M2 NPAC 2022/2023

QFT **Problem set n°4**

Feynman diagrams

Feynman rules

Start with a Lagrangian depending on a set of independent fields $\phi, \psi \ldots$ and containing interaction terms each described by a monomial of total degree n in the fields.

— To each independent field, associate a line (oriented, in the case of complex fields)

— To each interaction, associate a vertex with n incoming lines (one for each time a given field enters in the monomial).

Feynman rules for correlaction functions (momentum space)

To compute an N-point function at order k in perturbation theory :

1. Draw all diagrams with N external lines and k vertices which do not contain vacuum bubbles ;
2. Associate a momentum to each line ;
3. To each line, associate a factor of the corresponding Feynman propagator evaluated at the corresponding momentum ;
4. To each vertex, associate a factor of the coupling constant as it appears in the Lagrangian (multiplied by $p!$, where p is the number of identical fields coming into the vertex) and a factor $(-i)$.
5. Enforce momentum conservation at every vertex ;
6. Integrate over all momenta running in loops.
7. Sum over all terms obtained as above

Feynman rules for scattering amplitudes (momentum space)

For an amplitude involving N external particles (incoming or outgoing) :

1. Consider momentum-space N-point correlation functions as computed above, obtained by summing over *connected* diagrams only
2. Put all external momenta on-shell (i.e. enforce $p^2 = m^2$)
3. Remove the propagators associated with external lines.
4. Extract an overall momentum-conserving delta-function.

1. ϕ^3 theory

Consider the Lagrangian density for a single massive real scalar field, with cubic interaction :

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3 \tag{1}$$

1. Write down the Feynman rules.

2. Write down the diagrams which contribute to $2 \to 2$ scattering to lowest order in perturbation theory (tree level). From these diagrams, and dimensional analysis alone, estimate the differential cross section for $2 \to 2$ in the center-of-mass frame (in terms of the typical energy/transfered momentum and the coupling constant).
3. Evaluate the diagrams above and compute the $2 \to 2$ scattering amplitude in terms of the incoming and out-going momenta.
4. Write down (without computing them) the one-loop diagrams which contribut to the same process at the next order.

2. Scalar Yukawa theory

Cosinder a theory with one complex scalar ψ of mass M (the "nucleon") and one real scalar ϕ of mass m (the "meson"), with Lagrangian given by :

$$\mathcal{L} = (\partial^\mu \psi)^* \partial_\mu \psi - M^2 \psi^* \psi + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - g \phi \psi^* \psi \tag{2}$$

This is a toy-model for scalar electrodynamics, which is a toy model for QED.

1. Write down the Feynman rules.
2. Write down the diagrams which contribute to the following processes, at tree level, give the corresponding scattering amplitude, and estimate, by dimensional analysis, the corresponding cross section.
   — Nucleon-nucleon scattering ($\psi\psi \to \psi\psi$)
   — Nucleon-meson scattering ($\psi\phi \to \psi\phi$)
   — Nucleon-antinucleon annihilation into two mesons ($\psi\bar{\psi} \to \phi\phi$)
   — Nucleon-antinucleon annihilation into nucleon-antinucleon pair ($\psi\bar{\psi} \to \psi\bar{\psi}$)
3. Is meson-meson scattering ($\phi\phi \to \phi\phi$) possible at tree-level ? Draw the lowest order diagram contributing for this process.
4. Suppose that the meson is very heavy, $m \gg M$.
   i. Show that, for "low energy" processes, (i.e. $E_{CM} < m$), we can replace the Feynman diagrams which contribute to nucleon-nucleon interactions by those obtained by an effective pointlike interaction involving only nucleons. What is the corresponding term in the Lagrangian ?
   ii. What is the dimension of the corresponging operator/coupling constant ?
   iii From the validity of perturbation theory, up to which energy can we trust this "effective" theory with only nucleons ?