Quantum ChromoDynamics QCD Lagrangian and SU(3) structure

Particle Physics

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Overview

- QED Lagrangian, reminder
- Building the QCD Lagrangian
  - ✓ Recipe from QED and implications
  - ✓ Feynman rules
- QCD group structure
  - ✓ Gluon emission and gluon splitting
  - ✓ Infra red divergence, soft gluon emission probability
  - ✓ Hadron multiplicity measurements
  - ✓ 3jets / 4 jets measurements
- Color Factor
  - ✓ qq and $q\bar{q}$ interaction
  - ✓ QCD potential and meson stability
- Strong coupling constant measurements at the LHC
  - ✓ Total cross section at the LHC
  - ✓ Inclusive jet production
  - ✓ 3jets / 2jets ratio

Free field propagation (propagators)

Interaction quarks gluon (a la QED)

Vertex 3 gluons (due to the non-Abelian group)

Vertex 4 gluons (due to the non-Abelian group)

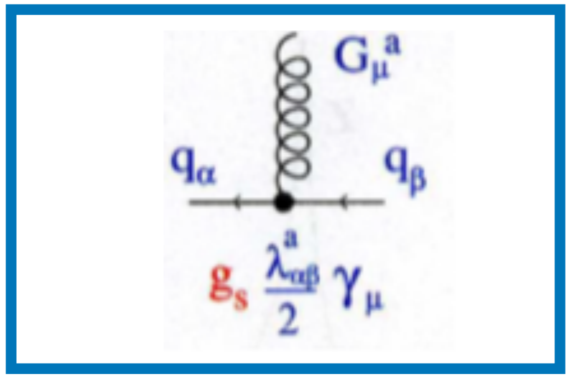
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) + \overline{\psi}(i\partial\!\!\!/ - m)\psi$$

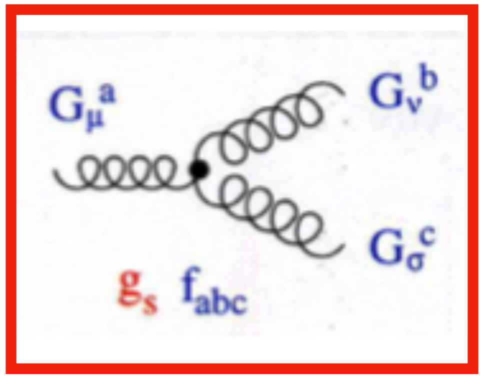
$$+ g_s \overline{\psi}_i \gamma^\mu (t^a)_{ij} \psi_j G^a_\mu$$

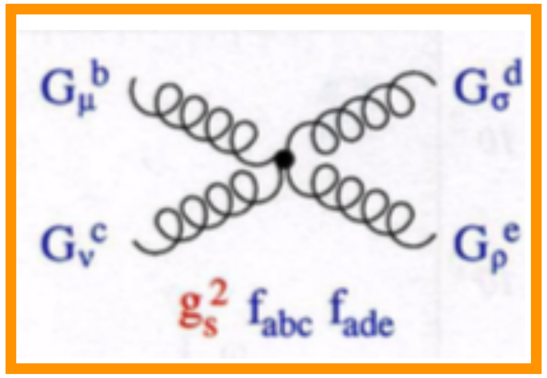
$$- \frac{g_s}{2} f^{abc}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) G^{b\,\mu} G^{c\,\nu}$$

$$- \frac{g_s^2}{4} f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d\,\mu} G^{e\,\nu}$$

$G^\mu = A^\mu$







QCD Feynman rules

Fermions lines carry a color charge!

$$c_i\, u_f^{(s)}(p^\mu) \quad \text{incoming}$$

$$c_i^\dagger\, \bar{u}_f^{(s)}(p^\mu) \quad \text{outgoing}$$

$$c_i^\dagger\, \bar{v}_f^{(s)}(p^\mu) \quad \text{incoming}$$

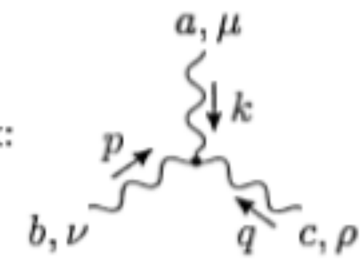
$$c_i\, v_f^{(s)}(p^\mu) \quad \text{outgoing}$$

Interaction (or completeness)

Fermion vertex:

