Midterm exam of Particle Physics
Tuesday November 15th 2022

Duration: 3 hours
6 printed pages

Allowed material: PDG booklet, simple calculator.
Solve on two separate sheets exercises I-II and exercises III-IV.

Approximate duration per exercise:

Ex. I:	10 min.	Ex. II:	80 min.
Ex. III:	20 min.	Ex. IV:	70 min.

Exercise I
Questions on the lectures

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. What was the motivation of Pauli for postulating the existence of neutrinos in the years 1930? Why are these particles difficult to detect? How and when were discovered the electron and muon neutrinos?

2. What is the baryon number and how is it defined? How is the fact that it is conserved by the strong, electromagnetic and weak interactions related to the allowed coupling (vertexes) of these interactions? Relate the conservation of the baryon number to the fact that the proton is a stable particle.

Exercice II

The BABAR experiment

The *BABAR* experiment was an electron-positron collider aimed at studying *CP* violation. The collider (PEP-II) properties are given in Tab. 1.

Table 1: Properties of the PEP-II collider (and naming conventions for the exercise)

Beam	Energy
Electron (oriented along $+\vec{z}$ axis and named A)	9 GeV
Positron (oriented along $-\vec{z}$ axis and named B)	3.1 GeV

We note throughout the exercise $\sqrt{s}$ the value of the total energy in the center of mass. For a given trivector $\vec{q}$, we note q_T its norm in the $(\vec{x}, \vec{y})$ plane, and θ the angle between $\vec{q}$ and the $\vec{z}$ axis. We add a * when these values are expressed in the center of mass.

1. *Kinematics of the reaction* $e^+e^- \to \overline{D}^0 D^{*0}$.

   We note m_0 and m_0^* the mass of $\overline{D}^0$ and D^{*0} particles (for numerical computations use $m_0 = 1864.8$ MeV and $m_0^* = 2006.9$ MeV).

   We note $k^* \equiv (E^*, \vec{k}^*)$ the four-vector of the $\overline{D}^0$ particle in the center-of-mass frame, $k \equiv (E, \vec{k})$ its corresponding four-vector in the lab. frame, and $p^{\text{cm}} \equiv (E^{\text{cm}}, \vec{p}^{\,\text{cm}})$ the four-vector of the center of mass (whole initial state) in the lab. frame.

   (a) Define s as a function of p^{cm}, then p^{cm} as a function of p_A, p_B. Find numerically the value of $\sqrt{s}$ assuming that electrons and positrons are massless. Justify the assumption a posteriori.

   (b) We consider the Lorentz boost transformation Λ^{cm} from the center-of-mass frame to the lab. frame. Express Λ^{cm} as a function γ^{cm} and β^{cm}, respectively the Lorentz factor and velocity along $\vec{z}$ of the center-of-mass frame (note that $\beta^{\text{cm}} > 0$). Express E^{cm} and p_z^{cm} as a function of $\sqrt{s}$, γ^{cm} and β^{cm} (hint: check that $p_z^{\text{cm}} > 0$). Find the numerical values of γ^{cm} and β^{cm}.

   (c) Express $|\vec{k}^*|$ as a function of $\sqrt{s}$, m_0 and m_0^*.

   (d) Compute the numerical values of $|\vec{k}^*|$ and E^* in GeV.

   (e) Using the boost Λ^{cm}, express the quantities k_T and k_z as a function of $|\vec{k}^*|$, E^*, $\sin\theta^*$, $\cos\theta^*$, γ^{cm}, β^{cm}.

   (f) Express $\tan\theta = \frac{k_T}{k_z}$ as a function of $\sin\theta^*$, $\cos\theta^*$, γ^{cm}, β^{cm} and the ratio $\frac{|\vec{k}^*|}{E^*}$. Taking $\sin\theta^* = \cos\theta^* = 1/\sqrt{2}$, find the numerical value of the angle θ.

2. *QED production* $e^+e^- \to F\bar{F}$.

   We now consider the production of fermion-antifermion pairs $(f\bar{f})$:

$$e^+e^- \to f\bar{f}.$$

   We remind you that the total LO QED cross-section (at tree level) is given by:

$$\sigma_{tot}\left(e^+e^- \to \mu^-\mu^+\right) = \frac{4\pi}{3}\,\frac{\alpha^2}{s}\sqrt{1 - \frac{4\,m_f^2}{s}}\,\left(1 + \frac{1}{2}\,\frac{4\,m_f^2}{s}\right) \tag{1}$$

Table 2: Total production cross section from various physics processes in e^+e^- collisions at $\sqrt{s}$ of PEP-II.

