Detector physics – NPAC 2022-2023

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Hello

- My name's Mat Charles.
  I work at the LPNHE lab at Sorbonne Université (at Jussieu)
- I'm an experimental particle physicist.
- I'm part of the LHCb collaboration, using the LHC at CERN.
  - We work on flavour physics (CP violation, rare decays, spectroscopy, etc), mostly of b and c hadrons.
- In the past, I also worked on the BABAR experiment, and on calorimeter reconstruction algorithms for a future linear collider detector.

Disclaimers & advice

- I'll probably make some mistakes during these lectures.
  - If you spot one, please let me know (in real time, or later).
  - If unsure, please ask! Or check a second source*.
- Please ask questions. (In English or French or Franglais.)
- Slides draw on work of previous lecturers (Jean-Paul Tavernet, Julien Bolmont, Sébastien Procureur), on textbook by Leo, on PDG review.
- These lectures aren't meant to be exhaustive, but to help you understand the physics and concepts behind detectors, and to give you the tools to understand what follows.
  - Will try to include key formulae for reference...
  - ... but honestly, if you need to (e.g.) calculate dE/dx to 1%, you should look up a specialised reference.

* Motto of Royal Society: "Nullius in verba"

Plan

- Second lecture (this one):
  - Some practicalities/admin
  - Interaction of charged particles in matter
- Third lecture: interaction of photons in matter, scintillators
- Fourth lecture: photodetectors, interactions of other neutral particles in matter

[This is a bit changed from last year, to try and get you some scintillator info during the first week of the TL. Please let us know in the end-of-semester feedback if this is useful or not.]

Practicalities: Mini-stages

Will be based on articles proposed by researchers (tutors) in Paris laboratories.

- The three lecturers (Philippe, Thomas, myself) will build a list of topics and send it to you.
- You will have to choose a topic by/on 11 October
- 2 students per topic (en binôme), or 3 if too few topics.

The mini-stage itself:

- Bibliography/TD session on Tue 25 October (14h-17h) at Orsay building 100, for reading the article, initial bibliography, Q&A to lecturers, discussion in binôme
- Then, two meetings with the tutors among these 3 slots:
  - Tue 8 November (14h-17h)
  - Tue 22 November (14h-17h)
  - Tue 29 November (14h-17h)
- Presentations (one group at a time) on Tue 13 + Wed 14 Dec [TBC]. Everybody must speak!

Practicalities: Mini-stages

Instructions:

- Preparation: read the article, discuss it with your tutor.
- Prepare a presentation. Your talk will be 20 min long, followed by 15 minutes of questions/discussion with the jury.
- Guidance for the presentation:
  - Place the article in context
  - Give the basic principles of the detector, how it works, advantages and drawbacks
  - Present in detail one of the results of the article, or all of the results (depending on the length of the article)
  - Discuss how the detector is used (or will be used)

Practicalities: Exams

I can't speak for the other lecturers, but for these three lectures*:

- I may set questions on the **physics** of anything we cover.
- I may set questions on the **operational principles** of any of the detector types we discuss.
- I won't ask for technical details of **specific detectors**.

For example, I may ask questions about scintillation detectors including how deep you would design one for a certain energy range, but I won't ask for the depth of the BABAR ECAL subdetector.

The exam will be in 3 parts, one part set by each lecturer, and each part accounting for roughly 1/3 of the exam marks.

The exam length will be 3 hours. Documents (including the PDG) are forbidden. Small ("college"-type) calculators are allowed, but not programmable calculators, smartphones, etc.

* Basic philosophy: there are some things that all HEP experimentalists should know -- you should know the jargon, be able to follow discussions of detectors, and be able to do quick, quantitative estimates of their performance (including knowing rough values for some key quantities). Memorising formulae is less important than understanding the principles behind them, and how they scale.

Practicalities: Evaluation

1/3 of the final mark will be based on the written exam (on 17 November)

2/3 of the final mark will be based on the oral exam in December (your presentation, your responses in the Q&A/ discussion with the jury)

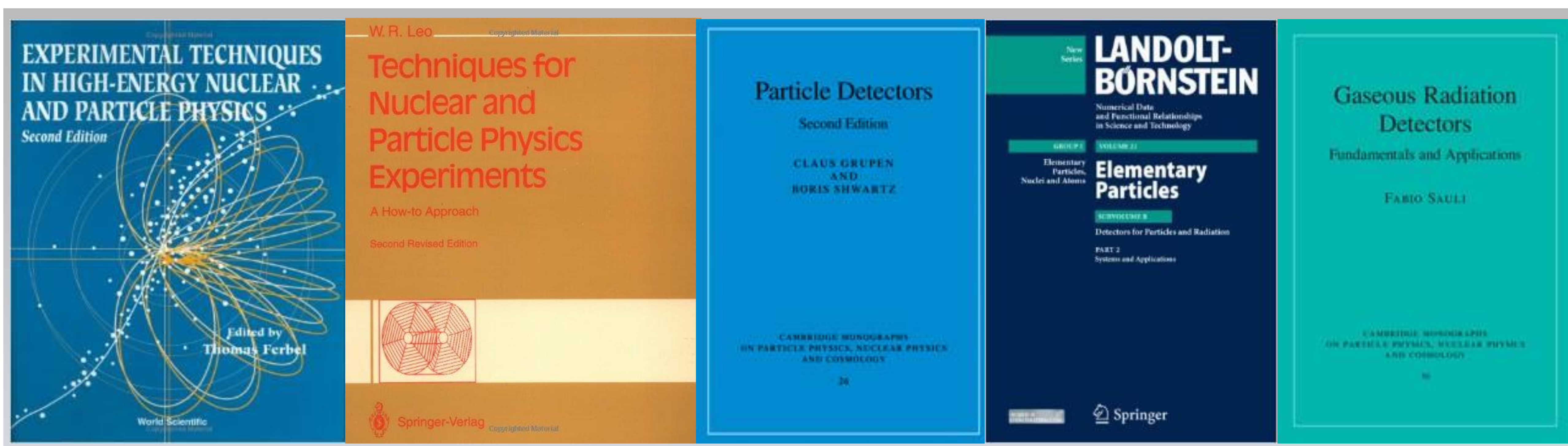
Practicalities: Where to learn more

There are lots of sources of information on detector physics, depending on what you need.

- A textbook if you need to get an overview, or understand the basic concepts. Should be copies in lab libraries.
  - Personal fave: "Techniques for Nuclear and Particle Physics Experiments" by W.R. Leo
- Slides/notes from CERN lectures on detector physics for summer students, or from summer schools, for a pedagogical overview.
- PDG reviews are a good place to go for more information on a topic and key formulae. (Lots of info, but very dense.)
- Technical papers (physics.ins-det) for latest on a particular technology.
- Detector performance papers and conference talks for a particular experiment's detector.

Practicalities: Textbooks

- Experimental Techniques in HEP, T. Ferbel, Ed. Frontiers in Physics (1991)
- Techniques for Nuclear and Particle Physics Experiments, 2nd edition, W.R. Leo, Springer (1994)
- Particle Detectors, 2nd edition, C. Grupen & B. Schwartz, CUP (2008)
- Gaseous Radiation Detectors, F. Sauli, CUP (2014)



List of references from Sébastien Procureur

Detector physics – NPAC 2021-2022

Detector physics – NPAC 2021-2022

- What do we mean by a "detector"?
- Why do we need/want one?
- What kind of detector do we want?
- How should it behave?
- How does it really behave?
- How does its performance affect our measurements?
- … and why should you care?

An undergrad/M1 PP exam question

4. The K^0 meson has a mass of $497.6\,\mathrm{MeV}/c^2$ and it decays into two charged pions of mass $139.6\,\mathrm{MeV}/c^2$. What is the energy of a pion as observed in the rest frame of the K^0?

The K^0 lifetime is $0.89 \times 10^{-10}\,\mathrm{s}$. State what interaction is responsible for the decay, and justify briefly your answer. [5]

(Question borrowed from Victoria Martin, Edinburgh ; source)

That's a fine L3/M1-level question. No doubt you can all do it easily!

But what would $K^0 \rightarrow \pi^+\pi^-$ look like experimentally?

- How would we make a K^0 meson? (→ accelerators lectures)
- How do we know we have a real K^0?
- What particles can we actually detect? How do we do so?
- What properties will we measure, and how well? How do we do that?
- What kind (technology) of detector could we use?
  What are the options, and what are the advantages/disadvantages of each?
- What sorts of backgrounds will there be? How do we suppress them?

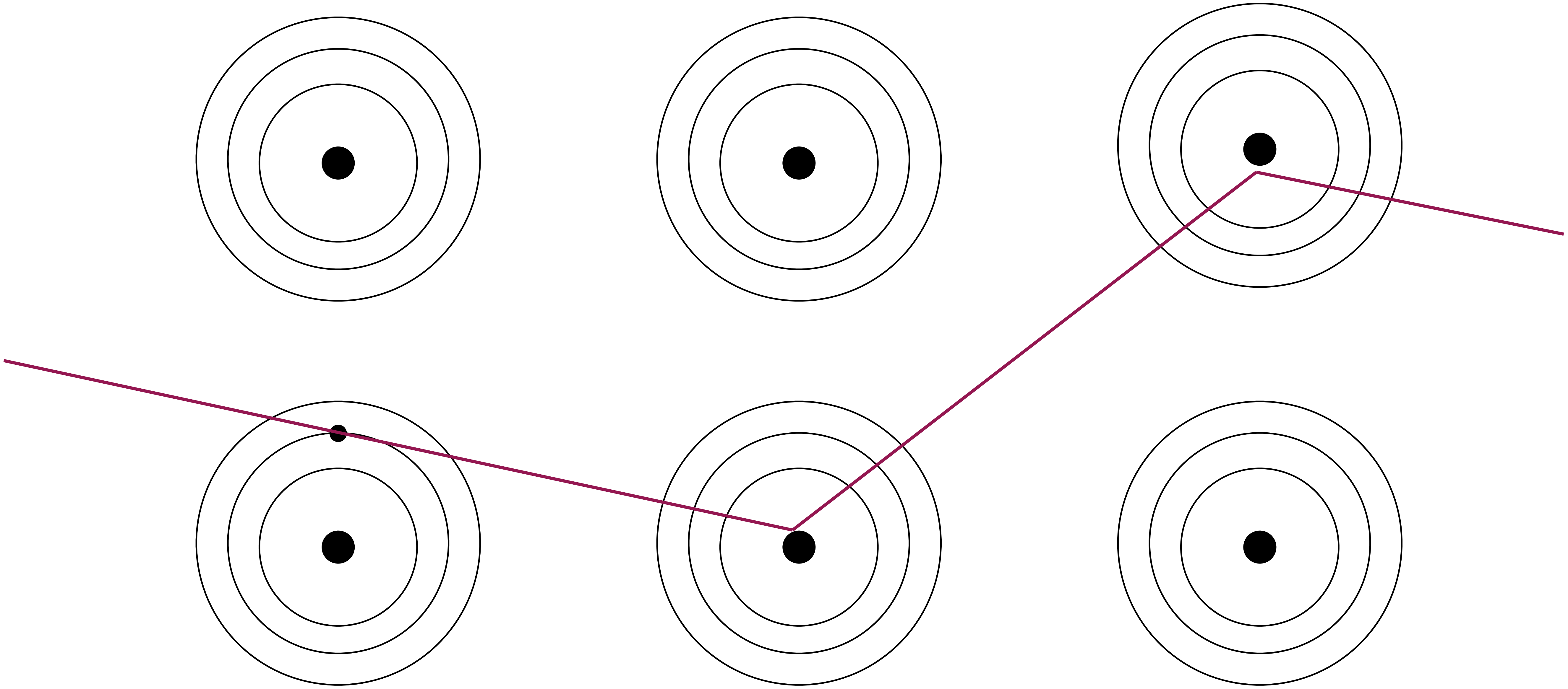
By the end of the course, you should be able to answer these kinds of question.

Detector physics

- There are many kinds of detectors, and many, many actual detectors…
- … but they all rely on the same basic physics, i.e. how particles interact with matter.
  - We'll go through this in the next lecture.
- You'll find that many detectors look different but actually rely on the same few tricks; once you understand the operating principles, you understand the detector.
- That said, there are a lot of subtleties, and some conventions/ jargon to learn.
  - RPC, GEM, RICH, APD, blablabla.
  - And some of the "basic physics" is not so simple/obvious.

Interactions of charged particles in matter

How can a charged particle interact in matter?



