NPAC
Particle Physics
Course 14 – Charged Weak Int. (part 3)
CKM Matrix and CP violation

Eli Ben-Haïm

Fabrice Couderc

CKM Matrix and CP Violation

1. **Cabibbo-Kobayashi-Maskawa (CKM) Matrix**
    1. Concept and definition
    2. Number of parameters and CP violation
    3. General parameterization
    4. Measurements, size and pattern of the CKM elements
    5. The Wolfenstein parameterization
    6. The unitarity triangles
2. **Flavour oscillations**
3. **Classification of CP violation effects**
4. **Experimental techniques for flavour tagging**
5. **Measurement of the CKM angle beta**
6. **Other unitarity triangle measurements**

The CKM (Cabibbo, Kobayashi, Maskawa) Matrix

In the **quark sector**: weak Int. eigenstates ≠ flavour eigenstates

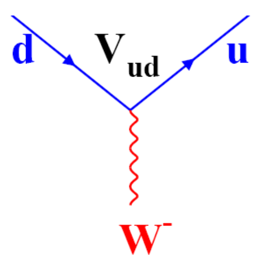
⇔ States that participate in weak processes are linear combinations of flavour eigenstates

⇔ Existence of 3X3 unitary matrix describing the **mixing of quarks**: the **CKM Matrix**

weak interaction eigenstates ≠ flavour/mass eigenstates (≡ strong interaction eigenstates)

Cabibbo

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{matrix}} & \begin{matrix} V_{ub} \\ V_{cb} \end{matrix} \\ \begin{matrix} V_{td} & V_{ts} \end{matrix} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



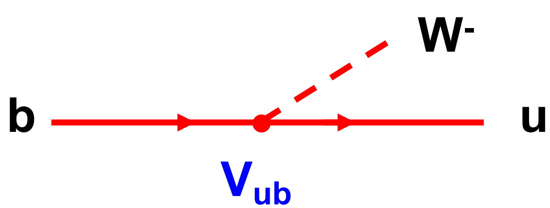
In the SM, the CKM matrix originates in the Higgs sector, where it is the product of two unitary matrices that diagonalize quark mass matrices arising from spontaneous breaking of electroweak symmetry.

It appears in the Lagrangien as:

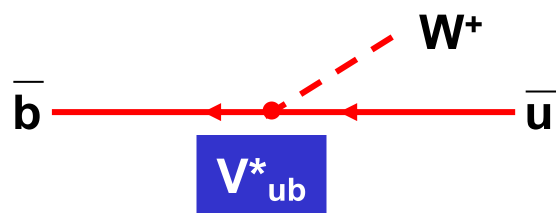
$$\begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The CKM Formalism and CP Violation

Transition amplitude between, e.g., b and u quarks



Transition amplitude between, e.g., $\bar{b}$ and $\bar{u}$ anti-quarks



If the CKM matrix is not real (CKM phase) $\Rightarrow V^*_{ub} \neq V_{ub}$

$\Rightarrow$ different behavior of matter and anti-matter

$\Rightarrow$ **CP violation!**

(question: why are we talking about CP and not simply about C?)

Obviously, this single amplitude cannot give an observable CP violation. We must have a sum of amplitudes → contribution from a few processes

But is the CKM matrix complex?

Number of paramaters in the CKM Matrix (I)

Number of physical (non-reducible) parameters corresponding to n quark generations

- The CKM matrix (V_{CKM}) is an n x n complex matrix
  $\Rightarrow$ in general, contains $2n^2$ real parameters.
- V_{CKM} is unitary $V^\dagger V$ ($= VV^\dagger$) $=$ **1** $\Rightarrow$ n^2 constraints $\Rightarrow$ n^2 real parameters.
- Each quark field has an **arbitrary phase**. As this phase cannot be observed and do not influence the system $\Rightarrow$ physics is invariant under the transformation:

$$V \rightarrow \begin{pmatrix} e^{i\Phi_1^U} & & 0 \\ & \ddots & \\ 0 & & e^{i\Phi_n^U} \end{pmatrix} V \begin{pmatrix} e^{i\Phi_1^D} & & 0 \\ & \ddots & \\ 0 & & e^{i\Phi_n^D} \end{pmatrix}$$

The overall phase cannot be fixed a-priori $\Rightarrow$ 2n-1 phases can be removed from V_{CKM}

$\Rightarrow$ n^2-(2n-1) = **$(n-1)^2$ independent meaningful parameters**

- A practical way to construct a unitary matrix with the smallest number of phases:
  - Start from an n x n (real) rotation matrix $\Rightarrow$ ½ n(n-1) rotation (mixing) angles
  - Take the other parameters as phases (non-reducible $\Leftrightarrow$ cannot be rotated away).

Number of paramaters in the CKM Matrix (II)

No phase for two generations!

# generations	# parameters	# angles	# non-reducible phases
n	$(n-1)^2$	$n(n-1)/2$	$n(n+1)/2 - (2n-1) = (n-1)(n-2)/2$
2	1	1	0
3	**4**	**3**	**1**
4	9	6	3

➔ **At least 3 generations are needed to have the CKM phase and CP violation!**

The fact that there are 3 families (with neutrino mass < ½ M_Z) has been proven at LEP from the width of the Z mass peak.

A comment about the quark sector of the standard model:

~ *half of the SM*

10 free parameters in the flavour sector of the SM:

6 quark masses | 4 CKM parameters

The 2008 Nobel Prize in Physics

was awarded to Kobayashi, Maskawa, and Nambu for their work on symmetry breaking and CP violation.

Photo: University of Chicago

Yoichiro Nambu

Photo: KEK

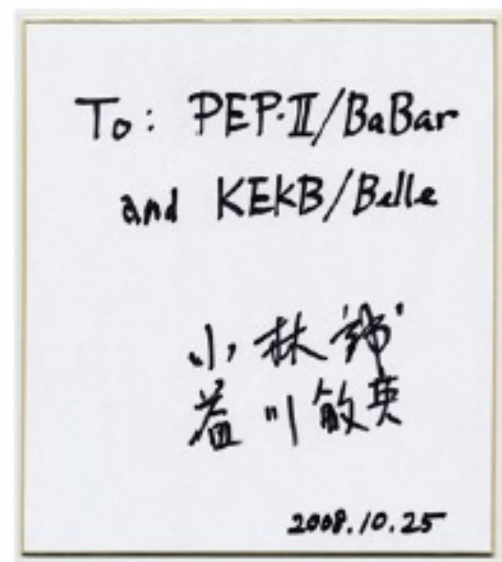
Makoto Kobayashi

Photo: Kyoto University

Toshihide Maskawa

From the BaBar statement following the Nobel Prize:

[…] They found that it was very hard to construct a plausible explanation of CP violation in quark decays working with only these two generations of four quarks. Their brilliant insight of **1972** *was* to realize that by extending the number of generations to three — and hence the number of quarks from four to six — CP violation appears quite naturally. Thus their description of CP violation entailed the very bold prediction of two entirely new and unobserved types of quark, now called "top" (t) and "bottom" (b). Quite remarkably, these new quarks were indeed discovered experimentally, the b in **1977** and the t in **1995**. More recently, Kobayashi and Maskawa's description of CP violation in quark decays was confirmed in detail by precision experiments at BaBar and Belle; their Nobel prize followed.