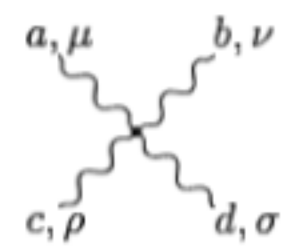
$$= ig\gamma^\mu t^a$$

3-boson vertex:



$$= g f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$

4-boson vertex:



$$= -ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})]$$

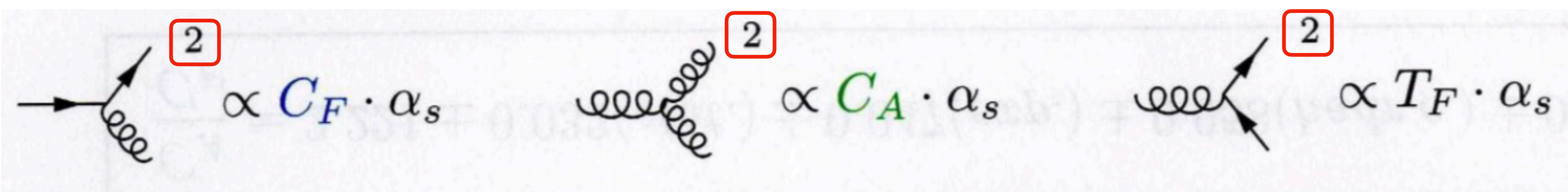
One trivial basis for color

$$c_r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad c_g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad c_b = \begin{pmatrix} 0 \\ \mathbf{0} \\ \mathbf{1} \end{pmatrix}$$

The group structure of QCD

From the PDG

Useful color-algebra relations include: $t^A_{ab}t^A_{bc} = C_F\delta_{ac}$, where $C_F \equiv (N_c^2-1)/(2N_c) = 4/3$ is the color-factor ("Casimir") associated with gluon emission from a quark; $f_{ACD}f_{BCD} = C_A\delta_{AB}$ where $C_A \equiv N_c = 3$ is the color-factor associated with gluon emission from a gluon; $t^A_{ab}t^B_{ab} = T_R\delta_{AB}$, where $T_R = 1/2$ is the color-factor for a gluon to split to a $q\bar{q}$ pair.



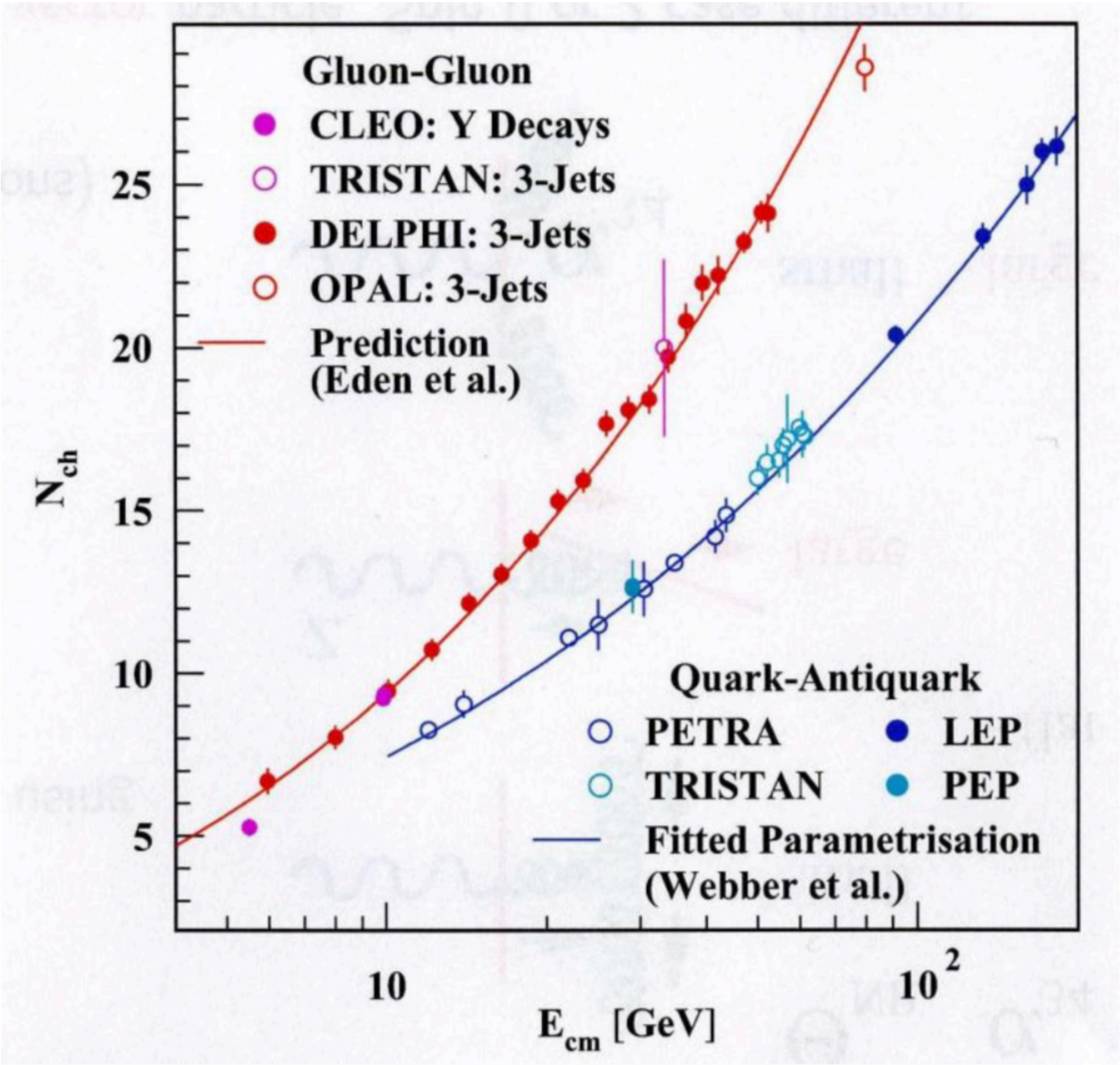
$$\sum_{a=1}^{N_A} \left(T^a T^{\dagger a}\right)_{ij} = \delta_{ij} C_F \qquad \sum_{a,b=1}^{N_A} f^{abc} f^{*abd} = \delta^{cd} C_A \, , \qquad \sum_{i,j=1}^{N_F} T^a_{ij} T^{\dagger b}_{ji} = \delta^{ab} T_F \, ,$$