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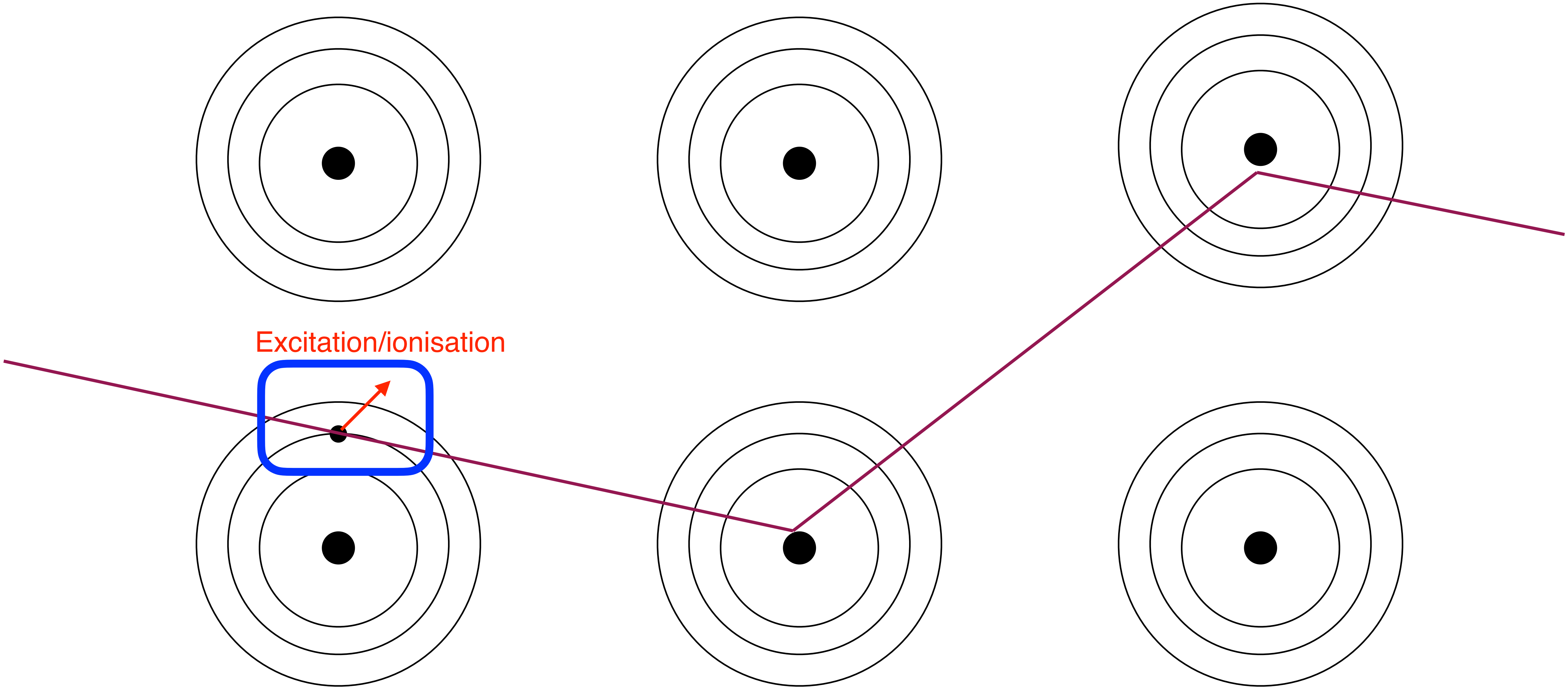
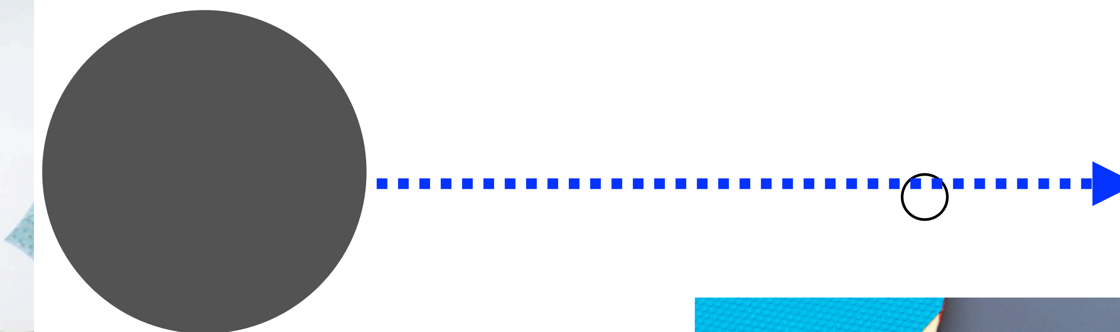


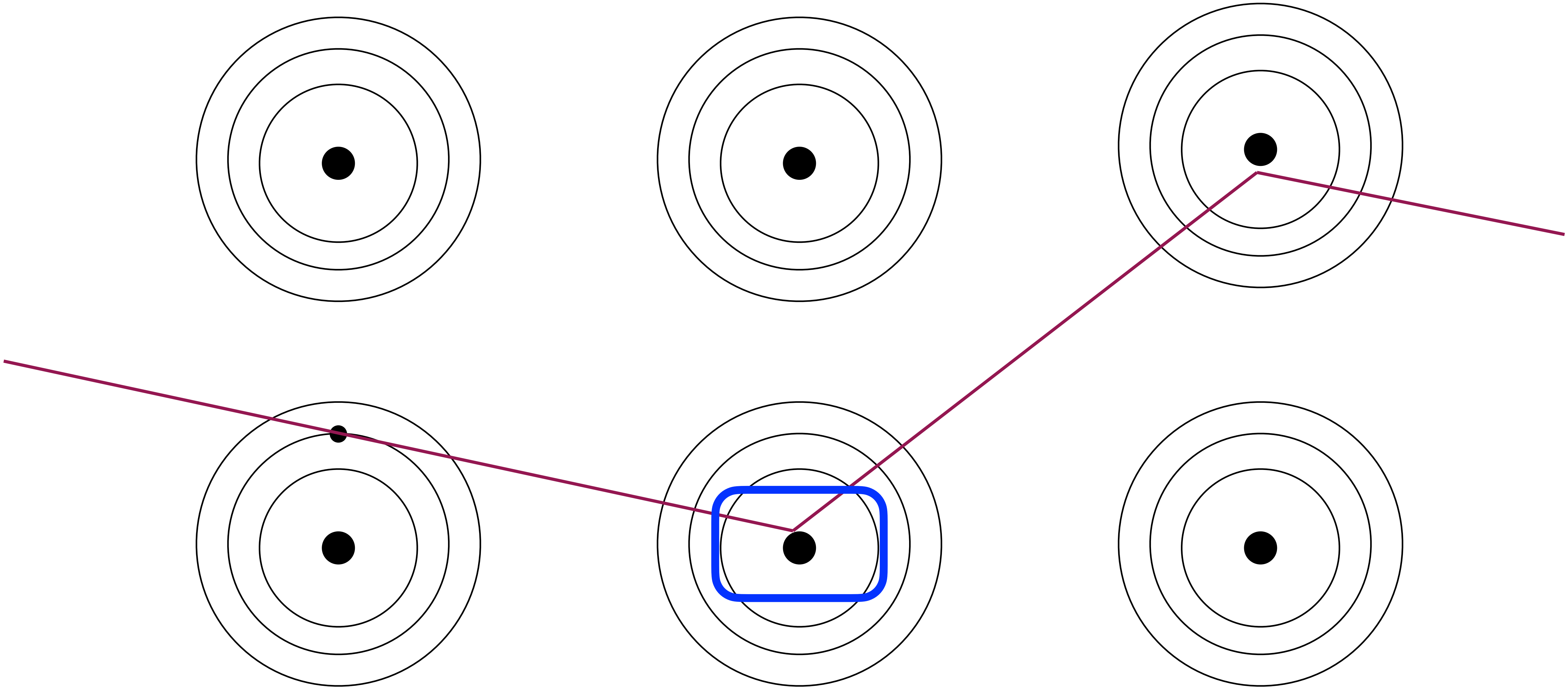
1) It can interact electromagnetically with an atomic electron and transfer energy to it. This can cause an excitation (electron moves to a higher orbital) or an ionisation (electron separated from the atom). Heavy particles will not be deflected much...

Heavy particle not significantly deflected by an electron.



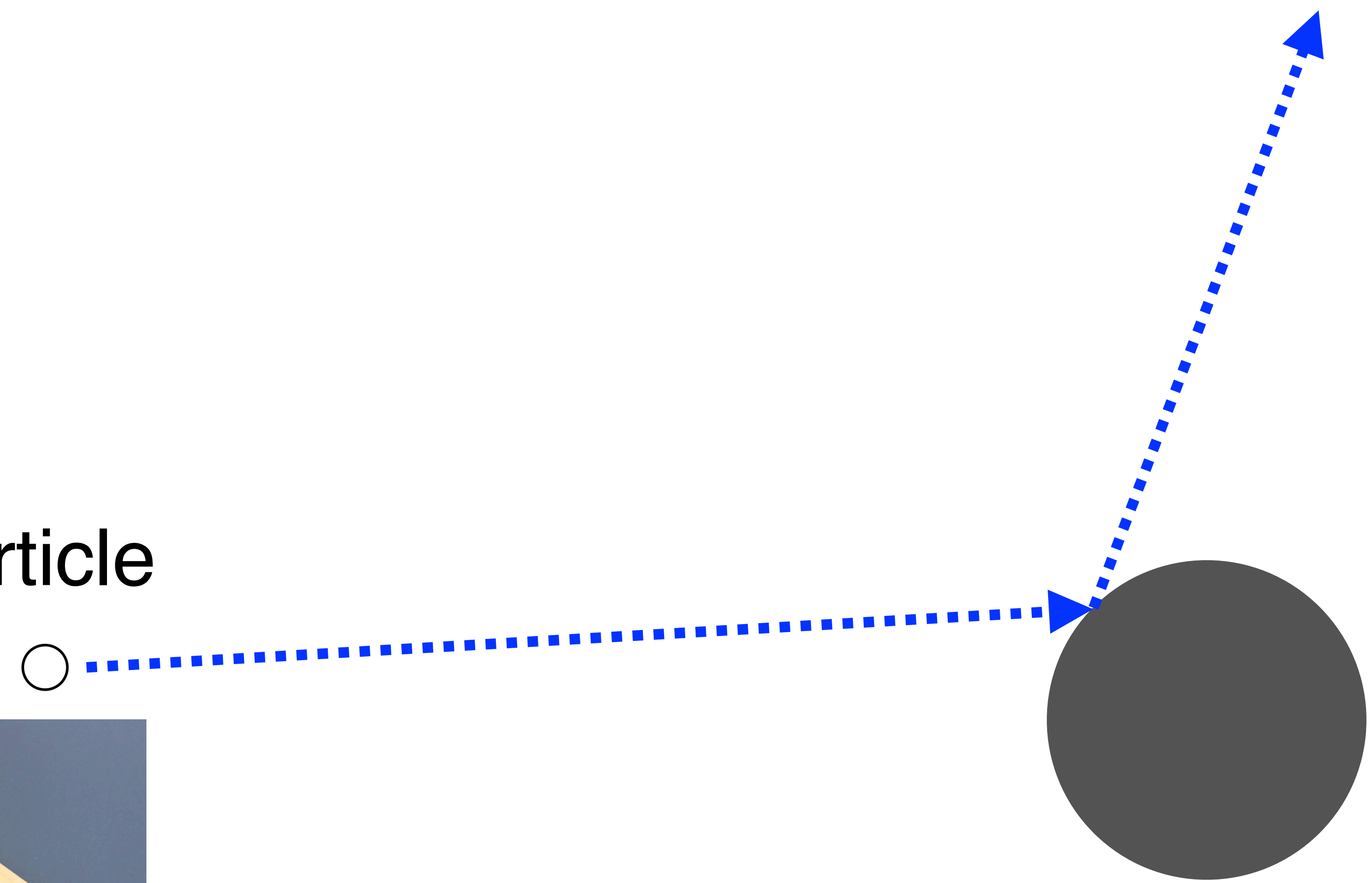
(Why is it not an elastic collision? Because the electron is in the electric field of the nucleus, so displacing/ejecting it means exchanging kinetic for potential energy. For a free electron this would be an elastic collision.)

How can a charged particle interact in matter?



2) It can interact electromagnetically with a nucleus and be deflected (scattered) in an ~ elastic collision...

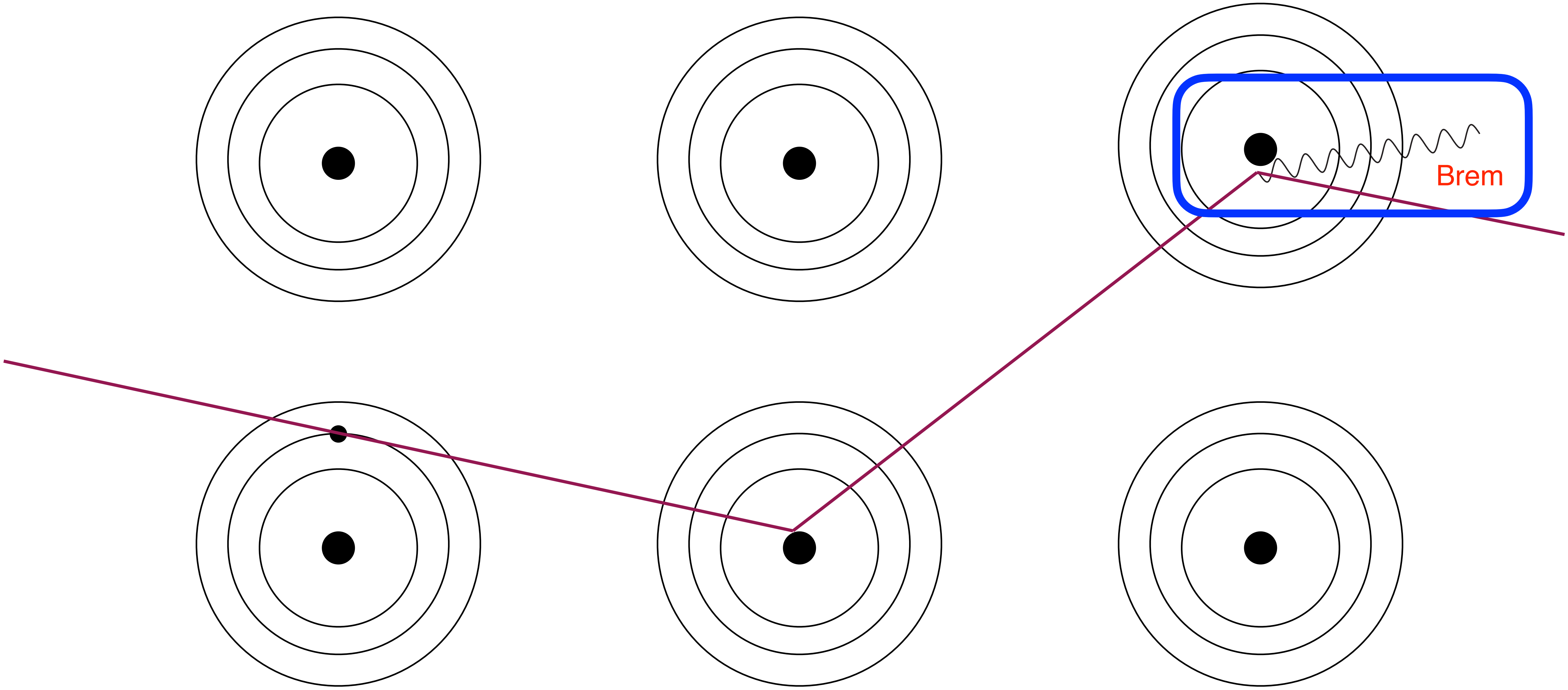
Incoming particle



Heavy nucleus

Particle significantly deflected by heavier nucleus, ~ elastic collision, almost no energy transfer due to difference in mass.