CKM Matrix Parameterization

There is no unique parameterization of the CKM matrix.
We can use, for example:

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \times \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}\,\mathrm{e}^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}\,\mathrm{e}^{i\delta} & 0 & \cos\theta_{13} \end{bmatrix} \times \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This representation is the one used by the PDG (p. 211 in PDG 2016, removed from PDG 2018, but can still be found in sec. 14 – Neutrino mixing)

$$\boxed{\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}}$$

Remark: the number of possibilities is

$$(3!)_{\text{rotation permutations}} \times 3_{\delta} \times 2_{\delta=\pm 1} = 36 \text{ possibilities}$$

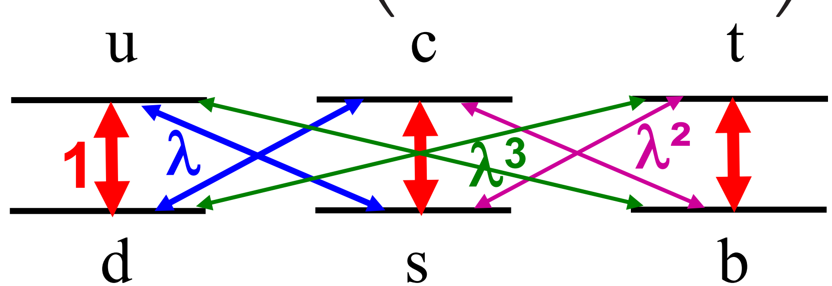
Size of the elements and pattern of the Matrix

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0411 \\ 0.00875 & 0.0403 & 0.99915 \end{pmatrix} \pm \begin{pmatrix} 0.00011 & 0.00050 & 0.00015 \\ 0.00050 & 0.00013 & 0.0013 \\ 0.00032 & 0.0013 & 0.00005 \end{pmatrix}$$

(diagonal terms dominate : d~d', s~s' et b~b')

We notice that, with $\lambda = \sin\theta_c \approx 0.22$

$$|V_{\mathrm{CKM}}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$



The SM does not provide an explanation for this numerical pattern!

Wolfenstein Parametrization

Power series of $\lambda=\sin(\theta_{cabibo})\approx 0.22$

At order λ^3:

$$V_{\mathrm{CKM}} = \underbrace{\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}}_{V_{\mathrm{CKM}}^{\mathrm{W3}}} + \mathcal{O}(\lambda^4)$$

$$\left.\begin{array}{c} \lambda = \sin\theta_c \\ A \sim 0.8 \\ \rho \sim 0.20 \\ \eta \sim 0.35 \end{array}\right\}\ \text{4 parameters}$$

At order λ^5:

$$V_{\mathrm{CKM}} = V_{\mathrm{CKM}}^{\mathrm{W3}} + \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5\left(1-2\left(\rho+i\eta\right)\right) & -\frac{1}{8}\lambda^4\left(1+4A^2\right) & 0 \\ \frac{1}{2}A\lambda^5(\rho+i\eta) & \frac{1}{2}A\lambda^4\left(1-2\left(\rho+i\eta\right)\right) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

Unitarity conditions

$$VV^{+}=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\times\begin{pmatrix} V_{ud}^{*} & V_{cd}^{*} & V_{td}^{*} \\ V_{us}^{*} & V_{cs}^{*} & V_{ts}^{*} \\ V_{ub}^{*} & V_{cb}^{*} & V_{tb}^{*} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Diagonal relations (unitarity)

$$V_{ud}V_{ud}^{*}+V_{us}V_{us}^{*}+V_{ub}V_{ub}^{*}=1$$
$$V_{cd}V_{cd}^{*}+V_{cs}V_{cs}^{*}+V_{cb}V_{cb}^{*}=1$$
$$V_{td}V_{td}^{*}+V_{ts}V_{ts}^{*}+V_{tb}V_{tb}^{*}=1$$

Off-diagonal relations (orthogonality)

from $VV^{+} = \mathbf{1}$ 3 independent relations

(3 are conjugates of 3 others)

$$V_{ud}V_{cd}^{*}+V_{us}V_{cs}^{*}+V_{ub}V_{cb}^{*}=0$$
$$V_{ud}V_{td}^{*}+V_{us}V_{ts}^{*}+V_{ub}V_{tb}^{*}=0$$
$$V_{cd}V_{ud}^{*}+V_{cs}V_{us}^{*}+V_{cb}V_{ub}^{*}=0$$
$$V_{cd}V_{td}^{*}+V_{cs}V_{ts}^{*}+V_{cb}V_{tb}^{*}=0$$
$$V_{td}V_{cd}^{*}+V_{ts}V_{cs}^{*}+V_{tb}V_{cb}^{*}=0$$
$$V_{td}V_{ud}^{*}+V_{ts}V_{us}^{*}+V_{tb}V_{ub}^{*}=0$$

from $V^{+}V = \mathbf{1}$

3 other independent relations

$$V_{ud}^{*}V_{us}+V_{cd}^{*}V_{cs}+V_{td}^{*}V_{ts}=0$$
$$V_{ud}^{*}V_{ub}+V_{cd}^{*}V_{cb}+V_{td}^{*}V_{tb}=0$$
$$V_{us}^{*}V_{ub}+V_{cs}^{*}V_{cb}+V_{ts}^{*}V_{tb}=0$$

The 6 orthogonality relations describe triangles in the complex plane

"The" Unitarity Triangle

CKM matrix — Wolfenstein parameterization:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

V_{CKM} Unitarity ⇒

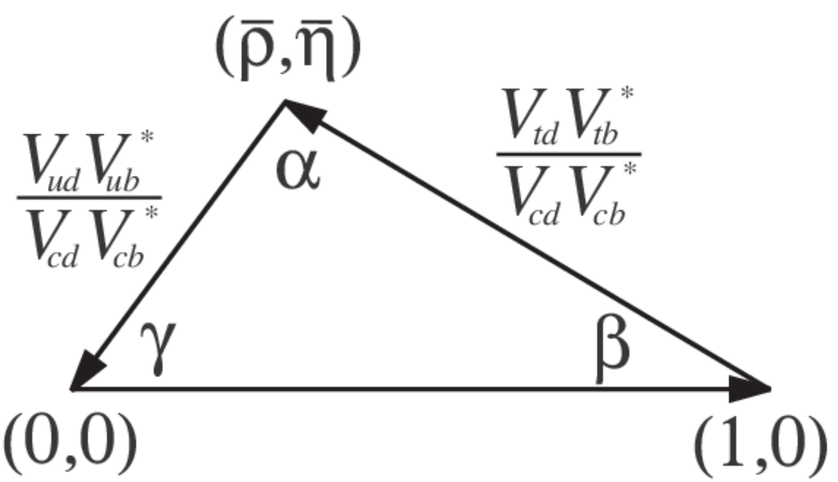
$$\underbrace{V_{ud}V_{ub}^*}_{\sim\lambda^3} + \underbrace{V_{cd}V_{cb}^*}_{\sim\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{\sim\lambda^3} = 0$$

This triangle is related to b-hadron decays

We notice that it's sides are comparable

It is usually divided by $V_{cd}V^*_{cb}$ (side of length 1)

Often called "the" unitarity triangle



CP Violation is possible in the Standard Model only if

V_{CKM} is complex ⇔ $\bar\eta \neq 0$ ⇔ Unitarity Triangle is not flat

We want to determine $\bar\rho$ and $\bar\eta$ experimentally by measuring the triangle sides and angles

