Master NPAC

Cosmology homework #1

November 24th, 2022

Answers are to be sent by email to nicolas.regnault-AT-lpnhe.in2p3.fr by Thursday Dec 1st, 2022. Please use the following format (given as an example):

```
<your name>
 1. abbc
 2. aaaab
 3. bbc
 4. d
 5. b
 6. aa
```

We give:

- $M_\odot = 1.98855\ 10^{30}$ kg
- $L_\odot = 3.828\ 10^{26}$ W
- mass of the proton: 938.272 MeV
- 1 julian year = 365.25 days

The scipy.constants module may be useful. It is available on our jupyterhub server.

```
>>> import scipy.constants as C
>>> print C.eV, C.c, C.year
```

1. It is useful to know how to express the value of the Hubble constant $H_0 = 100h$km s^{-1} Mpc^{-1} in different units (keeping the h symbol in the result). Express the value of H_0 in:

(A) s^{-1}

a) $3.24\ 10^{-18}\ h$ **b)** $2.22\ 10^{-24}\ h^{-1}$ **c)** $3.45\ 10^{-11}\ h$ **d)** None of that

(B) Gyr^{-1} (with 1 year = 365.25 days):

a) $1.2223\ 10^{-5}\ h$ **b)** $0.10227\ h$ **c)** $1.1987\ 10^{3}\ h$ **d)** None of that

(C) Mpc^{-1}

a) $3.336\ 10^{-4}\ h$ **b)** $2.1276h$ **c)** $7.7543\ 10^{12}h$ **d)** None of that

(D) GeV

a) $1.167\ 10^{-3}h$ **b)** $1.765\ 10^{-12}h$ **c)** $2.133\ 10^{-42}h$ **d)** None of that

2. Same thing with the critical density $\rho_c = \frac{3H_0^2}{8\pi G}$.

(A) g cm^{-3}:

a) $1.88\ 10^{-29}\ h^2$ **b)** $1.27\ 10^{-12}\ h^{-2}$ **c)** $7.22\ 10^{-42}\ h$ **d)** None of that

(B) eV cm^{-3}:

a) $10537\ h^2$ **b)** $1.272\ 10^{3}\ h^2$ **c)** $1.276\ 10^{-4}\ h^{-2}$ **d)** None of that

(C) GeV^4:

a) $1.42\ 10^{-12}\ h^2$ **b)** $8.09\ 10^{-47}\ h^2$ **c)** $3.23\ 10^{11}\ h^2$ **d)** None of that

(D) protons m^{-3}:

a) $11.2h^2$ **b)** $1.123\ 10^{3}\ h^2$ **c)** $5.223\ 10^{-5}\ h^2$ **d)** None of that

(E) $M_\odot Mpc^{-3}$:

a) $3.22\ 10^{15}\ h^2$ **b)** $2.78\ 10^{11}\ h^2$ **c)** $5.55\ 10^{3}\ h^2$ **d)** None of that

3. The luminosity of a typical galaxy is $2\ h_{70}^{-2}\ 10^{10}\ L_\odot$ and the mean energy of stellar photons is $\sim 2\ eV$.

(A) Evaluate the photon flux (in $m^{-2}s^{-1}$) of a galaxy at a redshift $z \ll 1$

a) $\approx 100z^{-2}$ **b)** $\approx 10^5 z^{-1}$ **c)** ≈ 1000 **d)** None of that

(B) Compute the ratio of the photon flux of the nearest large galaxies ($d \sim 1Mpc$) over the photon flux of the nearest stars $\sim 1pc$.

a) $2\ 10^{-2}$ **b)** $5\ 10^{-5}$ **c)** $4\ 10^{9}$ **d)** None of that

4. The luminosity density of the Universe is $\sim 1.2 h_{70} 10^8\ L_\odot Mpc^{-3}$. Assuming stellar light output has been relatively constant over time, and assuming a mean energy of stellar photons of $\sim 2\ eV$:

(A) estimate the density of photons (in m^{-3}) that have been produced by stars, since the formation of the first stars, about one Hubble time ago

a) $2\ 10^3$ **b)** $4\ 10^{-5}$ **c)** $7\ 10^9$ **d)** None of that

(B) Compare it to the density of CMB photons (≈ 400 cm^{-3})

a) about the same **b)** much larger **c)** negligible

(C) Estimate the corresponding reduced density $\Omega_{\gamma_{\rm vis}}$

a) 0.04 **b)** 0.005 **c)** $7.7\ 10^{-7}$ **d)** None of that

5. We have seen in class that the typical mass to light ratio of galaxies is $\frac{M}{L} \sim 2.5 \frac{M_\odot}{L_\odot}$. The typical galaxy luminosiy is $L \sim 10^{10} L_\odot$. The density of (bright) galaxies per cubic Mpc is about $0.015 h_{70}^3 {\rm Mpc}^{-3}$.

1. Estimate the density of visible matter (in $M_\odot {\rm Mpc}^{-3}$)

a) $5.2\ 10^{-4}$ **b)** $3.8\ 10^8$ **c)** $22\ 10^{10}$ **d)** None of that

2. Estimate the density of visible matter in units of the critical density

a) 0.32 **b)** 5.2E-5 **c)** 0.003 **d)** None of that

6. We have seen in class that models of primordial nucleosynthesis allow us to derive from the measurements of light element primordial abundances an estimate of the baryon-to-photon ratio $\eta \equiv \frac{N_b}{N_\gamma} \sim 6\ 10^{-10}$.

1. Derive from this estimate of η, an estimate of the baryon density $\Omega_b h^2$

a) 1.E-4 **b)** 0.05 **c)** 0.2 **d)** None of that

6. On a sphere of radius R, what is the circumference of a cicle of radius L ?

(A) The Earth may be idealized as a perfect sphere of radius $R = 6371$ km. If you could measure distances with a precision of ± 1m, how large should you draw a circle on Earth to convince yourself that the Earth is not flat ?

a) 2.3 km **b)** 34 km **c)** 570 km **d)** None of that

7. On a sphere of radius R

(A) what is the angular size of an object of width ℓ placed at a distance d from the observer ?

a) $\delta\theta = \frac{\ell}{R \sin d/R}$ **b)** $\delta\theta = \ell/d^2$ **c)** None of that

(B) What happens when $d \to \pi R$?

a) $\delta\theta \to \infty$ **b)** $\delta\theta \to 0$ **c)** None of that