Correction Exercise sheet № 8 - Around the Z boson

1 Corrections for QED $e^-e^+ \to f\bar{f}$

1. The matrix element can be written as

$$\begin{aligned} i\mathcal{M} &= \overline{v}(k_2)\, ieQ_e\gamma^\mu\, u(k_1)\ \frac{-i}{q^2}\eta_{\mu\nu}\overline{u}(p_1)\, ieQ_f\, v(p_2) \\ &= Q_eQ_f\ \ i\ \frac{e^2}{q^2}\overline{v}(k_2)\gamma^\mu u(k_1)\overline{u}(p_1)\gamma_\mu v(p_2) \end{aligned} \tag{1}$$

2. From the matrix element, we get:

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}|^2 &= \mathcal{M}\mathcal{M}^\dagger \\ &= Q_e^2Q_f^2\frac{e^4}{q^4}\overline{v}(k_2)\gamma^\mu u(k_1)\overline{u}(k_1)\gamma^\nu v(k_2)\ \ \overline{u}(p_1)\gamma_\mu v(p_2)\overline{v}(p_2)\gamma_\nu u(p_1) \\ &= Q_e^2Q_f^2\frac{e^4}{q^4}\ Tr(\gamma^\mu \not{k}_1\gamma^\nu \not{k}_2)\ Tr(\gamma_\mu \not{p}_2\gamma_\nu \not{p}_1) \\ &= Q_e^2Q_f^2\frac{e^4}{q^4}\ 4\,(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})k_{1\rho}k_{2\sigma} \\ &\quad\times 4\,(\eta_{\mu\rho'}\eta_{\nu\sigma'} + \eta_{\mu\sigma'}\eta_{\nu\rho'} - \eta_{\mu\nu}\eta_{\rho'\sigma'})p_{1\sigma'}p_{2\rho'} \\ &= Q_e^2Q_f^2\frac{e^4}{q^4}\ 16\ (k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - \eta^{\mu\nu}k_1.k_2)(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - \eta_{\mu\nu}p_1.p_2) \\ &= Q_e^2Q_f^2\frac{e^4}{q^4}\ 16\ \ (2\,k_1.p_1\,k_2.p_2 + 2\,k_1.p_2\,k_2.p_1 - k_1.k_2p_1.p_2(4 - \eta^{\mu\nu}\eta_{\mu\nu})) \\ &= Q_e^2Q_f^2\ \frac{e^4}{q^4}\ 32(k_1.p_1\,k_2.p_2 + k_1.p_2\,k_2.p_1), \end{aligned} \tag{2}$$

where we used the fact that $\eta_{\mu\nu}\eta^{\mu\nu} = \eta^\mu_\mu = 4$. The lab frame is the center of mass frame, therefore f and $\bar{f}$ have the same momentum and assuming massless fermions: $p_1 \equiv (E, E\sin\theta, 0, E\cos\theta)$, $p_2 \equiv (E, -E\sin\theta, 0, -E\cos\theta)$ we also have $p_f^* = E = \sqrt{s}/2$. Therefore

$$\begin{aligned} p_1.k_1 &= E^2(1-\cos\theta) \\ p_1.k_2 &= E^2(1+\cos\theta) \\ p_2.k_1 &= E^2(1+\cos\theta) \\ p_2.k_2 &= E^2(1-\cos\theta) \end{aligned} \tag{3}$$

giving

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}|^2 &= Q_e^2Q_f^2\frac{e^4}{s^2}\ 32\ E^4\left[(1-\cos\theta)^2 + (1+\cos\theta)^2\right] \\ &= Q_e^2Q_f^2\frac{e^4}{s^2}\ 32\frac{s^2}{16}2(1+\cos^2\theta) \\ &= Q_e^2Q_f^2\ \alpha^2\ 4\,(4\pi)^2(1+\cos^2\theta) \end{aligned} \tag{4}$$

Therefore the final cross section is

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2}{64\pi^2 s} \times 1 \\ &= Q_e^2\, Q_f^2\, \frac{\alpha^2}{4s}(1+\cos^2\theta)\end{aligned} \tag{5}$$