Angles and apex of "The" Unitarity Triangle

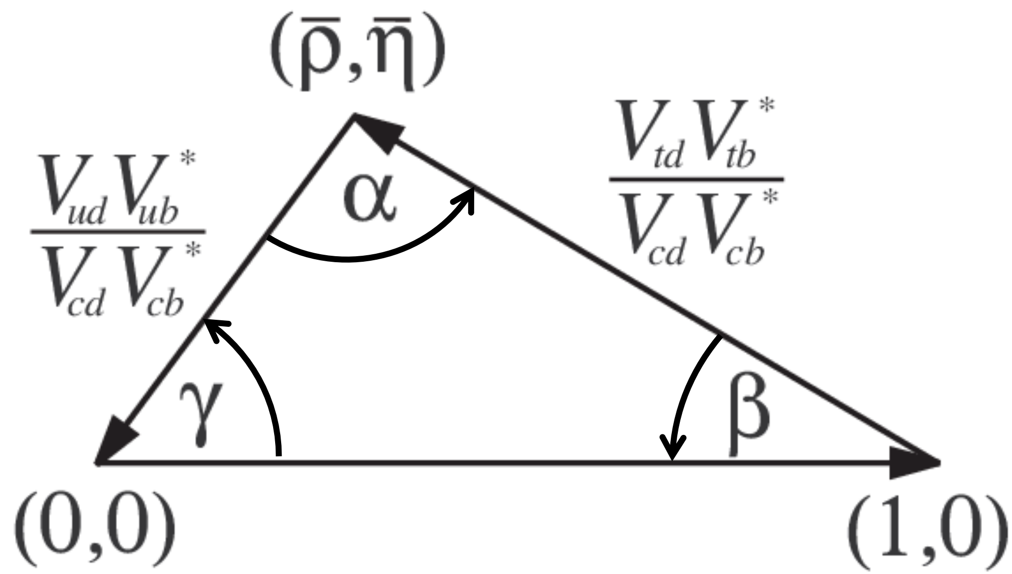
$$\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$$\alpha = \mathrm{Arg}\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}$$

$$\beta = \mathrm{Arg}\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}$$

$$\gamma = \mathrm{Arg}\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

← These are the exact coordinates of the apex
They slightly differ from of the
Wolfenstein parameterization, at O(λ^5)



By construction $\alpha + \beta + \gamma = \pi$
(only two independent angles)

Flavour oscillations in the neutral kaons system
(case with no CP violation)

- Amp. of an (instable) mass eigenstate (e.g. K_S): $a_S(t) = a_S(0) e^{-\left(\frac{im_S}{\hbar}\right)t} e^{-\left(\frac{\Gamma_S}{2\hbar}\right)t}$

describes "mass"

"lifetime" (exp. law) with $\Gamma = \hbar/\tau$

- Probability:

$$\Gamma(t) = a_s(t) a_s^*(t) = a_s(0) a_s^*(0) e^{-\left(\frac{\Gamma_s}{\hbar}\right)t} = \Gamma(0) e^{-\left(\frac{\Gamma_s}{\hbar}\right)t}$$

- For the K^0- $\overline{K}^0$ system:

$$K_S: \quad a_S(t) = a_S(0) e^{-\left(\frac{\Gamma_S}{2\hbar} + \frac{im_S}{\hbar}\right)t}$$

$$K_L: \quad a_L(t) = a_L(0) e^{-\left(\frac{\Gamma_L}{2\hbar} + \frac{im_L}{\hbar}\right)t}$$

CP conservation

t = 0: pure beam of K^0. Given that: $\left|K^0\right\rangle = \sqrt{\frac{1}{2}}\left(\left|K_S^0\right\rangle + \left|K_L^0\right\rangle\right) \Rightarrow a_L(0) = a_S(0) = \frac{1}{\sqrt{2}}$

At time t (in natural units):

$$\Gamma\left(\left|K^0\right\rangle(t)\right) = \frac{\left(a_S(t) + a_L(t)\right)}{\sqrt{2}} \cdot \frac{\left(a_S^*(t) + a_L^*(t)\right)}{\sqrt{2}} = \frac{1}{4}\left\{e^{-\Gamma_s t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S + \Gamma_L}{2}t} \cos \Delta m t\right\}$$

$-$ for $\overline{K}^0$ $\quad (\Delta m = |m_L - m_S|)$

$$\Gamma\left(\left|K^0\right\rangle\right) - \Gamma\left(\left|\overline{K}^0\right\rangle\right) = e^{-\left[\left(\Gamma_S + \Gamma_L\right)/2\right]t} \cos \Delta m t \qquad \Gamma\left(\left|K^0\right\rangle\right) + \Gamma\left(\left|\overline{K}^0\right\rangle\right) = \frac{1}{2}\left(e^{-\Gamma_S t} + e^{-\Gamma_L t}\right)$$

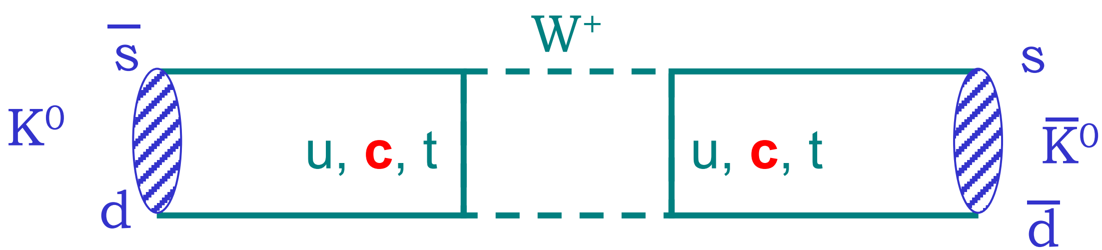
The K^0 - $\overline{K}^0$ oscillation frequency is Δm

Oscillation Frequency

The experimental measurement for neutral kaons gives:

$$\Delta m \cong 3.52 \cdot 10^{-6} eV \qquad m_{K_L} > m_{K_S}$$

Very small mass difference (due to weak interaction). We don't have to worry about it…



In this diagram the c quark gives the dominant contribution (similarly to the t quark in loop/box diagrams of b decays and oscillations)

Note that by measuring the frequency we can access experimentally a tiny mass difference $\Delta m/m \sim 0.7\ 10^{-14}$!!!

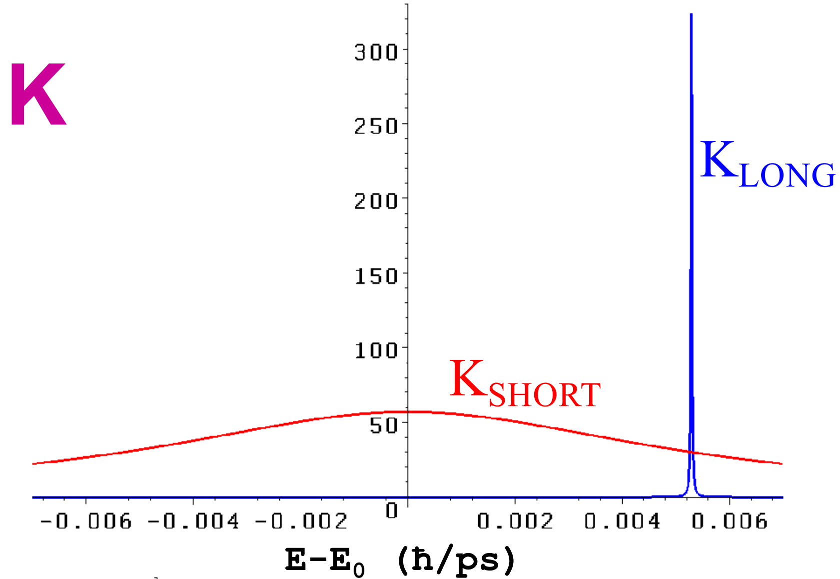
Recall that this measurement gives access to some of the parameters of the SM: CKM matrix elements.

