QFT – TD3

1] ϕ^3 Theory

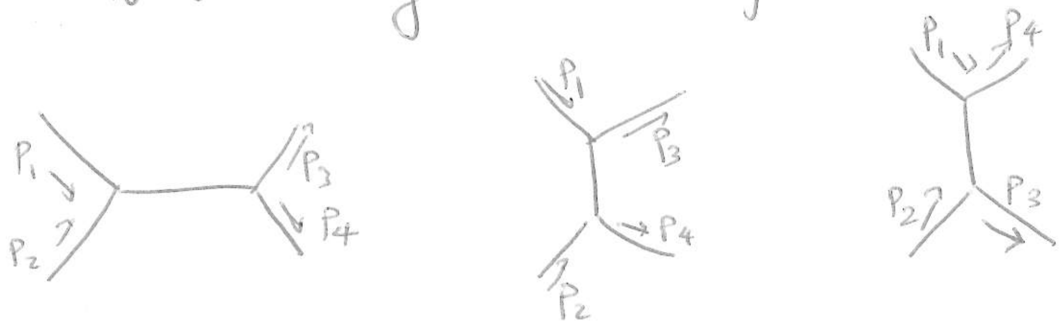
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3$$

1. 

$= \dfrac{i}{p^2 - m^2 + i\epsilon}$ propagator


$= -ig$ vertex

2. - No diagrams to order g.
   - To order g^2: 3 diagrams:



the amplitude of each of these is

$$A \propto g^2 \Rightarrow |A|^2 \propto g^4$$

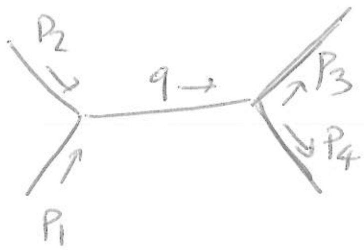
$\rightarrow \dfrac{d\sigma}{d\Omega} \propto g^4$. ~~Since~~ $[\sigma] = L^2 = \dfrac{1}{E^2}$ and

$$[g] = E$$

$\Rightarrow$ by dimensional analysis

$$\sigma \propto \frac{g^4}{E^6}$$

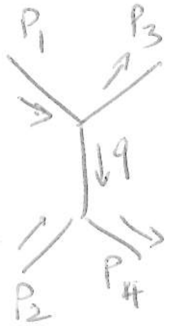
where E is the typical energy involved in the process (e.g. $(p_1^0 + p_2^0) = E_{cm}$)

3. 

$$q = p_1 + p_2 = p_3 + p_4$$

$$= \underbrace{(-ig)^2 \frac{i}{q^2 - m^2 + i\epsilon}}_{A_1} (2\pi)^4 \delta^{(4)}\left((p_1+p_2) - (p_3+p_4)\right)$$

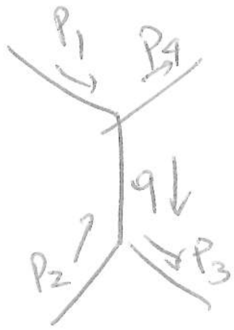
$$A_1 = -\frac{i g^2}{(p_1+p_2)^2 - m^2 + i\epsilon}$$



$$= (-ig)^2 \frac{i}{q^2 - m^2} (2\pi)^4 \delta^{(4)}\left((p_1+p_2) - (p_3+p_4)\right)$$

$$q = p_1 - p_3$$ =p4-p2

$$A_2 = -\frac{i g^2}{(p_1 - p_3)^2 - m^2 + i\epsilon}$$



$$= (-ig)^2 \frac{i}{q^2 - m^2} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - (p_3 + p_4))$$

$$q = p_1 - p_4$$

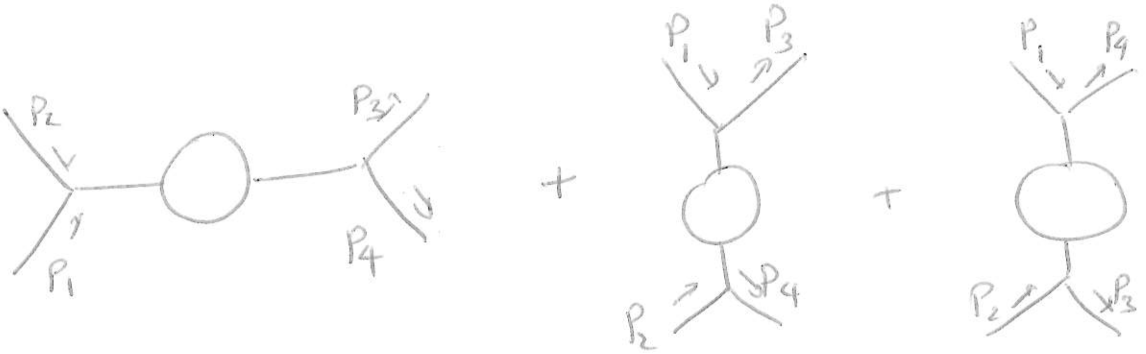
$$\mathcal{A}_3 = \frac{-ig^2}{(p_1 - p_4)^2 - m^2 + i\epsilon}$$

