**M2 NPAC - Particle physics (2022-2023)**

# Correction Exercise sheet № 5 - QED and PDFs

*Heavy photon production at the LHC*

A- *Cross section $q\bar{q} \to \mu^+\mu^-$*

1. There are 2 similar diagrams. The first one is the QED one from the course. The second one looks similar but with the exchange of a heavy photon rather than a QED photon. For the latter one, the 2 couplings are multiplied by $g_H$ and the propagator is different.
2. The QED matrix element can be written as

$$\begin{aligned} i\,\mathcal{M}_{QED} &= \bar{u}(k_1)\,(i\,(-1)e\,\gamma^\mu)\,v(k_2) \times \bar{v}(\hat{p}_B)(i\,Q_q\,e\,\gamma^\nu)u(\hat{p}_A) \times \frac{-i\,\eta_{\mu\nu}}{(\hat{p}_A+\hat{p}_B)^2+i\epsilon}\;, \\ i\,\mathcal{M}_{QED} &= \frac{Q_q}{\hat{s}+i\epsilon} \times (i\,\mathcal{M}_f)\;\;, \\ i\,\mathcal{M}_f &= -i\;(4\pi)\frac{e^2}{4\pi}\;\bar{u}(k_1)\gamma^\mu v(k_2) \times \bar{v}(\hat{p}_B)\gamma_\mu u(\hat{p}_A)\;, \end{aligned} \tag{1}$$

where we used $\sqrt{\hat{s}} = (\hat{p}_A + \hat{p}_B)^2$. For the heavy photon exchange, the changes to the matrix element are minor:

$$\begin{aligned} i\,\mathcal{M}_H &= \bar{u}(k_1)\,(i\,g_H\,(-1)e\,\gamma^\mu)\,v(k_2) \times \bar{v}(\hat{p}_B)(i\,g_H\,Q_q\,e\,\gamma^\nu)u(\hat{p}_A) \times \mathcal{P}_H(\hat{p}_A+\hat{p}_B), \\ i\,\mathcal{M}_H &= \frac{g_H^2\;Q_q}{\hat{s}-M_H^2-i\;M_H\,\Gamma_H} \times (i\,\mathcal{M}_f) \\ i\,\mathcal{M}_H &= \frac{g_H^2\;\hat{s}}{\hat{s}-M_H^2-i\;M_H\,\Gamma_H} \times (i\,\mathcal{M}_{QED}) \end{aligned} \tag{2}$$

3. The total matrix element is therefore the sum $\mathcal{M}_{tot} = \mathcal{M}_{QED} + \mathcal{M}_H$, which gives for the cross section (using the master formula)

$$\begin{aligned} d\sigma_{tot} &= \left|1\;+\;\frac{g_H^2\,\hat{s}}{\hat{s}-i\;M_H\,\Gamma_H}\right|^2 \times d\sigma_{QED} \qquad \text{with} \\ d\sigma_{QED} &= \frac{1}{2\,E_A\;2\,E_B|\beta_A-\beta_B|} \times |\mathcal{M}_{QED}|^2 \times (2\pi)^4\delta^{(4)}(p_A+p_B-\sum_{i=1}^{i\leq 2}k_i)\prod_{i=1}^{i\leq 2}\frac{d^3\vec{k}_i}{(2\pi)^3\;2\,E_i}\;. \end{aligned} \tag{3}$$

The prefactor in front of $d\sigma_{QED}$ does solely depend on $\hat{s}$ and therefore can be factorized from the integration over $d^3\vec{k}_1$, $d^3\vec{k}_2$. This gives:

$$\sigma_{tot}(\hat{s}) = F_H(\hat{s}) \times \sigma_{QED}(\hat{s}) \tag{4}$$

with

$$\begin{aligned} F_H(\hat{s}) &= \left|1\;+\;\frac{g_H^2\,\hat{s}}{\hat{s}-i\;M_H\,\Gamma_H}\right|^2 \\ F_H(\hat{s}) &= 1 + \frac{2\,g_H^2\hat{s}\,(\hat{s}-M_H^2)}{(\hat{s}-M_H^2)^2\;+\;M_H^2\,\Gamma_H^2} + \frac{g_H^4\;\hat{s}^2}{(\hat{s}-M_H^2)^2\;+\;M_H^2\,\Gamma_H^2} \end{aligned} \tag{5}$$

4. The cross section is in $1\sqrt{\hat{s}}$ (cf. course QED) with a resonant peak at $\hat{s} = M_H^2$ (cf course on Breit-Wigner).