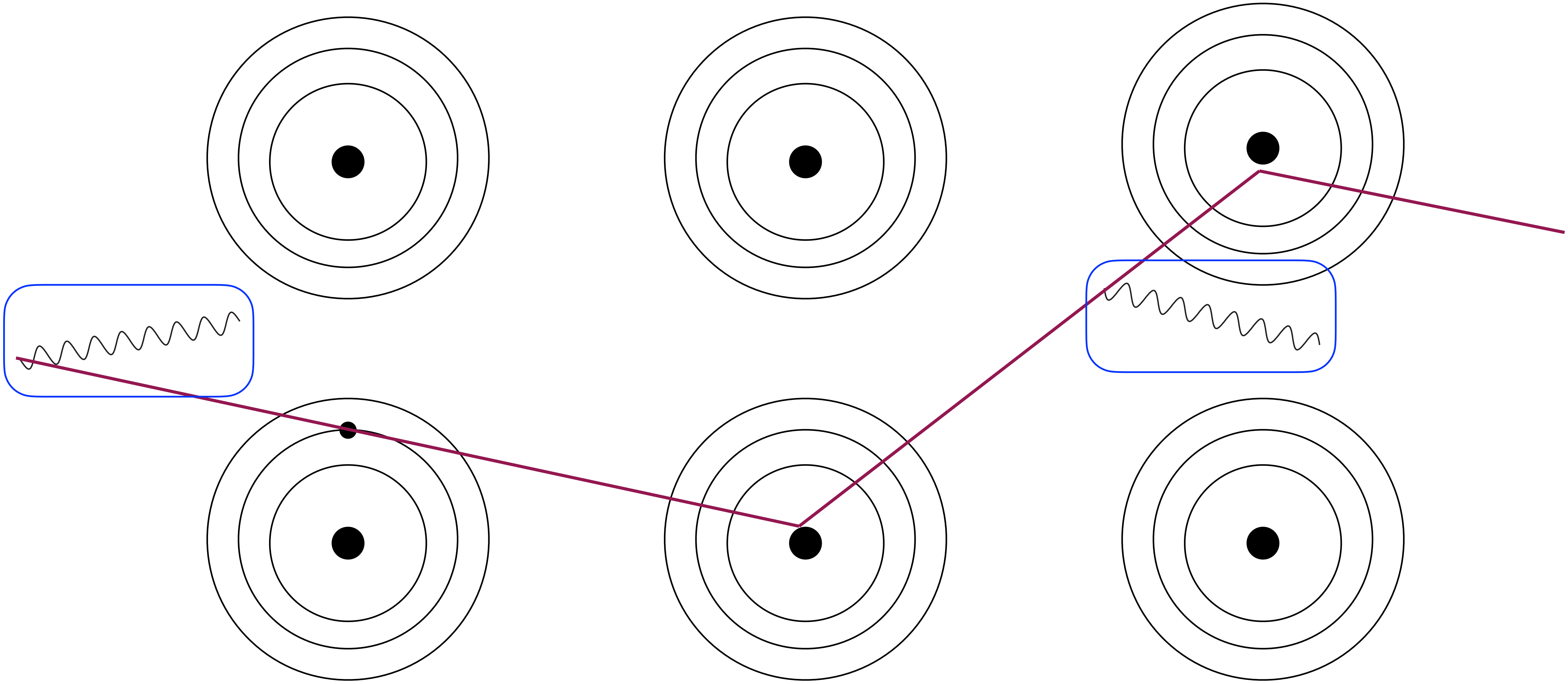
How can a charged particle interact in matter?



2) It can interact electromagnetically with a nucleus and be deflected (scattered) in an ~ elastic collision...

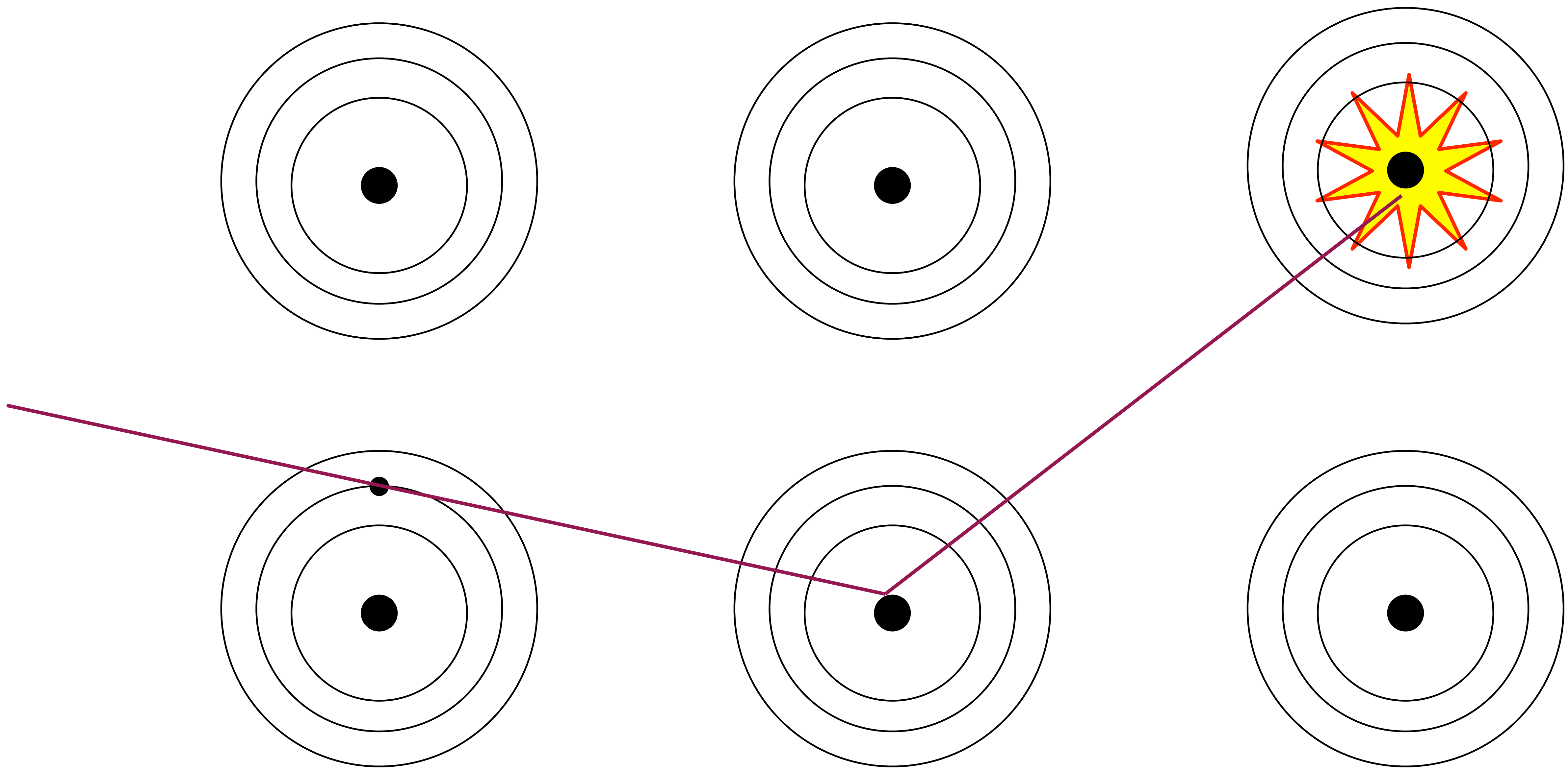
3) ... and it may emit radiation (bremsstrahlung, braking radiation) in the EM field of the nucleus, losing energy.

How can a charged particle interact in matter?



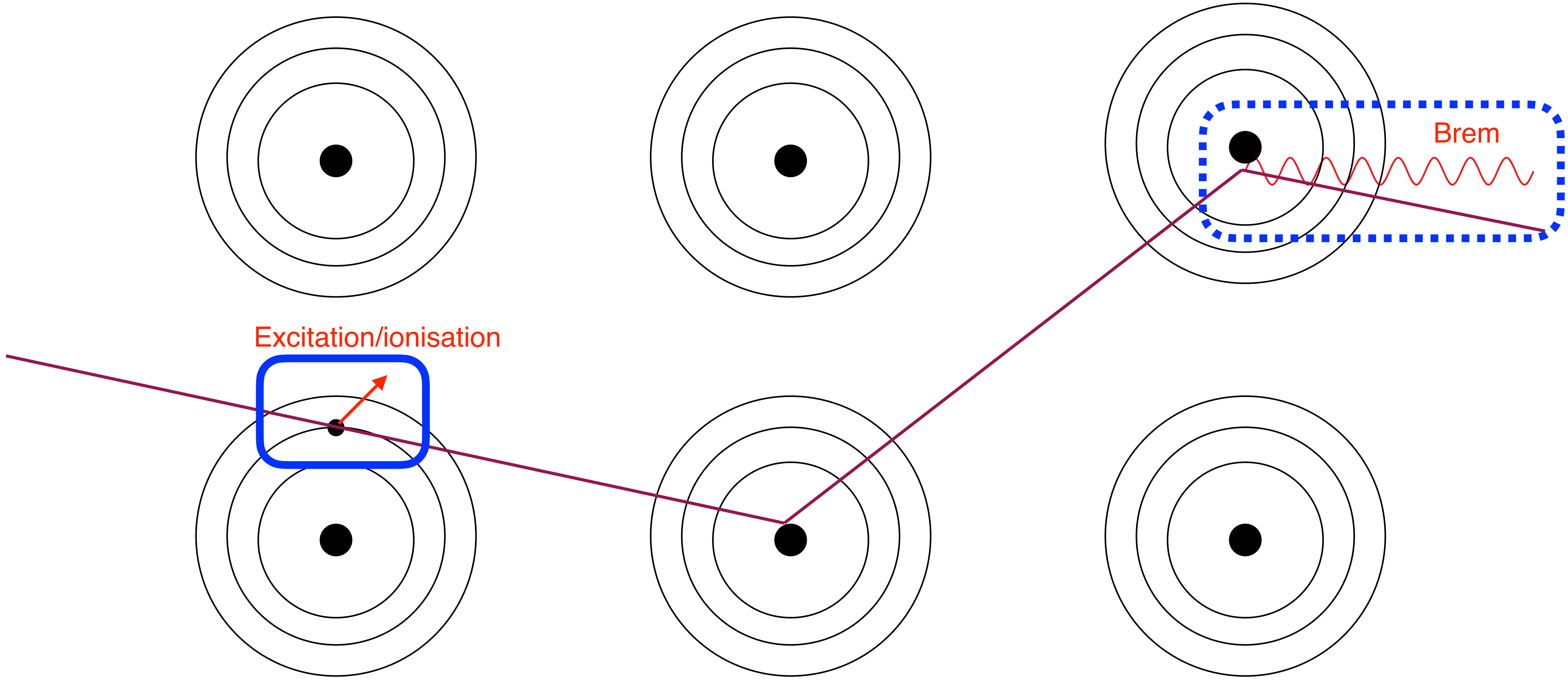
4) Depending on conditions, it might emit EM radiation inside the medium (Cherenkov radiation), or at an interface between media (transition radiation). These don't significantly affect the total energy loss.

How can a charged particle interact in matter?



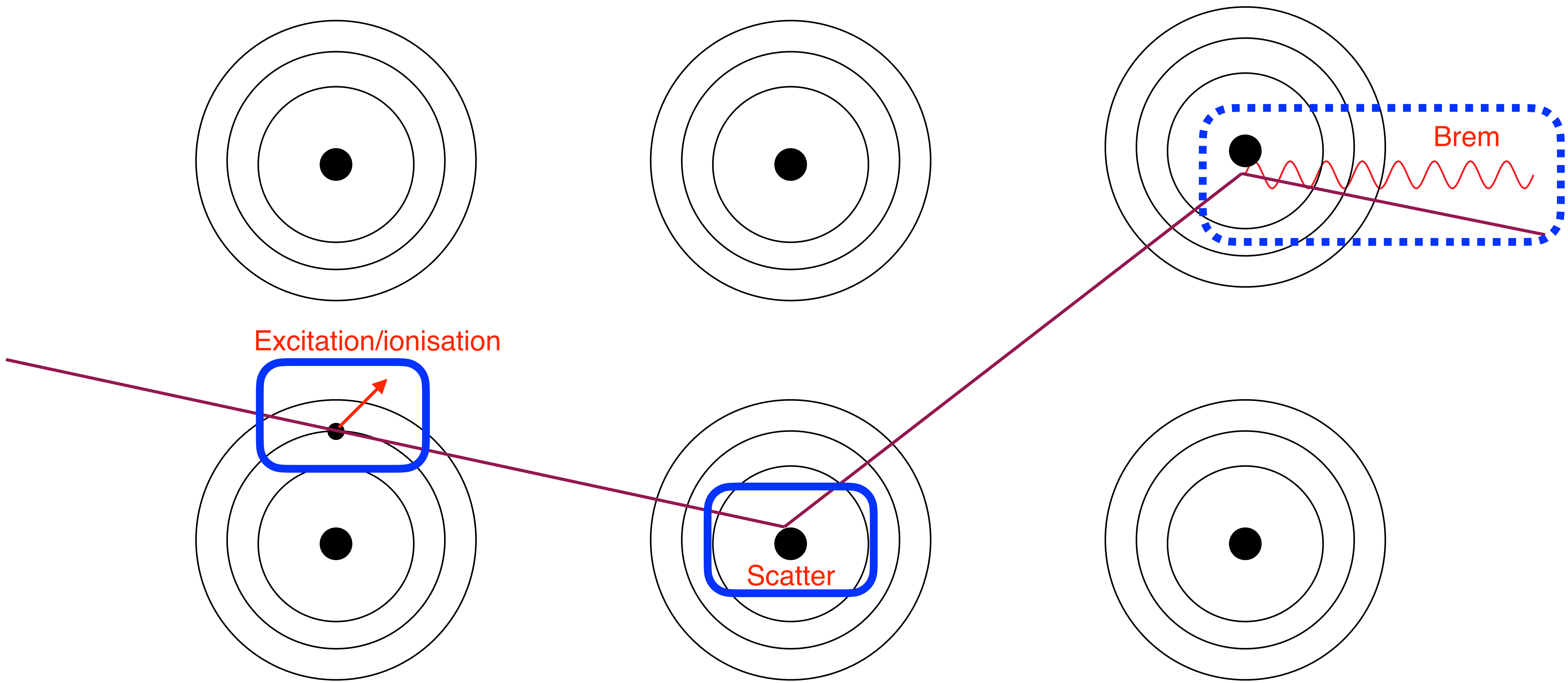
5) Other interactions are possible, depending on the particle type -- e.g. hadrons can interact strongly with the nucleus, or a positron can annihilate with an electron. These have a smaller cross-section but are often more violent. We'll come back to these later.

