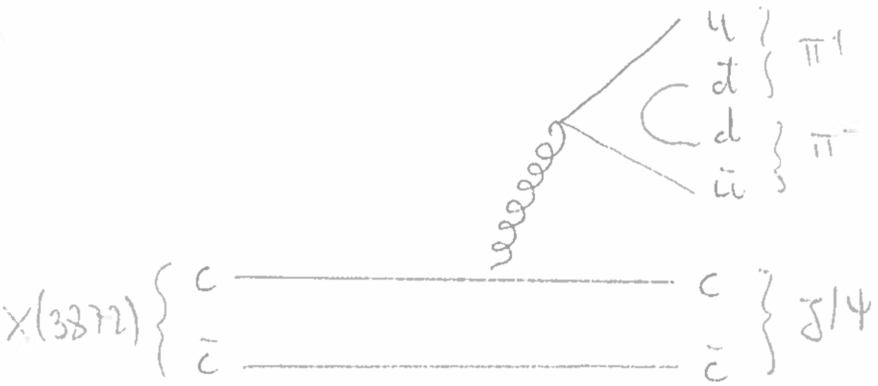
Exercise IV

1. The figure shows the spectrum of
$M(J/\psi\,\pi^+\pi^-) - M(J/\psi)$,
where $M(J/\psi\,\pi^+\pi^-)$ is the invariant mass of 3 final-state particles: J/ψ, π^+ and π^-, and $M(J/\psi)$ is the mass of the J/ψ meson.
The spectrum shows two peaks, corresponding to two resonances decaying to the $J/\psi\,\pi^+\pi^-$ final state. A zoomed view of the peaks is shown in the insets. The most pronounced peak, at $M(J/\psi\,\pi^+\pi^-) \simeq 3690$ MeV corresponds to the well-known $\psi(2S)$ $c\bar{c}$ meson while the second peak, at $\underline{M(J/\psi\,\pi^+\pi^-) \simeq 3875 \text{ MeV}}$, with roughly 17 times less events (according to the maxima of the two peaks) corresponds to the exotic hadron $X(3872)$.
The signal and background components in the fit to data are shown as the red and green dashed lines, respectively.

The FWHM of the $\psi(2S)$ peak is $\simeq 10$ MeV while according to the PDG its width is $\Gamma(\psi(2S)) \simeq 300$ keV. It is concluded that the width of the peak is mainly due to the experimental resolution.
The $X(3872)$ peak is very slightly larger. A reasonable estimate is
$\Gamma(X(3872)) \sim 1$ MeV.

2. As the SI conserves all the charges including flavors, the charges of the $X(3872)$ can simply be determined from those of the $J/\psi\,\pi^+\pi^-$ final state.
$Q(X) = Q(J/\psi) + Q(\pi^+) + Q(\pi^-) = 0 + 1 - 1 = 0$
In the same way
$\mathcal{B}(X) = 0$
$S(X) = C(X) = B(X) = 0$
$L_\ell(X) = 0$.

3.



4. $I(c) = I(\bar{c}) = 0$

$\Downarrow$

$I(X(3872)) = 0$

Furthermore:

$I(J/\psi) = 0 \Rightarrow \vec{I}(J/\psi\, \pi_1 \pi_2) = \vec{I}(\pi_1) + \vec{I}(\pi_2) \equiv \vec{I}_f$

with $I_f = 0, 1$ or 2.

$|i\rangle = |0,0\rangle$

with $\pi^+\pi^-$:

$|f\rangle_{\pi^+\pi^-} = |1\ 1\rangle \otimes |1\ -1\rangle = \frac{1}{\sqrt{6}}|2\ 0\rangle + \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle$

with $\pi^0\pi^0$:

$|f\rangle_{\pi^0\pi^0} = |1\ 0\rangle \otimes |1\ 0\rangle = \sqrt{\frac{2}{3}}\,|2\ 0\rangle - \frac{1}{\sqrt{3}}\,|0,0\rangle$

$\Downarrow$

$$\frac{\Gamma(X(3872) \to J/\psi\ \pi^+\pi^-)}{\Gamma(X(3872) \to J/\psi\ \pi^+\pi^-)} = \frac{|\langle 0 0|H_F|f\rangle_{\pi^+\pi^-}|^2}{|\langle 0 0|H_F|f\rangle_{\pi^0\pi^0}|^2} =$$

$$= \frac{\frac{1}{3} f_0}{\frac{1}{3} f_0} = 1$$

The Isospin is conserved as the decay occurs by SI. The matrix element of H_F only connects the same $|I, I_3\rangle$ states, (and does not depend on I_3). The computation above follows.

