NPAC
Particle Physics
Course 5 – Introduction to QCD

Eli Ben-Haïm
Fabrice Couderc

Program

1. General reminders
2. A few reminders about quarks and hadrons
3. The SU(3) group and its representations
4. Flavor-SU(3) and hadrons phenomenology
5. Probing the structure of neucleons
   5.1 Magnetic moments
   5.2 Scattering experiments
      Elastic scattering
      A first look into Deep Inelastic Scattering (DIS)
6. The QCD running coupling $\alpha_s(q^2)$
7. Hadronisation and jets
8. Color and color-SU(3)
   The 8 gluons
   Why baryons and mesons (confinement)
9. Experimental evidence for color
10. Soft and collinear divergences in QCD
11. Jets and "infrared safety"

2. A few reminders about quarks and hadrons

Gell-Mann first used the term quark, inspired from a citation from James Joyce's "*Finnegans Wake*": "**Three quarks for Muster Mark**".

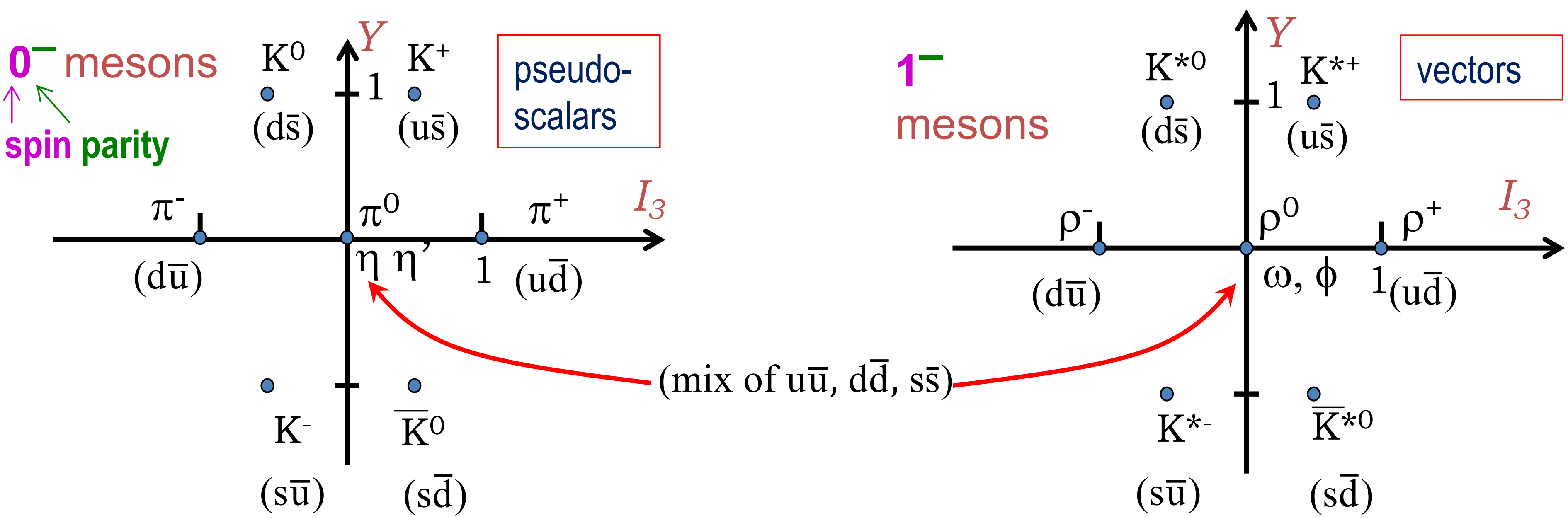
- A few properties of the quarks:

Name	Symbol	Mass (GeV)	Q	$\mathfrak{B}$	S	C	B	T
Down	d	~0.005	-⅓	⅓	0	0	0	0
Up	u	<~ m_d	+⅔	⅓	0	0	0	0
Strange	s	~0.100	-⅓	⅓	-1	0	0	0
Charmed	c	1.27	+⅔	⅓	0	1	0	0
Bottom / Beauty	b	4.18	-⅓	⅓	0	0	-1	0
Top (Truth…)	t	173.21	+⅔	⅓	0	0	0	1

- Hadrons are bound states of quarks: mesons ($q_1\bar{q}_2$) and baryons ($q_1 q_2 q_3$).

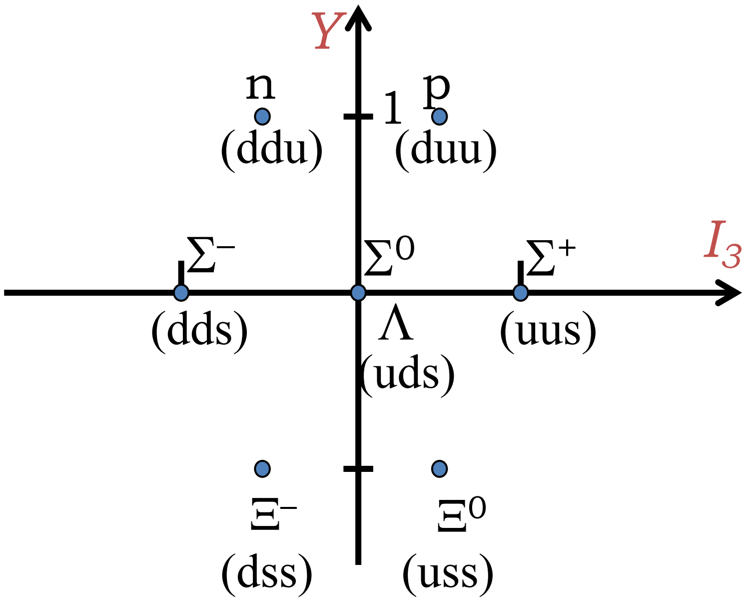
- In the beginning of the 1960s, many hadrons are found, in a complete disorder.
- Trying to make a theory to explain the "Zoo" of particles, Gell-Mann and Ne'eman (1961) notice that the lightest hadrons (only ones known by then) form structures in the (Y, I_3) plane ("The Eightfold Way")
- These particular structures, interpreted as an underlying SU(3) **symmetry**, suggest the **existence of the "quarks" u, d, and s**, and that these objects act as if they were the **same with respect to the (strong) interaction**.

- The structures and associated "quarks" for mesons:

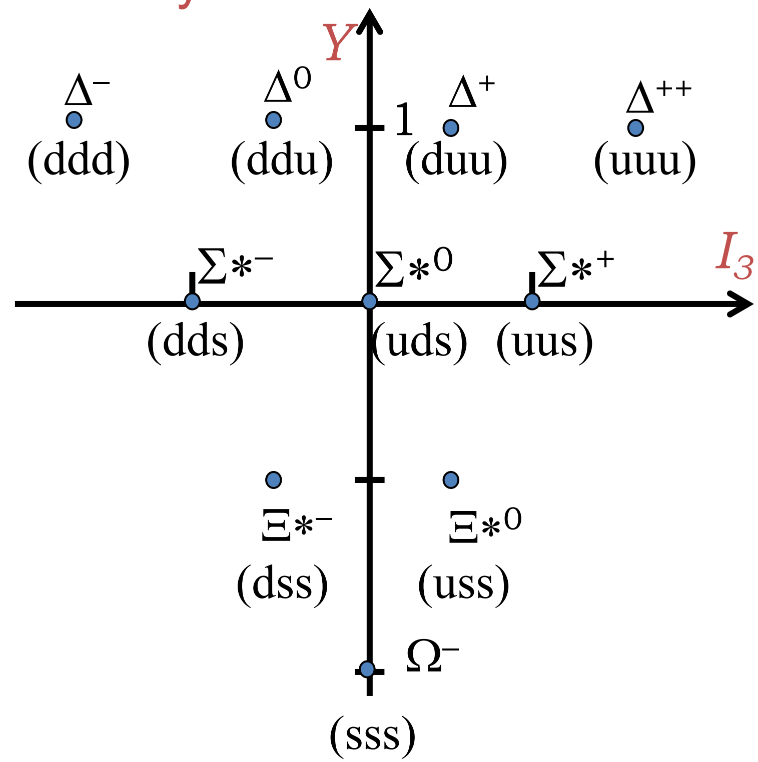


- Similar schemes for baryons:

$1/2^+$ baryons



$3/2^+$ baryons



Important to know:

