

# Master NPAC

## Cosmology homework #3

December 15, 2022

Answers are to be sent by email to nicolas.regnault-AT-lpnhe.in2p3.fr by Thursday January 5th, 2023. Exercices 1, 2, 6 and 9 are multiple choice and are to be answered the usual way:

<your name>

1. B
2. A
6. C
9. A

For questions **3**, **4**, **5**, **7** and **8** you will need a computer. You may use your favorite programming language – as long as it allows you to plot functions. Answers for each of these questions consist in plots. Save your plots in png or pdf format, name them from the question number (e.g. 3\_a.png, 3\_b.png, 4.png, 5.png, 7.png, 8.png) and attach them to your email.

We have seen in class that the expansion history is governed by the Friedmann equation:

$$H(a) = H_0 \left( \Omega_m \left( \frac{a_0}{a} \right)^3 + \Omega_r \left( \frac{a_0}{a} \right)^4 + \Omega_\Lambda + \underbrace{(1 - \Omega_m - \Omega_r - \Omega_\Lambda)}_{\Omega_k} \left( \frac{a_0}{a} \right)^2 \right)^{1/2} \quad (1)$$

which may be re-arranged as:

$$dt = \frac{1}{H_0} \frac{dx}{(\Omega_m/x + \Omega_r/x^2 + \Omega_\Lambda x^2 + \Omega_k)^{1/2}} \quad \text{with } x \equiv a/a_0 \quad (2)$$

We also have seen that the densities of the different types of fluids evolve differently as a function of the scale factor. The Universe was first dominated by radiation, then, by matter and now by vacuum-energy. In in homework, we will look into this a little more quantitatively.

Unless specified otherwise, we will consider a flat Universe, with  $\Omega_m = 0.31$ ,  $\Omega_\Lambda = 0.69$ ,  $\Omega_r = 9 \cdot 10^{-5}$ , and  $H_0 = 68 \text{ km/s/Mpc}$ .

**1.** Compute the redshift of matter-vacuum energy equality,  $z_{\text{eq}\Lambda}$ , and matter-radiation equality  $z_{\text{eq}}$ :

(A)  $z_{\text{eq}\Lambda} = 1.3$      $z_{\text{eq}} = 3444$

(B)  $z_{\text{eq}\Lambda} = 0.7$      $z_{\text{eq}} = 3443$

(C)  $z_{\text{eq}\Lambda} = 0.3$      $z_{\text{eq}} = 3443$

(D) None of the above

**2.** It is interesting to note that the transition between decelerated and accelerated expansion occurred slightly before matter-vacuum-energy equality. For a flat Universe, dominated by matter and vacuum-energy, compute the redshift at which cosmic expansion started accelerating (use the second Friedmann equation, and  $\ddot{a} = 0$ )

(A)  $1 + z_{\text{acc}} = \left(\frac{2\Omega_{\Lambda}}{\Omega_m}\right)^{1/3}$      $z_{\text{acc}} \approx 0.65$

(B)  $1 + z_{\text{acc}} = \left(\frac{\Omega_{\Lambda}}{\Omega_m}\right)^{2/3}$      $z_{\text{acc}} \approx 0.71$

(C)  $1 + z_{\text{acc}} = \left(\frac{\Omega_{\Lambda}}{\Omega_m}\right)^{1/2}$      $z_{\text{acc}} \approx 0.49$

(D) None of the above

**3.** Write a function that returns the value of  $\rho_c$  (in  $\text{kg}/\text{m}^3$ ) as a function of  $H$  (in  $\text{km}/\text{s}/\text{Mpc}$ ). Write a function that returns the value of  $H$  (in  $\text{km}/\text{s}/\text{Mpc}$ ) as a function of  $a/a_0$  and the cosmological parameters ( $\Omega_m, \Omega_r, \Omega_{\Lambda}, H_0$ ). Plot the evolution of  $\rho_c$  as a function of  $a/a_0$  – you will probably want to produce a log-log plot (use the function `pl.loglog()` instead of `pl.plot()`).

**4.** Plot the evolution of  $\rho_m, \rho_r$  and  $\rho_{\Lambda}$  (in  $\text{kg}/\text{m}^3$ ) as a function of  $a/a_0$  (again, you will quickly realize that it is convenient to use a log-log scale). Verify that the redshifts of matter-dark energy equality and matter-radiation equality are consistent with what you have found above.

**5.** Plot the evolution of  $\Omega_m(a), \Omega_r(a)$  and  $\Omega_{\Lambda}(a)$  as a function of  $a/a_0$  (use a semilog scale, e.g. `pl.semilogx()` instead of `pl.plot()`).

**6.** We can estimate the time since matter-dark energy equality by integrating analytically the Friedmann equation, neglecting the matter and radiation components. The

time of matter-radiation equality may be estimated in a similar way. Do the computation. What do you obtain ?

- (A)  $t_{\text{eq}\Lambda} \approx 12.5 \text{ Gyr}$     $t_{\text{eq}} \approx 300000 \text{ yr}$
- (B)  $t_{\text{eq}\Lambda} \approx 11.6 \text{ Gyr}$     $t_{\text{eq}} \approx 64000 \text{ yr}$
- (C)  $t_{\text{eq}\Lambda} \approx 11.5 \text{ Gyr}$     $t_{\text{eq}} \approx 300000 \text{ yr}$
- (D) None of the above

**7.** A better alternative is to integrate numerically the Friedmann equation. We have done that in the “introduction to python” class by using a differential equation solver. Here, we will just estimate numerically the integral in equation 2.

Write a function that returns the integrand of equation 2 (beware of the units !). Then, write a function that computes the integral and returns the age of the Universe (in Gyr) as a function of  $a/a_0$ , or  $z$ , if you prefer (to perform the integration, you may use, for example, `scipy.integrate.trapz`). Plot the age of the Universe as a function of  $z$ .

**8.** Draw the evolution of  $a/a_0$  as a function of time (in Gyr) (1) for our Universe (2) for  $(\Omega_m = 2, \Omega_\Lambda = 0.)$ ,  $(\Omega_m = 1, \Omega_\Lambda = 0.)$ , and a  $(\Omega_m = 0.3, \Omega_\Lambda = 0.)$ .

**9.** Now, that you have a function that gives  $t(z)$ , re-estimate the time of matter-dark energy equality and matter-radiation equality.

- (A)  $t_{\text{eq}\Lambda} \approx 10.2 \text{ Gyr}$     $t_{\text{eq}} \approx 49900 \text{ yr}$
- (B)  $t_{\text{eq}\Lambda} \approx 10.2 \text{ Gyr}$     $t_{\text{eq}} \approx 365400 \text{ yr}$
- (C)  $t_{\text{eq}\Lambda} \approx 11.5 \text{ Gyr}$     $t_{\text{eq}} \approx 420000 \text{ yr}$
- (D) None of the above