Measuring the group structure

N_{ch} = charge hadron multiplicity in a jet

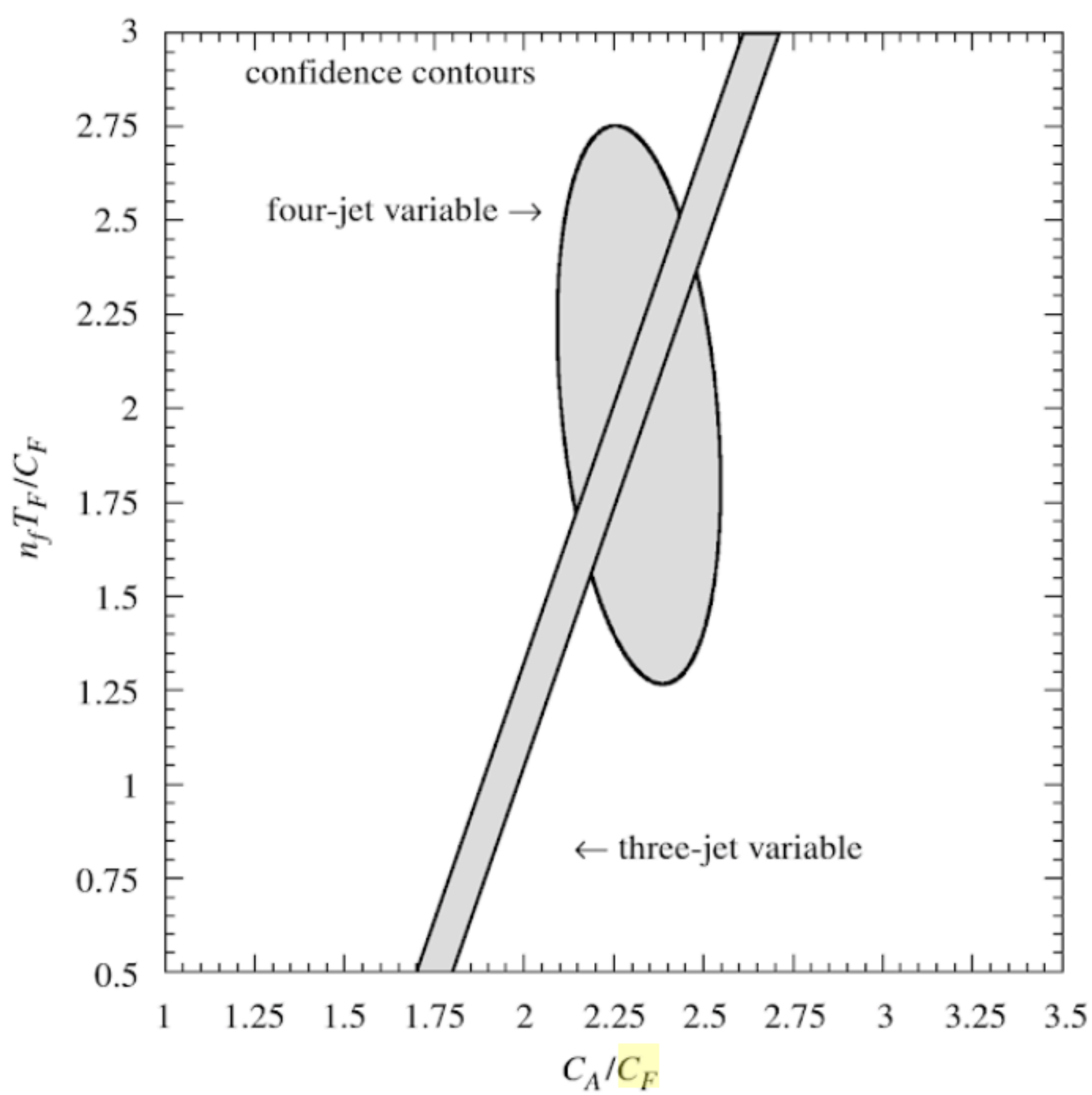
Ratio of the “slopes”