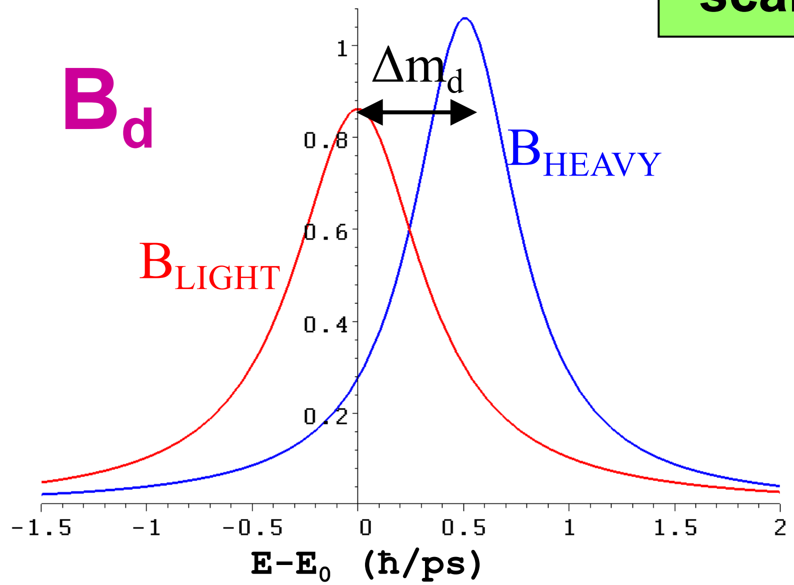
(it also provides information on the mass of the dominant virtual quark in the box, here: c-quark)

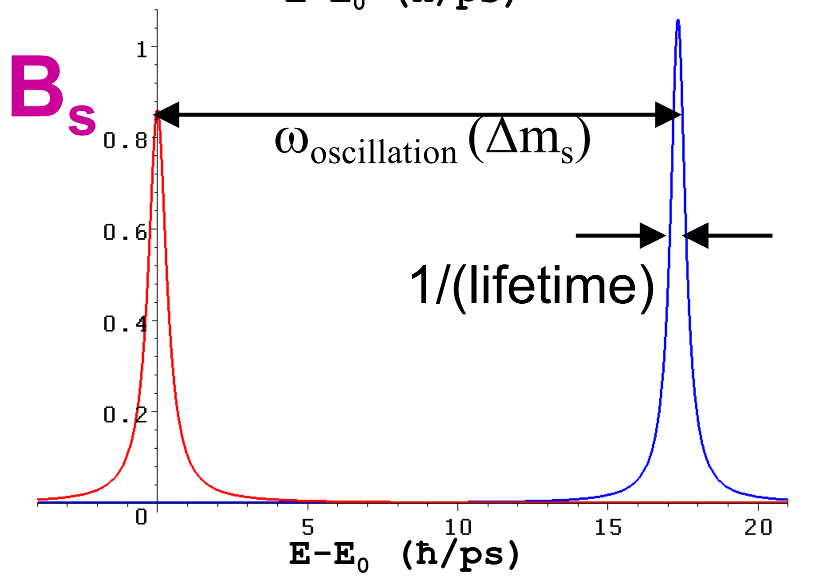
Comparison of K, B_d and B_s Oscillations

- Oscillations (mixing) characterized by mass and lifetime differences between the two eigenstates of weak interaction.
- Differences between flavours:
  - K: very different states (because of the phase space difference)
  - B_d: Oscillation and decay are comparable
  - B_s: Rapid oscillations

Mind the scales!



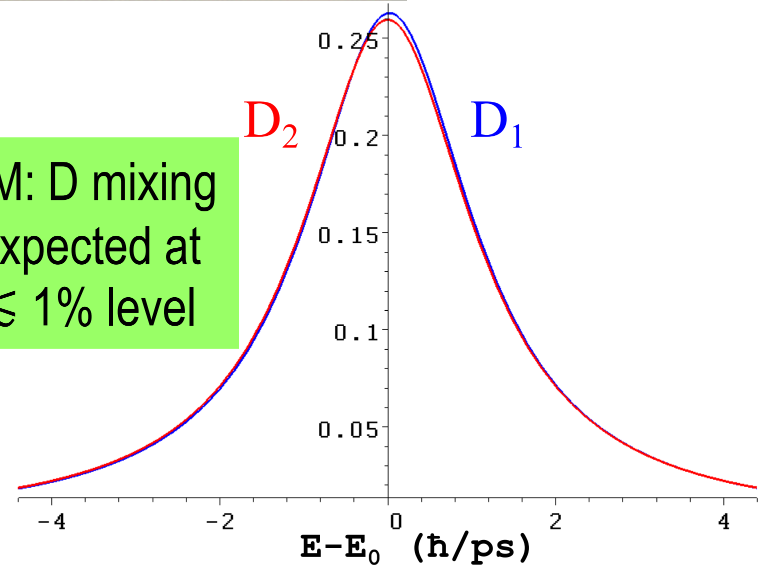




And Finally D-Oscillations

- Very slow oscilations
- An experimental challenge!
- Both BaBar and Belle observed mixing (Winter 2007)
- Results are consistent with SM
- Charm sector: only place where CP violation with down-type quarks in the mixing diagram can be explored.
- LHCb has now taken over these measurements
- CP violation in Charm decays was observed by LHCb in 2019

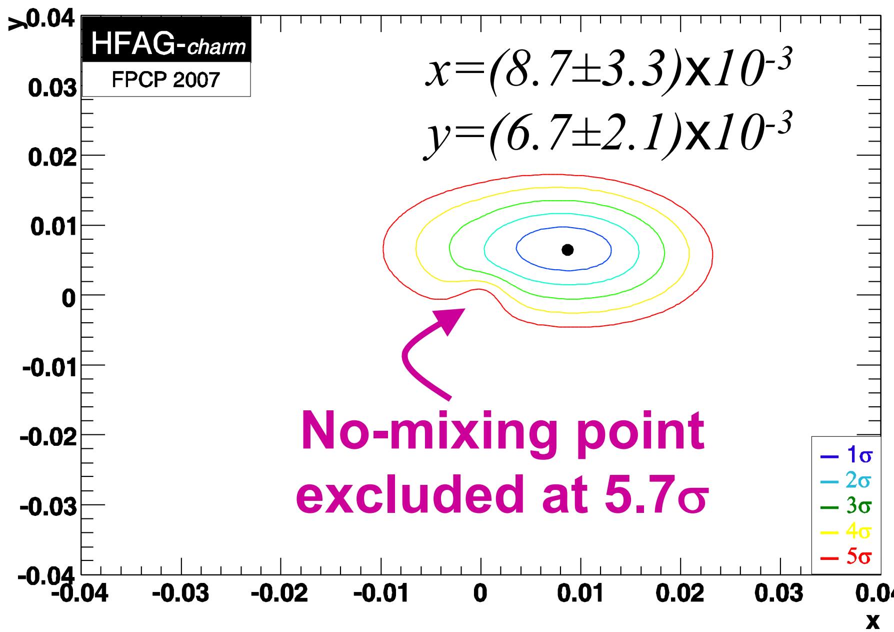
SM: D mixing expected at $\lesssim$ 1% level