Physics process	Cross section [nb]
$b\bar{b}$	1.11
$u\bar{u}$	1.61
$d\bar{d}$	0.40
$s\bar{s}$	0.38
$c\bar{c}$	1.30
$\mu^+\mu^-$	1.15

(a) From the Feynamn rules given in Fig. 2 (assuming $f \neq e^-$), draw the Feynamn diagram of the QED LO production and write the corresponding matrix element. Mention the different possibilities for f.

(b) From the PDG, what are the mass and width of the resonance $\Upsilon(4S)$? What is the quark content of the $\Upsilon(4S)$ and its dominant decay modes?

(c) If not done already in II.1.a, compute the value of $\sqrt{s}$.

(d) From the two previous questions, argue that the $\Upsilon(4S)$ resonance is produced in the *BABAR* experiment. Discuss qualitatively its impact on the total cross section with respect to the QED-LO prediction from Eq. 1?

(e) Simplify Eq. 1 in the case where $2\,m_f \ll \sqrt{s}$. Compute the value of the expected cross section $\sigma(e^+e^- \to \mu^-\mu^+)$ at QED LO using this approximation. Compare to the value from Tab. 2 and give one potential reason for the difference.

(f) Justify that $2\,m_u \ll \sqrt{s}$, $2m_d \ll \sqrt{s}$, and from QED-LO predict the ratios $\frac{\sigma(e^+e^- \to u\bar{u})}{\sigma(e^+e^- \to \mu^-\mu^+)}$ and $\frac{\sigma(e^+e^- \to d\bar{d})}{\sigma(e^+e^- \to \mu^-\mu^+)}$. Compare to the result in Tab. 2 and give one potential source for the difference.

(g) What do you expect for the ratio $\sigma(e^+e^- \to b\bar{b})/\sigma(e^+e^- \to d\bar{d})$ at LO in QED? Give a quantitative estimate.

(h) Compare your LO QED prediction of $\sigma(e^+e^- \to b\bar{b})/\sigma(e^+e^- \to d\bar{d})$ to the measured one from Tab. 2. Explain the difference if any.

3. *Available statistics.*

(a) The instantaneous luminosity of the machine was $\mathcal{L} \approx 10 \times 10^{33}$ cm^{-2}s^{-1}. Assuming a month is effectively only 20 days of data taking, what was the integrated luminosity collected per month (express your result in fb^{-1}/month)?

(b) The *BABAR* experiment collected 433 fb^{-1}. Using the cross sections given in Tab. 2, what is the total number of $b\bar{b}$ pairs recorded by the experiment? How many $B^0\overline{B}^0$ pairs and B^+B^- pairs were collected knowing the $b\bar{b}$ pairs were produced via the $\Upsilon(4S)$ resonance?

Change sheets here

Exercise III

Allowed and forbidden processes, Feynman diagrammes

For each of the processes below, determine whether it is allowed or forbidden. For the forbidden processes, explain why, giving *all the possible reasons* (here we do not require to take into account multiplicative quantum numbers, angular momentum or isospin). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Note on the diagram all the particles (including virtual particles).

1. $\Sigma^+ \longrightarrow \Lambda\mu^+\nu_\mu$ 2. $D^+ \longrightarrow p\tau^-\nu_\tau$
3. $e^-p \longrightarrow e^-n\pi^+$ 4. $B^0 \longrightarrow \tau^+\tau^-$
5. $e^+e^- \longrightarrow \gamma\gamma$

Exercise IV

The exotic hadron $X(3872)$

The exotic hadron $X(3872)$ was discovered by the Belle experiment in the year 2003 in the decay mode $J/\psi\,\pi^+\pi^-$. Since its discovery, other experiments, in particular LHCb, confirmed its existence, and physicists are trying to understand and determine its nature, properties and quantum numbers. **Since its discovery the $X(3872)$ was included in the PDG. Here we will <u>not use its properties as quoted in the PDG</u>.**

1. A spectrum published by the LHCb experiment in 2013 is given in Fig. 1. It shows a clear peak that corresponds to the X(3872). Briefly explain and comment the *whole* figure using relevant numbers from the PDG. Using the information in the figure, determine the mass of the $X(3872)$. Using the figure, what can you tell about its width?