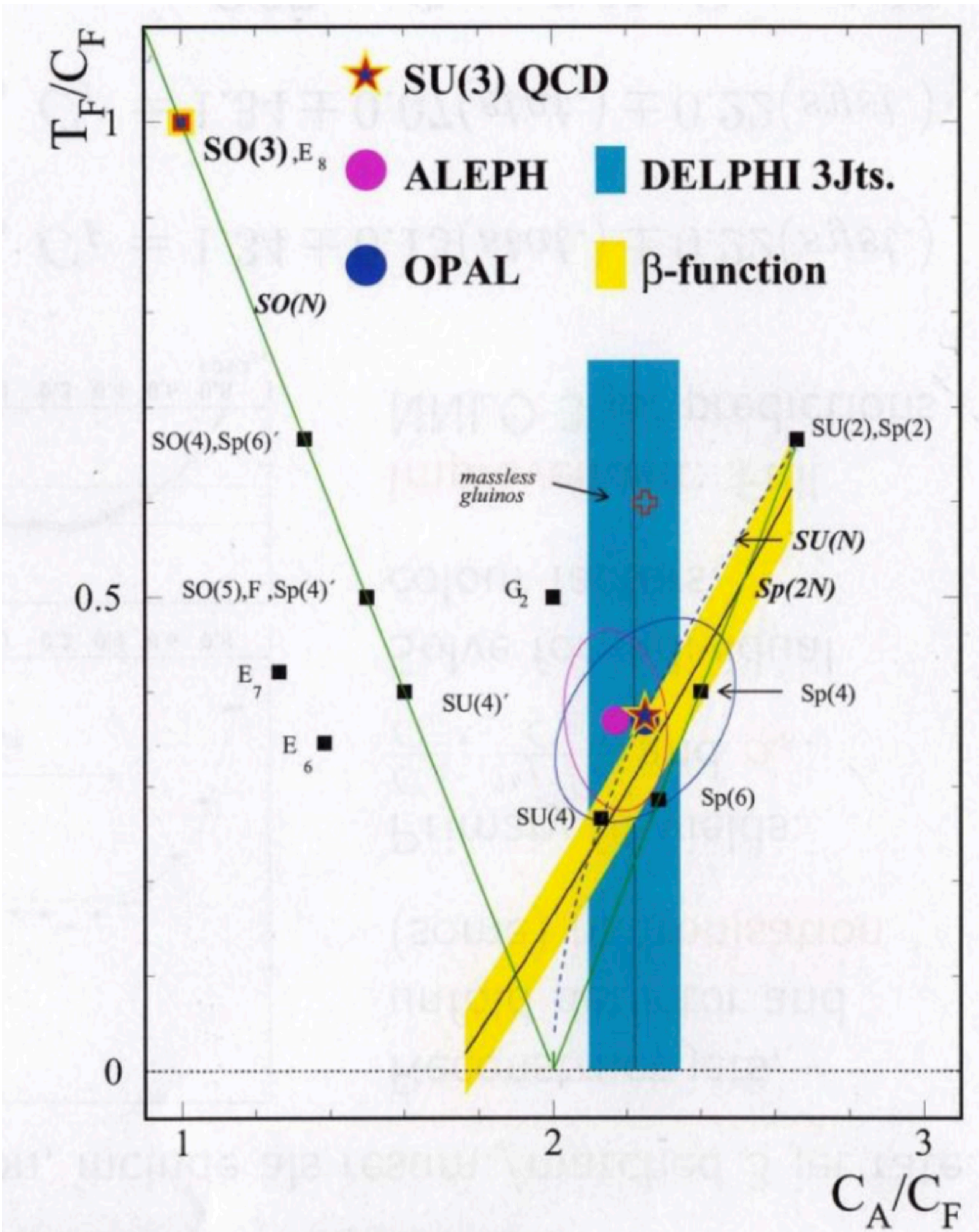
R = 2.22 ± 0.11



Measuring the group structure

- LEP events with 3 and 4 jets





$$t^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad t^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad t^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$t^4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad