Corrections Midterm exam of Particle Physics (Parts I-II)
Tuesday November 15th 2022

Exercise I

Questions on the lectures

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. What was the motivation of Pauli for postulating the existence of neutrinos in the years 1930? Why are these particles difficult to detect? How and when were discovered the electron and muon neutrinos?

   Under the hypothesis that the final state of the nuclear beta process has two bodies (the recoil nucleus and the electron[1]) the momenta of the recoil nucleus and the electron are back-to-back, and their energies do not have any degree of freedom. They are determined by the masses of the initial and final particles. Thus, under this hypothesis, the energy spectrum of electrons should have been a "Dirac" peak. But experimental observations showed that the electron energy spectrum is a distribution, indicating either that energy is not conserved in the process (Bohr's hypothesis) or that there is at least one more final-state neutral particle, which for some reason is not detected. Pauli postulated the existence of such particle and named it neutrino ("the little neutral").

   Neutrinos couple to other particles only via the weak interaction, and thus the corresponding cross sections are very small. Detecting them requires a large number of neutrinos (intense beams) and very massive detectors, in order to obtain a large-enough interaction probability.

   The electron neutrinos were discovered by Reines and Cowan in 1956. They used a massive detector, near a nuclear plant (naturally producing a big amounts of neutrinos).

   Muon neutrinos were discovered in 1962 in the Brookhaven accelerator (not required in the answer: by Lederman, Schwartz, Jack Steinberger, 1988 Nobel prize, who used a pion beam to create a muon neutrinos beam, isolated by passing by a shielding wall that let only neutrinos through, and a massive spark chamber).

[1] positron in beta plus decays

2. What is the baryon number and how is it defined? How is the fact that it is conserved by the strong, electromagnetic and weak interactions related to the allowed coupling (vertexes) of these interactions? Relate the conservation of the baryon number to the fact that the proton is a stable particle.

   The Baryon number is an additive quantum number (or a charge), which has the values of $+1$ *for baryons,* -1 *for anti-baryons (*$1/3$ *for quarks and* $-1/3$ *for anti-quarks), and is 0 for mesons, leptons and gauge bosons. It was initially introduced to account for the fact that observed processes always had the same number of baryons (minus anti-baryons) in their initial and final states.*
   Its conservation by all interactions is related to the fact that quarks are always produced and annihilated by pairs (and that the baryon numbers of all the quarks are the same).
   The proton is the lightest baryon, and thus it cannot decay to any particle without violating baryon-number conservation. It is thus a stable particle, at least according to the standard model.

Exercice II

The BABAR experiment

The *BABAR* experiment was an electron-positron collider aimed at studying *CP* violation. The collider (PEP-II) properties are given in Tab. 1.

Table 1: Properties of the PEP-II collider (and naming conventions for the exercise)

Beam	Energy
Electron (oriented along $+\vec{z}$ axis and named A)	9 GeV
Positron (oriented along $-\vec{z}$ axis and named B)	3.1 GeV

We note throughout the exercise $\sqrt{s}$ the value of the total energy in the center of mass. For a given trivector $\vec{q}$, we note q_T its norm in the $(\vec{x}, \vec{y})$ plane, and θ the angle between $\vec{q}$ and the $\vec{z}$ axis. We add a * when these values are expressed in the center of mass.

1. *Kinematics of the reaction* $e^+e^- \to \overline{D}^0 D^{*0}$.

   We note m_0 and m_0^* the mass of $\overline{D}^0$ and D^{*0} particles (for numerical computations use $m_0 = 1864.8$ MeV and $m_0^* = 2006.9$ MeV).

   We note $k^* \equiv (E^*, \vec{k}^*)$ the four-vector of the $\overline{D}^0$ particle in the center-of-mass frame, $k \equiv (E, \vec{k})$ its corresponding four-vector in the lab. frame, and $p^{\mathrm{cm}} \equiv (E^{\mathrm{cm}}, \vec{p}^{\,\mathrm{cm}})$ the four-vector of the center of mass (whole initial state) in the lab. frame.

   (a) Since we have $p_A = (9, 0, 0, 9)$ and $p_B = (3.1, 0, 0, -3.1)$ (we assume electrons and positrons to be massless):

$$\begin{aligned} s &= (p^{\mathrm{cm}})^2 = (p_A + p_B)^2 \\ &= p_A^2 + p_B^2 + 2\, p_A.p_B \\ &= 0 + 0 + 2(E_A.E_B - \vec{p}_A \vec{p}_B) \\ &= 2 \times (9 \times 3.1 + 9 \times 3.1) \\ \sqrt{s} &= 10.5641 \text{ GeV} \end{aligned} \tag{1}$$

   The electron and positron masses are indeed negligible with respect to 10 GeV.

