Exercise sheet № 5 - QED and PDFs

Heavy photon production at the LHC

A heavy photon field could result from an additional $U(1)$ symmetry to the SM. Heavy photons behave very much like QED photons except that they are mediated via a heavy neutral vector-boson particle γ_H. Its couplings to the standard model particles are similar to SM photons modified by a coupling factor g_H, while the dark photon propagator is

$$\mathcal{P}_H = \frac{-i\,\eta_{\mu\nu}}{p^2 - M_H^2 - i\Gamma_H\,M_H},$$

where M_H is the mass of the heavy photon and Γ_H its total natural width. The Feynman rules for QED and the heavy photon sector are given in the appendix (page 5).

We consider the production of muon-anti-muon pairs at the LHC via QED + heavy photon (*i.e.* we neglect the weak interaction). The processes are therefore

$$q + \bar{q} \to \mu^- + \mu^+,$$

where q and $\bar{q}$ are the two incoming partons in p-p collisions and we neglect the mass of all fermions. We define:

- 4-momentum proton 1: $p_A \equiv (E, 0, 0, +E)$, 4-momentum proton B: $p_B \equiv (E, 0, 0, -E)$,
- 4-momentum q: $\hat{p}_A \equiv (E_A, 0, 0, +E_A)$, 4-momentum $\bar{q}$: $\hat{p}_B \equiv (E_B, 0, 0, -E_B)$,
- 4-momentum μ^-: k_1, 4-momentum μ^+: k_2,
- $\hat{s} \equiv (\hat{p}_A + \hat{p}_B)^2$, $s \equiv (p_A + p_B)^2$.

To ease the notation, we have considered that q is from proton 1 (along $+\vec{z}$) while $\bar{q}$ is from proton 2 (along $-\vec{z}$). Of course, it could be the opposite and the actual process is symmetric with respect to the protons.

For numerical applications, we consider:

- a heavy photon with mass $M_H = 1.0$ TeV,
- the LHC beam energy to be $E = 6.5$ TeV,
- thus, the LHC center of mass energy is $\sqrt{s} = 13$ TeV.

We remind the master formula for cross-section and partial-width computations:

$$d\sigma = \frac{1}{2\,E_A\;2\,E_B|\beta_A - \beta_B|} \times |\mathcal{M}(A + B \to f)|^2 \times (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum_{i=1}^{i\leq f} k_i) \prod_{i=1}^{i\leq f} \frac{d^3\vec{k}_i}{(2\pi)^3\;2\,E_i},$$
$$d\Gamma = \frac{1}{2\;M_A} \times |\mathcal{M}(A \to f)|^2 \times (2\pi)^4 \delta^{(4)}(p_A - \sum_{i=1}^{i\leq f} k_i) \prod_{i=1}^{i\leq f} \frac{d^3\vec{k}_i}{(2\pi)^3\;2\,E_i}. \qquad (1)$$

This exercise is divided into 4 parts, A, B, C and D, which are mostly independent from one another.

A- *Cross section $q\bar{q} \to \mu^+\mu^-$*

1. Considering QED + heavy sector, draw the possible diagrams (without loops) for $\mu^+\mu^-$ production at the LHC.
2. Write the matrix elements using the Feynman rules given in the appendix. Note $\mathcal{M}_{QED}$ the matrix element corresponding to QED and $\mathcal{M}_H$ the matrix element corresponding to the heavy sector. Note that all the matrix elements can be separated into a fermion part $\mathcal{M}_f$ and a propagator part. What is the common $\mathcal{M}_f$?
3. We remind that in the Standard Model (QED):

$$\sigma_{QED}(\hat{s}) = \frac{4\pi}{3} Q_q^2 \frac{\alpha^2}{\hat{s}},$$

where Q_q is the electric charge of the parton q and α the fine-structure constant. Using this expression, the cross-section master formula and the matrix elements from previous questions, show (without lengthy calculations) that the total cross section can be written:

$$\sigma_{tot} = \sigma_{QED}(\hat{s}) \times F_H(\hat{s}) \, .$$

What is the expression of $F_H(\hat{s})$?

4. Draw schematically the total cross section as a function of $\hat{s}$.
5. At the pole, *i.e.* for $\sqrt{\hat{s}} \equiv M_H$, show that there is a term which is zero in the expression of F_H. We denote this term $I_H(\hat{s})$. What does this term correspond to? What are the other terms corresponding to?
6. In the vicinity of M_H, *i.e.* for $\delta M \equiv \sqrt{\hat{s}} - M_H \in [-\Gamma_H, +\Gamma_H]$, find a relation between g_H, M_H and Γ_H so that $I_H(\hat{s})$ is always negligible compared to the heavy photon production.

B- *Total width of heavy photons*

1. Draw the Feynman diagram for the decay $\gamma_H \to \mu^+\mu^-$.
2. Neglecting the mass of the muons (*i.e.* $m_\mu << M_H$), the matrix element for the decay $\gamma_H \to \mu^+\mu^-$ is given by

$$|\mathcal{M}(\gamma_{\mathcal{H}} \to \mu^-\mu^+)|^2 = \frac{4\pi}{3} \alpha g_H^2 M_H^2 \, .$$

Using the master formula for the partial width and neglecting the muon mass, write the partial width for $\gamma_H \to \mu^+\mu^-$ as

$$\Gamma_\mu \equiv \Gamma(\gamma_H \to \mu^-\mu^+) = \mathcal{N}_\mu \; \alpha \; g_H^2 \; M_H, \tag{2}$$

specifying the expression of the normalisation factor $\mathcal{N}_\mu$.