5. The term

$$I_H(\hat{s}) = \frac{2\, g_H^2 \hat{s}\, (\hat{s} - M_H^2)}{(\hat{s} - M_H^2)^2 \ + \ M_H^2\, \Gamma_H^2}$$

is exactly zero for $\hat{s} = M_H^2$. This represents the interference between the QED production, the first term (*i.e.* the factor 1 in $F_H$) in $F_H$, and a pure heavy photon production, which is the last term in $F_H$.

6. In the vicinity of the peak, this term is negligible if $2\ (\hat{s} - M_H^2) \ll g_H^2 \hat{s}$, with $\hat{s} = (M_H + \delta M)^2 \approx M_H^2 + 2\, M_H\, \delta M$ (to first order in $\delta M$). This gives the constraint that $|4\, \delta M| \ll g_H^2 M_H^2$ to first order in $\delta M$, which is satisfied for all $\delta M \in [-\Gamma_H, \Gamma_H]$ if

$$4\ \frac{\Gamma_H}{M_H} \ll g_H^2 \qquad (6)$$

B- *Total width of heavy photons*

1. The Feynman diagram is just the vertex with a heavy photon as in-coming particle.

2. The partial width to $\mu^+\mu^-$ is given by

$$\begin{aligned}
\Gamma_\mu &= \frac{1}{2\ M_H} \times |\mathcal{M}_\mu|^2 \times (2\pi)^4 \delta^{(4)}(k_1 + k_2 - q_H) \frac{d^3\vec{k}_1}{(2\pi)^3\ 2\,E_1}\ \frac{d^3\vec{k}_2}{(2\pi)^3\ 2\,E_2}, \\
\Gamma_\mu &= \frac{1}{2\ M_H} \times |\mathcal{M}_\mu|^2 \times (2\pi)\delta(2 \times k_* - M_A) \frac{k_*^2\ d\,k_*\ d\varOmega_1}{(2\pi)^3\ 2\,k_*}\ \frac{1}{2\,k_*},
\end{aligned} \qquad (7)$$

computed in the center of the heavy photon given by $q_H \equiv (M_H, \vec{0})$. For the second line, we integrated over $(2\pi)^3 \delta(3)(\vec{k}_1 + veck_2)\, d^3\vec{k}_2$, which is fixing $\vec{k}_2 = -\vec{k}_1$. In addition since the masses are neglected we used in the center of mass$E_1 = E_2 = |\vec{k}_1| = |\vec{k}_2| \equiv k_*$. Since $|\mathcal{M}_\mu)|^2$ is independent on $k_*$ and $\varOmega_1$, we can integrate (remember that $\delta(2\,k_* - M_A) = \delta(k_* - M_A/2)/2$).

$$\begin{aligned}
\Gamma_\mu &= \frac{1}{2\ M_H} \times |\mathcal{M}_\mu|^2 \frac{1}{2} \frac{4\pi}{(2\pi)^2 \times 4}, \\
\Gamma_\mu &= \frac{1}{2\ M_H} \frac{4\pi}{3}\, \alpha\, g_H^2\ M_H^2\ \frac{1}{8\pi}, \\
\Gamma_\mu &= \frac{1}{12}\ \alpha\ g_H^2\ M_H.
\end{aligned} \qquad (8)$$

Thus, we find that $\mathcal{N}_\mu = \frac{1}{12}$ which is a constant number (independent of $M_H$ in particular).