3. Using the Wigner formalism, one find easily that:

   - $d\sigma_{LL} = Q_e^2 Q_f^2\ \mathcal{A}_{LL}(1+\cos\theta)^2$,
   - $d\sigma_{RR} = Q_e^2 Q_f^2\ \mathcal{A}_{RR}(1+\cos\theta)^2$,
   - $d\sigma_{LR} = Q_e^2 Q_f^2\ \mathcal{A}_{LR}(1-\cos\theta)^2$,
   - $d\sigma_{RL} = Q_e^2 Q_f^2\ \mathcal{A}_{RL}(1-\cos\theta)^2$

   Because the EM interaction is symmetric under parity we find out that $d\sigma_{LL} = d\sigma_{RR}$ and $d\sigma_{RL} = d\sigma_{LR}$.

   The total cross section can be written as:

$$\begin{aligned}d\sigma &= Q_e^2 Q_f^2\ \frac{1}{4}\left(d\sigma_{LL} + d\sigma_{RR} + d\sigma_{LR} + d\sigma_{RL}\right) \\ &= Q_e^2 Q_f^2\ \frac{1}{2}\left(\mathcal{A}_{LL}(1+\cos\theta)^2 + \mathcal{A}_{LR}(1-\cos\theta)^2\right) \\ &= Q_e^2 Q_f^2\ \frac{1}{2}\left((\mathcal{A}_{LL}+\mathcal{A}_{RL})(1+\cos^2\theta) + 2\cos\theta(\mathcal{A}_{LL}-\mathcal{A}_{RL})\right)\end{aligned} \tag{6}$$

   We conclude to get the proper angular distribution that:

$$\mathcal{A}_{LR} = \mathcal{A}_{RL} = \mathcal{A}_{RR} = \mathcal{A}_{LL} \equiv A = \frac{\alpha^2\pi}{2\,s}$$

4. The matrix element with P_X and P_Y is simply (remember that a L anti-fermion is the right chirality of the bi-spinor$v$ so $f_R\bar{f}_L$ raises a term $P_R v(p_2)$ and Y correspond to the chirality of the f.

$$\begin{aligned}i\mathcal{M}_{XY} &= Q_e Q_f\quad i\ \frac{e^2}{q^2}\ \overline{v}(k_2)\gamma^\mu P_X u(k_1)\ \overline{u}(p_1)\gamma_\mu P_Y\, v(p_2) \\ &= Q_e Q_f\quad i\ \mathcal{M}^{EM}_{XY}\end{aligned} \tag{7}$$

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2 Corrections for the Z boson partial widths $\Gamma(Z \to \; f\bar{f})$

1. Diagram ok...

2. The matrix elements are given by

$$
\begin{aligned}
i\mathcal{M}_L &= i\mathcal{A}_L \; \frac{g}{c_w}(T_3^f - Q_f\, s_w^2) \\
i\mathcal{M}_R &= i\mathcal{A}_R \; \frac{g}{c_w}(-Q_f\, s_w^2)
\end{aligned}
\tag{8}
$$

3. $|A_L|^2$ is given by, summing over the initial polarisation of the boson and final spins of the fermions.

$$
\begin{aligned}
|\mathcal{A}_L|^2 &= \sum_{\text{polar}} \sum_{\text{helicities}} \overline{u}(p_1)\gamma^\mu \frac{1-\gamma_5}{2} v(p_2)\overline{v}(p_2)\gamma^\nu \frac{1-\gamma_5}{2} u(p_1)\epsilon_\mu \epsilon^*_\nu \\
&= \left(-\eta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m_Z^2}\right) Tr\left((\not{p}_1 - m_f)\gamma^\mu \frac{1-\gamma_5}{2} (\not{p}_2 + m_f)\gamma^\nu \frac{1-\gamma_5}{2}\right)
\end{aligned}
\tag{9}
$$