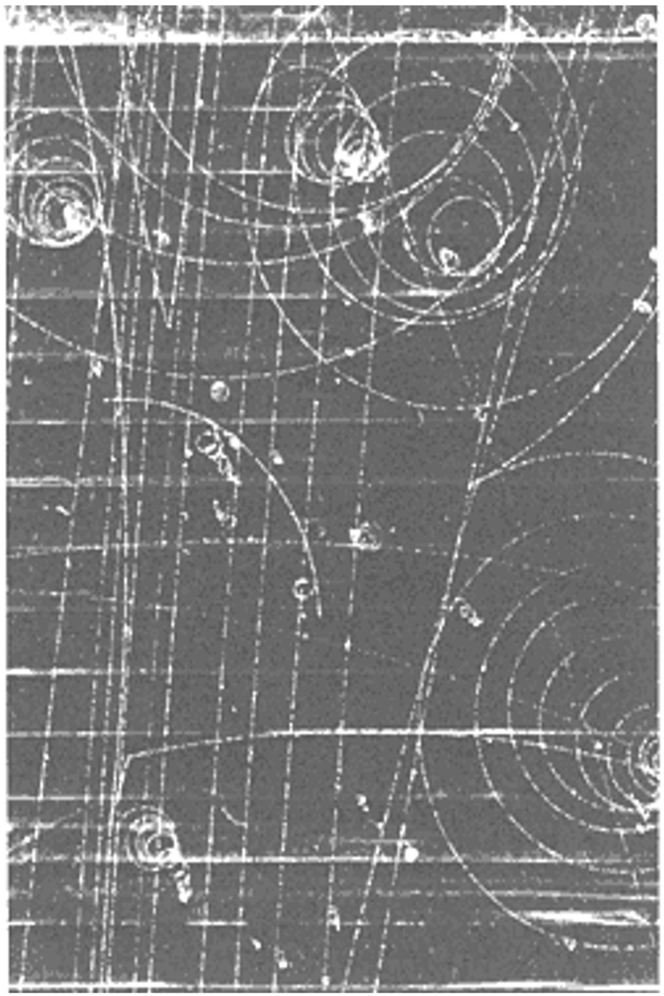
$I_3 = Q - Y/2$

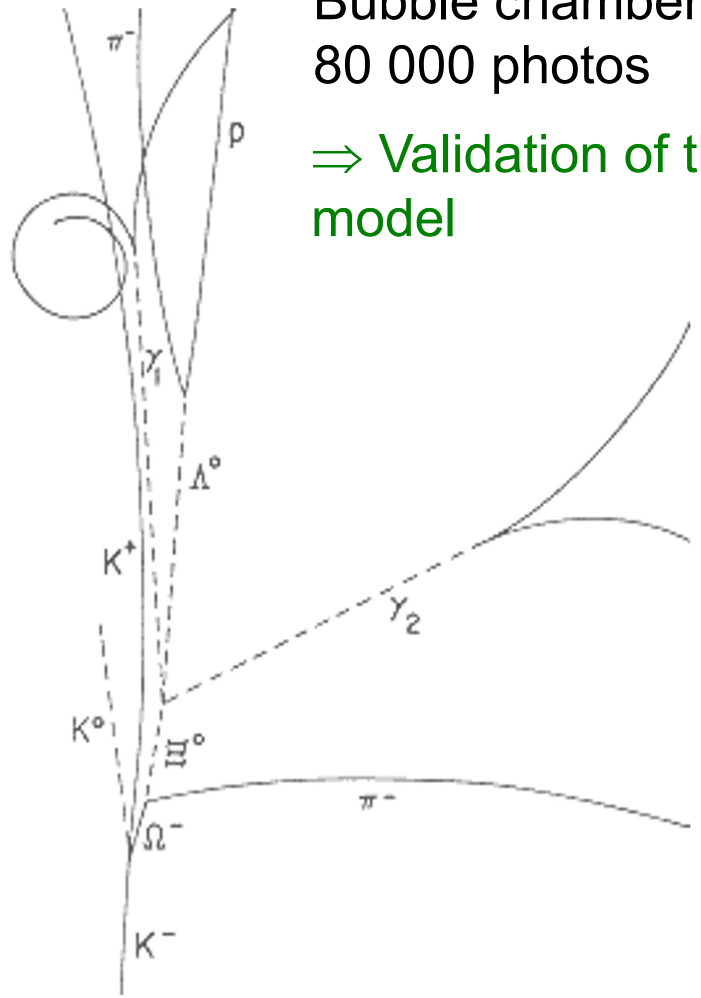
$Y = \mathcal{B} + S$

(These schemes and the quark compositions are important to memorize)

February 1964: discovery of the Ω^-

By the time of "The Eightfold Way" the Ω^- baryon has not yet been discovered.





Bubble chamber in Brookhaven;
80 000 photos

$\Rightarrow$ Validation of the SU(3) quark model

$$K^- p \rightarrow \Omega^- K^+ K^0$$

$$\Omega^- \rightarrow \Xi^0 \pi^-$$

$$\Xi^0 \rightarrow \Lambda \pi^0$$

$$\Lambda \rightarrow p \pi^-$$

3. The SU(3) group and its representations

(First of all let us remind ourselves what is SU(2), its generators, representations...)

- Ensemble of 3x3 matrices (U), unitary with determinant = 1
- Generators of the group: $3^2 - 1 = 8$ independent hermitian matrices with trace = 0

  $U = e^{i\vec{\theta}\bullet\vec{T}}$ (T are the generators, and θ real rotation angles)

- Only 2 (= 3 - 1) of the 8 generators can be simultaneously diagonal

  = maximum number of commuting generators and number of Casimir operators (operators that commute with all the generators) ➔ SU(3) is of rank 2.

  Reminder: in SU(2) there is only one Casimir operator (J^2, commutes with the Pauli matrices)

- Under SU(3) symmetry, we can "rotate" linear combinations of 3 states and leave the system unchanged

  $$\left|q_i'\right\rangle = \sum_{j=1}^{3} U_{ij}\left|q_j'\right\rangle,$$

  where U_{ij} are the elements of any SU(3) matrix U

  By analogy with SU(2) (where the generators are $J_i = \sigma_i / 2$) we define for SU(3):

The 8 λ matrices in the standard form introduced by Gell-Mann:

$$\underbrace{\lambda_1 = \begin{pmatrix} \boxed{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} \boxed{\begin{matrix} 0 & -i \\ i & 0 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix},}_{\text{The } \sigma \text{ matrices. SU(2) is a sub-group of SU(3)...}}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

☆ The 3 symmetric states:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

☆ $\lambda_{4..7}$ easily understandable in terms of σ.

☆ Verify the properties of the matrices, e.g.0-trace, linear independence.

2 diagonal matrices:
- additive quantum numbers
- simultaneously measured quantities

- λ_1 exchanges states 1 and 2
- λ_4 exchanges states 1 and 3
- λ_6 exchanges states 2 and 3
- λ_2 , λ_5 , λ_7: same action, with a complex factor
- By analogy with SU(2) (where $J_i = \sigma_i / 2$) we define for SU(3): $T_i = \dfrac{\lambda_i}{2}$

Raising and lowering operators $\longrightarrow$ (move among states in a representation)

$I_\pm = ½(\lambda_1 \pm i\, \lambda_2)$ states $1 \leftrightarrow 2$

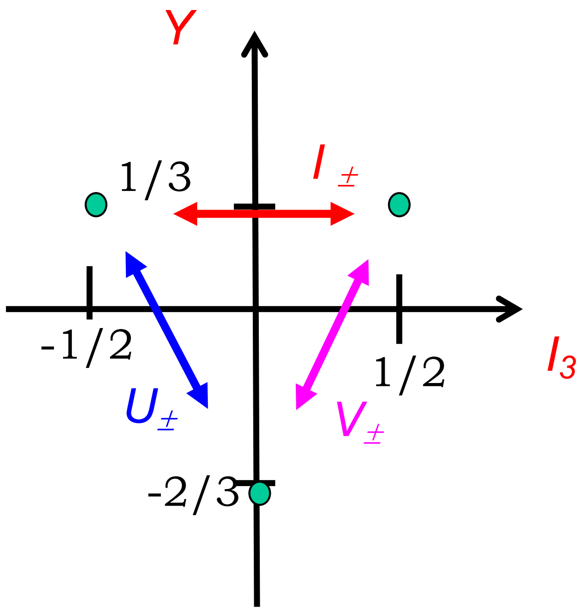
$V_\pm = ½(\lambda_4 \pm i\, \lambda_5)$ states $1 \leftrightarrow 3$

$U_\pm = ½(\lambda_6 \pm i\, \lambda_7)$ states $2 \leftrightarrow 3$

The three symmetric states are defined by the additive quantum numbers given by the eigenvalues of the diagonal generators (with a slight change for T8…)

I_3 = Q – Y/2 = T_3

$Y = \mathscr{B} + S \quad \equiv (2/\sqrt{3})T_8$



It is easy to verify the action of the raising/lowering operators on the electric charge and the strangeness. They allow to move among the states in a multiplet.