t^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

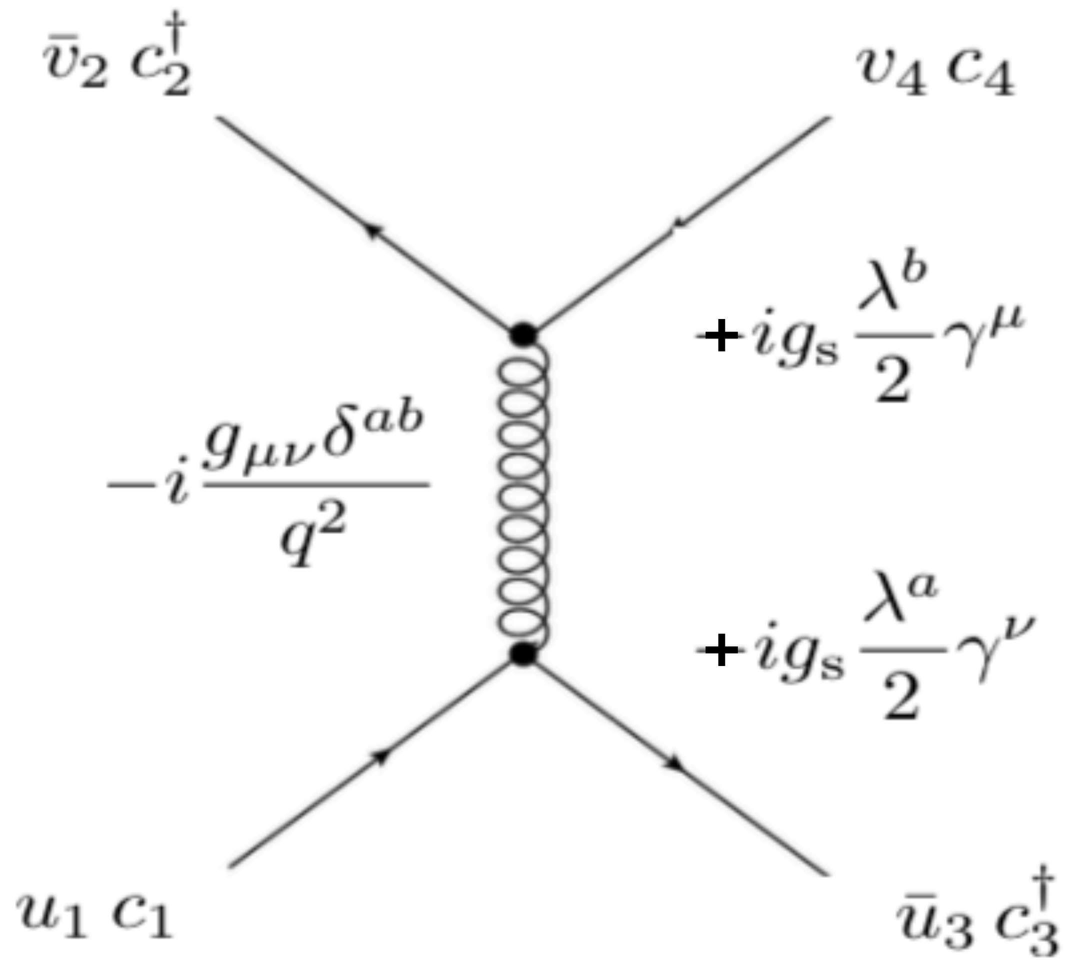
$$t^6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t^7 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad t^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Color factors