Significant sources of energy loss:



- Coulomb interactions with atomic electrons (excitation/ ionisation)
- At higher $\beta\gamma$ (meaning ≥ medium energy for electrons, very high energy for anything else), bremsstrahlung.

Significant sources of deflection:



- Excitation/ionisation
- Bremsstrahlung (at high $\beta\gamma$)
- Elastic scattering from nuclear

Energy loss of charged particles

- A particle moves a short distance dx through material.
- It started with energy E_i and finished with energy E_f
- Change in energy is $\mathrm{d}E = E_f - E_i < 0$
- The energy loss per unit length is
  $$-\frac{\mathrm{d}E}{\mathrm{d}x} > 0$$
- But that's just for one particle, and could fluctuate. We usually work with the mean/expected energy loss:
  $$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle$$
- We'll often drop the $\langle\ \rangle$ here for simplicity, but they're still implied; using expected/average/mean energy loss.

Energy loss and stopping power

- As we'll see shortly, it's convenient to work with stopping power, which is just dE/dx divided by the density:

$$\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle$$

- Units of dE/dx are: MeV cm^{-1}
  Units of stopping power are: MeV g^{-1} cm^2

- Here we'll write

$$\frac{\mathrm{d}E}{\mathrm{d}X} \equiv \frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x}$$

  to distinguish them, but in the literature you'll often see stopping power written as "dE/dx". Watch the units closely to see which is which.

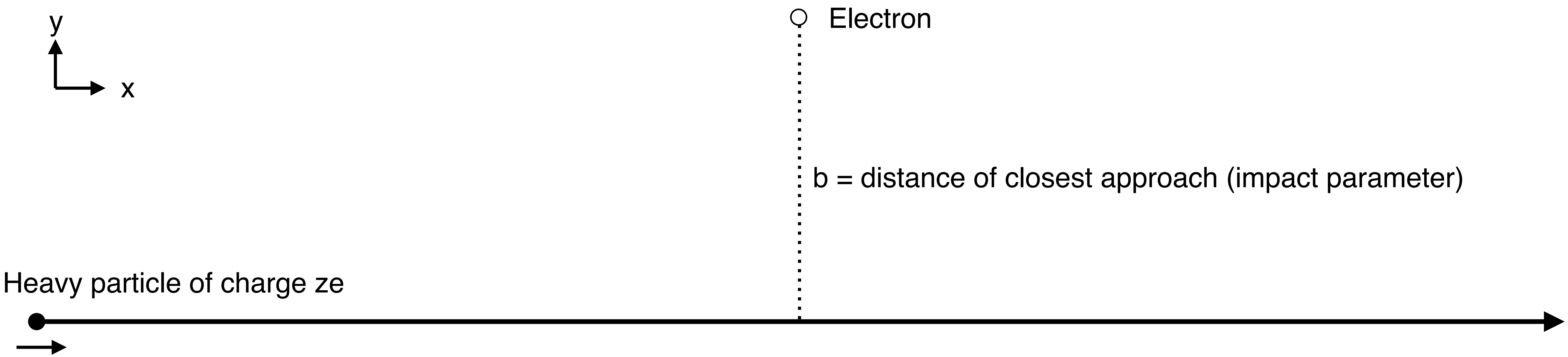
Energy loss for a heavy particle

- A particle moves a short distance dx through material. How do we calculate the expected energy loss?
- It should depend on:
  - The parameters of the incoming particle
  - The properties of the target material
- The general solution is a really hard problem, but we can make physics arguments to simplify the problem:
  - Assume the particle is heavy compared to the electrons, but not heavy compared to the nuclei.
  - Assume bremsstrahlung is negligible (for now).
  - Then the energy loss is dominated by inelastic Coulomb interactions with an atomic electron.
  - We know how electromagnetism works, so we can calculate the energy transfer (for a simple case).

Coulomb energy transfer to one electron

- Initial assumptions:
  - Assume the electron is free and at rest.
    (It's not actually free, but just to calculate the energy transfer.)
  - Assume the electron does not move significantly during the interaction.
    (Thus, we only need to calculate the electric field at its initial position.)
  - Assume the incident heavy particle is undeviated from its initial path
    (since its mass $M \gg m_e$)
- With these assumptions, we can do a calculation using classical electromagnetism (per Bohr).
  - I will hand-wave some steps; if you want more, see Leo or Jackson.
  - You don't need to memorise this! But you should understand it.
- It's going to give us the wrong answer, because these are not really classical pointlike objects and QM has an effect...
- ... but it's close, and it's useful to see where things come from (and the limits of the assumptions' validity).
- The real formula (Bethe-Bloch) follows shortly.

Coulomb energy transfer to one electron



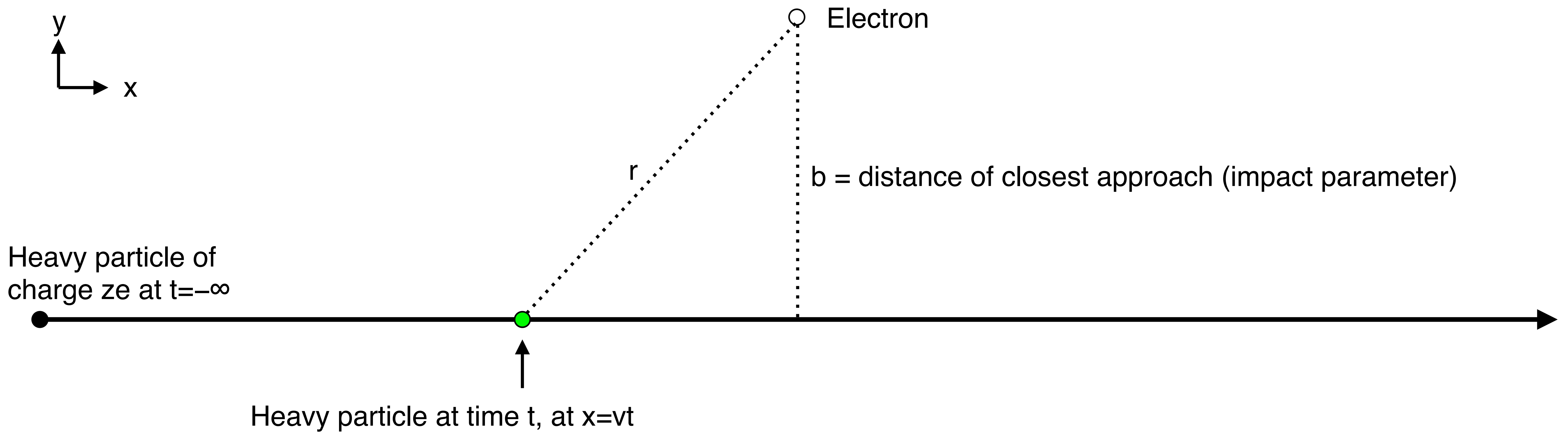
The incoming particle starts at x=–∞ and travels to x=+∞ (i.e. far enough away that its field at the electron is negligible at start & end of path).

The electron is at x=0.

We wish to calculate the momentum transfer (impulse) to the electron.

Because the electron is assumed to be static during the interaction, the problem has a forward-backward symmetry and there will be no net impulse in the x direction. Thus, we only need to calculate the force in the y direction (perpendicular to the incoming particle's path).

Coulomb energy transfer to one electron



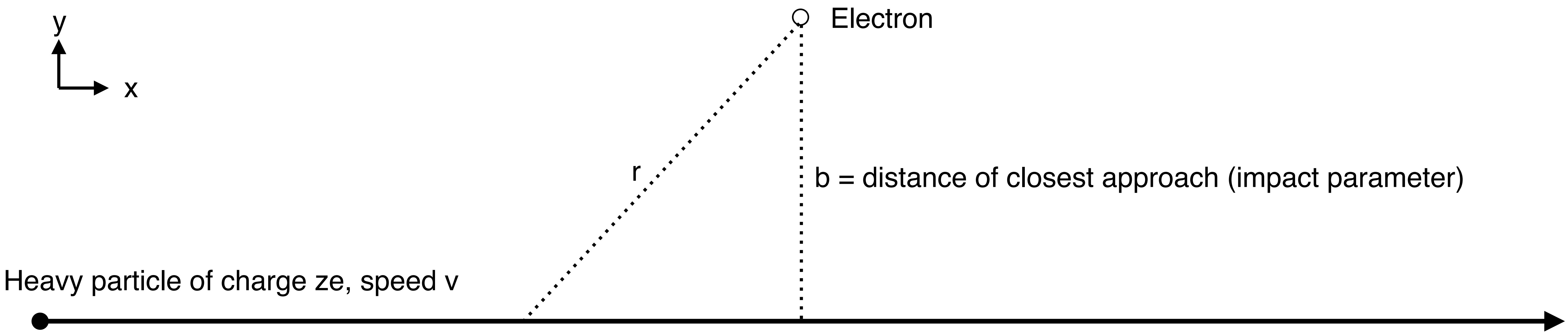
The impulse transfer can be calculated by brute force:

For symmetry, suppose the heavy particle passes through x=0 at t=0:

$$F(t) = \frac{q_e q_h}{r^2} = \frac{ze^2}{b^2 + (vt)^2}$$ (dropping factor of $4\pi\epsilon_0$, ignoring signs)

$$\int_{-\infty}^{+\infty} F_\perp(t)\,\mathrm{d}t = 2\int_0^{+\infty} F_\perp(t)\,\mathrm{d}t = 2\int_0^{+\infty} F(t)\frac{b}{\sqrt{b^2+(vt)^2}}\,\mathrm{d}t = \frac{2\,z\,e^2}{bv}$$

Coulomb energy transfer to one electron



Alternatively, there's a nice proof using Gauss's law, applied to a cylinder whose axis is the particle's path and whose surface includes the electron:

$$I = \int F \, \mathrm{d}t = e \int E_\perp \, \mathrm{d}t = e \int E_\perp \frac{\mathrm{d}t}{\mathrm{d}x} \, \mathrm{d}x = e \int \frac{E_\perp}{v} \, \mathrm{d}x = \frac{e}{v} \int E_\perp \, \mathrm{d}x$$

only need perp component of field

change variables

v assumed constant

But Gauss $\Rightarrow \oiint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = 4\pi z e$ and $\mathrm{d}A = (2\pi b)\,\mathrm{d}x$

(dropping factor of $4\pi\epsilon_0$, ignoring signs)

$$\Rightarrow \int E_\perp 2\pi b \, \mathrm{d}x = 4\pi z e \Rightarrow \int E_\perp \, \mathrm{d}x = \frac{2ze}{b} \Rightarrow I = \frac{e}{v} \int E_\perp \, \mathrm{d}x = \frac{2ze^2}{bv}$$

Coulomb energy transfer to one electron

- Either way, we get:

$$I = \frac{2ze^2}{bv}$$

where I is the momentum transfer from a heavy particle of charge ze, speed v, impact parameter b w.r.t. the electron.

- The electron was initially free and at rest, so its KE gained is

$$T = \frac{I^2}{2m_e} = \left(\frac{4z^2e^4}{b^2v^2}\right)\frac{1}{2m_e} = \boxed{\frac{2\,z^2\,e^4}{m_e\,v^2\,b^2}}$$

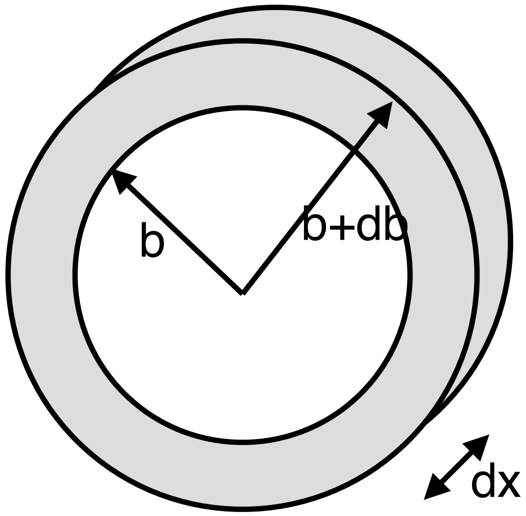
- This is the classical energy transfer for a single interaction.
- Note that it depends on the particle charge (z^2) and speed ($1/v^2$), and on the impact parameter ($1/b^2$), but not on the particle's mass (so long as $M \gg m_e$).

Coulomb energy transfer to electrons

- We've got the energy transfer for a heavy particle interacting with a single electron it passes in the material.
- But there are many electrons! In principle our charged particle sees them all. How do we take them all into account?

Coulomb energy transfer to electrons

- We've got the energy transfer for a heavy particle interacting with a single electron it passes in the material.
- But there are many electrons! In principle our charged particle sees them all. How do we take them all into account?
- Let's consider a thin ring of material around the particle's path, of radial thickness db and longitudinal thickness dx:



- The number of electrons in this ring is $(2\pi b\, \mathrm{d}b\, \mathrm{d}x)n_e$, where n_e is the density of electrons in the material.
- Each gets an energy transfer $T = 2\, z^2\, e^4 / m_e\, v^2\, b^2$
- So total energy transfer to the element is

$$\mathrm{d}E = \frac{4\pi z^2 e^4}{m_e v^2} n_e \left(\frac{\mathrm{d}b}{b} \right) \mathrm{d}x$$

... so we just need to integrate db from 0 to ∞, right?

