Exercise sheet № 3 - Collisions and decays

1 Cockroft-Walton experiment: course complement

In 1932, Cockroft and Walton smashed protons on to a lithium7 target. A scheme of the experiment is provided on Fig. 1, the z axis is oriented along the proton bram. The kinetic energy of the protons was adjustable in the actual experiment and in this exercise, we use a proton kinetic energy of $K[p] = 800keV$. Lithium atoms capture proton which in turn decay to 2 $\alpha \equiv {}^2_4\text{He}$

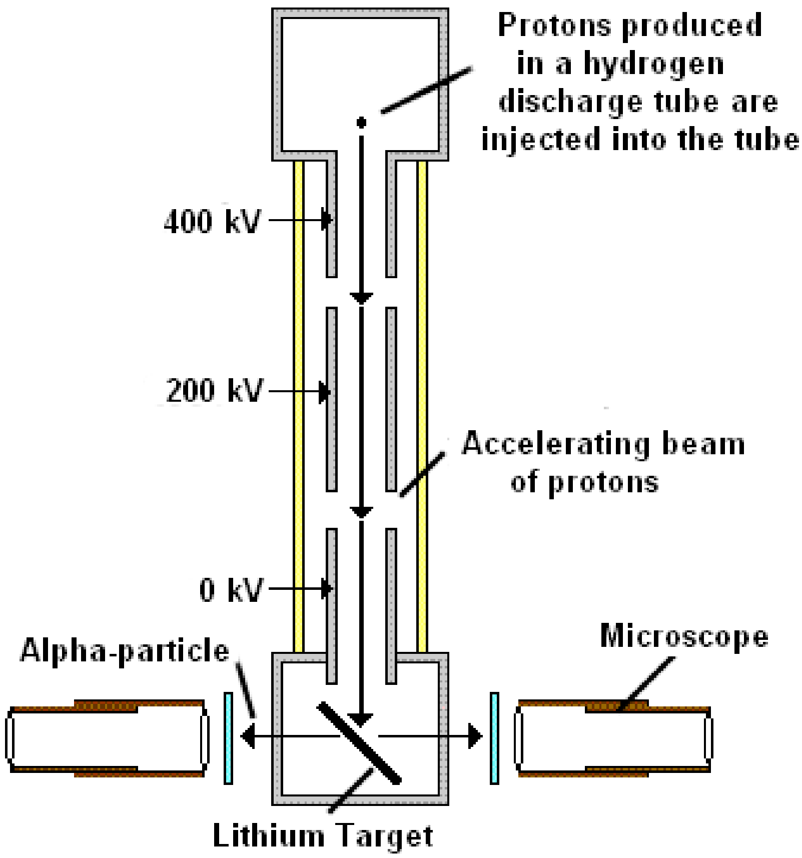
$$^7_3Li + p \to \alpha + \alpha$$

. The masses of the different particles are:

- $m[^7_3Li] = 6535.4$ MeV,
- $m[p] = 938.3$ MeV,
- $m[\alpha] = 3728.4$ MeV.

We note $\vec{p}_A$ the 3-momentum of A particle and q_A its 4-momentum. We remind you that the kinetic energy of a particle A can be defined as $K_A = E_A - m_A$. The lithium in the target is considered at rest in the lab and the proton 3-momentum is given by $\vec{p}_p = (0, 0, p_z)$.

Figure 1: Scheme of the Cockroft-Walton experiment



1. Show that when $|\vec{p}_A| \ll m_A$, the total energy can be expressed as $E_A \approx m_A + \frac{p_A^2}{2\,m_A}$ and that therefore the kinetic energy takes the classical mechanic form $K_A = \frac{p_A^2}{2\,m_A}$. Show that $K_A = (\gamma - 1)\,m_A$.

2. Express the total mass of the colliding particles M_* as a function of m_{Li}, m_p and K_p. Can you neglect K_p in this expression?

3. Express the boost β_* from the lab to the rest frame of the colliding particles as a function of m_{Li}, m_p and E_p. What is the expression of the factor $\gamma_* \times \beta_*$ as a function of p_z, m_{Li}, m_p.

4. Compute the total energy E_{p*} of the proton in the rest-frame of the colliding particles by boosting the proton four momentum to the center-of-mass rest frame. Deduce from that the kinetic energy K_{p*} of the proton in the rest-frame of the colliding particles (Reminder $K_{p*} = E_{p*} - m_p$). Can it be neglected ?

5. **General two body decay of a particle M to two particles p_1, p_2 : $M \to p_1\ p_2$.** We consider the aforementioned two body decay in the center of mass frame In the center of mass frame, *i.e.*the quadri vector of M is $q_M \equiv (M_*, 0, 0, 0)$. Justify that $|\vec{p}_1| = |\vec{p}_2|$. We note $p_* \equiv |\vec{p}_1| = |\vec{p}_2|$. Compute p_* with respect to m_1, m_2 and M_*.

6. Apply the previous question to the case of the process $Li + p \to \alpha + \alpha$. Find the p_* of the α particles as well as their kinetic energy $K_*[\alpha]$ as expected from classical mechanics $K_*[\alpha] = p_*^2/(2\,m_\alpha)$, find out the numerical value.

7. Find the same result using the conservation of the energy in the center of mass frame, what is the numerical value? Why is it slightly different from the previous question?

8. Justify this is the same kinetic energy as in the lab frame. The measured energy by Cockroft-Walton was about 8 MeV. Does it correspond to your result ?

9. Epilogue. This was the first proof that mass can actually be transformed in actual kinetic energy and earned the Nobel prize to Cockroft and Walton and 1951.

2 Luminosity: course complement

In Tab. 1 are presented the design parameter of the Large Hadron Collider.

Table 1: Large Hadron Collider parameters.

parameter	nominal value
transverse emittance	3.75 μm
β^*	0.55 m
number of bunches	2808
number of proton/bunch	1.1×10^{11}
bunch spacing	25 ns
crossing angle	285 μrad
Piwinski angle, ϕ	0.64

1. Assuming the transverse emittance and β^* are identical in the x and y dimension, compute the instantaneous luminosity $\mathcal{L}$ in cm^{-2}.s^{-1}.

2. Convert your result in nb.s^{-1} .

3. During the course of a year, the LHC runs for approximately 10^7 s. What is the total integrated luminosity per year, assuming the instantaneous luminosity to be constant with time.

4. The Higgs boson cross section at the LHC (13 TeV) is approximaltely $\sigma(pp \to H) = 50$ pb. How many Higgs boson are produced per year at the LHC ?

5. The Higgs boson branching ratio $\mathcal{B}(H \to \gamma\gamma) = 2 \times 10^{-3}$. How many Higgs boson are recorded at the LHC during a year, assuming the experimental efficiency to record a Higgs boson decaying in the diphoton channel to be $\epsilon \approx 50$ % ?