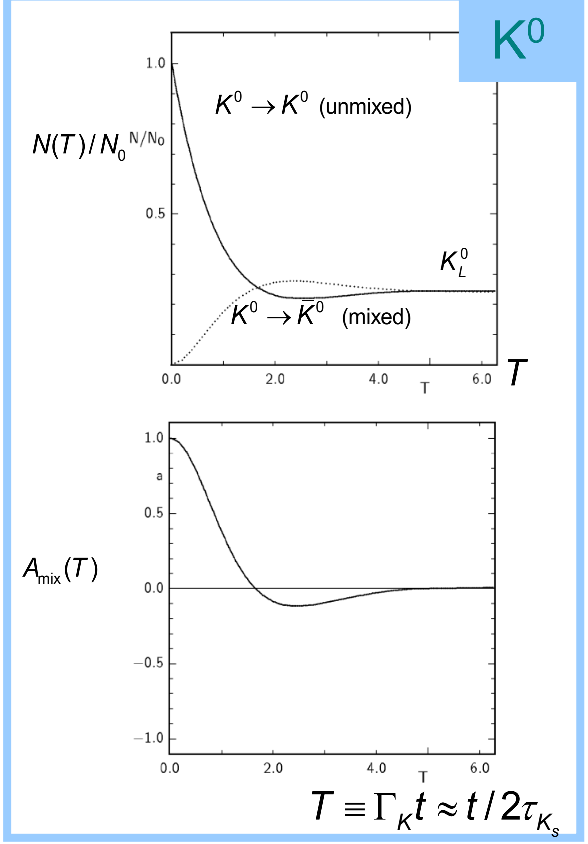
$$x = \frac{m_1 - m_2}{\Gamma}$$

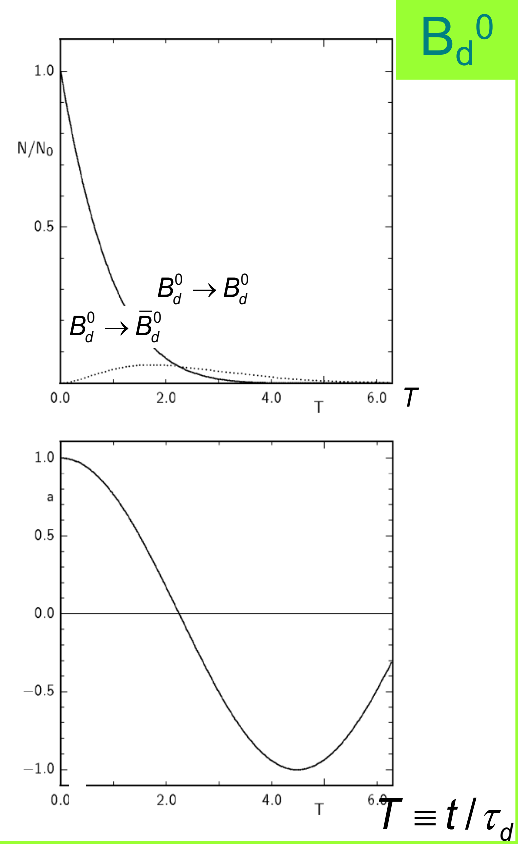
$$y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}$$

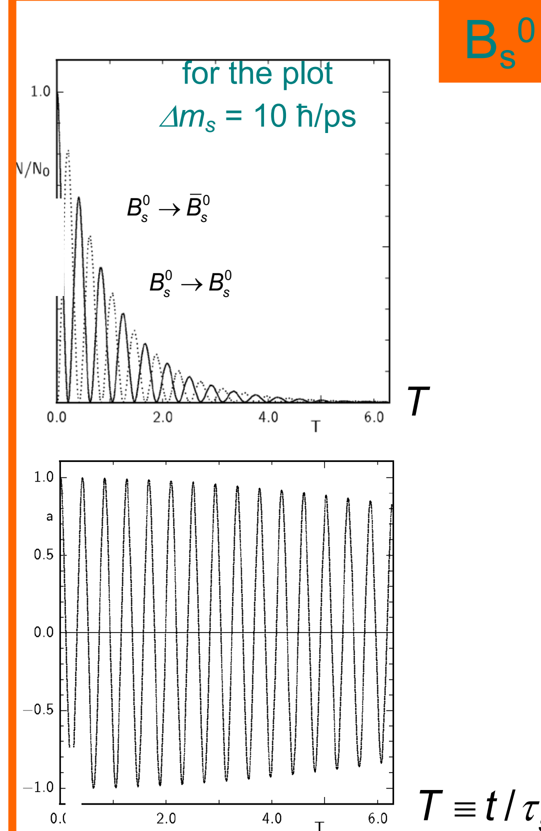
$$\Gamma = \frac{1}{2}(\Gamma_1 + \Gamma_2)$$



Time Evolution Plots (I)







	τ_L	τ_H	$\Delta m = m_H - m_L$ ($\hbar s^{-1}$)
K^0 ($s\bar{d}$)	~0.9 10^{-10} s	~0.5 10^{-7} s	0.53 10^{10} (~ a few 10^{-6} eV)
D^0 ($c\bar{u}$)	$\tau_H \sim \tau_L$ ~0.41 10^{-12} s		0.95 10^{10}
B_d ($b\bar{d}$)	$\tau_H \sim \tau_L$ ~1.5 10^{-12} s		0.51 10^{12} (3.4 10^{-4} eV)
B_s ($b\bar{s}$)	$\tau_H \sim \tau_L$ ~1.5 10^{-12} s		17.76 10^{12} (1.12 10^{-2} eV)

CDF, D0
2006

Time evolution plots (II)

From "Physics of B Factories" book (arXiv:1406.6311)

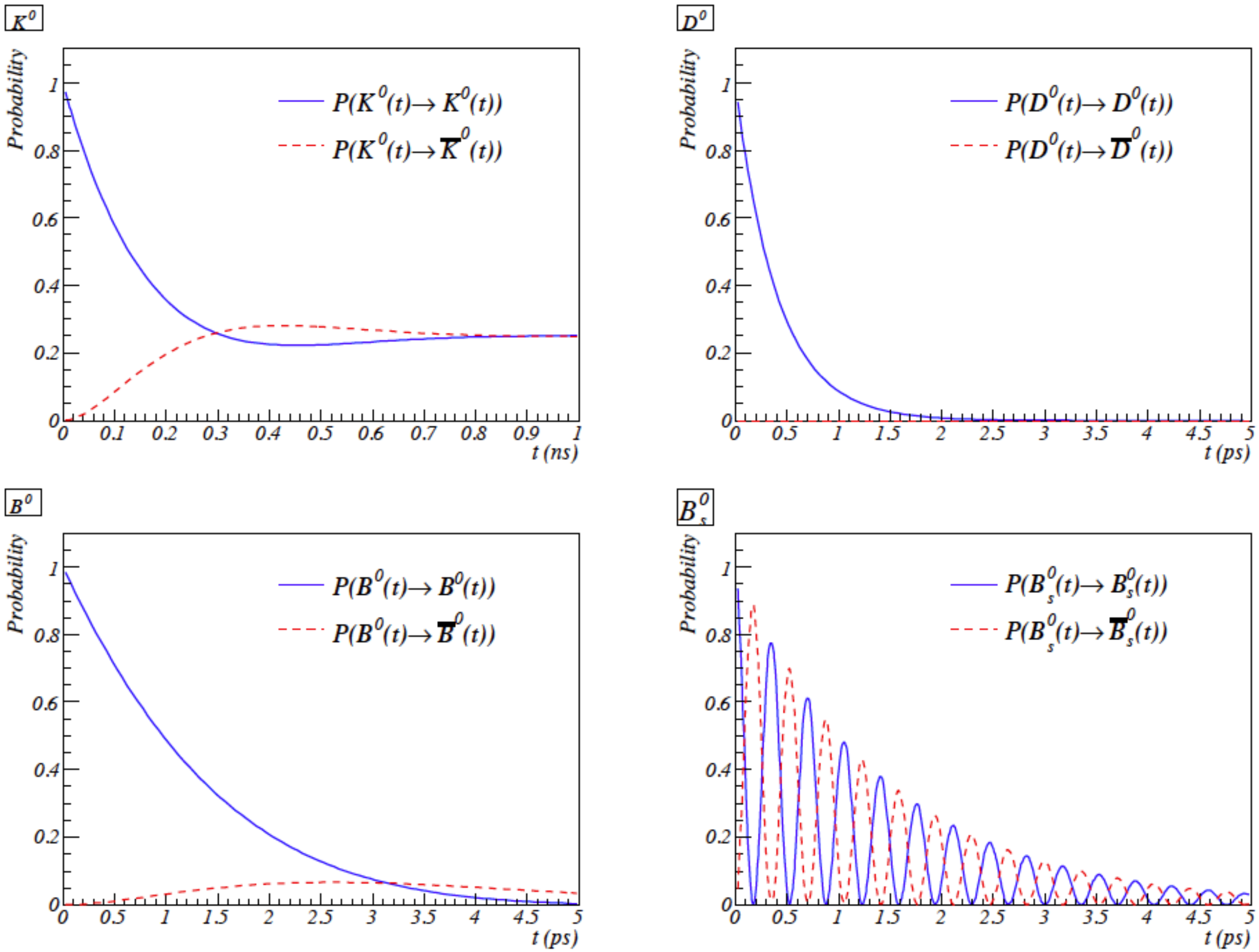
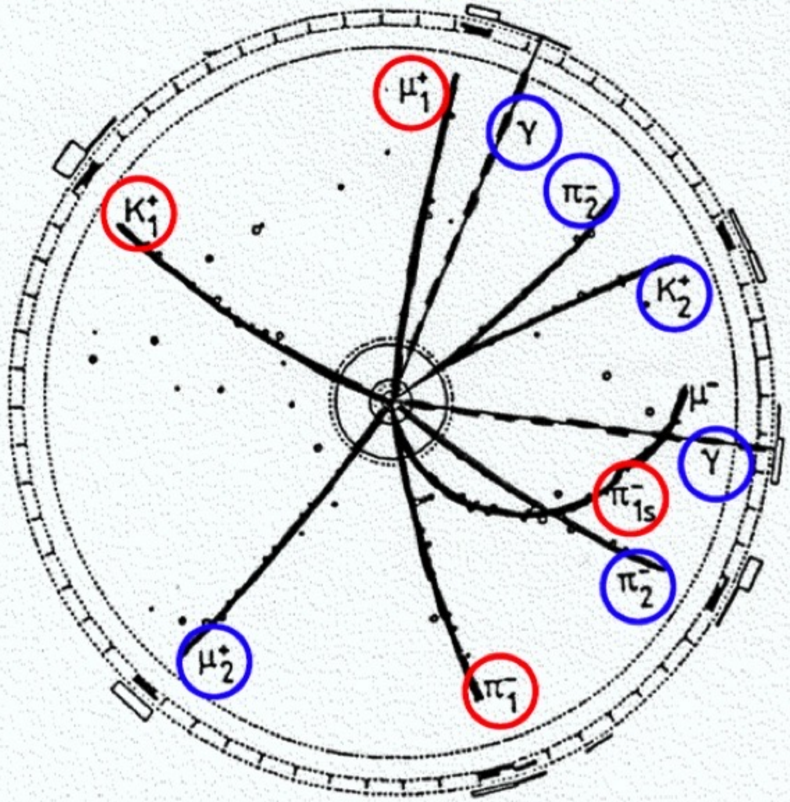