Coulomb energy transfer to electrons

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Coulomb energy transfer to electrons

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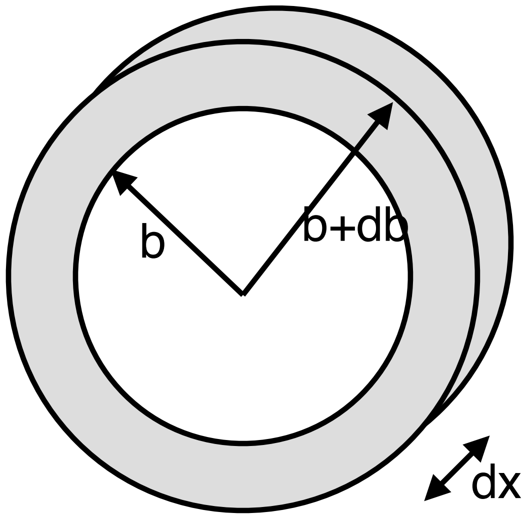

- If you try to integrate that from b=0 to b=∞, you will have divergence at both ends.
- What went wrong? We didn't respect the initial assumptions.
  - b → 0: energy transfer is large (and thus electron would move during the interaction); also electrons not perfectly localised in QM
  - b → ∞: assumption that electron is static and interaction time is short is not valid; also screening; also electron orbital energy is quantised and we can't transfer an arbitrarily small amount.
- How do we fix it? We integrate only over a physical range $(b_{\mathrm{min}}, b_{\mathrm{max}})$. This will give us a term like $\ln(b_{\mathrm{max}}/b_{\mathrm{min}})$.
- We'll estimate b_{min} and b_{max} shortly, but keep in mind that the energy loss only depends logarithmically on them, so the estimate just needs to be of the right order of magnitude.

Physical limits of Coulomb energy transfers

- For b_{min}, there is an upper limit on the kinematically allowed energy transfer => a lower limit on b.
  - Classically, limit in head-on collision is $2m_e v^2$ or $2\gamma^2 m_e v^2$ taking relativity into account
  - This sets a cut-off on dE, which implies a cut-off on b of

$$b_{\text{min}} = \frac{ze^2}{\gamma m_e v^2}$$

- For b_{max}, there is a lower limit on the physically allowed energy transfer, associated with the energy scale of the electron-nucleus binding (or the orbital frequency).
  - This sets a cut-off on b of

$$b_{\text{max}} = \frac{\gamma v}{\bar{\nu}}$$

where $\bar{\nu}$ is an average/typical value of the orbital frequency of the atomic electrons; $I = h\bar{\nu}$ is known as the mean excitation potential [not related to momentum transfer I from earlier!]

The Bohr Formula

Putting all of this together:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi z^2 e^4}{m_e v^2} n_e \ln\left(\frac{\gamma^2 m_e v^3}{z e^2 \bar{\nu}}\right)$$

Use a trick to relate electron density n_e to the mass density ρ.

- To a good approximation, 1 mol of C-12 weighs 12g, and the mass of an atomic scales almost linearly with its mass number A, so 1 mol of any atoms weighs about A grams.
- Thus, 1g contains about (1/A) mol = (N_A/A) atoms.
- 1cm^3 of any material weighs ρ grams, and thus contains about $(\rho N_A/A)$ atoms, and thus about $(Z\rho N_A/A)$ electrons.

So, finally, Bohr's (classical, wrong) formula is:

[Formulae from Leo]

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi z^2 e^4}{m_e v^2}\frac{Z}{A}\rho N_A \ln\left(\frac{\gamma^2 m_e v^3}{z e^2 \bar{\nu}}\right)$$

Aside: $-\frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x} \simeq \frac{Z}{A} \times f(z, v)$ and (Z/A) is very roughly constant across all materials. This is why we use stopping power: to first order, it's independent of the material.

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and (Z/A) is very roughly constant across all materials. This is why we use stopping power: to first order, it's independent of the material.

The Bethe-Bloch Formula

- That's nice, but it's wrong. What does a less wrong formula look like?

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi z^2 e^4}{m_e v^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

Completely equivalent, alternative form:

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi z^2 r_e^2 m_e c^2}{\beta^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

where the classical electron radius $r_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e c^2}$ (though be careful of convention for $4\pi\epsilon_0$!)

[Formulae from Leo. You don't need to memorise them, but you should understand them and know what terms are present and the rough scaling with key quantities such as Z, A, β.]

The Bethe-Bloch Formula

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi z^2 e^4}{m_e v^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

- The main functional dependence (left) is very similar to Bohr! Some new terms and corrections:
  - I is the mean excitation potential of the material (~ $h\bar{\nu}$) -- n.b. not impulse
  - $W_{\max}$ is the maximum energy transfer in a single collision:

$$W_{\max} = \frac{2m_e c^2(\beta\gamma)^2}{1 + 2\frac{m_e}{M}\sqrt{1+(\beta\gamma)^2} + \left(\frac{m_e}{M}\right)^2} \approx 2m_e c^2\beta^2\gamma^2 \text{ if } M \gg m_e$$

  - δ is the density correction (polarisation/shielding effect that limits force exerted on distant electrons; important at high energies)
  - C is the shell correction parameter (taking into account that atomic electrons are not really at rest; important at low energies)

[Formulae from Leo. You don't need to memorise them, but you should understand them and know what terms are present and the rough scaling with key quantities such as Z, A, β.]

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- This is a complex formula. Different parts dominate in different kinematic regions; we'll go through them shortly.
- As before, it's roughly constant across all materials (Z/A ~ const) and independent of the mass of the incoming heavy particle.
- Note the prefactor of z^2 : particles with more charge (e.g. α) will lose a lot more energy.
- Approximate domain of validity:
  - Compact particles, light ions up to alpha [or heavier, with corrections]
  - $\beta\gamma \gtrsim 0.05$ (particle velocity larger than electron velocity)
  - $\beta\gamma \lesssim 10^3$ (bremsstrahlung negligible)
  - Special case: channeling (particles following crystal channels in lattice) => much lower dE/dx at very low angles of incidence

The Bethe-Bloch Formula

$$-\frac{1}{\rho}\left\langle\frac{dE}{dx}\right\rangle = \frac{2\pi z^2 e^4}{m_e v^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

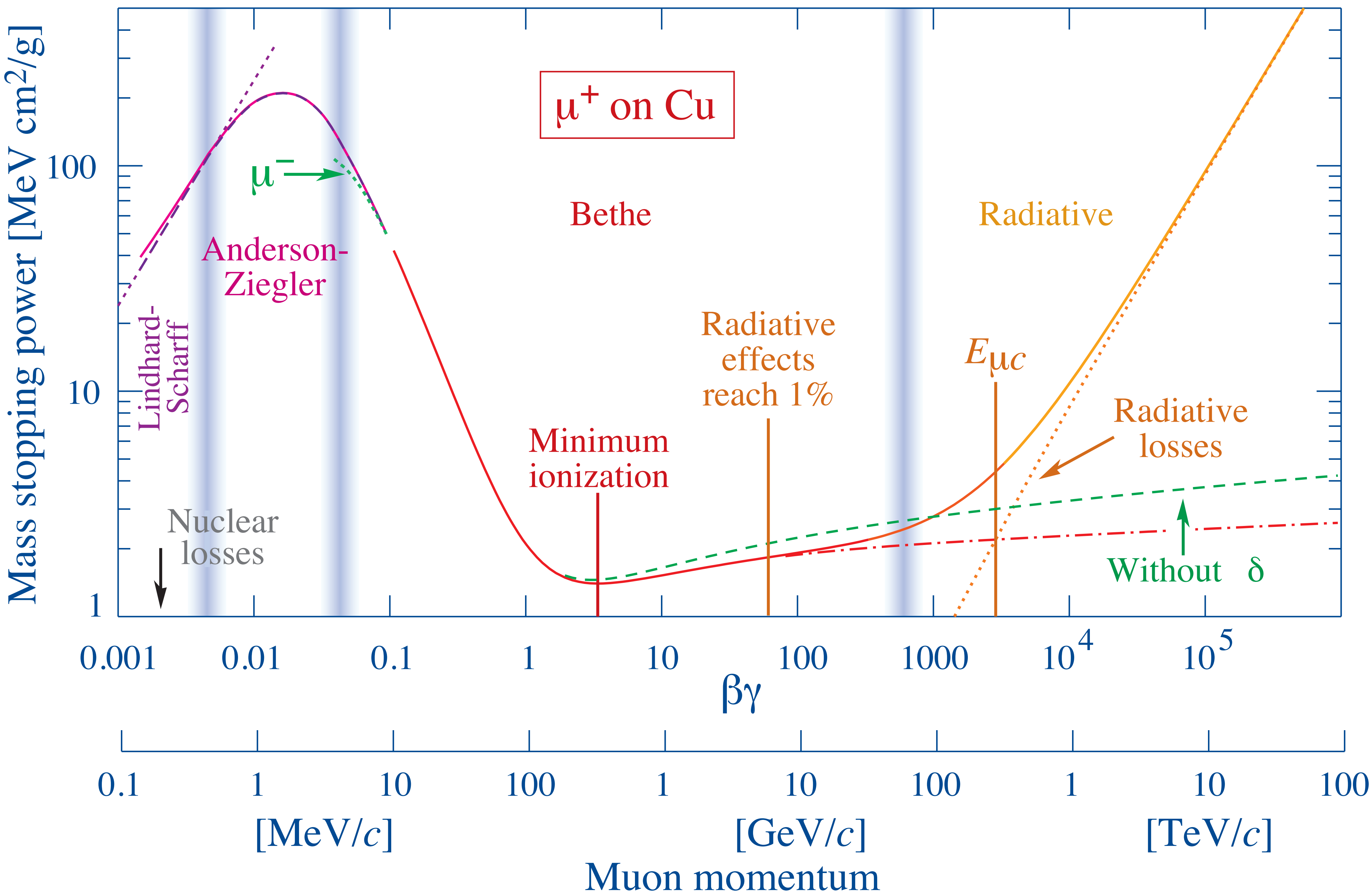

Figure from PDG; you should understand it and be able to describe its key features.