The term proportional to m_f are zero because the trace contains an odd number of γ's. The term in m_f^2 is proportional to $Tr\left(\gamma^\mu \frac{1-\gamma_5}{2}\gamma^\nu \frac{1-\gamma_5}{2}\right) = Tr\left(\gamma^\mu\gamma^\nu \frac{1+\gamma_5}{2}\frac{1-\gamma_5}{2}\right) = 0$ because $P_R\, P_L = 0$. We are left with:

$$
\begin{aligned}
|\mathcal{A}_L|^2 &= \sum_{\text{polar}} \sum_{\text{helicities}} \overline{u}(p_1)\gamma^\mu \frac{1-\gamma_5}{2} v(p_2)\overline{v}(p_2)\gamma^\nu \frac{1-\gamma_5}{2} u(p_1)\epsilon_\mu \epsilon^*_\nu \\
&= \left(-\eta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m_Z^2}\right) Tr\left(\not{p}_1\gamma^\mu \frac{1-\gamma_5}{2} \not{p}_2\gamma^\nu \frac{1-\gamma_5}{2}\right) \\
&= \left(-\eta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m_Z^2}\right) p_{1\rho}\, p_{2\sigma} \times Tr\left(\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu \frac{1-\gamma_5}{2}\right) \\
&= \left(-\eta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m_Z^2}\right) p_{1\rho}\, p_{2\sigma} \times 2\left(\eta^{\mu\rho}\eta^{\sigma\nu} + \eta^{\mu\sigma}\eta^{\rho\nu} - \eta^{\rho\sigma}\eta^{\mu\nu} + i\; \epsilon^{\rho\mu\sigma\nu}\right)
\end{aligned}
\tag{10}
$$

where we used the traces expressions given in the introduction to the exercise. The terms corresponding to γ_5 are proportional to $\epsilon^{\rho\mu\sigma\nu}$ (anti-symmetric under $\mu\nu$ exchange) which is multipled by a symmetric tensor in $\mu\nu$ and are therefore zero. This means that $|\mathcal{A}_L|^2 = |\mathcal{A}_R|^2$. We pursue the computation:

$$
\begin{aligned}
|\mathcal{A}_L|^2 &= \left(-\eta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m_Z^2}\right) 2\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - \eta^{\mu\nu}\, p_1.p_2\right) \\
&= 2\;\left(-2\,p_1.p_2 + 4\,p_1.p_2 + 2\,\frac{(p_1.k)(p_2.k)}{m_Z^2} + \frac{k^2}{m_Z^2} p_1.p_2\right) \\
&= 2\;\left(p_1.p_2 + 2\,\frac{(p_1^2 + p_1.p_2)(p_2^2 + p_1.p_2)}{m_Z^2}\right) \\
&= 2\,p_1.p_2\left(1 + \frac{2\,p_1.p_2}{m_Z^2}\right),
\end{aligned}
\tag{11}
$$

in the last line we have neglected the terms proportional to $p_1^2 = p_2^2 = m_f^2$ (assuming massless fermions). We also have used $k = p_1 + p_2$ which implies that $k^2 = m_Z^2 = p_1^2 + p_2^2 + 2p_1.p_2 = 2\,p_1.p_2$ again for $m_f = 0$. Eventually, we get:

$$
|\mathcal{A}_L|^2 = |\mathcal{A}_R|^2 = 2\,m_Z^2
\tag{12}
$$

4. The matrix element being independent of the angle, we have directly $\Gamma = |\mathcal{M}|^2 \frac{p_f^*}{8\pi m_Z^2}$ since the integral over Ω gives a factor of 4π. Using the fact that $e = g s_w$, we have (averaging over the initial Z polarisations brings in a factor 1/3, and noting the fine structure constant $\alpha = e^2/4\pi$):

$$\begin{aligned}
\Gamma_L &= \frac{1}{3}|\mathcal{A}_L|^2 \frac{e^2}{s_w^2 c_w^2}\left(T_3^f - Q_f s_w^2\right)^2 \frac{m_Z/2}{8\pi m_Z^2} \\
\Gamma_L &= \frac{\alpha\, m_Z}{6 c_w^2\, s_w^2}\left(T_3^f - Q_f s_w^2\right)^2 \\
\Gamma_R &= \frac{\alpha\, m_Z}{6 c_w^2\, s_w^2}\left(-Q_f s_w^2\right)^2
\end{aligned} \tag{13}$$

5. The total partial width is therefore given by

$$\Gamma = \Gamma_L + \Gamma_R = \frac{\alpha\, m_Z}{6 c_w^2\, s_w^2} \times \left[\left(T_3^f - Q_f s_w^2\right)^2 + Q_f^2 s_w^4\right] \times N_c(f) \tag{14}$$

where $N_c(f)$ is a color factor that includes QCD corrections, it is 1 for leptons and $N_c(f) = 3(1 + \alpha_s/\pi)$ for quarks. Using $\alpha_s(m_Z) = 0.118$, $s_w^2 = 0.232$ and $m_Z = 91.2$ GeV, we find the partial widths and branching ratios in Tab. 1).

