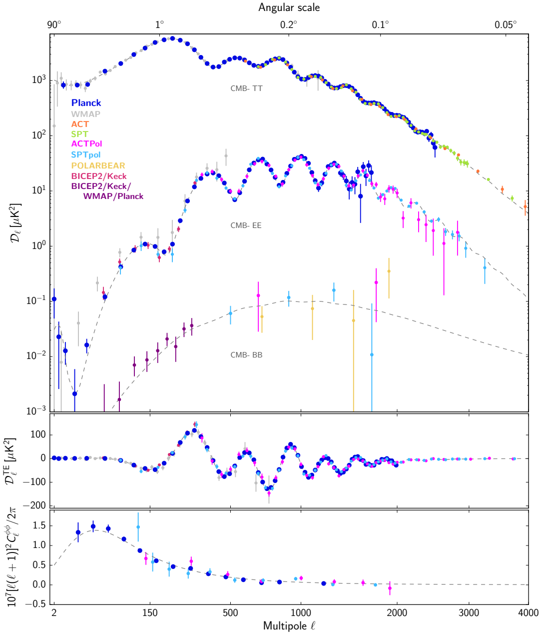
plasma at high ϕ

Response of free particles to G-wave



Two modes; Movement perpendicular to $\vec{k}$

$\Rightarrow$ mixture of E and B modes



TT, EE, BB power spectra

sources of B-modes:

$\ell > 200$ gravitational lensing on foreground structures (E to B-mode leakage)

$\ell < 100$ G waves on LSS (inflationary prediction)

Galactic dust