Figure 3.3: *If one starts with a pure P^0-meson beam the probability to observe a P^0 or a $\bar{P}^0$-meson at time t is shown,* $\mathrm{Prob}(t) = \frac{e^{-\Gamma t}}{2}\left(\cosh \frac{1}{2}\Delta\Gamma t \pm \cos \Delta m t\right)$.

The B^0 mixing was observed for the first time in 1987 by the Argus collaboration:

B^0-mixing: First Observation at Argus, DESY, 1987 PLB192, 245 (1987)

Fig. 11: The fully reconstructed ARGUS event [26]
$e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0 \to B^0B^0$
as the first evidence for the occurance of $B^0\overline{B}^0$ oscillations.
$B^0 \to D_1^{*-}\mu_1^+\nu$,
$D_1^{*-} \to \pi_{1s}^-\overline{D}^0$, $\overline{D}^0 \to K_1^+\pi_1^-$.
$\overline{B}^0 \to B^0 \to D_2^{*-}\mu_2^+\nu$,
$D_2^{*-} \to \pi^0 D_2^-$,
$\pi^0 \to \gamma\gamma$, $D_2^- \to K_2^+\pi_2^-\pi_2^-$.



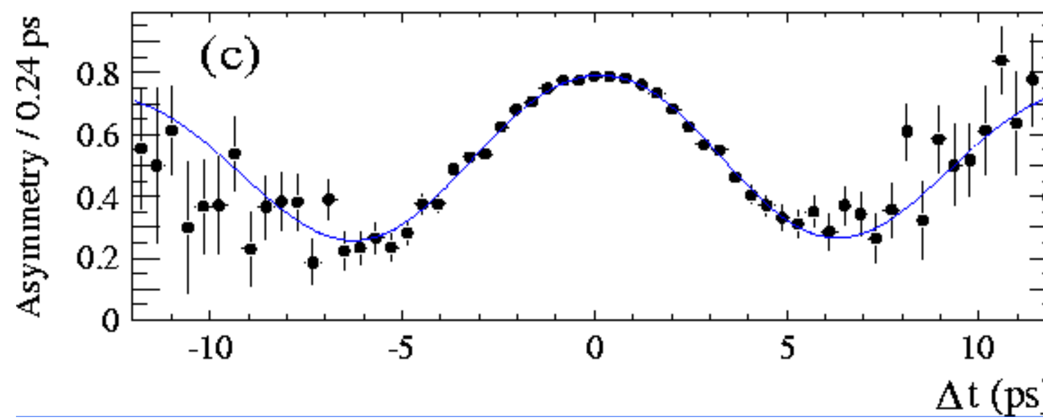
$B^0 \to D^{*-}\mu^+\nu$

$B^0 \to D^{*-}\mu^+\nu$

This predicted that m(top) > 50 GeV !

B factories:
(2005)

asymétrie: $\propto \cos(\Delta m_d t)$



Classification of CP Violation effects

- **Direct CP Violation (CP Violation in Decay):**

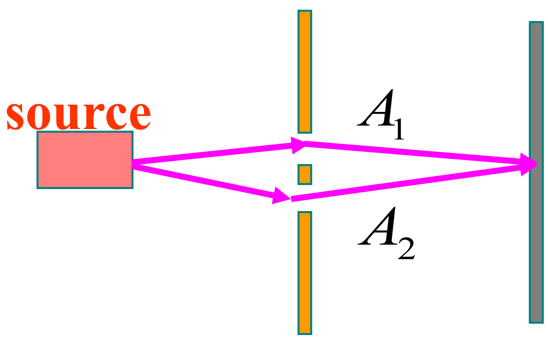
  $$\Gamma(X \to \mathrm{f}) \neq \Gamma(\bar{X} \to \bar{\mathrm{f}}) \qquad (|\bar{A}_{\bar{\mathrm{f}}}| \neq |A_{\mathrm{f}}|)$$

  - To measure it, only need to count events (e.g. for $B^0 \to K^+\pi^-$)
    Rates are different ⇔ CP is violated
  - This is the only possible type CP violation in charged-particle and baryon decays
- **CP violation in mixing:** $\Gamma(B^0 \to \bar{B}^0) \neq \Gamma(\bar{B}^0 \to B^0)$ $(|q/p| \neq 1)$
  N.B. unlike in neutral kaons, for B^0 and B^0_s decays $|q/p| \simeq 1$
- **CP violation in the interference between decay and mixing:**