   (b) We note $p^{\mathrm{cm}*}$ the 4-vector of the center-of-mass in the center-of-mass. Therefore, by construction, $p^{\mathrm{cm}*} = (\sqrt{s}, 0, 0, 0)$. The Lorentz-boost transformation is given by:

$$\Lambda^{\mathrm{cm}} = \begin{pmatrix} \gamma^{\mathrm{cm}} & 0 & 0 & +\gamma^{\mathrm{cm}}\beta^{\mathrm{cm}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma^{\mathrm{cm}}\beta^{\mathrm{cm}} & 0 & 0 & \gamma^{\mathrm{cm}} \end{pmatrix} \tag{2}$$

   We can verify the sign in front of the $\gamma^{\mathrm{cm}}\beta^{\mathrm{cm}}$ factor by boosting the center-of-mass 4-vector in the lab:

$$p^{\mathrm{cm}} = \Lambda^{\mathrm{cm}} p^{\mathrm{cm}*} = (\gamma^{\mathrm{cm}}\sqrt{s}, 0, 0, \gamma^{\mathrm{cm}}\,\beta^{\mathrm{cm}}\,\sqrt{s}).$$

   Since $\beta^{\mathrm{cm}} > 0$, $p_z^{\mathrm{cm}} = \gamma^{\mathrm{cm}}\,\beta^{\mathrm{cm}}\,\sqrt{s} > 0$ which is indeed what we want since $p_z^{\mathrm{cm}} = p_{A_z} + p_{B_z} > 0$. Thus, we get that:

$$E^{\mathrm{cm}} = 9 + 3.1 \text{ GeV} = \gamma^{\mathrm{cm}} \times \sqrt{s} \tag{3}$$

$$p_z^{\mathrm{cm}} = 9 - 3.1 \text{ GeV} = \gamma^{\mathrm{cm}}\beta^{\mathrm{cm}} \times \sqrt{s} \tag{4}$$

(5)

and as a consequence

$$\gamma^{\rm cm} = \frac{9+3.1}{10.56} = 1.1454 \quad (6)$$

$$\beta^{\rm cm} = \frac{9-3.1}{9+3.1} = 0.4876 \quad (7)$$

$$(8)$$

(c) This computation was done in the course. Quick reminder:

$$\begin{aligned}
\sqrt{s} &= \sqrt{|\vec{k}^*|^2 + m_0^2} + \sqrt{|\vec{k}^*|^2 + m_0^{*2}} \\
|\vec{k}^*|^2 + m_0^2 &= s + |\vec{k}^*|^2 + m_0^{*2} - 2 * \sqrt{|\vec{k}^*|^2 + m_0^{*2}}\sqrt{s} \\
|\vec{k}^*|^2 + m_0^{*2} &= \frac{1}{4s}\left(s^2 + m_0^4 + m_0^{*4} - 2sm_0^2 + 2sm_0^{*2} - 2m_0^2 m_0^{*2}\right) \\
|\vec{k}^*| &= \frac{1}{2\sqrt{s}}\sqrt{s^2 + m_0^4 + m_0^{*4} - 2s\,m_0^2 - 2s\,m_0^{*2} - 2m_0^2 m_0^{*2}}
\end{aligned} \quad (9)$$

(d) Compute the numerical values of $|\vec{k}^*|$ and E^* in GeV. Numerically we find that:

$$\begin{aligned}
|\vec{k}^*| &= 4.9141 \text{ GeV} \\
E^* &= \sqrt{m_0^2 + |\vec{k}^*|^2} = 5.2560 \text{ GeV}
\end{aligned} \quad (10)$$

(e) In the c-o-m, the 4-vector of the D^0 particle is by definition

$$k^* \equiv (E^*, |\vec{k}^*|\sin\theta^*\cos\varphi^*, |\vec{k}^*|\sin\theta^*\sin\varphi^*, |\vec{k}^*|\cos\theta^*)$$

Thus, in the lab. $k = \Lambda^{\rm cm} k^*$:

$$\begin{pmatrix} E \\ k_T\cos\varphi \\ k_T\sin\varphi \\ k_z \end{pmatrix} = \begin{pmatrix} \gamma^{\rm cm} & 0 & 0 & -\gamma^{\rm cm}\beta^{\rm cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma^{\rm cm}\beta^{\rm cm} & 0 & 0 & \gamma^{\rm cm} \end{pmatrix} \times \begin{pmatrix} E^* \\ |\vec{k}^*|\sin\theta^*\cos\varphi^* \\ |\vec{k}^*|\sin\theta^*\sin\varphi^* \\ |\vec{k}^*|\cos\theta^* \end{pmatrix} \quad (11)$$