3. Neglecting all fermions masses (which is not a good approximation for the top quark but for all the other fermions this is perfectly fine), all the partial widths to fermions are given by

$$\Gamma_f = N_c\, Q_f\ \times \Gamma_\mu,$$

where $N_c$ is the number of colors, 3 for quarks and 1 for leptons, $Q_f$ the electric charge of fermion $f$. Summing over all fermions

$$\begin{aligned}
\Gamma_H &= \Gamma_e + \Gamma_\mu + \Gamma_\tau + \sum_{q\in[u,d,s,c,b,t]} \Gamma_q \\
\Gamma_H &= 3\,\Gamma_\mu + N_c \times (3\,Q_u^2 + 3\,Q_d^2)\Gamma_\mu \\
\Gamma_H &= 3 \times (1 + 3\,Q_u^2 + 3\,Q_d^2)\,\Gamma_\mu \\
\Gamma_H &= \frac{1 + 3\,Q_u^2 + 3\,Q_d^2}{4}\ \alpha\ g_H^2\ M_H.
\end{aligned} \tag{9}$$

Thus,

$$\mathcal{N}_H \equiv \frac{1 + 3\,Q_u^2 + 3\,Q_d^2}{4} = \frac{2}{3}.$$

4. For $g_H \equiv 1,\ \alpha = 1/100$, we find that

$$\Gamma_H = \frac{2}{3}\frac{1}{100}\ 1\ 1000 \approx 6.7 \text{ GeV} .$$

5. $I_H$ is negligible if $4\ \frac{\Gamma_H}{M_H} \ll g_H^2$ which is equivalent to $\frac{8}{3}\alpha \ll 1$ which is indeed always true since $\alpha \approx 1/100$.

C- *Kinematics of the heavy photon*

1. $\hat{p}_i \approx x_i \times p_i$
2. Since $\hat{s} = (\hat{p}_A + \hat{p}_B)^2 = 2\,\hat{p}_A\,\hat{p}_B$ and $s = (p_A + p_B)^2 = 2\,p_A\,p_B$, we immediatly obtain

$$\hat{s} = x_A\,x_B\ s$$

3. The energy and longitudinal momentum of the produced heavy photon are obtained by the 4-momentum conservation

$$\begin{aligned}
E_H &= x_A E + x_B E = (x_A + x_B)\,E \\
pz_H &= x_A E - x_B E = (x_A - x_B)\,E
\end{aligned} \tag{10}$$

4. This gives for the rapidity

$$\begin{aligned}
Y_H &= \frac{1}{2}\ln\frac{(x_A + x_B)E + (x_A - x_B)E}{(x_A + x_B)E - (x_A - x_B)E} \\
Y_H &= \frac{1}{2}\ln\left(\frac{x_A}{x_B}\right)
\end{aligned} \tag{11}$$

5. First, we need $\hat{s}$ to be higher than $M_H$ to have enough energy to produce a heavy photon. Then, because of the phase space, the total production is dominated by the production of a heavy photon at rest. Therefore

$$\sqrt{s} \approx M_H$$

6. From the previous questions, we obtain the system

$$\begin{aligned} x_A\, x_B &= \frac{M_H^2}{s} \\ \frac{x_A}{x_B} &= e^{2Y_H} \end{aligned} \tag{12}$$

which gives by multiplying and dividing the two equations

$$\begin{aligned} x_A^2 &= \frac{M_H^2}{s} e^{2Y_H} \\ x_B^2 &= \frac{M_H^2}{s} e^{-2Y_H} \end{aligned} \tag{13}$$

Taking the square root, one obtains the desired result.

D- *Heavy photon production at the LHC*

1. To obtain a heavy resonance we need $x$ to be large. $q$ will tend to come from a valence while $\bar{q}$ is necessary a sea quark. Sea quarks are produced at low $x$ while valence quark can be produced with $x_v \geq 0.1$. Thus we expect $x_A > x_B$, *i.e.* a valence quark vs a sea quark.
2. In order to have $x_A > x_B$, we expect $Y_H > 0$ from the Eq. 4 in the text. The numerical application gives
$$x_A \approx 0.13 \qquad \text{and} \qquad x_B \approx 0.05 \tag{14}$$
3. $q$ is indeed mostly a valence quark ($x_A \approx 0.13$) since it the valence quark peak, while $\bar{q}$ is a sea quark (can not be a valence quark).
4. Given the pdf for $u$ and $d$ in Fig. 1 , the production will be dominated by $u\bar{u}$ and then $d\bar{d}$. In addition the charge of $Q_u$ is equal to $2 \times Q_d$, giving an additional boost (from the matrix element) to the cross section compared to $d\bar{d}$. The other $q\bar{q}$ productions are much smaller since for $x = 0.13$, the pdf for sea quarks (only possibility for $s$, $c$, $b$) are very small while all the other parameters in the productions (especially the couplings) are identical.