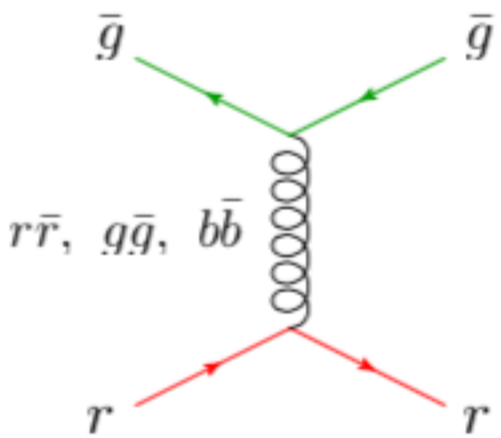
Example of qq and qqbar scattering

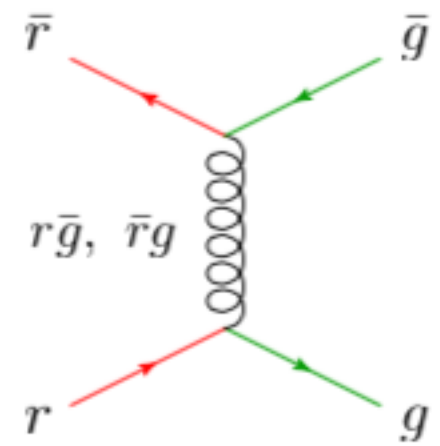
$q\bar{q} \to q\bar{q}$	$qq \to qq$	f(ijkl)
xx→xx	xx→xx	f(xxxx) = +1/3
xx→yy	xy→yx	f(yxxy) = +1/2
xy→xy	xy→xy	f(xxyy) = -1/6
xy→yx	xx→yy	f(yxyx) = 0