The Bethe-Bloch Formula

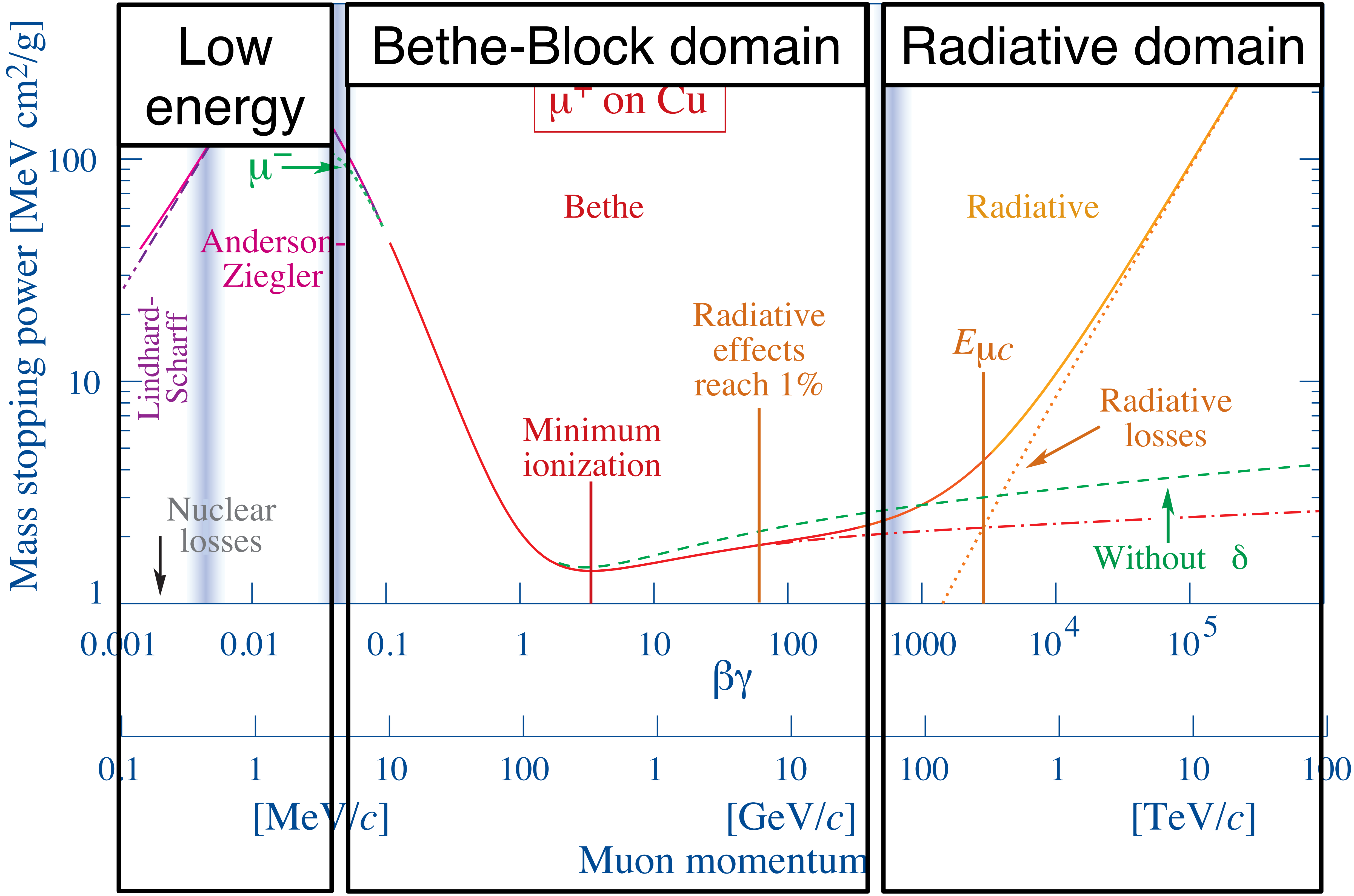

Figure from PDG

The Bethe-Bloch Formula

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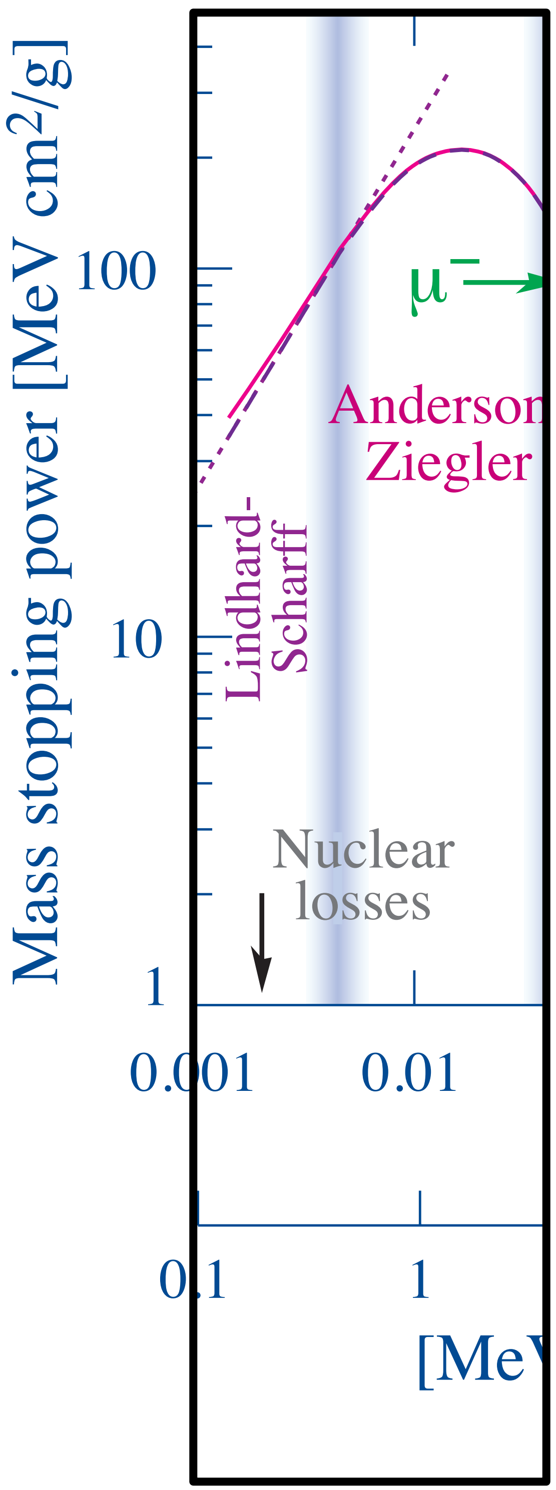
Shell correction



Complex low-energy region (not crucial to NPAC domain physics) in which one needs to take in account the form and the shielding of the nucleus, the valence electron velocity, ...

The Bethe-Bloch Formula

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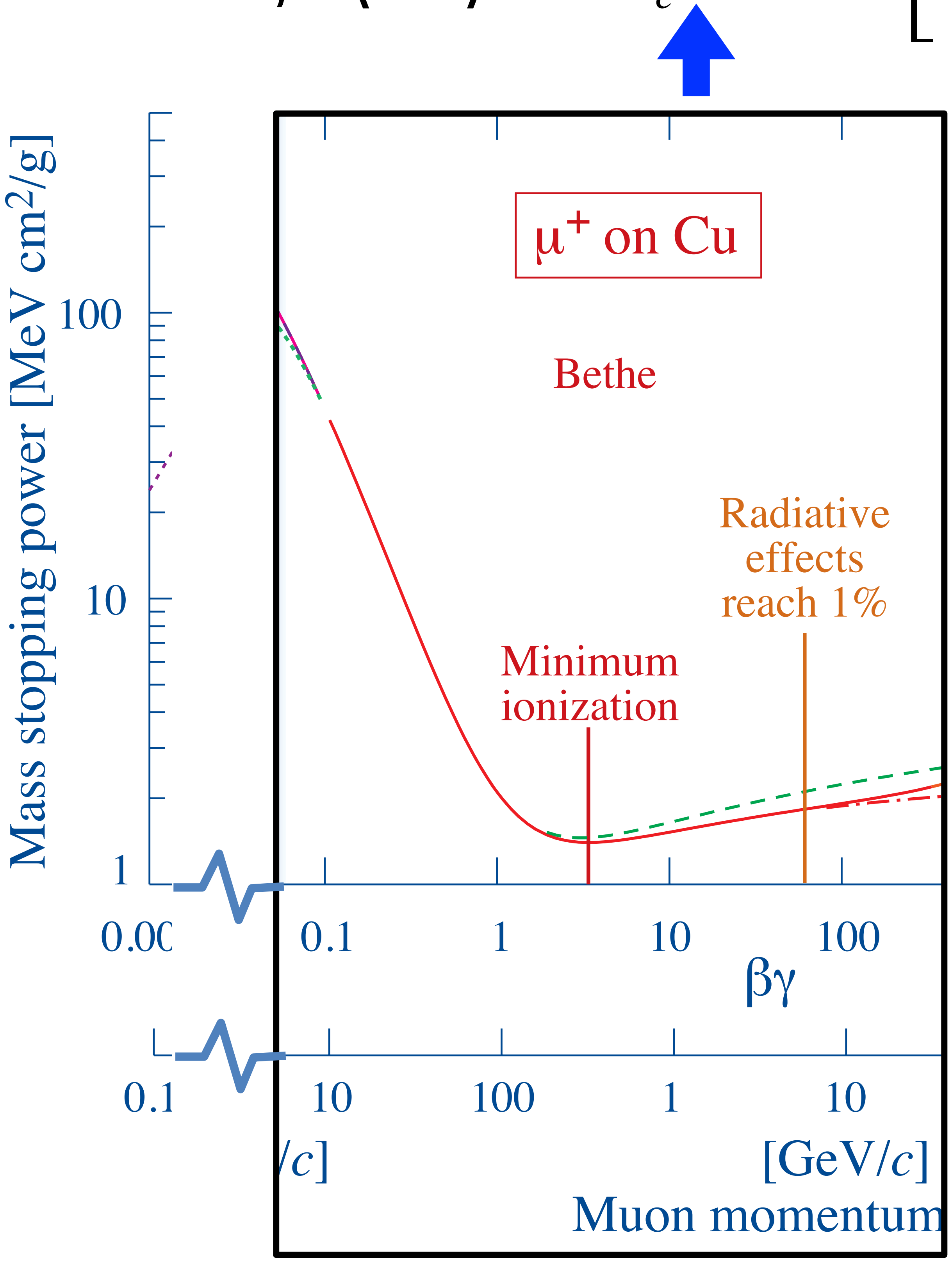

First, dE/dx falls like β^{-2} (kinematic term)

- Faster particles feel electric force of atomic electron for shorter time

... then minimum ionizing particle: $\beta\gamma \approx 3$

- Occurs around v = 0.96 c.
- Min value of stopping power is ~ same for all particles of same charge.

... then rises like ln(βγ) (relativistic rise)

- High energy particle: transverse electric field increases due to Lorentz transform $E_y=\gamma E_y^*$. Thus interaction cross section increases.

... saturates at large βγ (density effect)

- Transverse electric field increased, interaction 'far' from the particle track but polarization of the matter increases, thus the electric field is shielded.

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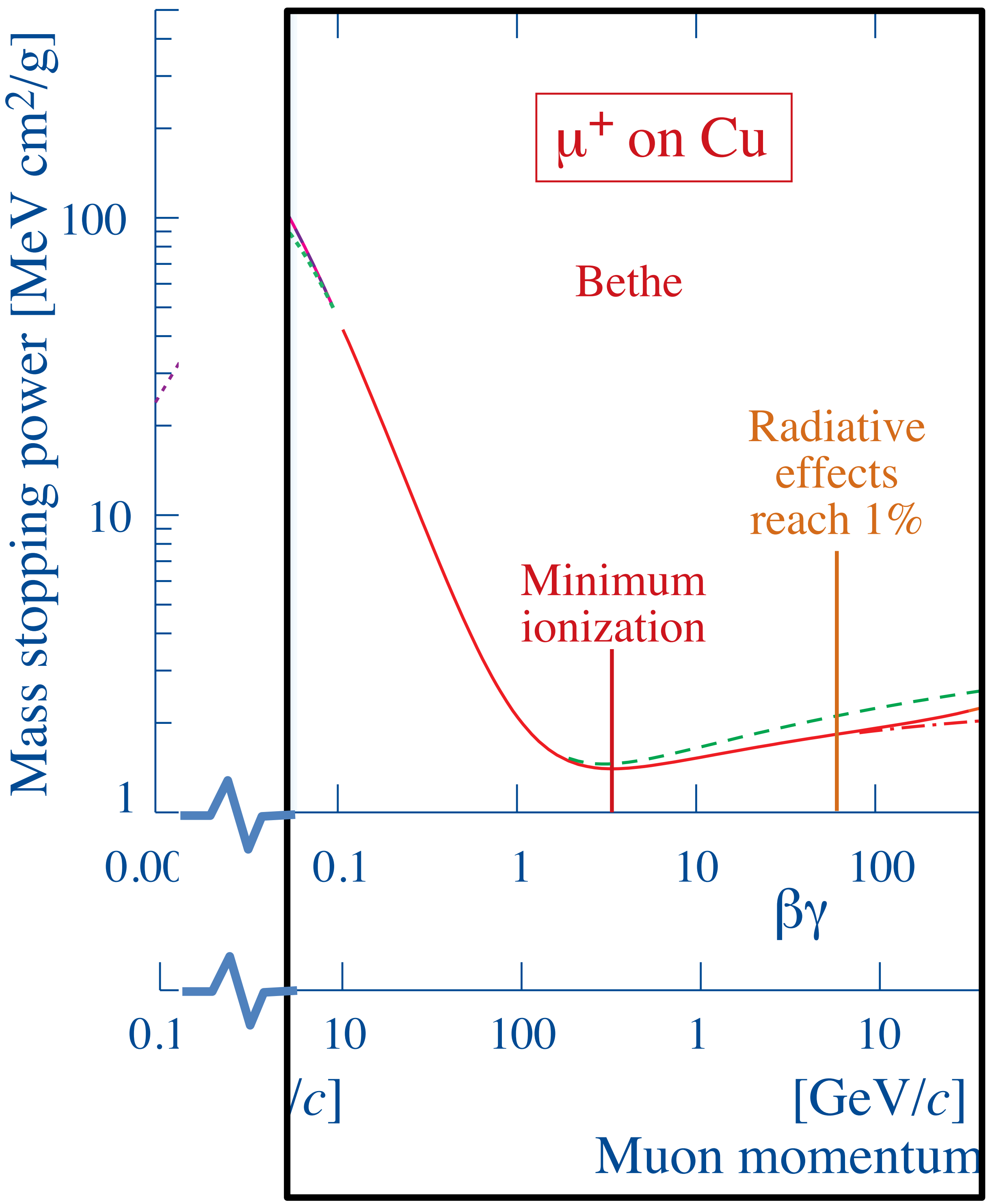
$\beta \to 1$, $\gamma \sim$ few, everything plateaus



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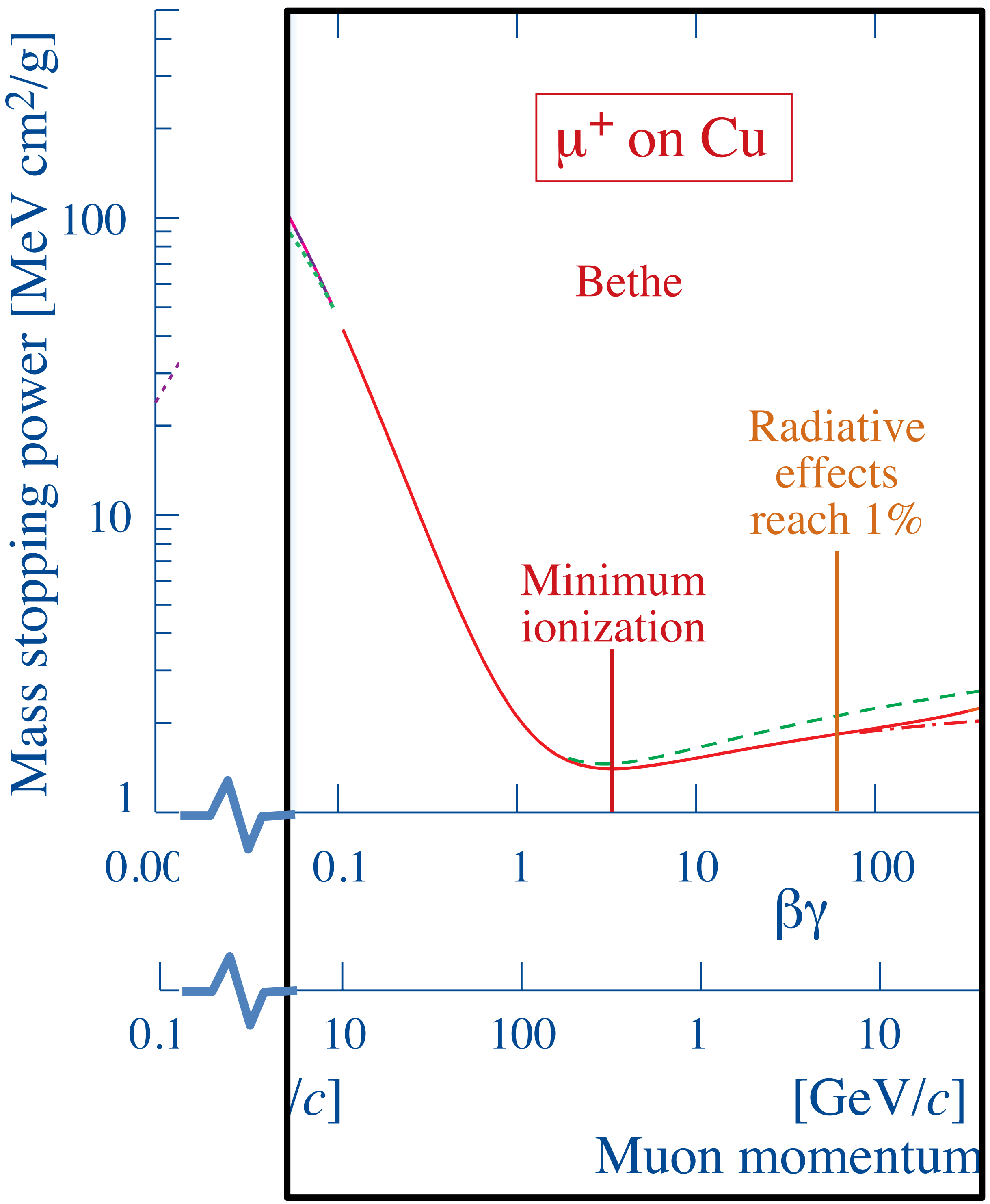