5. $X(3872) \to J/\psi\,\gamma$

Here the decay occurs by EM interaction, as nothing forbids it and as a γ is radiated. C conservation applies.

$$C_i = C_f = C_{J/\psi} \cdot C_\gamma = (-1)(-1) = +1$$

$$\Downarrow \quad C_X = +1$$

6. In the SI process $X(3872) \to J/\psi\,\pi^+\pi^-$ C is also conserved.
The C eigenstate $\pi^+\pi^-$ replaces the γ in (5.) above.

$$\Downarrow \quad C_1 \equiv C(\pi^+\pi^-) = C_\gamma = -1$$

7. The decay $R \to \pi^+\pi^-$ (SI) conserves C.
Thus $C_R = C_1 = -1$
This is verified for the $\rho(770)$ and $\omega(782)$, but not for other low mass resonances (η, $f_0(500)$, η', $f_0(980)$, $a_0(980)$).
Resonances with larger masses cannot contribute as

$$m_R \lesssim m_X - m_{J/\psi} \simeq 775$$

(even taking into account their width).

$\Downarrow$ Only $\rho(770)$ and $\omega(782)$ contribute.

Furthermore:
$BR(\rho \to \pi^+\pi^-) \simeq 100\%$
while
$BR(\omega \to \pi^+\pi^-) \sim 1\%$ (contribution from ω is small)

8. The $\pi^+\pi^-$ system is a particle - antiparticle system. Therefore:

$C_1 = P_1 \cdot E_{spin}$

$\hookrightarrow = 1$ as $J(\pi) = 0$

$\Downarrow$

$C_1 = P_1 = -1$

$P_1 = [P(\pi)]^2 \times (-1)^{l_1} = (-1)^{l_1} = -1$

$\hookrightarrow$ l_1 is odd

$\vec{J}_1 = \underbrace{\vec{J}_{\pi^+} + \vec{J}_{\pi^-}}_{0} + L_1$

$J_1 = l_1$

9. Parity is conserved by SI

$\Downarrow$

$P_X = \underbrace{P_{J/\psi}}_{-1} \cdot \underbrace{P_1}_{-1} \cdot (-1)^{l_2} = (-1)^{l_2}$

10. The phase space of the decay is small, and thus also the momenta of the final-state particles. It follows that l_2, which can only take integer values, will be most probably the smallest allowed integer.

We obtained

$P_X = (-1)^{l_2}$

$P_X = +1 \Rightarrow l_2$ is even

$\Rightarrow l_2 = 0$

11. Parity conservation in the SI process:

$$\underbrace{P_X}_{+1} = \underbrace{P_{J/\psi}}_{-1} \times P_1 \times \underbrace{(-1)^{l_2}}_{+1}$$

$\Rightarrow P_1 = -1$ as found in question 8 using other arguments

12. $\vec{J}_X = \vec{J}_1 + \vec{J}_{J/\psi} + \underbrace{\vec{L}_2}_{0} = \vec{J}_1 + \underbrace{\vec{J}_{J/\psi}}_{1}$

$J_X = 1 \Rightarrow |\vec{J}_1 + \underbrace{\vec{J}_{J/\psi}}_{1}| = 1$

$\Rightarrow J_1 = 0, 1$ or 2

As $J_1 = l_1$ must be odd, only $J_1 = 1$ is possible.

13. The resonances $\rho(770)$ and $\omega(778)$ have $P = -1$ and $J = 1$ and thus they are allowed. Nothing forbids a non-resonant state that have the right quantum numbers.