6. the Left-Right asymmetry is only given by the vertex factor since $|\mathcal{A}_R| = |\mathcal{A}_L|$, we have:

$$A_{LR}^f = \frac{\left(T_3^f - Q_f s_w^2\right)^2 - Q_f^2 s_w^4}{\left(T_3^f - Q_f s_w^2\right)^2 + Q_f^2 s_w^4} \tag{15}$$

Propery	Theo. (tree level)	Measurement
Γ_Z	2.49 GeV	2.4952 ± 0.0023 GeV
$\mathcal{B}(Z \to \nu\bar{\nu})$	20.04 %	20.00 ± 0.06 %
$\mathcal{B}(Z \to \ell\ell)$	3.36 %	3.3658 ± 0.0023%
$\mathcal{B}(Z \to \mathrm{up-type})$	11.9 %	11.6 ± 0.6 %
$\mathcal{B}(Z \to \mathrm{down-type})$	15.4 %	15.6 ± 0.4 %

Table 1: Predictions (from this exercise) and measurements of different Z boson parameters

3 Correction for QED + Weak neutral currents $e^-e^+ \to f\bar{f}$

1. Diagram...
2. We have only 4 cross-sections because the only couplings existing are $\bar{\psi}_L\gamma^\mu\psi_L$ and $\bar{\psi}_R\gamma^\mu\psi_R$ which is also true for pseudo-vector (axial) current. In this case the matrix elements can be written as:

$$\begin{aligned} i\mathcal{M}_{XY} = \overline{v}(k_2)\, ieQ_e\gamma^\mu\, P_X u(k_1)\, \frac{-i}{q^2}\eta_{\mu\nu}\overline{u}(p_1)\, ieQ_f\, P_Y v(p_2) \\ +\overline{v}(k_2)\, \frac{ie}{c_w\, s_w}(T_3^e - Q_e s_w^2)\gamma^\mu\, P_X u(k_1) \\ \times\, \frac{-i}{q^2-m_Z^2}\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}\right) \\ \times\, \overline{u}(p_1)\, \frac{ie}{c_w\, s_w}(T_3^f - Q_f s_w^2)P_Y v(p_2) \end{aligned} \tag{16}$$

3. The terms in $\overline{v}(k_2)\gamma^\mu\, P_X u(k_1) q^\mu = 0$ because

$$\begin{aligned} \overline{v}(k_2)\gamma^\mu\, P_X u(k_1) q^\mu &= \overline{v}(k_2)\gamma^\mu\, \frac{1\pm\gamma_5}{2} u(k_1)(k_1^\mu + k_2^\mu) \\ &= \overline{v}(k_2)\frac{1\mp\gamma_5}{2}\not{k}_1 u(k_1) + \overline{v}(k_2)\not{k}_2\frac{1\pm\gamma_5}{2}u(k_1) \\ &= m_e\overline{v}(k_2)\frac{1\mp\gamma_5}{2}u(k_1)\ -\ m_e\,\overline{v}(k_2)\frac{1\pm\gamma_5}{2}u(k_1) \\ &= \mp m_e\overline{v}(k_2)\gamma_5 u(k_1) \\ &= \epsilon_X\, m_e\,\overline{v}(k_2)\gamma_5 u(k_1) \end{aligned} \tag{17}$$

   with $\epsilon_L = +1$ and $\epsilon_R = -1$. So the terms in $q_\mu\, q_\nu/m_Z^2 \propto \frac{m_e\, m_f}{m_Z^2} \ll 1$ can be neglected.