$I_{\pm}$ (S→S) (Q→Q±1)

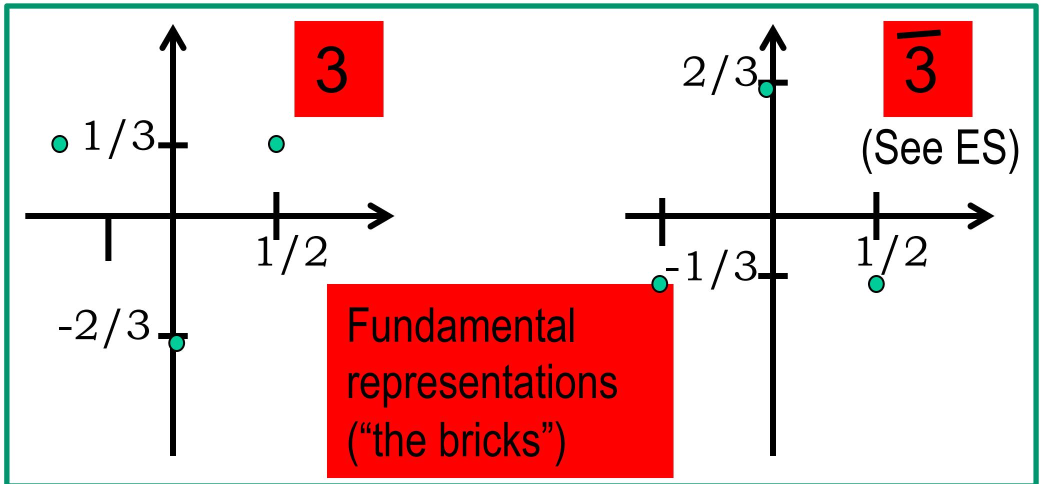
$U_{\pm}$ (Q→Q ; S→S±1)

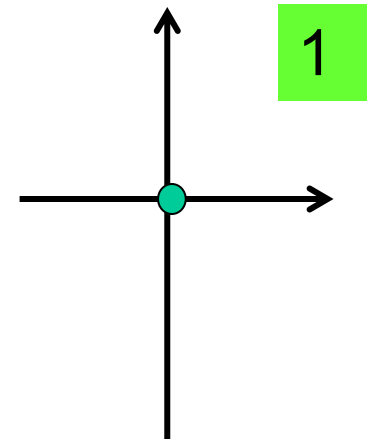
$V_{\pm}$ (Q→Q ±1 ; S→S±1)

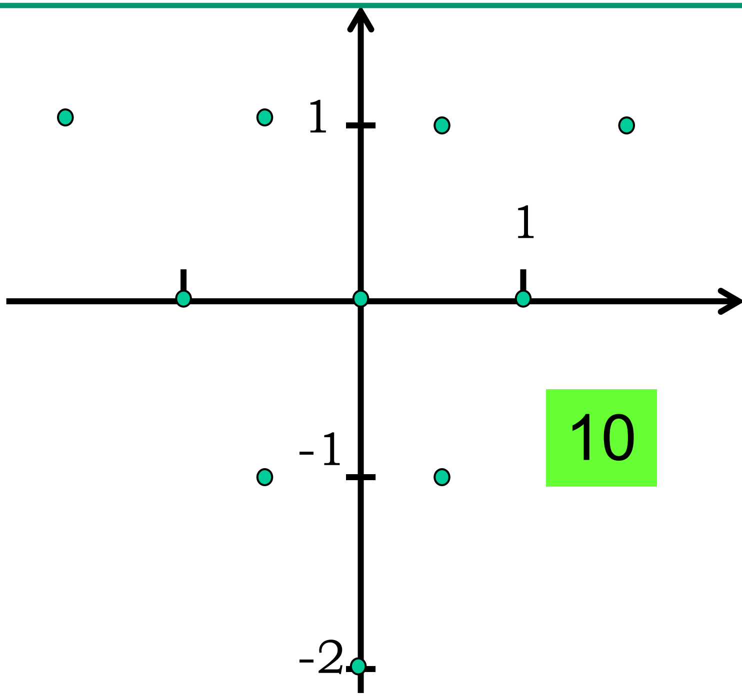
If applied on an element lying in the extremity of a multiplet to make a “step outside”, these operators yield 0, like $J_{\pm}$ of SU(2). They provide a way to construct the “allowed” multiplets.

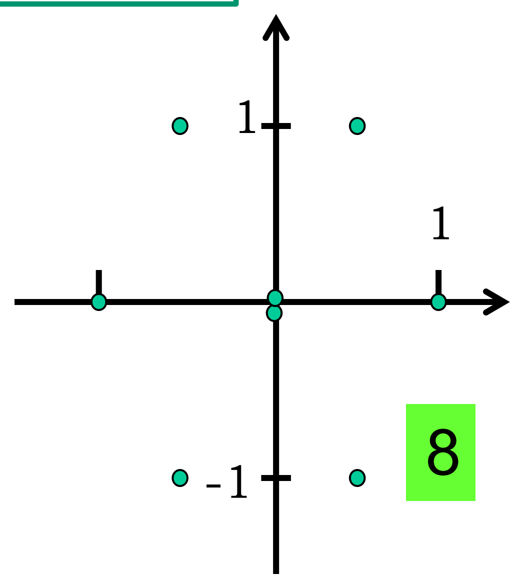
Multiplets (representations) of SU(3)

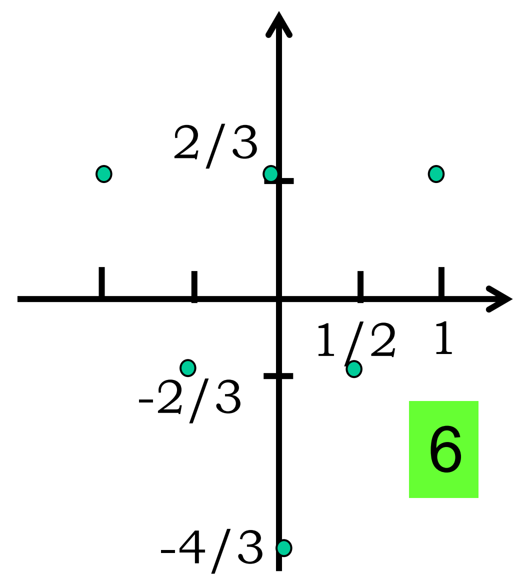
Like SU(2) multiplets, they are completely determined by the group's algebra: all the members of the same multiplet have the same eigenvalues of the Casimir operators (two numbers). This is understandable by the fact that the Casimir operators commute with the generators, and thus with the rising/lowering operators.







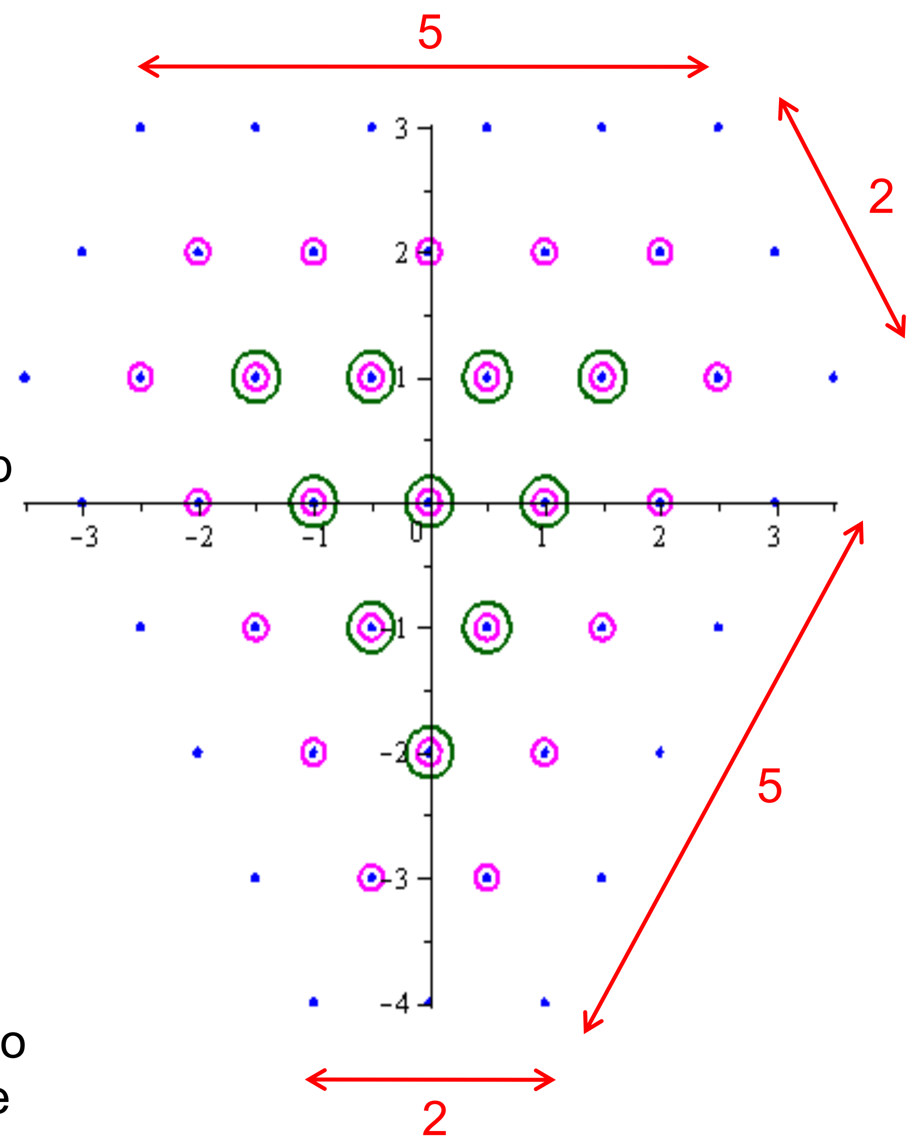




The construction of a general multiplet:

The example of D(5,2)

- The multiplet is characterized by two numbers (these are not the Casimir operators).
- Generally, the multiplet is a hexagon (a triangle is a hexagon with one side =0)
- The multiplicity is incremented in each step towards the center.
- Maximum multiplicity: number of members on the shorter side.



