Exercise sheet № 6 - QCD: color factors and parton density functions

1 Color factors

We consider $q\bar{q}'$ scattering at low energy. In this case the classical potential between the pair quark anti-quark is the classical limit of the matrix element:

$$V_{strong} = \langle q\bar{q}'| \mathcal{M}_{int} |q\bar{q}'\rangle$$

where $\mathcal{M}$ is the matrix element of the scattering diagram.

1. Draw the Feynman diagram of the scattering $q\bar{q}$ considering only the strong interaction.

2. Find out that the classical potential $V_{q\bar{q}'} = F_{col}\alpha_w V_{EM}$ where V_{EM} is the classical electromagnetic potential (without the hyperfine structure constant α_{EM}). Give the expression of the color factor F_{col} as a function f_{ijkl} the color factors defined in the course. In this case the two partons have an opposite color charge (color vs anti-color) therefore V_{EM} is an attractive potential unless F_{col} is negative.

3. We consider $q\bar{q}'$ state in the "colorless" configuration of the octet, *i.e.*

$$|8\rangle = \frac{1}{\sqrt{6}} |r\bar{r} + g\bar{g} - 2\, b\bar{b}\rangle$$

   Find the color factor F_{col} for the scattering $|8\rangle \to |8\rangle$. What is its sign ? What do you conclude on the stability of this "colorless" state.

4. What color factor do you expect for the scattering $|8\rangle \to |0\rangle$, where

$$|0\rangle = \frac{1}{\sqrt{3}} |r\bar{r} + g\bar{g} + b\bar{b}\rangle$$

   is the singlet meson state? Compute this color factor and conclude.

5. Eventually find the color factor of the scattering $|0\rangle \to |0\rangle$? What is its sign and what does it mean ?

2 Nucleon momentum

We note f^p_q (resp. f^n_q) the probability density function of parton q in the proton (resp. in the neutron). In this exercise we will only consider high x and low Q^2 therefore limit ourselves to valence quarks and gluons.

1. Write the total momentum of the proton as a function of the integrals of f^p_u, f^p_d and f^p_g. We note $\epsilon^p_q = \int x\, f^p_q(x)\mathrm{d}x$, find out the conservation rule:

$$\epsilon^p_u + \epsilon^p_d + \epsilon^p_g = 1$$

2. Using the fact that strong interaction conserves isospin, write the same expression for the neutron, and simplify your expression using the notation for all partons q

$$\epsilon_q^p \equiv \epsilon_q$$

3. We remind you that the form factor of the proton involved in photon-nucleon scattering is $F_2^p(x) = \sum_q Q_q^2 f_q^p(x)$ where Q_q is the charge of parton q in unit of the electron charge. Write the expression of $\int x\, F_2^p(x)\mathrm{d}x$ as a function of ϵ_q's

4. Similarly for the neutron, we define $F_2^n(x) = \sum_q Q_q^2 f_q^n(x)$. Write this integral as a function of ϵ_q^n's and use again the isospin to get the expression as a function of the ϵ_q's

5. Experimentally, we measured:

$$\begin{aligned} \int_0^1 x\ F_2^p(x)\mathrm{d}x &= 0.18\ , \\ \int_0^1 x\ F_2^n(x)\mathrm{d}x &= 0.12\ . \end{aligned} \tag{1}$$

   From these two measurements conclude on the value of ϵ_g the fraction of the nucleon momentum carried out by gluons. This was one of the first evidence for the presence in the nucleon of something else that valence quarks.