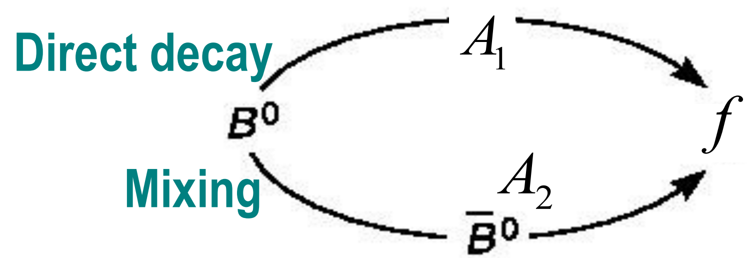
  $$\Gamma(B^0 \to \mathrm{f}) \neq \Gamma(\bar{B}^0 \to \mathrm{f}) \qquad (\text{e.g. for } B^0 \to J/\psi\, K_S)$$

  may occur even if $|q/p|=1$ due to the phase of q/p

Analogy to "Double-Slit" experiment

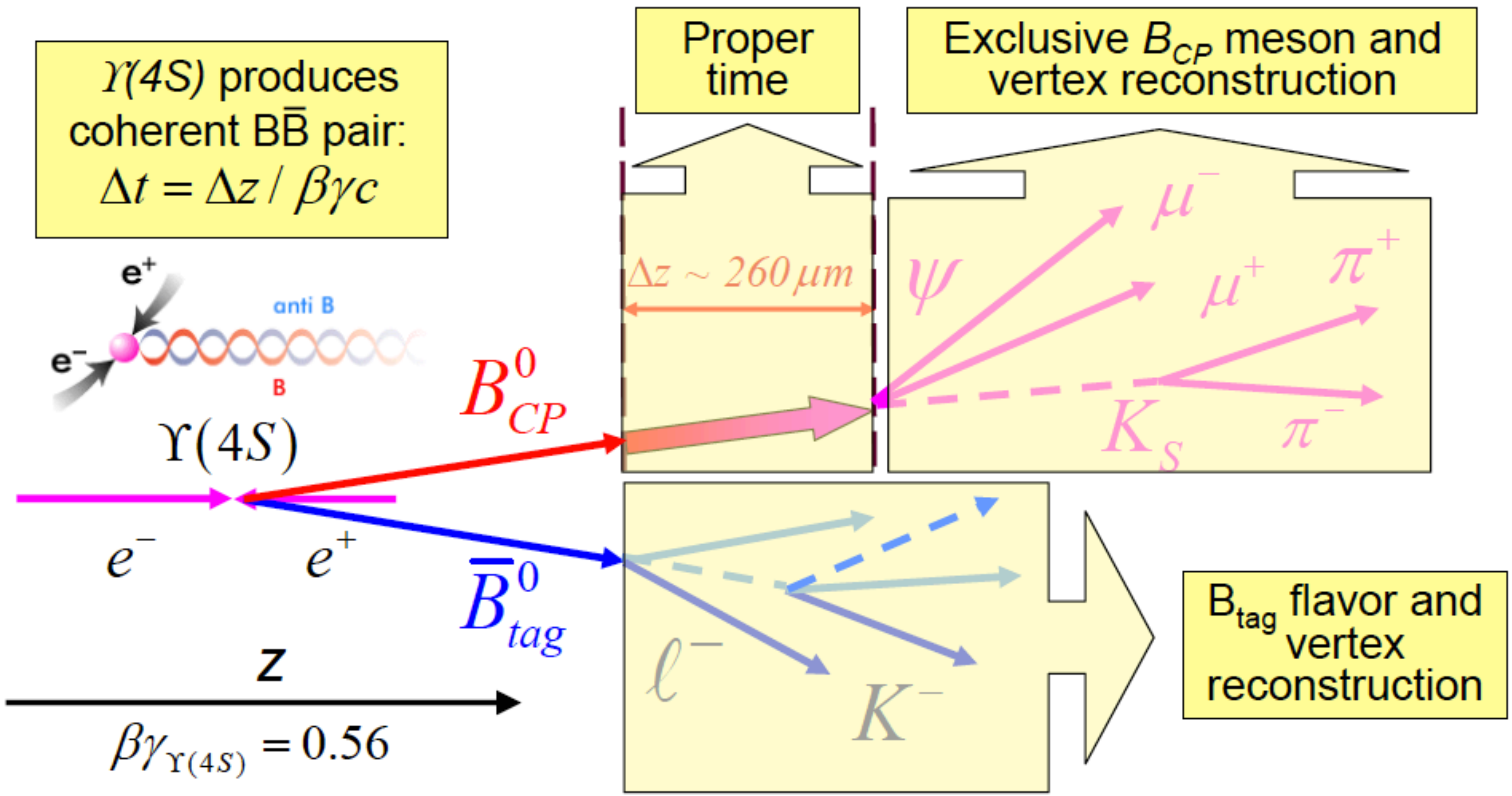


In the double-slit experiment, there are two paths to the same point on the screen.

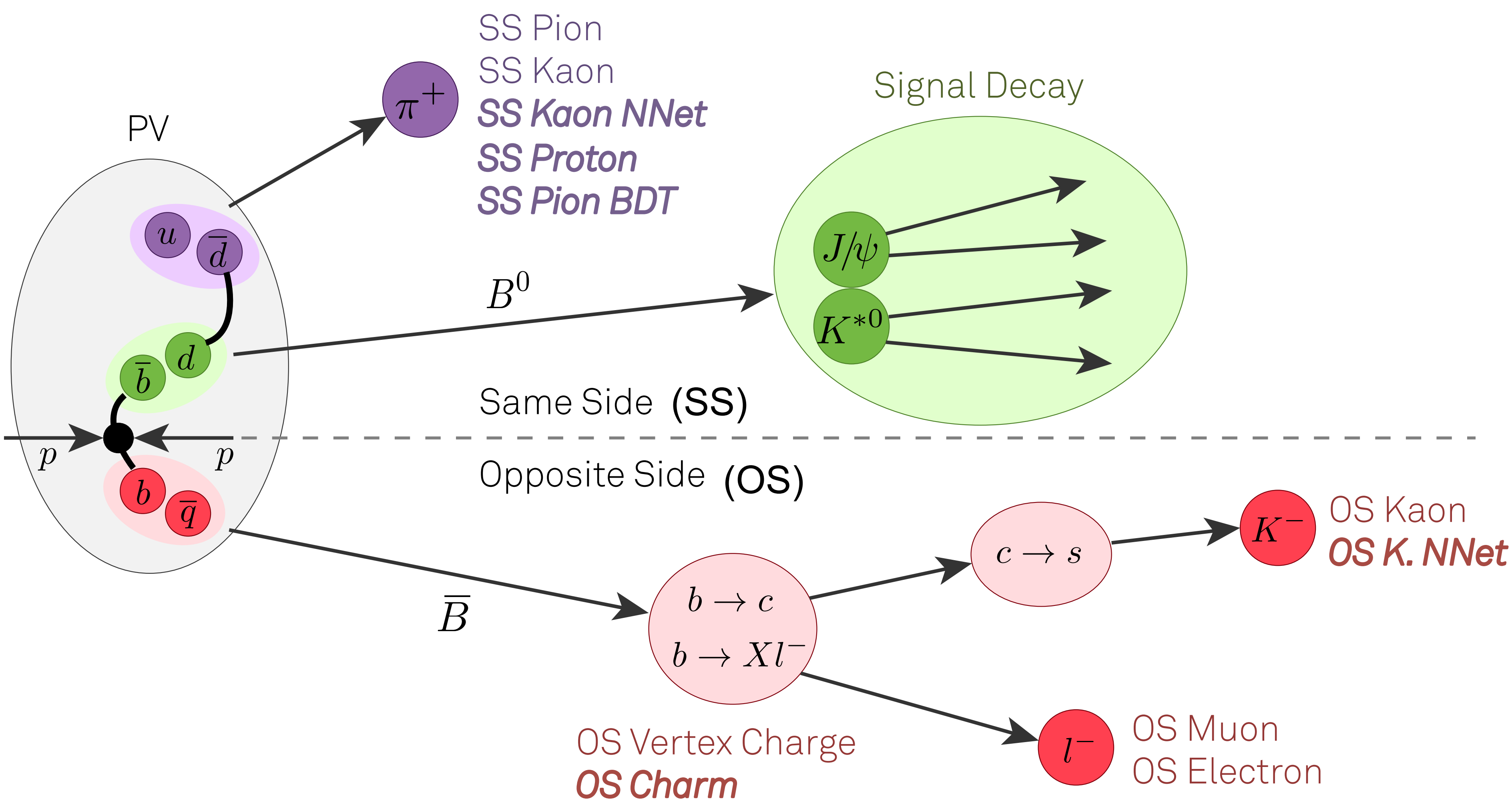


Here (B-meson decay), we must choose final states into which both a $\bar{B}^0$ and a B^0 can decay. *Logic: "perform the experiment twice"* (starting from B^0 and from $\bar{B}^0$), then compare the results.