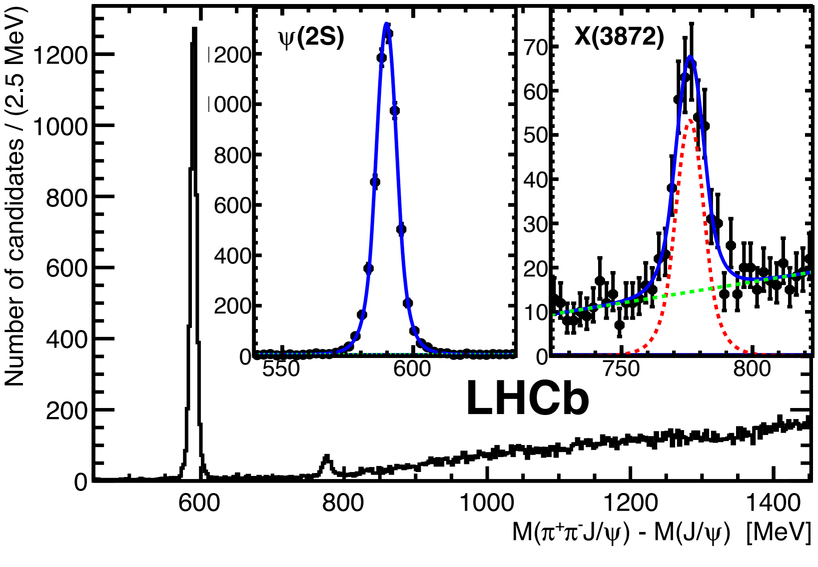


Figure 1: Spectrum of the variable $M(J/\psi\,\pi^+\pi^-) - M(J/\psi)$, published by LHCb in 2013. The peak corresponding to the $X(3872)$ is clearly visible. It is enlarged in the right-hand side inset. The dashed line shows the signal component in the fit to data.

In the following we will make the hypothesis that the decay $X(3872) \rightarrow J/\psi\,\pi^+\pi^-$ occurs by the strong interaction.

2. Deduce, with a brief explanation, all the additive quantum numbers (electric charge, baryon number, flavours...) of the $X(3872)$.

3. Draw the Feynman diagram of the decay, supposing that $X(3872)$ is a $c\bar{c}$ state.

4. Under the hypothesis that the $X(3872)$ is a $c\bar{c}$ state, what is its isospin? Determine the ratio of partial widths of its decay to $J/\psi\,\pi^+\pi^-$ and $J/\psi\,\pi^0\pi^0$. Explain.

There exist several indications that the $X(3872)$ is not a meson or a baryon. It could be a molecule of two $c\bar{q}$ mesons (where q is a u, d or s quark), or a $c\bar{c}q\bar{q}$ state, bound by strong interaction, which is called tetraquark. To determine the nature of the $X(3872)$, it is useful to measure its spin, parity and charge conjugation.

5. Knowing that the decay $X(3872) \to J/\psi\,\gamma$ was observed, determine the quantum number C (charge conjugation) of the $X(3872)$. Briefly explain.

6. We now go back to the decay $X(3872) \to J/\psi\,\pi^+\pi^-$. Using the answer to the previous question, find the quantum number C of the $\pi^+\pi^-$ pair, noted C_1.

7. Using all the information that you have until now, conservation laws and relevant information from the PDG, show that *if* the $\pi^+\pi^-$ pair originates in a decay of a resonance, this resonance can be either the $\rho^0(770)$ or the $\omega(782)$ *and nothing else*. In this case, why can the contribution from the $\omega(782)$ be neglected?

If the π^+ and the π^- do not originate in a resonance, we say that they constitute a non-resonant state. Even in this case, we can consider the $\pi^+\pi^-$ pair as a system and obtain its quantum numbers.

8. Establish a relation between the eigenvalue C_1 of the $\pi^+\pi^-$ pair and its parity P_1. Deduce a constraint on the relative angular momentum, ℓ_1, between the two pions, and the total angular momentum, J_1, of the pair.

The LHCb experiment, exploiting the angular distributions of all the final-state particles, showed that the J^{PC} of the $X(3872)$ is 1^{++}. We will now deduce all the relevant quantum numbers of final state $J/\psi\,\pi^+\pi^-$.

