

## Exercise sheet № 9 - CKM Matrix and $CP$ violation

- For each of the processes below, determine whether it is allowed or forbidden in the standard model. For the forbidden processes, explain why, giving *all the possible reasons* (here we do not require to take into account multiplicative quantum numbers and angular momentum). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Note on the diagram the names of all real and virtual particles. When relevant, indicate near the vertex the CKM matrix elements that contribute and give their orders of magnitude in terms of  $\lambda = \sin \theta_c$  ( $\theta_c$  is the Cabibbo angle). Then, give the total order of magnitude of the diagram in terms of  $\lambda$ . In case of Penguin or box diagrams, do this for the dominant intermediate quarks. As in general, tree processes are favoured comparing to penguin or box processes. We will try to privilege it when several topologies are possible. Also, if possible, we will try to privilege colour allowed to colour suppressed diagrams.

- $B^0 \rightarrow D^{*-} \pi^+$ ,
- $B^0 \rightarrow D^{*+} \pi^-$ ,
- $B^0 \rightarrow \bar{p} e^+ \nu_e$ ,
- $B^0 \rightarrow D^{*-} K^+$ ,
- $B^0 \rightarrow \bar{D}^{*0} \pi^0$ .

- Order the decay modes above by decreasing order of probability and explain your arguments.
- Estimate the ratio of the branching fractions

$$\frac{BF(B^0 \rightarrow D^{*+} \pi^-)}{BF(B^0 \rightarrow D^{*-} \pi^+)} \quad (1)$$

in terms of the  $\mathcal{O}(\lambda^3)$  Wolfenstein parameterization. Compute approximately and comment.

- The decays  $B^0 \rightarrow D^{*-} \pi^+$  can be used to measure  $CP$  violation. To observe  $CP$  violation there must be (at least) two interfering paths, or amplitudes, allowing the transition between the initial and the final state. What are these two paths in the present case?
- What is the type of  $CP$  violation involved in that case:  $CP$  violation in decay,  $CP$  violation in mixing or  $CP$  violation in interferences between decay and mixing?
- For this type of  $CP$  violation, the time dependent asymmetry is given by

$$A(t) = \frac{\Gamma(B^0 \rightarrow f)(t) - \Gamma(\bar{B}^0 \rightarrow f)(t)}{\Gamma(B^0 \rightarrow f)(t) + \Gamma(\bar{B}^0 \rightarrow f)(t)} = -\text{Im}(\lambda_f) \sin(\Delta mt), \quad (2)$$

with  $\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$ , using the notations used in the lectures. Express  $\frac{\bar{A}_f}{A_f}$  as a ratio of the CKM matrix elements, and briefly justify the fact that  $\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$ .

7. We recall that

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5) \quad (3)$$

Find the expression of  $A_{CP}(t)$  as a function of the angles of the Unitary Triangle and the  $\mathcal{O}(\lambda^3)$  Wolfenstein parameterization, at leading order in  $\lambda$ . Would you say that the decay modes  $B^0 \rightarrow D^{*\pm}\pi^\mp$ , are advantageous for the measurement of  $CP$  violation? Explain.

8. To perform the actual measurement, the decays have to be experimentally reconstructed. Looking at the PDG, find the decay modes (of the  $D^{*+}$  and of its decay products) which you think are the best for this measurement, justifying your choice.