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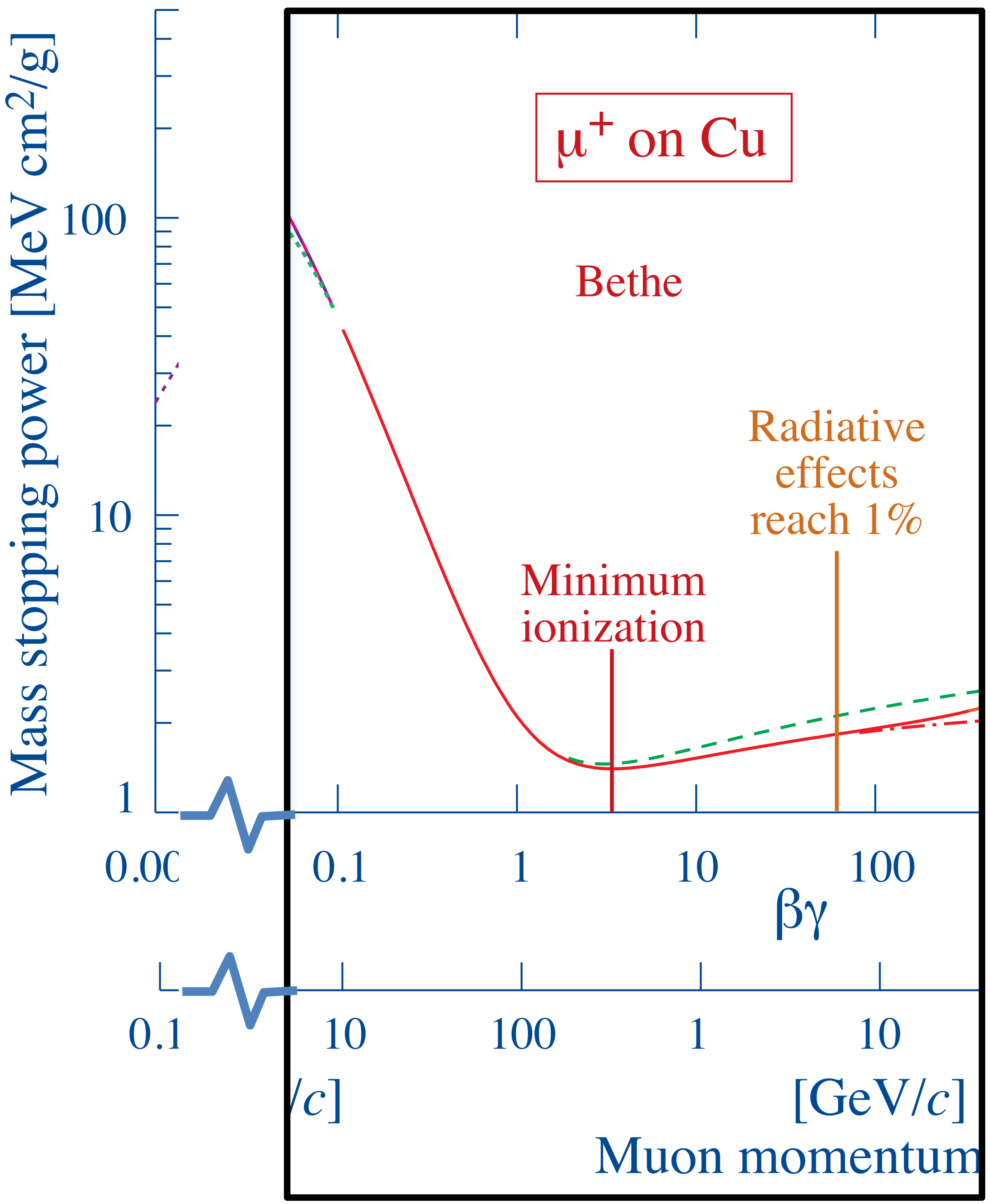
Density correction



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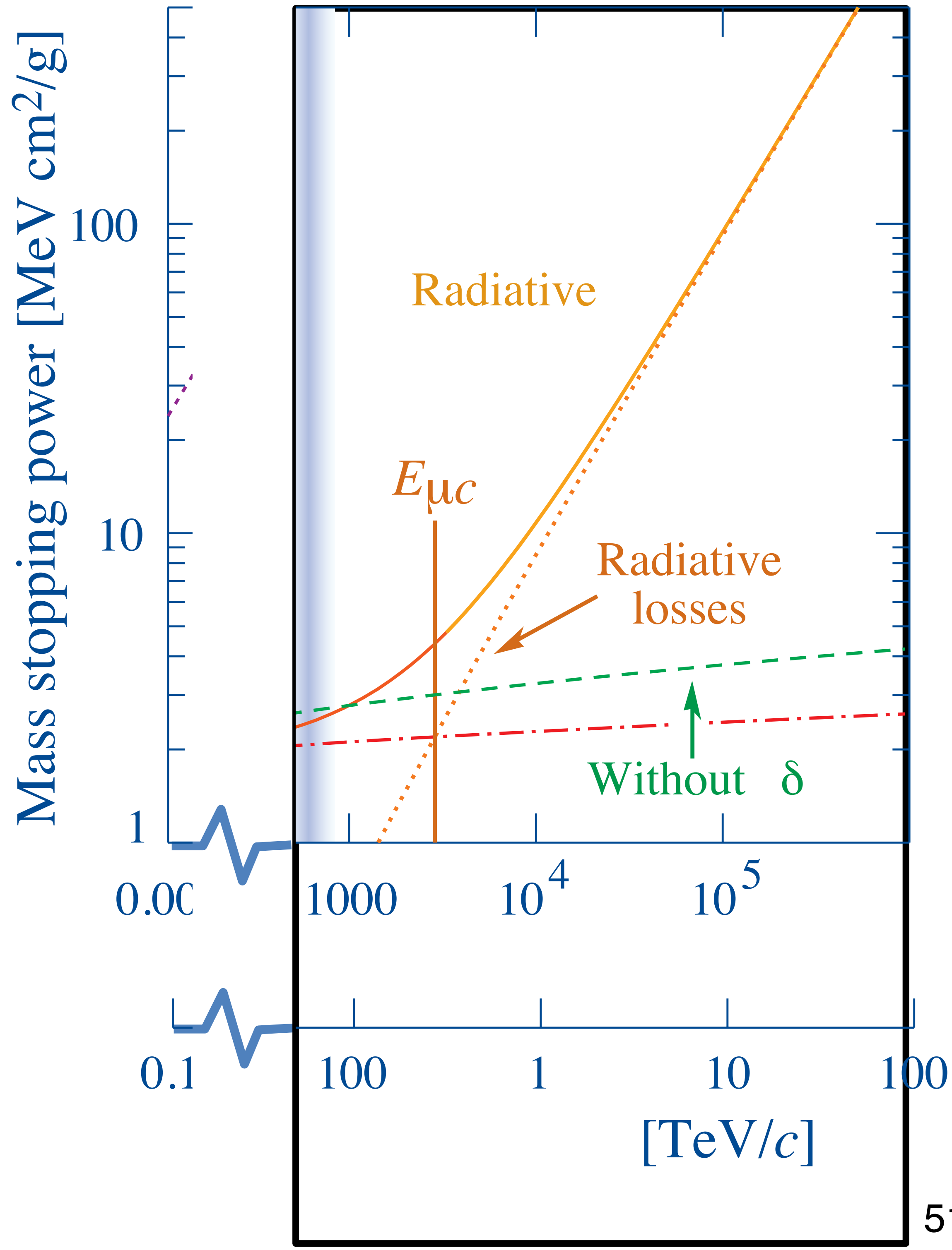
- Above "critical energy", radiative processes dominate [more on these soon].
- Remember: Bethe-Bloch ONLY covers energy loss by Coulomb interactions with atomic electrons, not (e.g.) radiative interactions in field of nuclei.



Minimum Ionising Particles (MIPs)

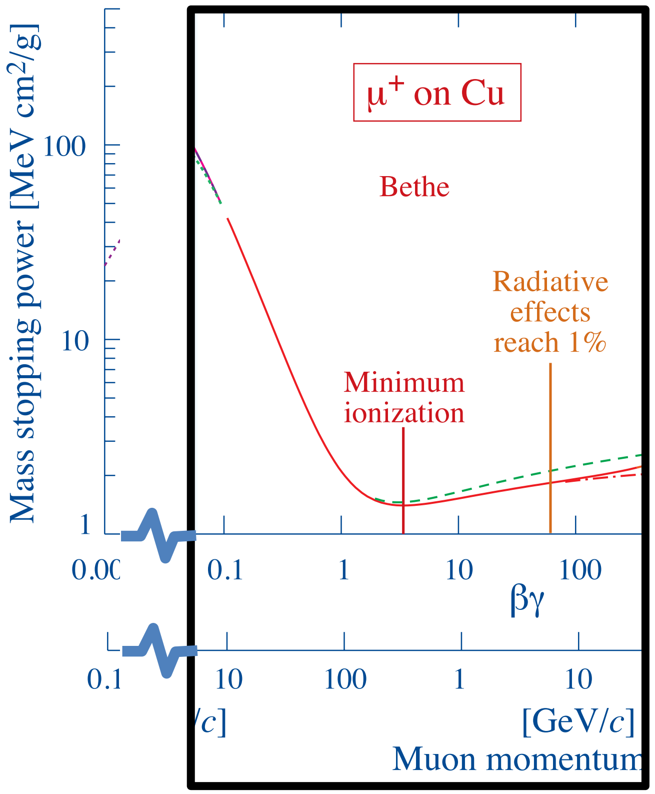
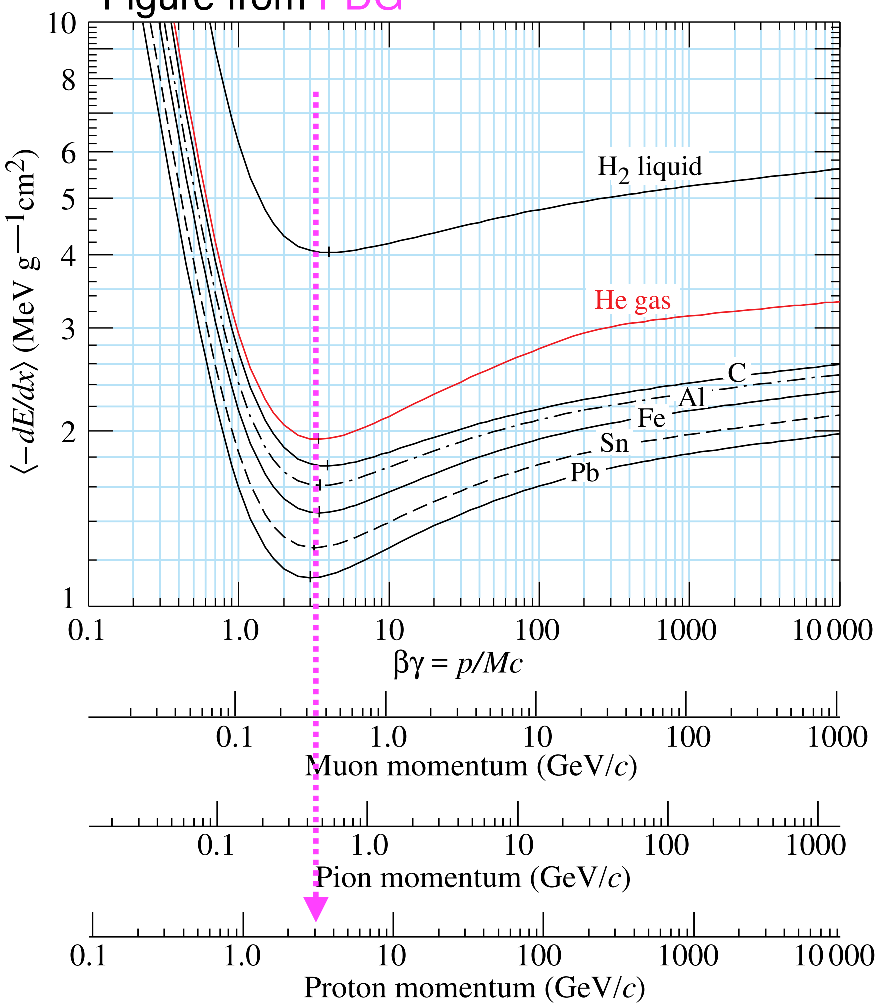


Figure from PDG



Concept of MIP is useful: particles in this range lose energy slowly, and mean stopping power (thus dE/dx) is well known. Often use them for calibration.

Minimum is around $\beta\gamma \approx 3$ ($\beta \approx 0.96$), i.e.