Using this notation:

3 = D(1,0) ; $\bar{3}$ = D(0,1) ; 8 = D(1,1) ; 10 = D(3,0)…

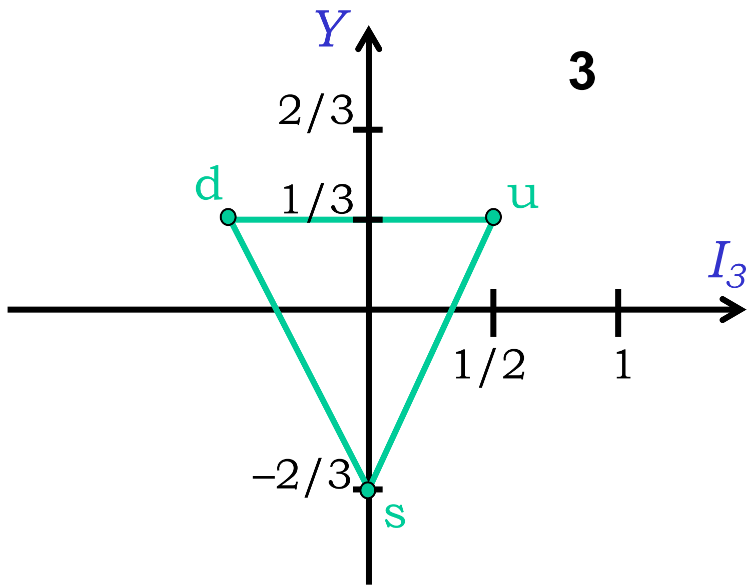
(The first number is the number of “3” used to construct the multiplet, and the second is the number of “$\bar{3}$”)

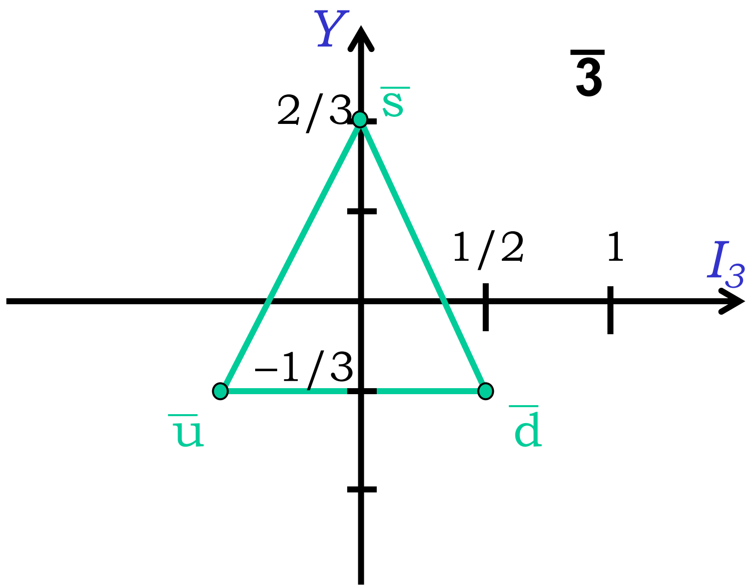
4. Flavor SU(3) and hadrons phenomenology

$|u\rangle, \ |d\rangle, \ |s\rangle$ — $\mathbf{3}$

$|\bar{u}\rangle, \ |\bar{d}\rangle, \ |\bar{s}\rangle$ — $\bar{\mathbf{3}}$

The fundamental representations





$I_3 = Q - Y/2 \quad ; \quad Y = \mathscr{B} + S$

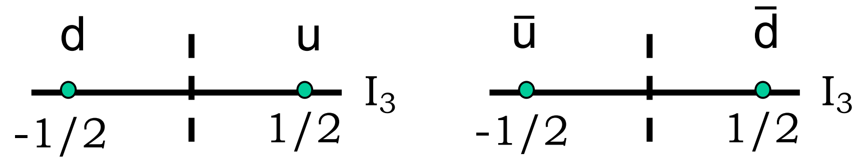
Electric charge:	Baryon number:	Isospin:	Strangeness:
Q(u) = 2/3	B(u,d,s) = 1/3	I_3(u,d) = ±1/2	S(u,d) = 0
Q(d,s) = –1/3		I_3(s) = 0	S(s) = –1

Construction of meson multiplets:

- Mesons are $q_1\bar{q}_2$ states.
- With only u and d quarks, combinations of SU(2): $2 \otimes \bar{2} = 1 \oplus 3$ (we only consider isospin)

Multiplet of quarks

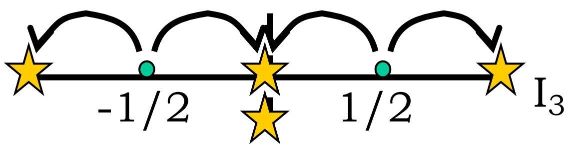
Multiplet of anti-quarks (see exercise)



$$I_3 = \frac{1}{2} \quad \begin{cases} (2) & |u\rangle \\ (\bar{2}) & -|\bar{d}\rangle \end{cases}$$

$$I_3 = -\frac{1}{2} \quad \begin{cases} (2) & |d\rangle \\ (\bar{2}) & |\bar{u}\rangle \end{cases}$$

- Graphically:



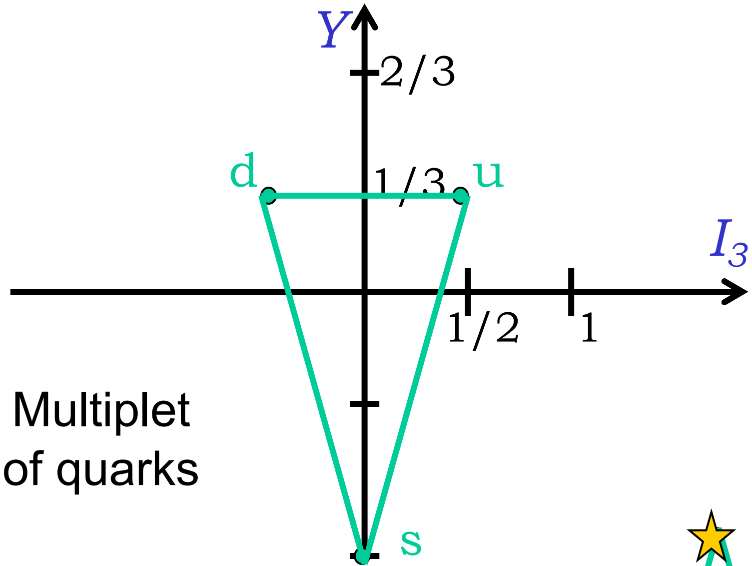
We obtain the singlet and the triplet

- Analytically (see exercise) we can obtain (pay attention to signs):

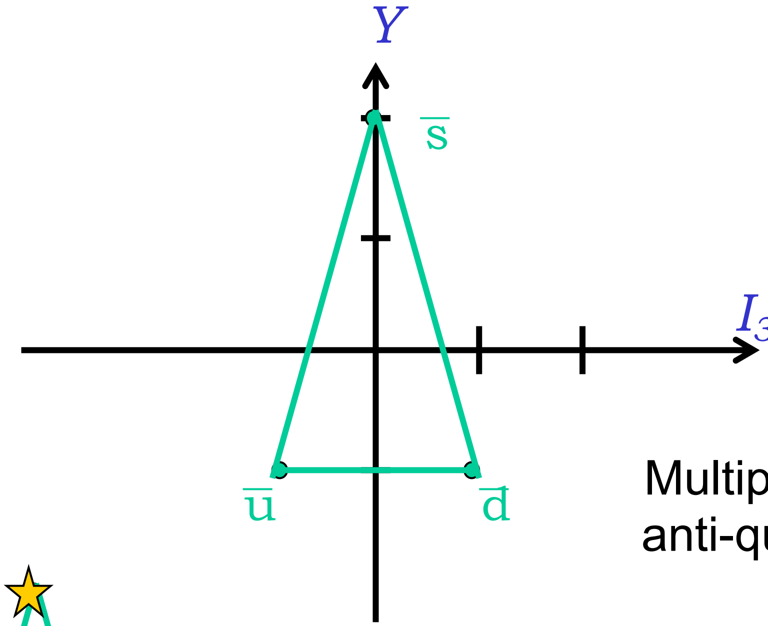
$$\begin{cases} |I=1, I_3=1\rangle = -u\bar{d} \\ |I=1, I_3=0\rangle = \sqrt{1/2}\left(u\bar{u} - d\bar{d}\right) \\ |I=1, I_3=-1\rangle = d\bar{u} \\ |I=0, I_3=0\rangle = \sqrt{1/2}\left(u\bar{u} + d\bar{d}\right) \end{cases}$$

(Same coefficient for the two members of the singlet)

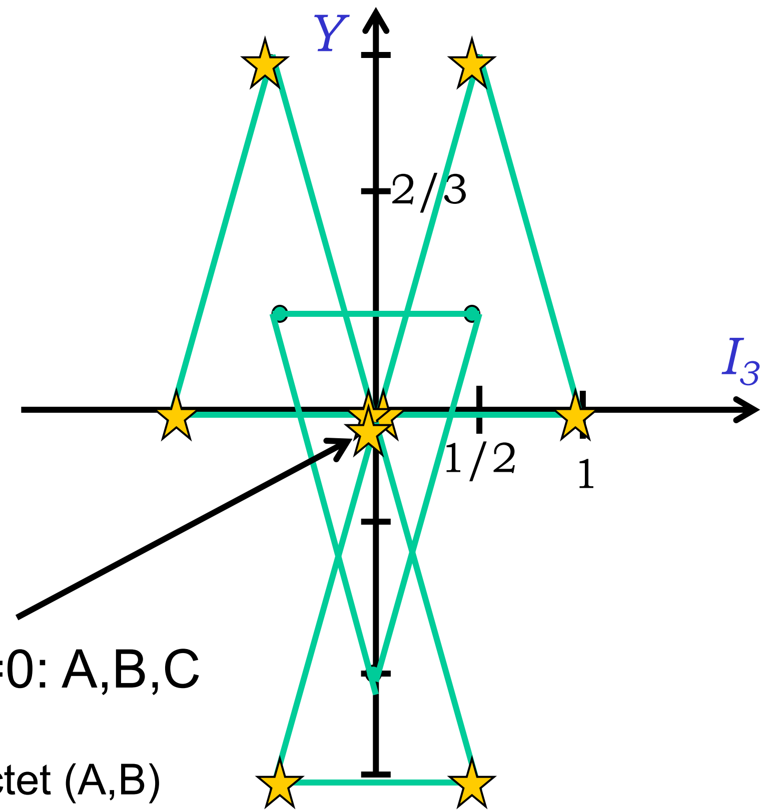
With u,d,s (triangles are equilateral)