$$\mathcal{A}_{tot} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 =$$

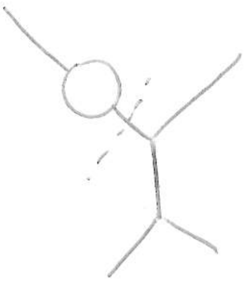
$$= -ig^2 \left[\frac{1}{(p_1+p_2)^2 - m^2} + \frac{1}{(p_1-p_3)^2 - m^2} + \frac{1}{(p_1-p_4)^2 - m^2} \right]$$

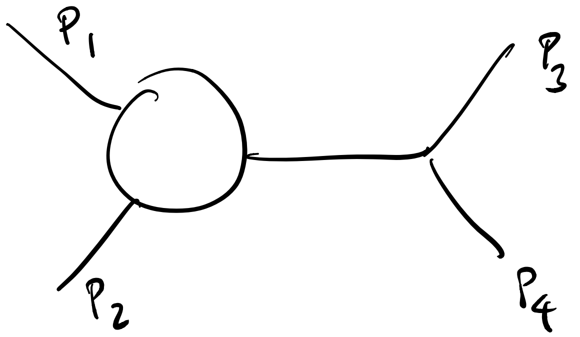
$$= -ig^2 \left[\frac{1}{s - m^2} + \frac{1}{t - m^2} + \frac{1}{u - m^2} \right]$$

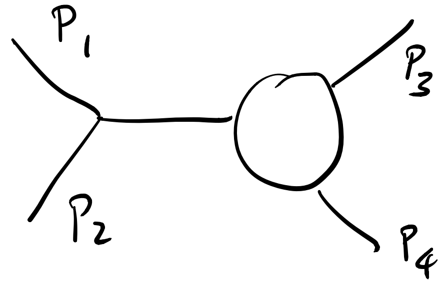
$$s \equiv (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

4. 

$(\sim g^4)$

[this :  does not count, since we can amputate the loop by cutting one external leg]

+ 

+ 

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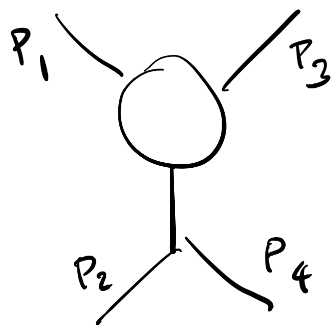
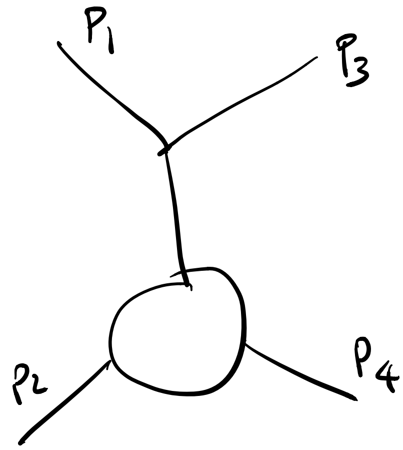
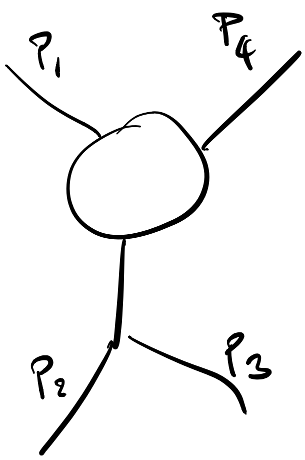
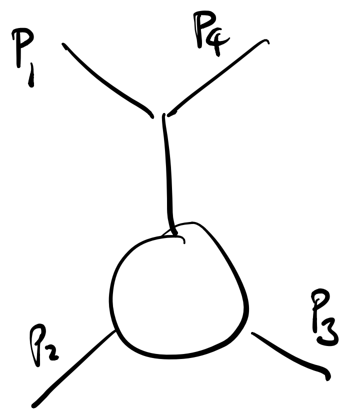
+ 