B tagging technique at B factories (Υ(4S))



B tagging technique at hadron colliders



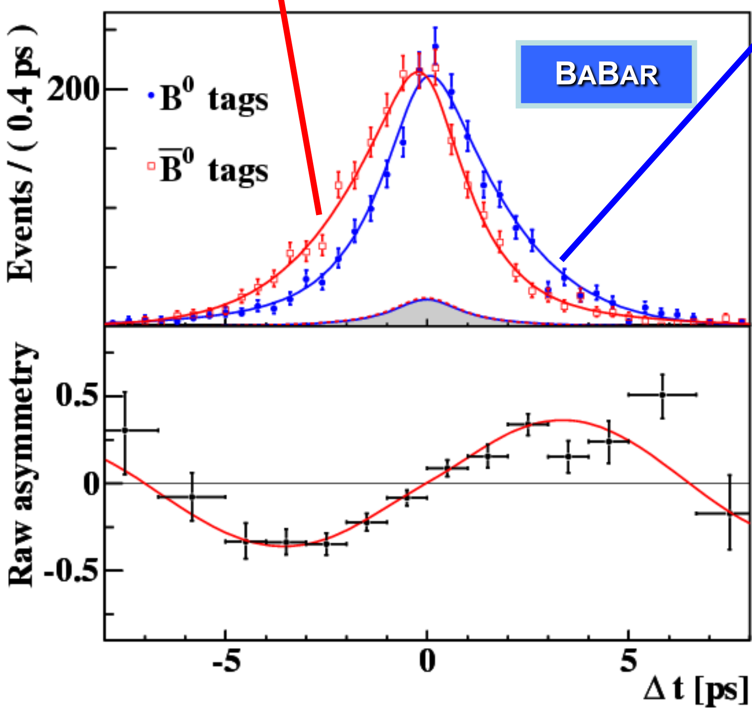
Measurement of sin(2β) with $B^0 \rightarrow J/\psi\, K^0_S$

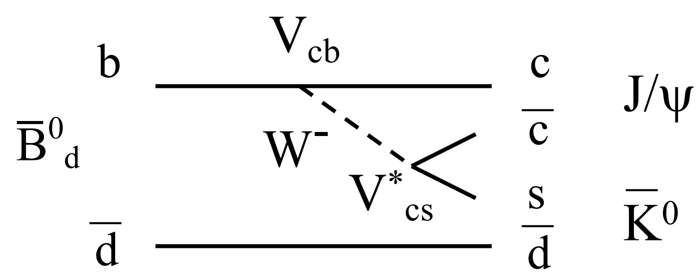
- Final state accessible to B^0 and $\bar{B}^0$ → Time dependent asymmetry:

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) - \Gamma(B^0(t) \rightarrow J/\psi K_S)}{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) + \Gamma(B^0(t) \rightarrow J/\psi K_S)} = \boxed{S}\sin(\Delta m_d t) - \boxed{C}\cos(\Delta m_d t)$$

indirect

direct





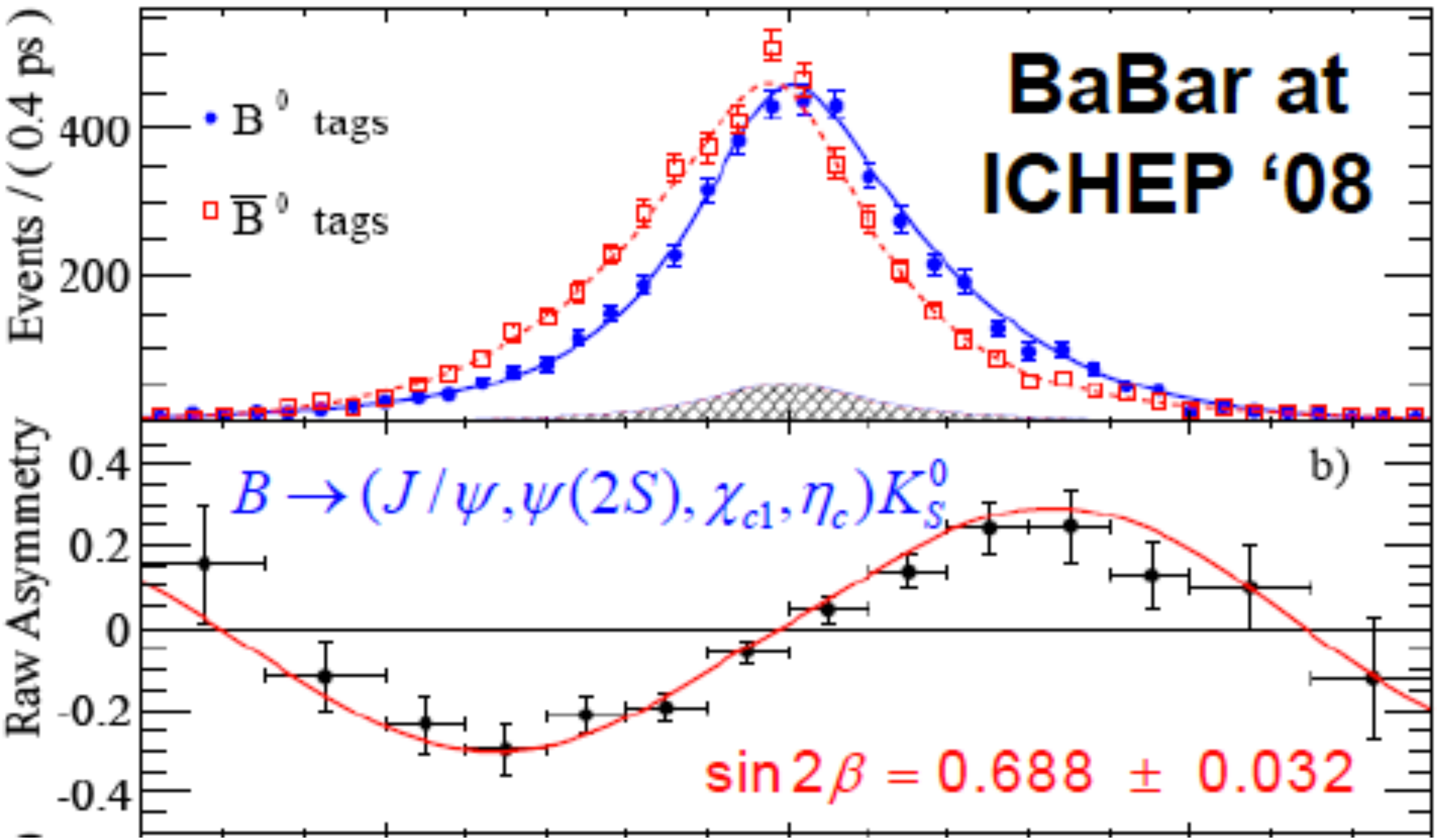
~only one amplitude

$$C_f = 0$$

$$S_f = -\eta_{CP} \sin 2\beta$$

⇒ Extraction of sin(2β) from A_{cp}

$\sin 2\beta$ measurement
[BABAR, PRD79, 072009 (2009)]



Unitarity triangle measurements (2018)