Multiplet of quarks



Multiplet of anti-quarks



$\boxed{3 \otimes \bar{3}}$

=

$\boxed{1 \oplus 8}$

3 states I_3=0, Y=0: A,B,C

One singlet (C),

2 members of the octet (A,B)

The states I_3=0, Y=0 of $3\otimes\bar{3}$

- The states A, B and C are linear combinations of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$

- The singlet C of SU(3) must contain a combination with the same weights of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ (same as for isospin singlet: $|I=0, I_3=0\rangle = \sqrt{1/2}\left(u\bar{u}+d\bar{d}\right)$)

$$\eta_1 = C = \sqrt{\frac{1}{3}}\left(u\bar{u}+d\bar{d}+s\bar{s}\right)$$

- A is defined as a part of the isospin triplet ($d\bar{u}$, A, $-u\bar{d}$) :

$$\pi^0 = A = \sqrt{\frac{1}{2}}\left(u\bar{u}-d\bar{d}\right)$$

  It is a real particle because isospin-SU(2) is almost an exact symmetry

- A, B and C must be orthogonal with respect to each other (eigenstates of a hermitian operator are orthogonal and have real eigenvalues). From this condition, the isospin singlet B is:

$$\eta_8 = B = \sqrt{\frac{1}{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)$$

- As quarks have spin ½ , mesons with $\ell=0$ can have $J^P = 0^-$ or 1^-
- For the state with isospin I = 0
  - η,η' are linear combinations of η_1, η_8 that can mix because flavor-SU(3) is not an exact symmetry, and because they have the same quantum numbers (I=0, I_3=S=0).
  - Same for the physical states ω and ϕ, which are mixtures of ϕ_1 et ϕ_8 (identical argumentation for the $J^P = 1^-$ meson multiplet)

$$|\eta\rangle = |\eta_1\rangle \sin\chi - |\eta_8\rangle \cos\chi$$
$$|\eta'\rangle = |\eta_8\rangle \sin\chi + |\eta_1\rangle \cos\chi$$

χ is the mixing angle. It has to be measured experimentally, e.g. with $\eta\to\gamma\gamma$ (explain…)

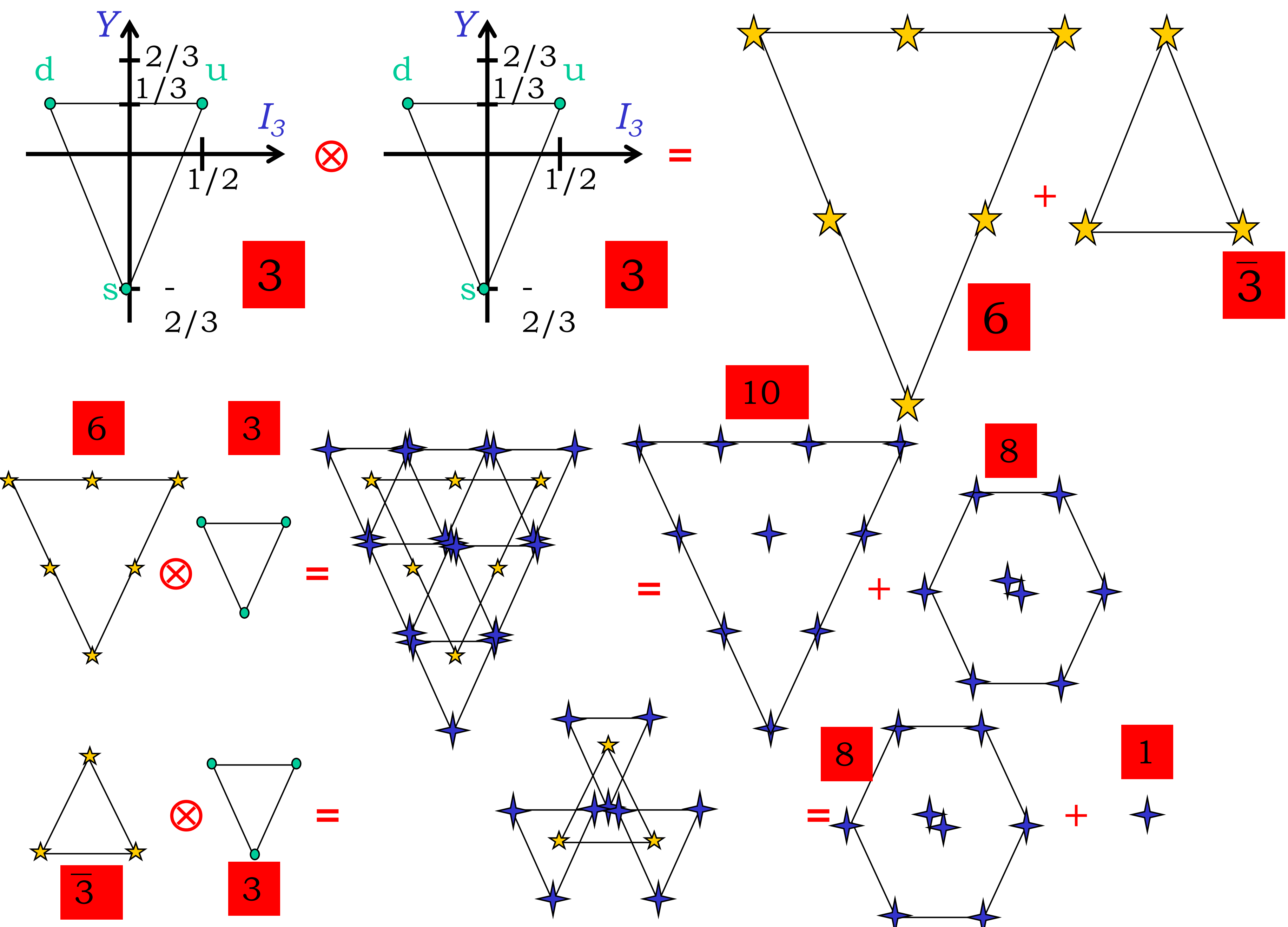
$$|\omega\rangle = \sqrt{\frac{1}{2}}\left(u\bar{u} + d\bar{d}\right)$$