+ 

+ 

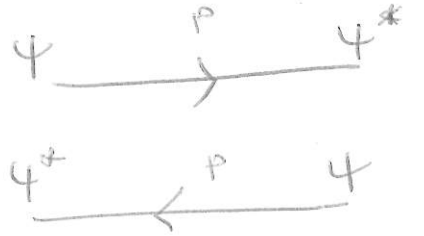
+ 

All are $O(g^4)$

2. Scalar Yukawa theory

$$\mathcal{L} = (\partial^\mu \psi)^* \partial_\mu \psi - M^2 \psi^* \psi + \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \underbrace{g \phi \psi^* \psi}_{\text{interaction part.}}$$

1. • ψ now is a complex field, so there are two kinds of lines:

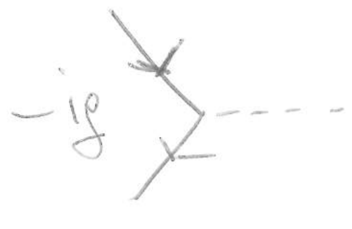


$$= \frac{i}{p^2 - M^2 + i\epsilon}$$

• ϕ is a real field so the lines are unoriented

$$= \frac{i}{p^2 - m^2 + i\epsilon}$$

• The vertex joins one ϕ, one ψ and one ψ^*

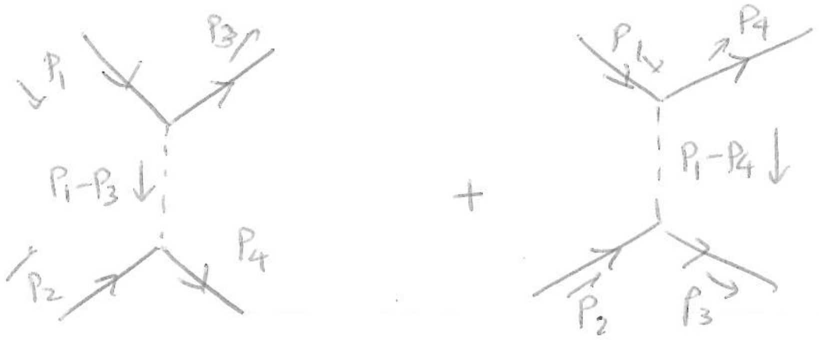


(notice direction of the arrows)

The arrows may be thought of the direction the <u>charge</u> flows (i.e. the conserved $U(1)$ charge of the theory)

$\Rightarrow$ the net charge entering a vertex must add up to zero (charge conservation).

2. $- \ \psi\psi \longrightarrow \psi\psi$



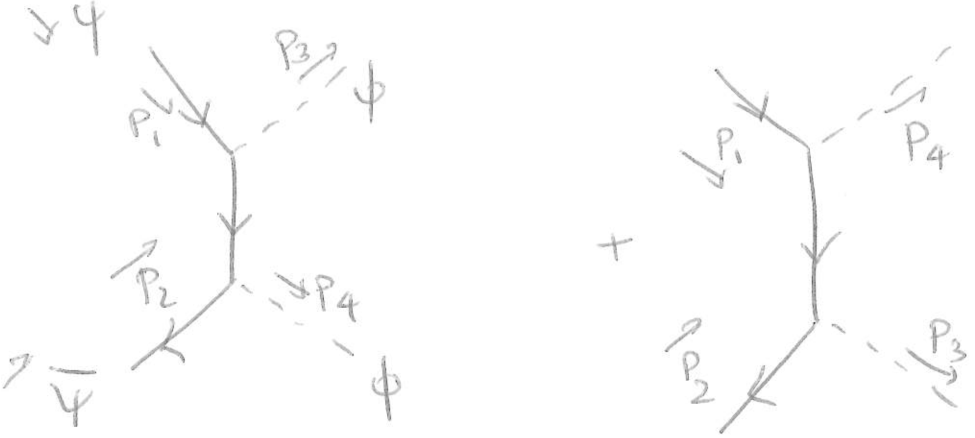
$$= (-ig)^2 \left[\frac{i}{(p_1-p_3)^2 - m^2} + \frac{i}{(p_1-p_4)^2 - m^2} \right]$$

$$\frac{d\sigma}{d\Omega} \sim \frac{g^4}{Q^6} \qquad Q = \text{momentum transferred}$$