4. The form or the matrix element from the question is straightforward after removal of the terms $q^\mu q^\nu$. Indeed Eq. 16 becomes:

$$\begin{aligned} i\mathcal{M}_{XY} &= (i\,e)^2\overline{v}(k_2)\gamma^\mu\, P_X u(k_1)\, \frac{-i}{q^2}\eta_{\mu\nu}\overline{u}(p_1)\, P_Y v(p_2) \\ &\times \left(Q_e\, Q_f + \frac{(T_3^e - Q_e s_w^2)\,(T_3^f - Q_f s_w^2)}{c_w^2\, s_w^2}\frac{q^2}{q^2-m_Z^2}\right) \\ &= i\mathcal{M}_{XY}^{EM} \times \left(Q_e\, Q_f + \frac{(T_3^e - Q_e s_w^2)\,(T_3^f - Q_f s_w^2)}{c_w^2\, s_w^2}\frac{q^2}{q^2-m_Z^2}\right) \end{aligned} \tag{18}$$

5. We find the different partial cross sections from the QED result, indeed

$$\begin{aligned} \frac{d\sigma_{XY}}{d\cos\theta} &= \frac{1}{Flux}|\mathcal{M}_{XY}^{EM}|^2 d\Phi_2\ \times\ |F_{XY}(f)|^2 \\ \frac{d\sigma_{XY}}{d\cos\theta} &= \frac{1}{Q_e^2 Q_f^2}\frac{d\sigma_{XY}^{EM}}{d\cos\theta}\ \times\ |F_{XY}(f)|^2 \end{aligned} \tag{19}$$

6. The total cross section is the averaged sum of the polarised ones

$$\begin{aligned} \sigma_{tot} &= \frac{1}{4}\frac{\pi\alpha^2}{2\,s}\int_{-1}^{1} d\cos\theta\left((|F_{LR}|^2 + |F_{LR}|^2)(1-\cos\theta)^2 + (|F_{RR}|^2 + |F_{LL}|^2)(1+\cos\theta)^2\right) \\ &= \frac{\pi\alpha^2}{3\,s}\left(|F_{LR}|^2 + |F_{LR}|^2 + |F_{LL}|^2 + |F_{RR}|^2\right) \end{aligned} \tag{20}$$

7. The F_{XY} can be written as $F_{XY} = -Q_f + A_{XY}\frac{s}{s-m_Z^2+i\Gamma_Z\, m_Z}$ with $A_{XY} = \frac{(T_3^e+s_w^2)(T_3^f-Q_f s_w^2)}{c_w^s\, s_w^2}$ which gives:

$$|F_{XY}|^2 = Q_f^2 + A_{XY}^2 \frac{s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} - Q_f A_{XY} \frac{s(s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \tag{21}$$

The interference term $A_{XY}Q_f$ is nul at the Z pole ($s = m_Z^2$), the typical ratio between the EM contribution and the Z contribution is given by:

$$\frac{\sigma_{EM}}{\sigma_Z}|_{\sqrt{s}=m_Z} = \frac{Q_f^2}{A_{XY}^2}\frac{\Gamma_Z^2}{m_Z^2} \approx \frac{\Gamma_Z^2}{m_Z^2} \approx \frac{1}{1000} \tag{22}$$

In fact for the muon for example:

$$\frac{\sigma_Z}{\sigma_{EM}}|_{\sqrt{s}=m_Z} = \frac{2(1/2 - s_w^2)^2 + s_w^4 + (1/2 - s_w^2)^4/s_w^4}{4c_w^2} \approx 170 \tag{23}$$

8. For the LR asymmetry, Γ_L and Γ_R have the same flux and phase space factor, the matrix element are very similar but differ only by the vertex factor hence (see the exercise on the Z boson width for the full proof):

$$A_{LR}^f = \frac{(T_3^f - Q_f s_w^2)^2 - Q_f^2 s_w^4}{(T_3^f - Q_f s_w^2)^2 + Q_f^2 s_w^4} \tag{24}$$