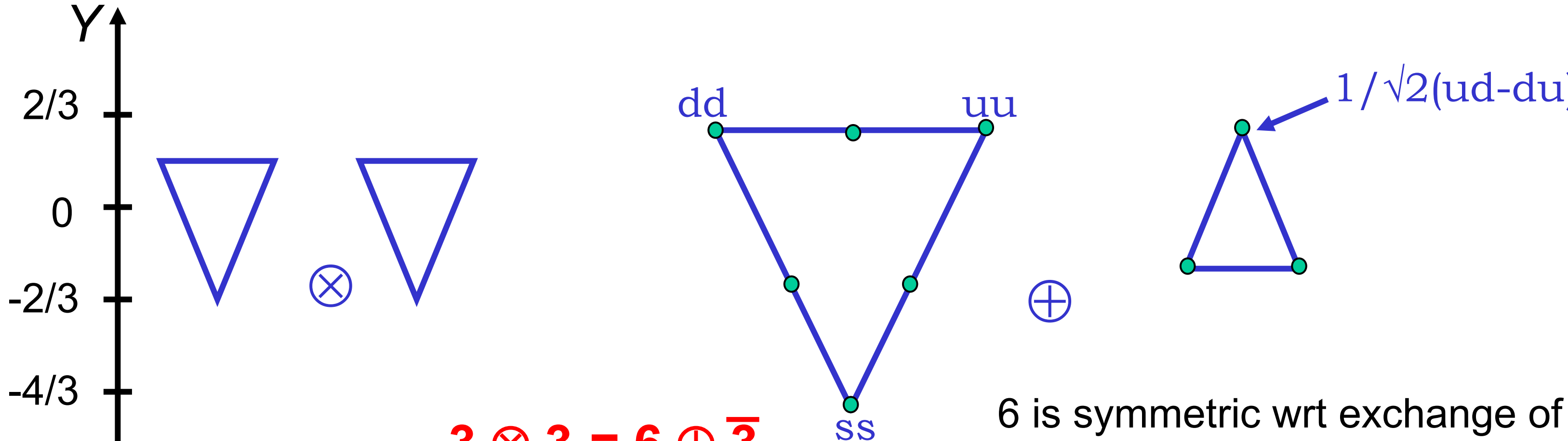
$$|\phi\rangle = s\bar{s}$$

Pure $s\bar{s}$ state

particular case with a nearly maximal mixing angle χ = 45°

Construction of baryon multiplets: 3⊗3⊗3



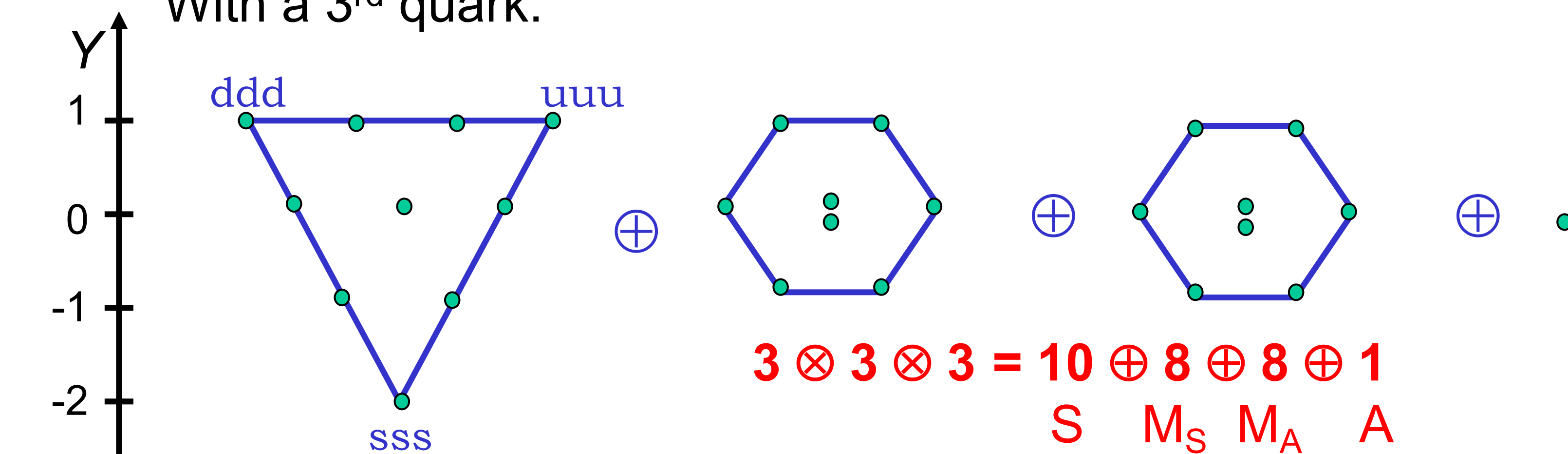


$\mathbf{3 \otimes 3 = 6 \oplus \bar{3}}$

6 is symmetric wrt exchange of 2 quarks

$\bar{3}$ is antisymmetric wrt exchange of 2 quarks

With a 3rd quark:



$\mathbf{3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1}$

$\mathrm{S} \quad \mathrm{M_S} \quad \mathrm{M_A} \quad \mathrm{A}$

M_S et M_A indicate the symmetry (S) or anti-symmetry (A) by exchange of the first 2 quarks

Spin:
$2 \otimes 2 \otimes 2$
$= 4 \oplus 2 \oplus 2$
$\mathrm{S} \quad \mathrm{M_S} \quad \mathrm{M_A}$

For info. only We can define categories wrt flavor-SU(3) and spin-SU(2). For example:

S: (10,4) + (8,2), M_S : (10,2) + (8,4) + (8,2) + (1,2), A: (1,4) + (8,2)

We find for baryons with $3/2^+$ S (10,4)), $1/2^+$ (S (8,2)), but things get complicated…

5. Probing the structure of the proton

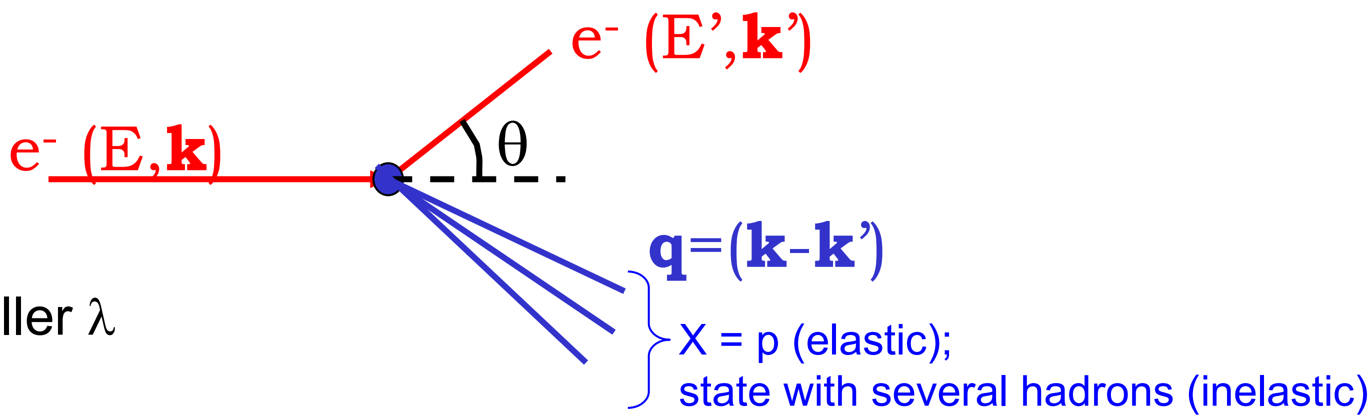
Scattering experiments

- The idea is similar to the Rutherford experiment:

  a pointlike projectile on an object that is supposed to have internal structure

  ⇒ use e^-p scattering, this time with larger energies:

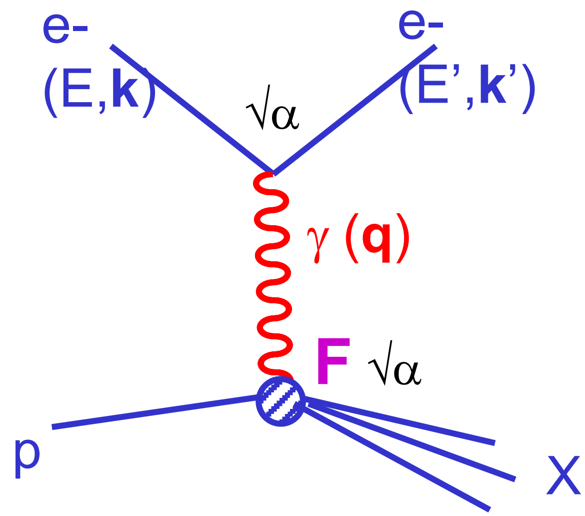
$$\lambda = \frac{2\pi\hbar}{|\mathbf{k}|}$$



Larger E ⇔ smaller λ

λ gives the order of de magnitude of the size of structures probed by the electron inside the proton.

- With a diagram:



$$\vec{q} = \vec{k} - \vec{k}' \qquad q^0 = E - E'$$

$$Q^2 = -q^2$$

In fact, the γ probes the proton

F: form factor. It is constant (=1) for a pointlike object.
This is what we want to measure…

- q^2 experimentally accessible, by measuring E', θ (quantities of the lepton!)

$$\boxed{\begin{aligned} -q^2 &= \left(\vec{k} - \vec{k}'\right)^2 - \left(E - E'\right)^2 = -2m^2 - 2kk'\cos\theta + 2EE' \\ &\approx 2EE'\left(1 - \cos\theta\right) = 4EE'\sin^2\left(\theta/2\right) \end{aligned}}$$

- Remark:

$$\lambda = \frac{2\pi\hbar}{|q|} \cong \frac{2\pi\hbar}{2\sqrt{EE'}\sin\left(\theta/2\right)}$$

Clearly, for a given *s*, λ decreases when θ increases.

➔ Large-angle scattering probes the small structures inside the proton.

Elastic e⁻p scattering

This is a purely EM process!

First Elastic e⁻ p scattering: McAllister and Hofstadter, using 188 MeV electrons on hydrogen target (SLAC, 1956), before the quark model [the plot here].

Interpretation was not easy without quarks.

Experiments with higher energies were performed in the late 1960 (next slide)