$- \ \psi\phi \longrightarrow \psi\phi$

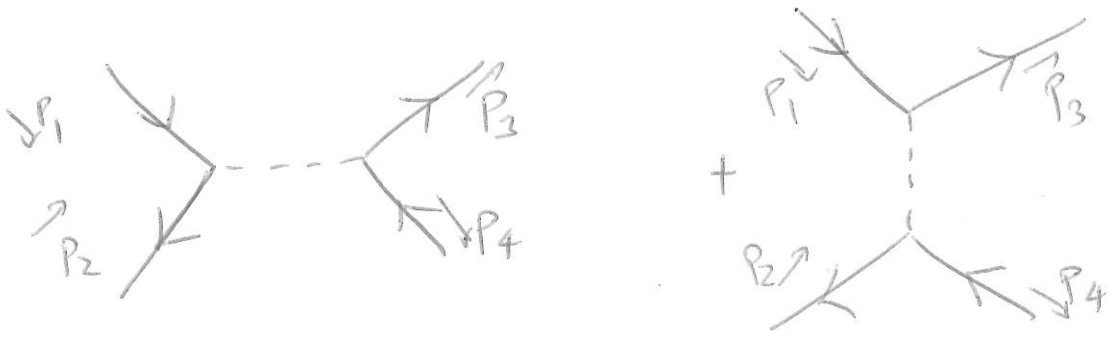
$$= (-ig)^2 \left[\frac{i}{(p_1-p_3)^2 - M^2} + \frac{i}{(p_1+p_2)^2 - M^2} \right]$$

- $\psi\bar{\psi} \longrightarrow \phi\phi$



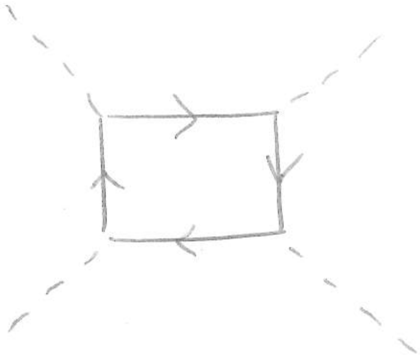
$$= (-ig)^2\left[\frac{i}{(p_1-p_3)^2-M^2}+\frac{i}{(p_1-p_4)^2-M^2}\right]$$

- $\psi\bar{\psi} \longrightarrow \psi\bar{\psi}$



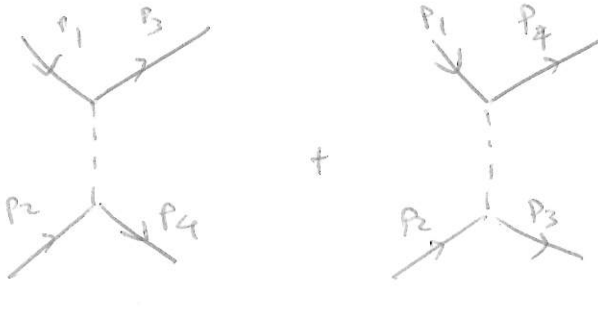
$$= (-ig)^2\left[\frac{i}{(p_1+p_2)^2-m^2}+\frac{i}{(p_1-p_3)^2-m^2}\right]$$

3. $\phi\phi \to \phi\phi$ cannot happen at tree-level because there are no vertices involving only ϕ (ϕ^4 or ϕ^3) so we need some ψ's and $\bar{\psi}$'s somewhere. Since they are not in the final states, they must be in loops. the lowest order process is:



$$\sim g^4$$

4. take for example $\psi\psi \to \psi\psi$ scattering: from point 2



$$\sim \frac{g^2}{(p_1-p_3)^2-m^2} + \frac{g^2}{(p_1-p_4)^2-m^2}$$

now suppose that all the external legs have energies $\ll m \Rightarrow (p_1-p_3)^2 - m^2 \simeq -m^2$ etc

$$\simeq \frac{g^2}{m^2}$$ = same vertex one would have with $\frac{g^2}{m^2}\psi\psi^*\psi\psi^*$

$$\mathcal{L}_{eff}(p^2 \ll m^2) = \partial_\mu \psi^* \partial^\mu \psi - M^2 \psi^* \psi - \underbrace{\frac{g^2}{m^2}}_{\lambda} \psi \psi^* \psi \psi^*$$

$\lambda = \dfrac{g^2}{m^2}$ effective quartic coupling of a theory with <u>nucleons</u> <u>only</u>

ii. λ is dimensionless, because $[g] = 1$
$[m] = 1$

iii. since $[\lambda] = 0$ perturbation theory is OK, in principle, even at energies higher than m.

However the predictions of the "effective" theory differ from those of the original theory which includes ϕ, at energies $E > m$.