Therefore we get that:

$$\begin{aligned}
E &= \gamma^{\rm cm}(E^* + \beta^{\rm cm}|\vec{k}^*|\cos\theta^*) \\
k_T &= \sqrt{k_x^2 + k_y^2} = |\vec{k}^*|\sin\theta^* \\
k_z &= \gamma^{\rm cm}(\beta^{\rm cm}E^* + |\vec{k}^*|\cos\theta^*)
\end{aligned} \quad (12)$$

(f) From the previous result we get that:

$$\begin{aligned}
\tan\theta &\equiv \frac{k_T}{k_z} = \frac{|\vec{k}^*|\sin\theta^*}{\gamma^{\rm cm}\ (\beta^{\rm cm}E^* + |\vec{k}^*|\cos\theta^*)} \\
\tan\theta &= \frac{\frac{|\vec{k}^*|}{E}\sin\theta^*}{\gamma^{\rm cm}\ (\beta^{\rm cm} + \cos\theta^*\frac{|\vec{k}^*|}{E})}
\end{aligned} \quad (13)$$

By taking the arctan of the previous equation, we find that

$$\theta \equiv \arctan(\tan\theta) = 0.4656 \text{ rad} = 26.7\ ^\circ$$

2. *QED production $e^+e^- \to F\bar{F}$.*

We remind you that the total LO QED cross-section (at tree level) is given by:

$$\sigma_{tot}\left(e^+e^- \to \mu^-\mu^+\right) = \frac{4\pi}{3}\frac{\alpha^2}{s}\sqrt{1-\frac{4\,m_f^2}{s}}\left(1+\frac{1}{2}\frac{4\,m_f^2}{s}\right) \tag{14}$$

(a) See the course for the s diagram and the corresponding matrix element. The fermions that are allowed in this reaction are f in $[\,\mu^-, \tau_\mu, u, d, s, c, b\,]$. The $t\bar{t}$ pairs can not be produced because the c-o-m energy is below the threshold.

(b) The $\Upsilon(4S)$ is a $b\bar{b}$ resonance. Its mass is around 10.58 GeV and its width is $\Gamma_{\Upsilon(4S)} \approx$ 20.5 MeVand its main decays are $\Upsilon(4S)$ what are the mass and width of the resonance $\Upsilon(4S) \to B^+B^-$ and $\Upsilon(4S) \to B^0\overline{B}^0$

(c) $\sqrt{s} = 10.56$ GeV

(d) Despite the fact that $\sqrt{s} < M_{\Upsilon(4S)}$ we have $\sqrt{s} > M_{\Upsilon(4S)} - \Gamma_{\Upsilon(4S)}$. Thus, the resonance can be produced in the BABARexperiment. As usual, when a resonance is produced this will boost the cross-section around the resonance mass compared to the tree-level expectation from QED given in Eq. 15?

(e) For $2\,m_f \ll \sqrt{s}$, Eq. 15 can be simplified as:

$$\sigma_{tot}\left(e^+e^- \to f\bar{f}\right) = \frac{4\pi}{3}\frac{\alpha^2}{s} \tag{15}$$

This approximation is justified for muons $m_\mu = 0.1$ GeV $\ll 10.56$ GeV. Since the $\Upsilon(4S)$ resonance does not decay to $\mu^+\mu^-$ we do not expect this resonance to contribute to the cross section and we can predict the result from LO QED formula. Since $\sigma(e^+e^- \to \mu^+\mu^-)s$ is in GeV, we need to convert it a to an area using $\hbar c$. This gives:

$$\begin{aligned}
\sigma(e^+e^- \to \mu^+\mu^-) &\approx \frac{4\pi}{3}\frac{(1/137)^2}{10.56^2}(\hbar c)^2 \\
\sigma(e^+e^- \to \mu^+\mu^-) &\approx \frac{4\pi}{3}\frac{(1/137)^2}{10.56^2}\,(200\ 10^{-3}\ 10^{-15})^2\ m^2 \\
\sigma(e^+e^- \to \mu^+\mu^-) &\approx \frac{4\pi}{3}\frac{(1/137)^2}{10.56^2}\,(200\ 10^{-3}\ 10^{-15})^2 10^{28}\ \text{b} \\
\sigma(e^+e^- \to \mu^+\mu^-) &\approx \frac{4\pi}{3}\frac{(1/137)^2}{10.56^2}\,4\ 10^{-32}\ 10^{28}\ \text{b} \\
\sigma(e^+e^- \to \mu^+\mu^-) &\approx 7.7\ 10^{-6} 10^{-32} 10^{28}\ \text{b} \\
\sigma(e^+e^- \to \mu^+\mu^-) &\approx 7.7\ 10^{-10}\ \text{b} \\
\sigma(e^+e^- \to \mu^+\mu^-) &\approx 0.77\ \text{nb}
\end{aligned} \tag{16}$$