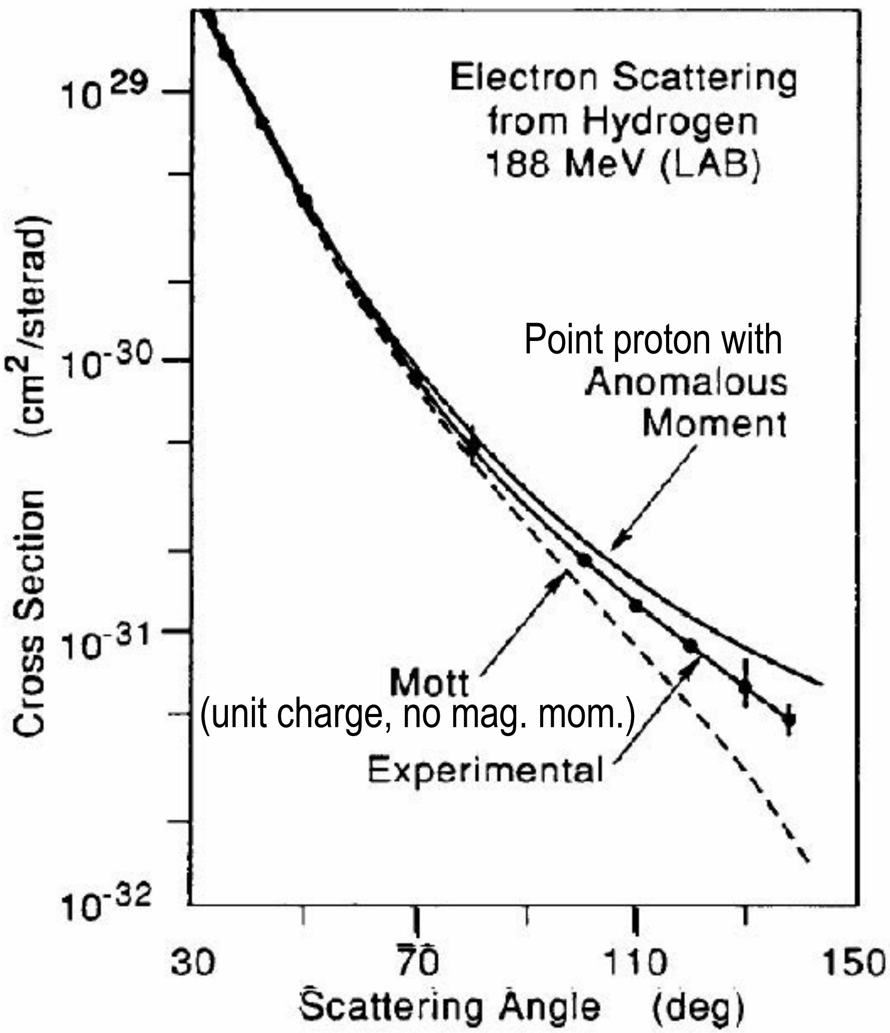


Fig. 5. Elastic electron scattering cross sections from hydrogen compared with the Mott scattering formula (electrons scattered from a particle with unit charge and no magnetic moment) and with the Rosenbluth cross section for a point proton with an anomalous magnetic moment. The data falls between the curves, showing that magnetic scattering is occurring but also indicating that the scattering is less than would be expected from a point proton.

A first look into Deep Inelastic Scattering (DIS), $e^- p \to e^- X$

Definitions:

$$Q^2 = -q^2$$

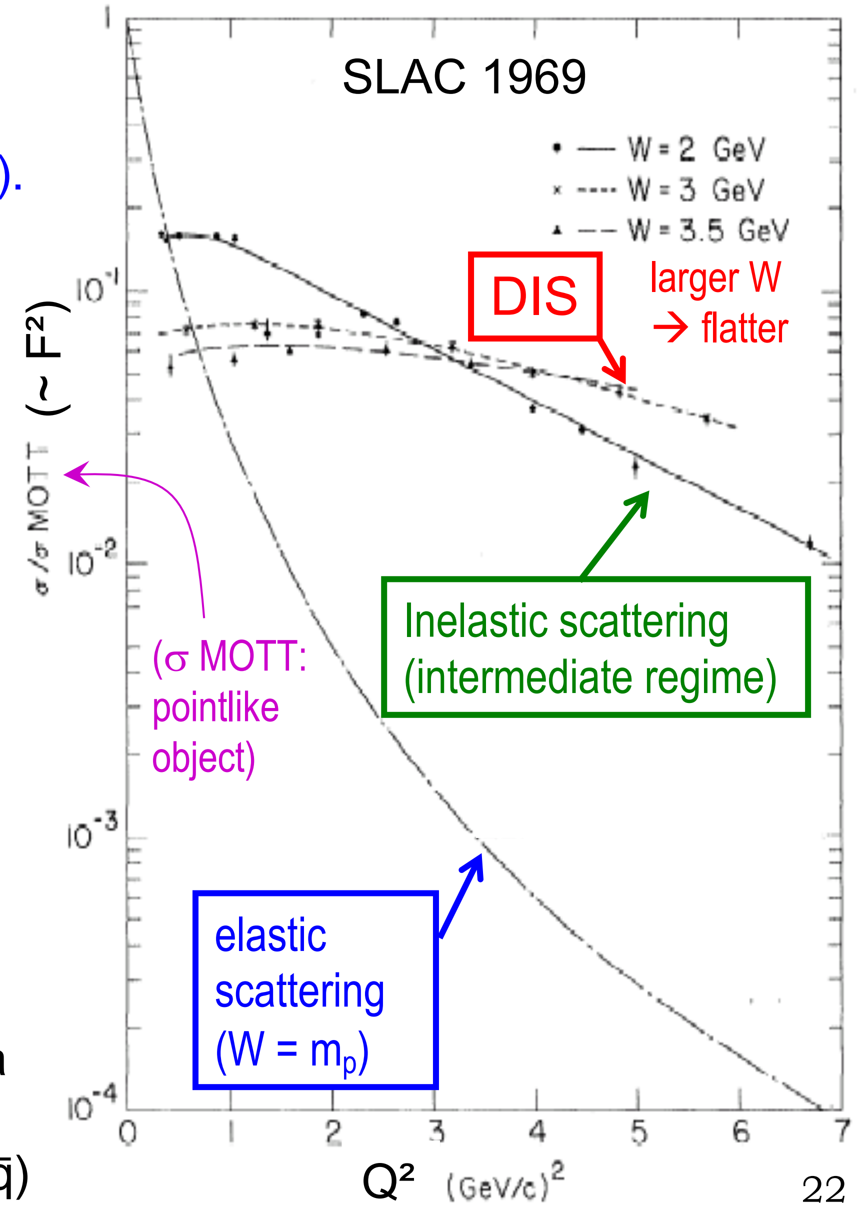
$$W^2 = M_X^2 = (p_p + q)^2 \quad \text{(mass² of the hadronic system)}$$

Elastic scattering: non-pointlike object ($F(Q^2)<1$).
➔ probability for coherent scattering with sub-structures strongly reduces with Q^2

DIS:
Spectacular behavior of F!

- **F ~ constant (Q^2)** $\Rightarrow$ collision with pointlike particles inside the proton that behave as if they were free!!!
- **F<1** $\Rightarrow$ the pointlike objects carry a fraction of the proton mass

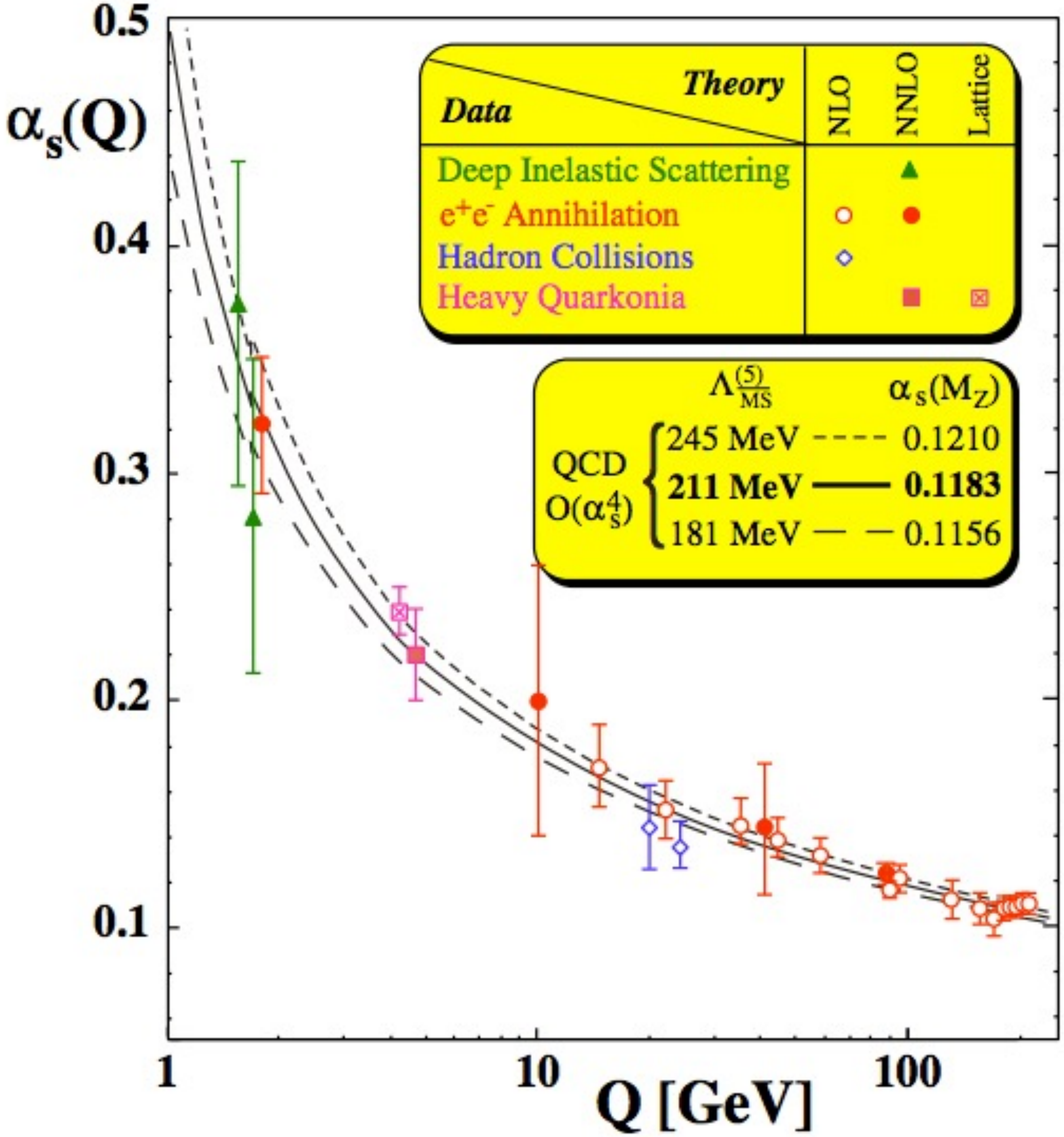
These pointlike objects, initially called *partons*, are quarks (and gluons…)