9. For the forward and backward cross section, we first needs the integrals:

$$\begin{aligned} \int_0^1 d\cos\theta\ (1-\cos\theta)^2 &= \frac{1}{3} \qquad & \int_0^1 d\cos\theta\ (1+\cos\theta)^2 &= \frac{7}{3} \\ \int_{-1}^0 d\cos\theta\ (1-\cos\theta)^2 &= \frac{7}{3} \qquad & \int_{-1}^0 d\cos\theta\ (1+\cos\theta)^2 &= \frac{1}{3} \end{aligned} \tag{25}$$

which gives (remember $\frac{1}{4}$ is for the average over the initial polarisation):

$$\begin{aligned} \sigma_F &= \frac{1}{4}\frac{\pi\alpha^2}{2\,s}\int_0^1 d\cos\theta \left[(|F_{LR}(f)|^2 + |F_{RL}(f)|^2)(1-\cos\theta)^2 + (|F_{LL}(f)|^2 + |F_{RR}(f)|^2)(1+\cos\theta)^2\right] \\ &= \frac{1}{4}\frac{\pi\alpha^2}{2\,s}\left[(|F_{LR}(f)|^2 + |F_{RL}(f)|^2)\frac{1}{3} + (|F_{LL}(f)|^2 + |F_{RR}(f)|^2)\frac{7}{3}\right] \\ \sigma_B &= \frac{1}{4}\frac{\pi\alpha^2}{2\,s}\left[(|F_{LR}(f)|^2 + |F_{RL}(f)|^2)\frac{7}{3} + (|F_{LL}(f)|^2 + |F_{RR}(f)|^2)\frac{1}{3}\right] \end{aligned} \tag{26}$$

10. The forward-backward asymmetry is thus:

$$A_{FB} = \frac{3}{4}\frac{|F_{LL}(f)|^2 + |F_{RR}(f)|^2 - |F_{LR}(f)|^2 - |F_{RL}(f)|^2}{|F_{LL}(f)|^2 + |F_{RR}(f)|^2 + |F_{LR}(f)|^2 + |F_{RL}(f)|^2} \tag{27}$$

11. At the Z pole, the EM contribution is negligible, we are left with the vertex number:

$$A_{FB}^0 = \frac{3}{4}\frac{Q_e^2 s_w^4\ Q_f^2 s_w^4 + (T_3^e - Q_e s_w^2)^2 (T_3^f - Q_f s_w^2)^2 - (T_3^e - Q_e s_w^2)^2\, Q_f^2 s_w^4 - (T_3^f - Q_f s_w^2)^2 Q_e^2 s_w^4}{Q_e^2 s_w^4\ Q_f^2 s_w^4 + (T_3^e - Q_e s_w^2)^2 (T_3^f - Q_f s_w^2)^2 + (T_3^e - Q_e s_w^2)^2\, Q_f^2 s_w^4 + (T_3^f - Q_f s_w^2)^2 Q_e^2 s_w^4} \tag{28}$$

which is precisely

$$A_{FB}^0 = \frac{3}{4}\ A_{LR}^e\, A_{LR}^f \tag{29}$$

12. A^0_{FB} does only depend on s_w^2, so it is indeed a measurement of the Weinberg angle.

13. In the SM, we find the values for leptons up-type quarks and down-type quarks, using $s_w^2 = 0.23$:

$$
\begin{aligned}
A^\ell_{LR} &= \frac{(-1/2 + s_w^2)^2 - s_w^4}{(-1/2 + s_w^2)^2 + s_w^4} = 0.159 \\
A^u_{LR} &= \frac{(1/2 - 2/3 s_w^2)^2 - 4/9\, s_w^4}{(1/2 - 2/3\, s_w^2)^2 + 4/9 s_w^4} = 0.672 \\
A^d_{LR} &= \frac{(-1/2 + 1/3 s_w^2)^2 - 1/9\, s_w^4}{(-1/2 + 1/3\, s_w^2)^2 + 1/9 s_w^4} = 0.934
\end{aligned}
\tag{30}
$$

which gives the forward-backward results in Tab. 2

Propery	Theo. (tree level)	Measurement		
f	A^f_{LR}	A^f_{FB} (pred. $s_w^2 = 0.23$)	A^f_{FB} (pred. $s_w^2 = 0.232$)	Meas.
$f \equiv \ell$	0.159	0.019	0.0154	0.0171 ± 0.0010
$f \equiv u$	0.672	0.080	0.0715	0.0699 ± 0.0036
$f \equiv d$	0.934	0.1110	0.1004	0.1000 ± 0.0017

Table 2: Predictions (with 2 values of s_w^2) and measurements of different asymmetries.