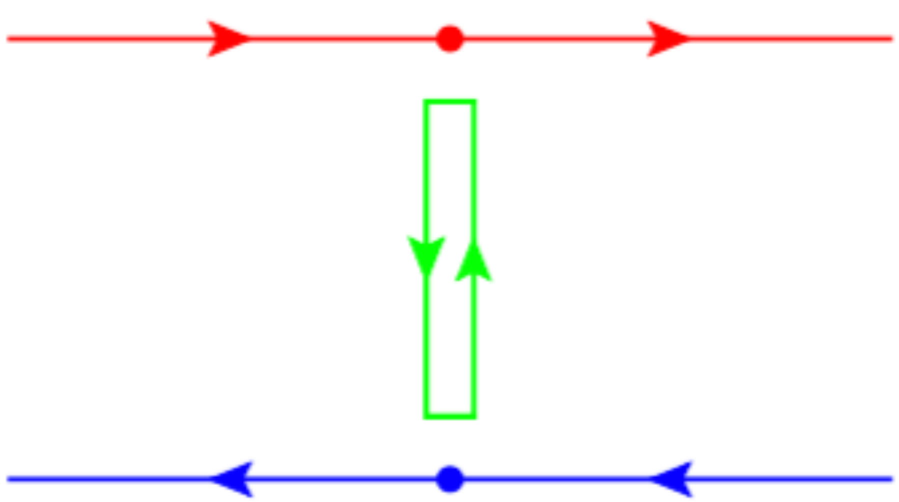
Quark - antiquark interaction

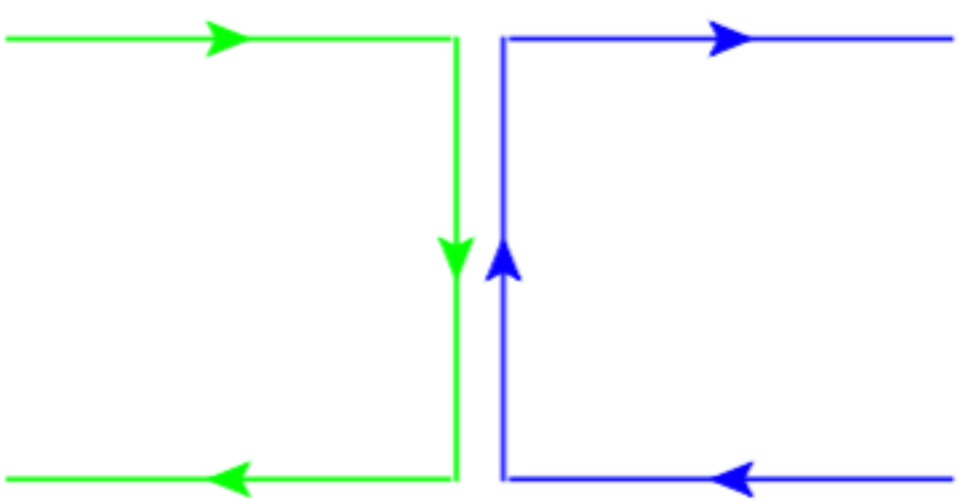


Color flow q q̄



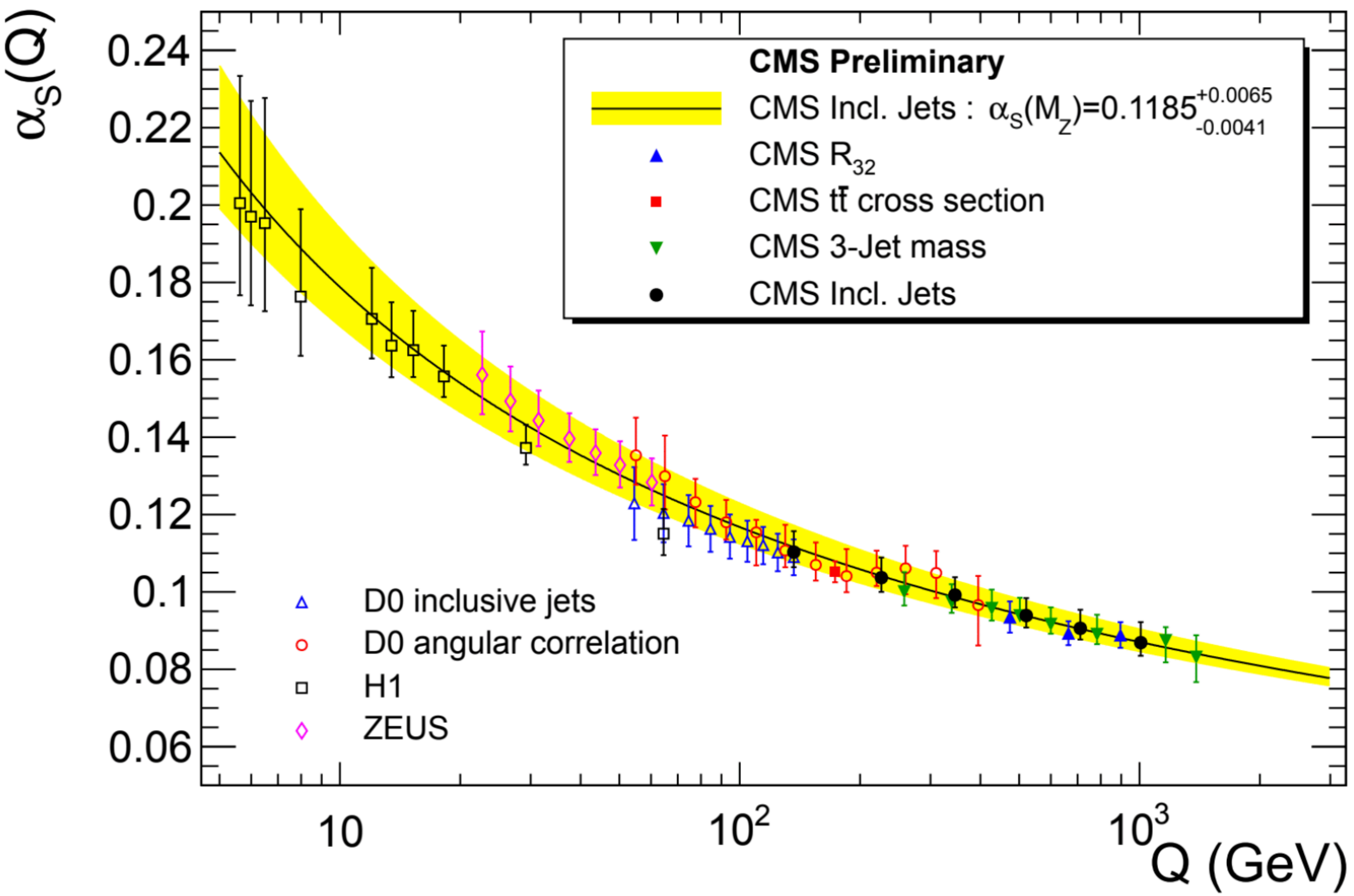




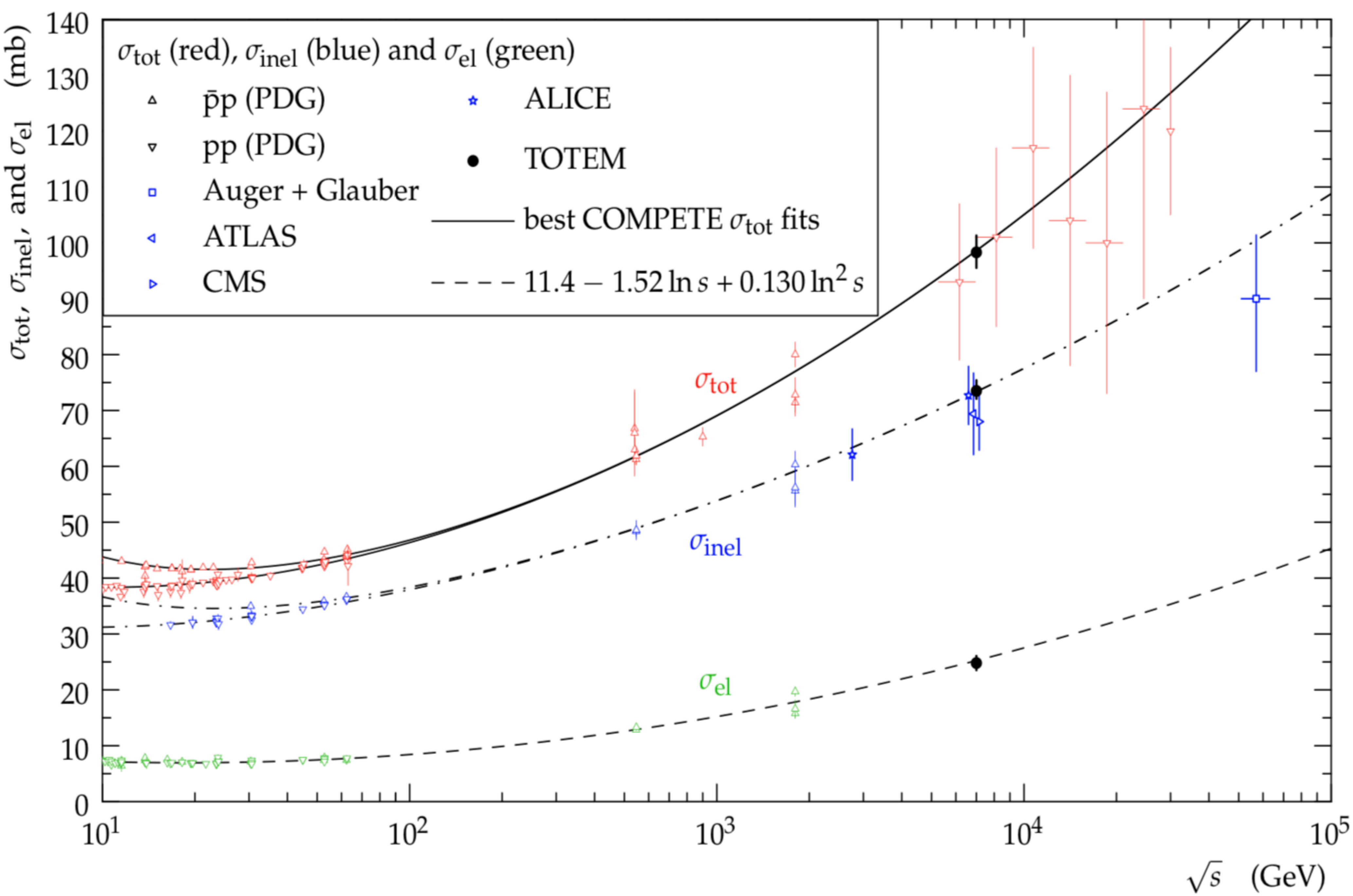


QCD running measurement

Measuring the running of α_s is still a very active field at LHC
Important to predict any cross section!!!



Total cross section at the LHC



R32 : 3jets to 2jets production

Naively $\sigma[2 \text{ jets}] \propto \alpha_s^2$; $\sigma[3 \text{ jets}] \propto \alpha_s^3 \implies \sigma[3 \text{ jets}] / \sigma[2 \text{ jets}] \propto \alpha_s$

several systematics cancel in the ratio
(*e.g.* pdf uncertainty greatly reduced)

$\mu_R = (p_{T1} + p_{T2}) / 2$

World average:
$\alpha_s(M_Z) = 0.1184 \pm 0.0007$

CMS 3/2 jets:
$\alpha_s(M_Z) = 0.1148 \pm 0.006$