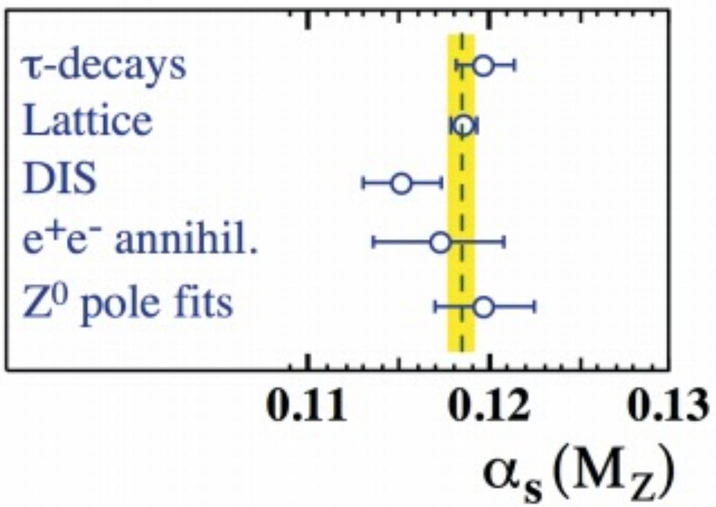


Remarks:

- Quarks have been shown to have spin ½ (in a few slides…)
- We have other proofs of quarks (e.g. $e^+e^- \to q\bar{q}$)

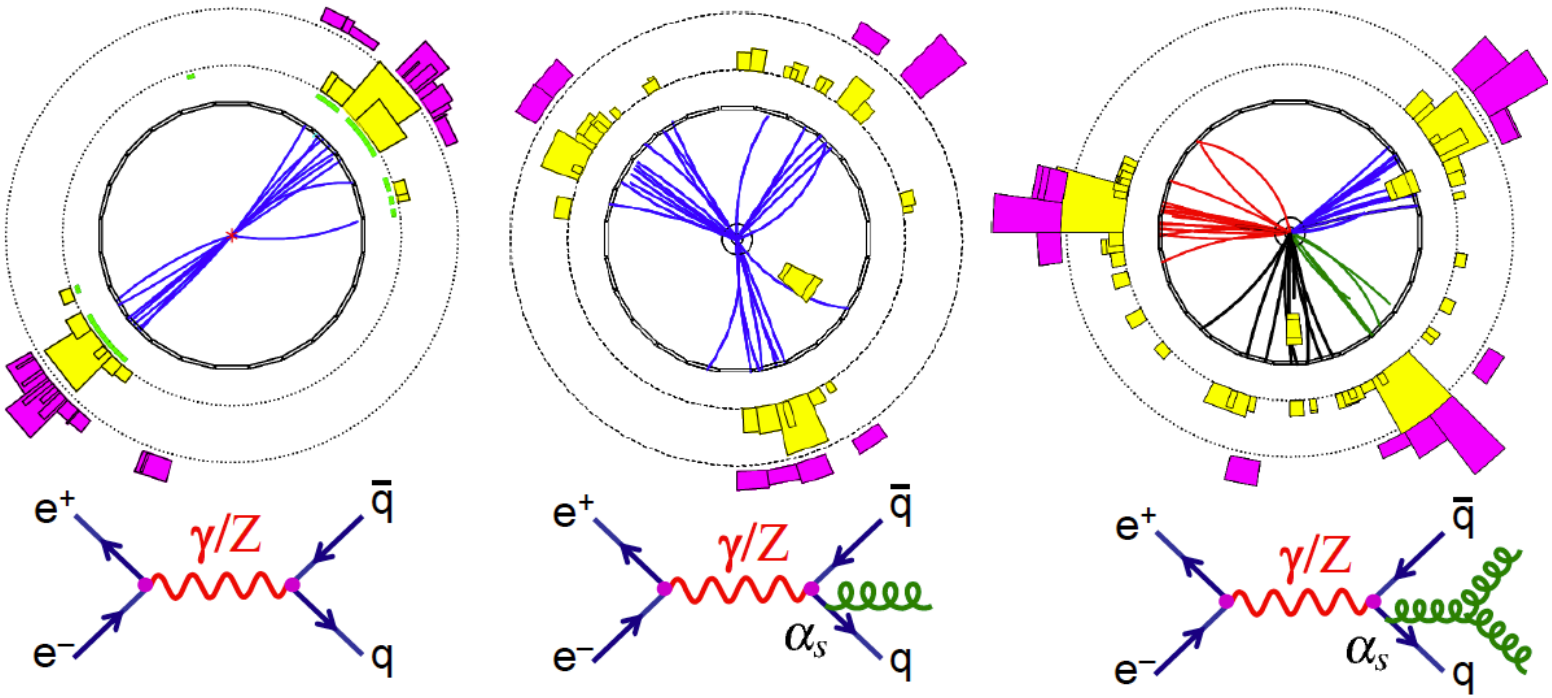
α_s running: experimental results



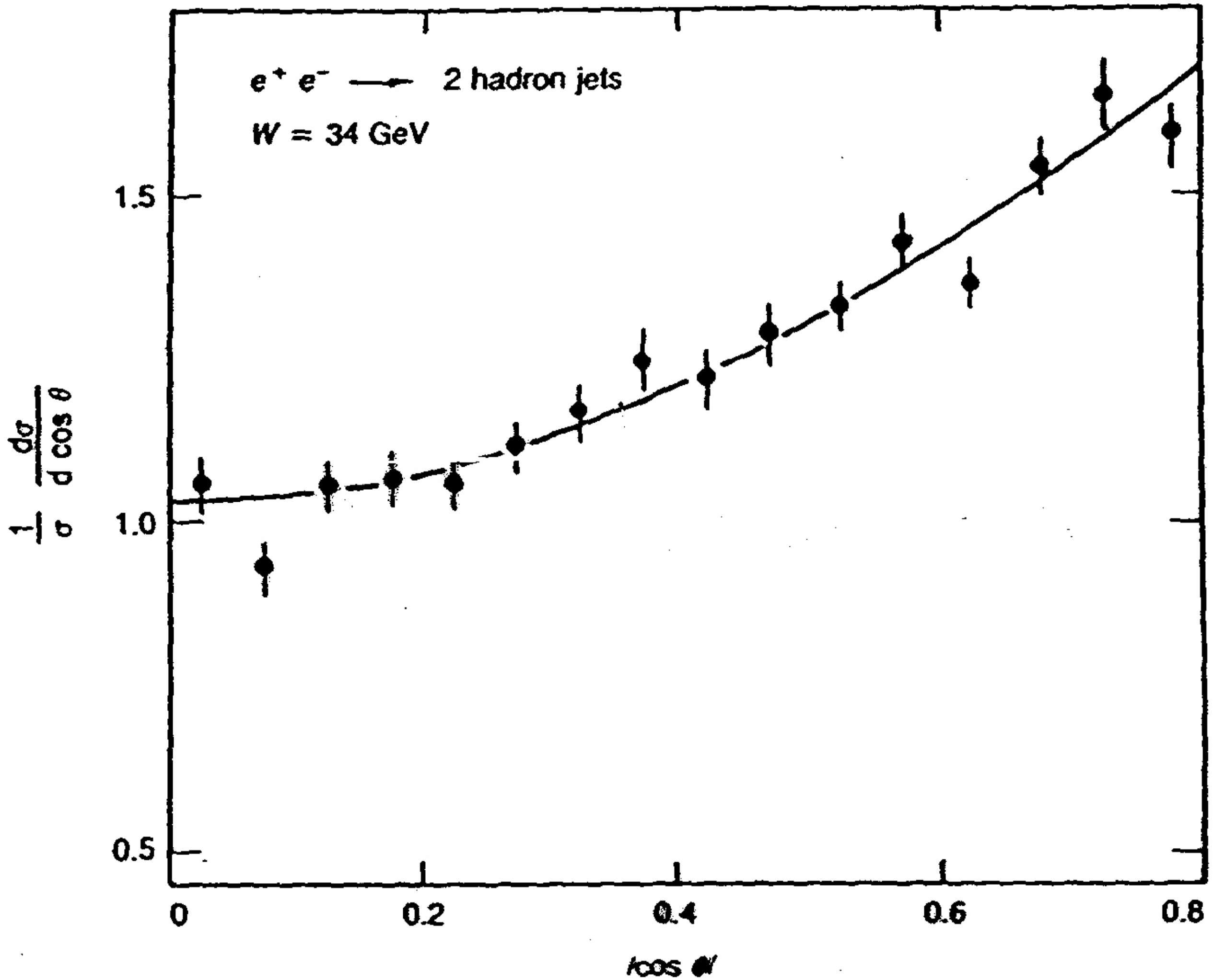


Event displays with jets

$e^+e^- \rightarrow q\bar{q} \rightarrow 2\text{jets}$ $e^+e^- \rightarrow q\bar{q}g \rightarrow 3\text{jets}$ $e^+e^- \rightarrow q\bar{q}gg \rightarrow 4\text{jets}$



Angular distributions of $e^+e^- \to q\bar{q}$



$$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}(\sqrt{s})$$