3. We consider that γ_H can solely decay to e^-, μ^-, τ^- leptons, u, d, s, c, b, t quarks. We neglect the mass of the outgoing particles. Write the total width Γ_H under the form

$$\Gamma_H = \mathcal{N}_{\mathcal{H}} \; \alpha \; g_H^2 \; M_H \; . \tag{3}$$

What is the expression of the normalisation factor $\mathcal{N}_H$ as a function of the quarks charges Q_u (up-type quark electric charge) and Q_d (down-type quark electric charge)? Give the numerical value of $\mathcal{N}_H$.

4. Assuming $g_H \equiv 1$ and $\alpha(1 \text{ TeV}) \approx 0.01$, what is the numerical value of Γ_H in GeV? Is it a narrow resonance (*i.e.* $\Gamma_H \ll M_H$)?

5. Using the expression of Γ_H demonstrate that I_H is indeed always negligible in the expression of F_H (use the relation established in the question A-6).

C- *Kinematics of the heavy photon*

We note x_A (resp. x_B) the fraction of the proton momentum p_A (resp. p_B) carried by q (resp. $\bar{q}$). We also assume that heavy photons are produced along the z axis, $p_{\gamma_H} \equiv (E_H, 0, 0, pz_H)$

1. Give the expression of $\hat{p}_i$ as a function p_i.
2. Neglecting the proton and quark masses, give the expression of $\hat{s}$ as a function of s.
3. Give the expressions of E_H and pz_H as functions of x_A, x_B and E (the proton-beam energy).
4. Obtain the expression of the rapidity Y_H of the heavy photon as a function of x_A, x_B and E. We remind that Y_H is given by

$$Y_H = \frac{1}{2}\ln\left(\frac{E_H + pz_H}{E_H - pz_H}\right)$$

5. What is the typical value of $\hat{s}$ to produce a heavy photon?
6. Using the expression of $\hat{s}$ vs s (question C-2) and Y_H (question C-4), demonstrate that:

$$x_A = \frac{M_H}{\sqrt{s}} e^{Y_H} \quad \text{and} \quad x_B = \frac{M_H}{\sqrt{s}} e^{-Y_H} . \tag{4}$$

D- *Heavy photon production at the LHC*

Fig. 1 shows the parton distribution function for $\sqrt{\hat{s}} = 1$ TeV.

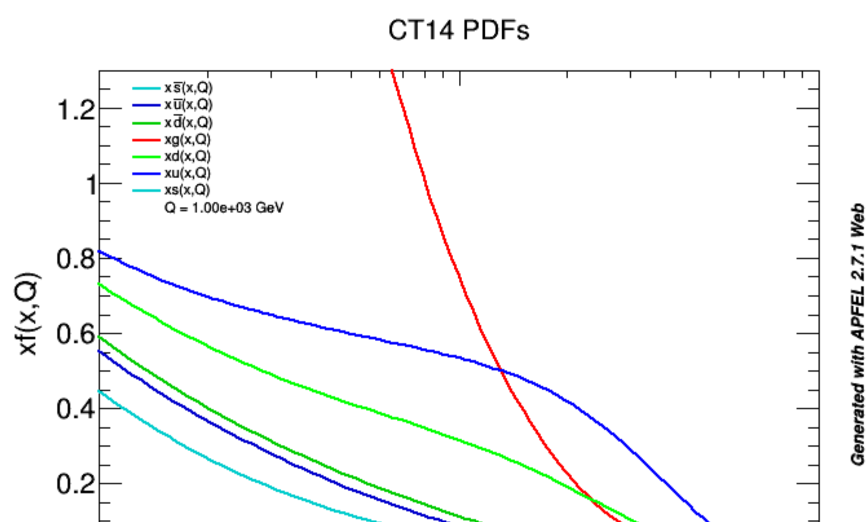

Figure 1: parton density function at $\sqrt{\hat{s}} = 13$ TeV

1. We remind that x_A corresponds to incoming quark q (along $+\vec{z}$) and x_B to incoming anti-quark $\bar{q}$ (along $-\vec{z}$). Based on your physical sense and on your knowledge of parton distribution functions (pdf), do you expect $x_A > x_B$ or the opposite? Justify.
2. The absolute average rapidity of heavy photon production is $|Y_H| = 0.5$. From Eq. 4, what are the numerical values of x_A and x_B ? What is the sign of Y_H?

3. Using Fig. 1 and given the value of x_A, is q more likely a valence or a sea quark? Given the value of x_B, is $\bar{q}$ is more likely a valence quark or a sea quark?
4. Using Fig. 1 and the values of x_A and x_B what is the quark flavor q dominating the total cross section?

Figure 2: QED Feynman rules. s refers to the (anti-)fermion spin and λ to the photon helicities.