9. What is the relation between the parity of the $X(3872)$ and the relative angular momentum, ℓ_2, between the pair $\pi^+\pi^-$ and the J/ψ?

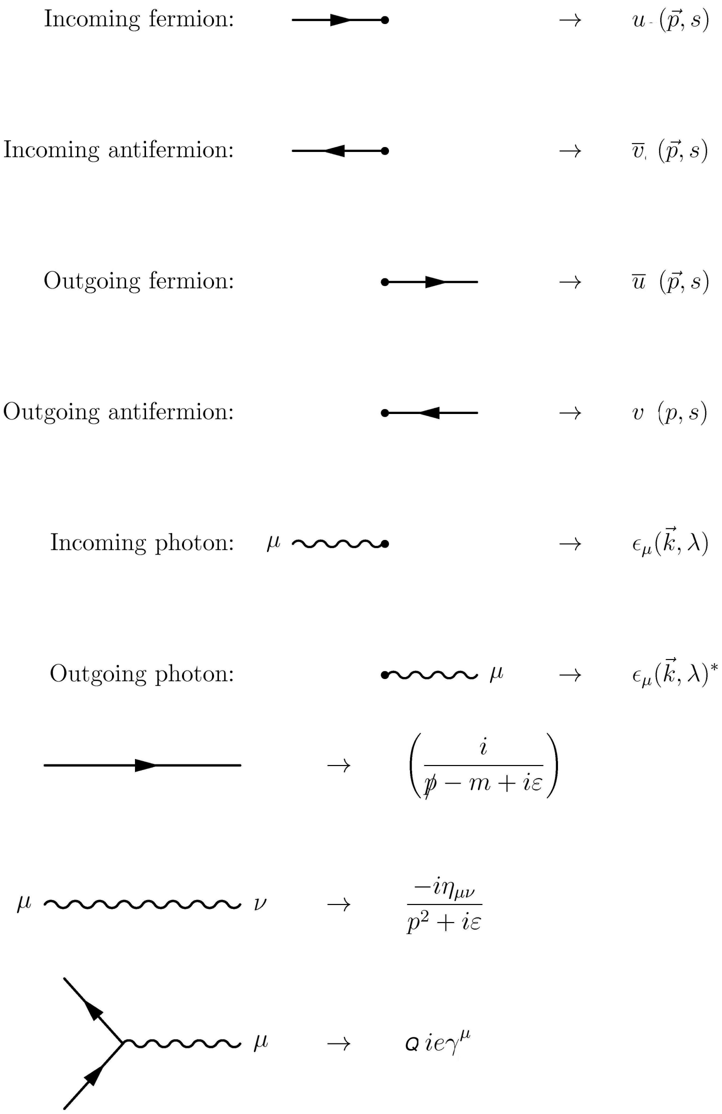
10. Using a simple argument, explain why it is reasonable to suppose that ℓ_2 takes the smallest possible value? What is this value?

11. Examine the coherence of this result with parity conservation in the process $X(3872) \to J/\psi\,\pi^+\pi^-$ and confirm the parity P_1 of the $\pi^+\pi^-$ pair found in question 8.

12. Give all the possibilities for the combination of the angular momenta $\vec{J_1} + \vec{J}_{J/\psi}$, where J_1 is the total angular momentum of the $\pi^+\pi^-$ pair and $J_{J/\psi}$ is the spin of the J/ψ.

13. Finally, is a resonant state of the pair $\pi^+\pi^-$ possible? Same question for the non-resonant state. Briefly argue.

Figure 2: QED Feynman rules. s refers to the (anti-)fermion spin and λ to the photon helicities.



Incoming fermion: →	$u(\vec{p}, s)$
Incoming antifermion: →	$\overline{v}(\vec{p}, s)$
Outgoing fermion: →	$\overline{u}(\vec{p}, s)$
Outgoing antifermion: →	$v(p, s)$
Incoming photon: μ →	$\epsilon_\mu(\vec{k}, \lambda)$
Outgoing photon: μ →	$\epsilon_\mu(\vec{k}, \lambda)^*$
→	$\left(\frac{i}{\not{p} - m + i\varepsilon}\right)$
μ ν →	$\frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$
μ →	$Q\, ie\gamma^\mu$