Therefore the order of magnitude from QED LO only is correct. The difference with the total cross section (1.15 nb) is due to NLO corrections. Note: in this case, this is mostly due to QED corrections with the emission of additional photons in the intial/final state.

(f) m_u and m_d are a few mev at most so negligible w/r to $\sqrt{s}$. Neglecting QCD effect we can predict from the QED approximated formula (the quark charge intervene only in the final vertex):

$$\begin{aligned}
\sigma(u\bar{u}) &= Q_u^2 N_c \sigma(\mu^+\mu^-) = 4/3\sigma(\mu^+\mu^-) = 1.53\ \text{nb} \\
\sigma(d\bar{d}) &= Q_d^2 N_c \sigma(\mu^+\mu^-) = 1/3\sigma(\mu^+\mu^-) = 0.38\ \text{nb}
\end{aligned} \tag{17}$$

where Q_q is the charge of quark q and $N_c = 3$ is the number of colors. This predictions are decently matching the cross section from the table. The differences coming essentially from QCD NLO effect.

(g) For the b quark, $m_b \approx 4.5$ GeV can not be neglected. Thus, at LO QED we expect

$$\frac{\sigma(e^+e^- \to b\bar{b})}{\sigma(e^+e^- \to d\bar{d})} = \sqrt{1 - \frac{4\,m_b^2}{s}}\left(1 + \frac{1}{2}\frac{4\,m_b^2}{s}\right)$$

where the last part is due to m_b mass effect in the phase space and matrix element. Using $m_b = 4.5$ GeV, we expect:

- b mass in phase space only: $\sqrt{1 - \frac{4\,m_b^2}{s}} = 0.52$
- b mass in matrix element only: $\left(1 + \frac{1}{2}\frac{4\,m_F^2}{s}\right) = 1.36$
- we therefore would expect

$$\frac{\sigma(e^+e^- \to b\bar{b})}{\sigma(e^+e^- \to d\bar{d})} = 0.71$$

(h) The actual ratio is $\frac{\sigma(e^+e^- \to b\bar{b})}{\sigma(e^+e^- \to d\bar{d})} = 2.775$ much larger than our QED prediction of 0.71. The difference is due to the production of the $\Upsilon(4S)$ resonance which is a NLO effect in the photon propagator. Since the $\Upsilon(4S)$ decays exclusively to $b\bar{b}$ pairs, this enhances only the $b\bar{b}$ production.

3. *Available statistics.*

(a) The instantaneous luminosity of the machine was $\mathcal{L} \approx 10 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}$. So for a month:

$$\begin{aligned}
L &= \mathcal{L} \times 20 \times 24 \times 3600 \\
L &= 4.8 \times 3.6 \times 10^{34} 10^{5}\text{cm}^{-2}/\text{month} \\
L &= 17.28\ 10^{39} 10^{-24}\ \text{b}^{-1}/\text{month} \\
L &= 17.28\ 10^{39} 10^{-24} 10^{-15}\ \text{fb}^{-1}/\text{month} \\
L &\approx 17.3\ \text{fb}^{-1}/\text{month}
\end{aligned} \tag{18}$$

(b)

$$N_{b\bar{b}} = \sigma(b\bar{b}) \times L_{tot} = 1.11\ 10^6\text{fb} \times 433\text{fb}^{-1} = 480\ 10^6 \tag{19}$$

Plugging the branching fractions:

$$\begin{aligned}
N_{B^+B^-} &== \sigma(b\bar{b}) \times L_{tot}\ \mathcal{B}(\Upsilon(4S) \to B^+B^-) = 480 \times 0.514 \times 10^6 = 246.7\ 10^6 \\
N_{B^0\overline{B}^0} &== \sigma(b\bar{b}) \times L_{tot}\ \mathcal{B}(\Upsilon(4S) \to B^0\overline{B}^0) = 480 \times 0.486 \times 10^6 = 233.3\ 10^6
\end{aligned} \tag{20}$$