- Electrons/positrons: p ~ 1-2 MeV
- Muons: p ~ 0.3 GeV
- Pions: p ~ 0.4 GeV
- Protons: p ~ 2-3 GeV

Around this $\beta\gamma$, stopping power for all singly charged particles in nearly any material is:

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle_{\mathrm{MIP}} \approx (1-2)\ \mathrm{MeV\,g^{-1}\,cm^{2}}$$

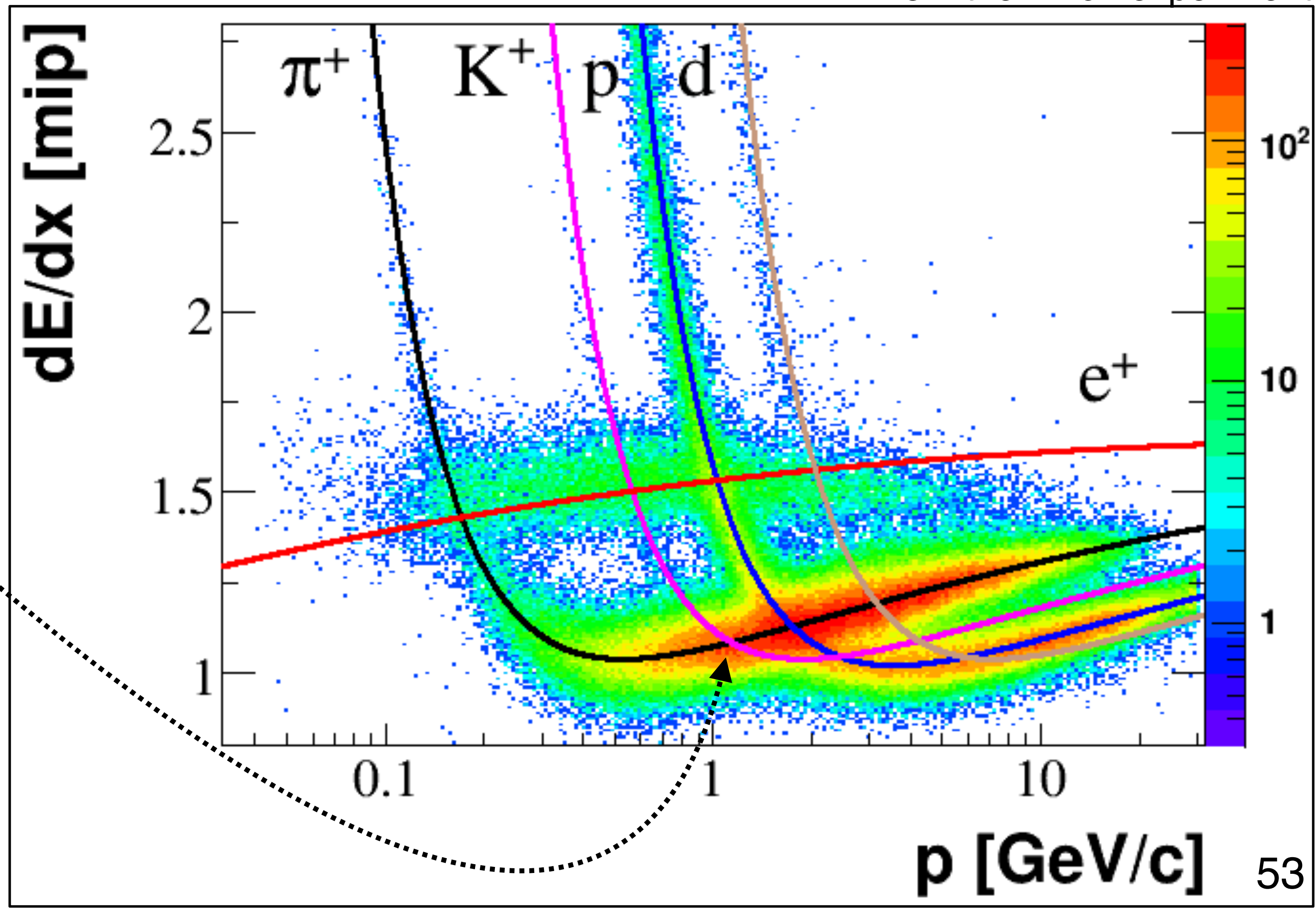
[You should know approximately the numbers in blue!]

Identifying particles with dE/dx

$$-\frac{1}{\rho}\left\langle \frac{dE}{dx} \right\rangle = \frac{2\pi z^2 e^4}{m_e v^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

- Often, we have one piece of kinematic information about a particle but we don't know its identity.
  - e.g. we measured its momentum from curvature in B-field
- Bethe-Bloch relates energy loss to kinematics, so by measuring dE/dx we get a second piece of information.
  - e.g. at O(GeV), dE/dx gives βγ, and with |p| => mass
- dE/dx depends on βγ and z of particle, but not on its mass.
- Discrete ambiguities, e.g. here
- ... and $m_\pi \sim m_\mu$ so this won't separate those so well.

From the NA61 experiment



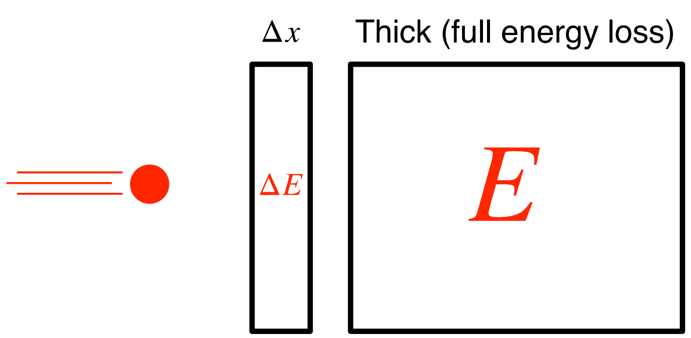
Identifying particles with dE/dx

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi z^2 e^4}{m_e v^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

- Another example: at lower energy ($\beta \ll 1$), we have

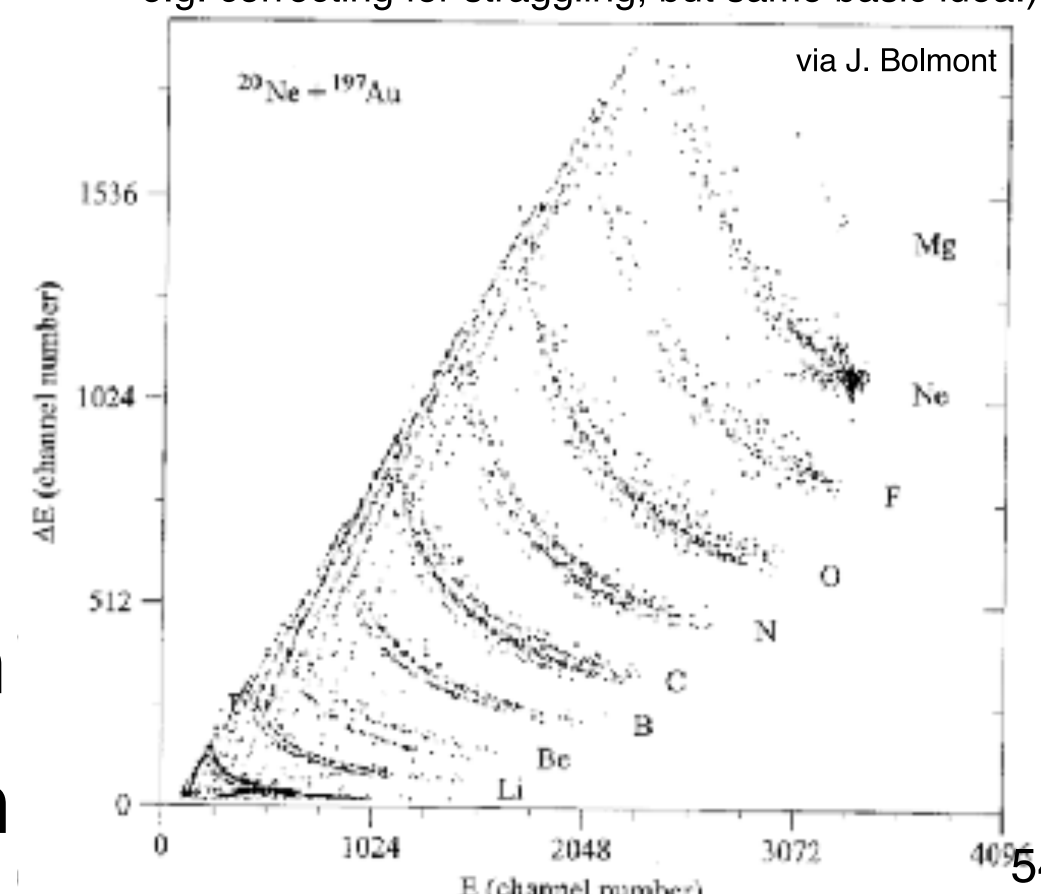
$$\frac{\mathrm{d}E}{\mathrm{d}x} \propto \frac{z^2}{v^2} \propto \frac{z^2 M}{T}, \text{ with classical kinetic energy } T = \frac{1}{2}Mv^2$$

- To identify heavy ions, can use thin + thick detectors:



- Energy in thin detector $\Delta E \propto z^2 M/T$
- Energy in thick detector $E \sim T - \Delta E$
- Plot ΔE vs E, get hyperbolas (~1/E) with coefficient giving $z^2 M$, i.e. identity of ion

(Modern versions uses more exact expressions, e.g. correcting for straggling, but same basic idea.)



Bethe-Bloch: An important limitation

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi z^2 e^4}{m_e v^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

- This formula gives the mean (expected) energy loss.
- It says nothing about the variation in energy loss.
  - Energy transfer of individual collisions varies (in classical model, because of variation in impact parameter b to electrons)
    - Occasional very large energy transfers: δ electrons (δ-rays)
  - Number of collisions can vary (~ Poisson)
- This is a difficult problem to solve in general, especially when collisions are not independent
  - (that is, if dE/dx in the Nth collision depends on particle energy, which depends on how much energy was lost in prior collisions)
- There are some regimes where we have good approximations or models.

Energy loss distribution

- Simplest case: absorber is thick but not very thick.
  - Thick => number of collisions is large => fluctuations in per-collision energy loss average out.
  - Not too thick => energy and $\langle \mathrm{d}E/\mathrm{d}x \rangle$ are constant
  - Under these conditions, energy loss distribution is Gaussian (and function is known, see Leo but roughly $\sigma^2 \propto \Delta x\, \rho Z/A$).
- If absorber is thin but not very thin, energy loss distribution is asymmetric. Well-known model by Landau (figure).

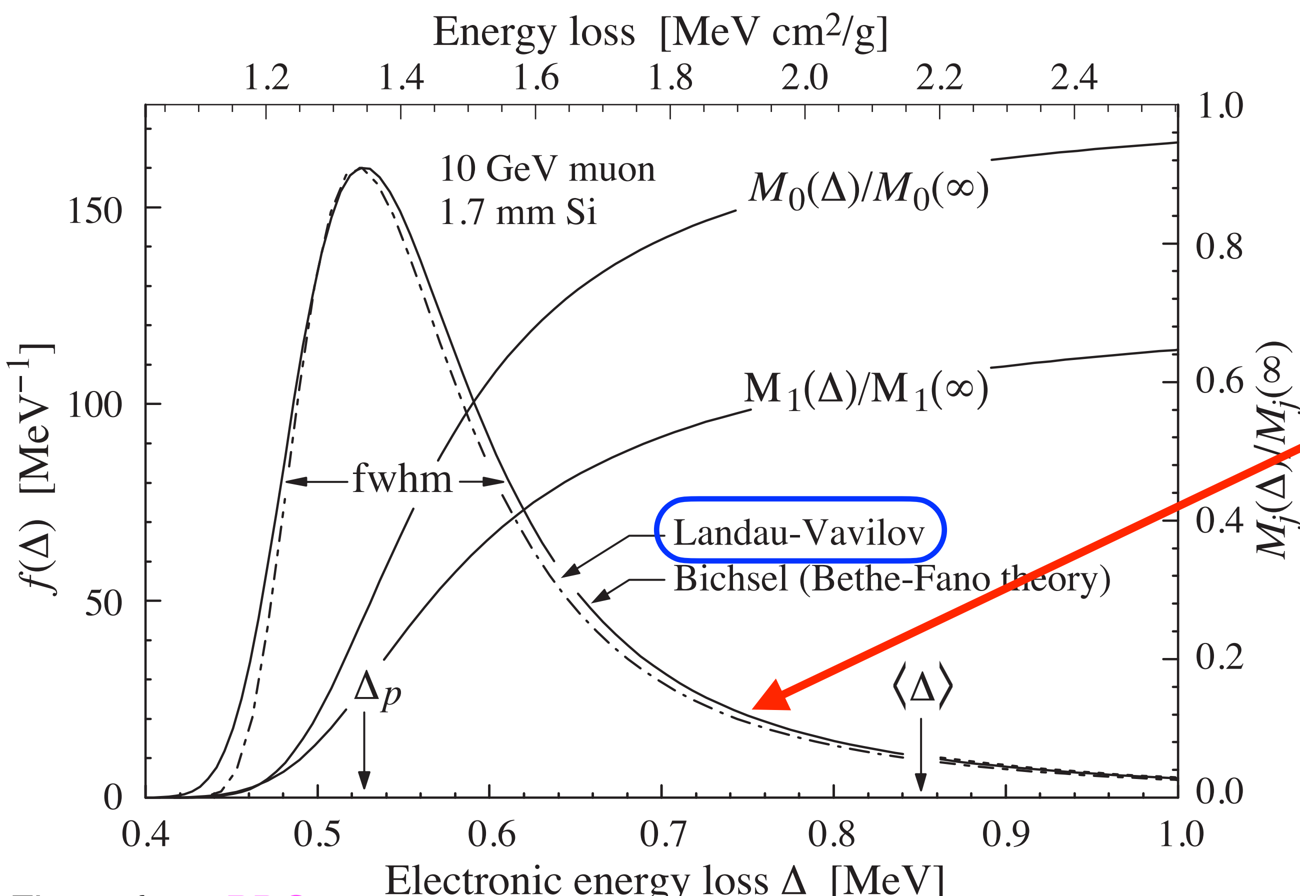


Figure from PDG

- Occasionally, an interaction has very large energy transfer => long tail
- Note that the most probable value Δ_p (peak) is very different from the mean $\langle \Delta \rangle$.
- If very thin, Landau fails too.

(Gaseous detectors need special treatment.)

Calculations vs measurements

- Bethe-Bloch => we know how to calculate the mean energy loss, but because of those fluctuations it's hard to measure precisely.
- Experimentally, it's often much easier to measure the most probable value.

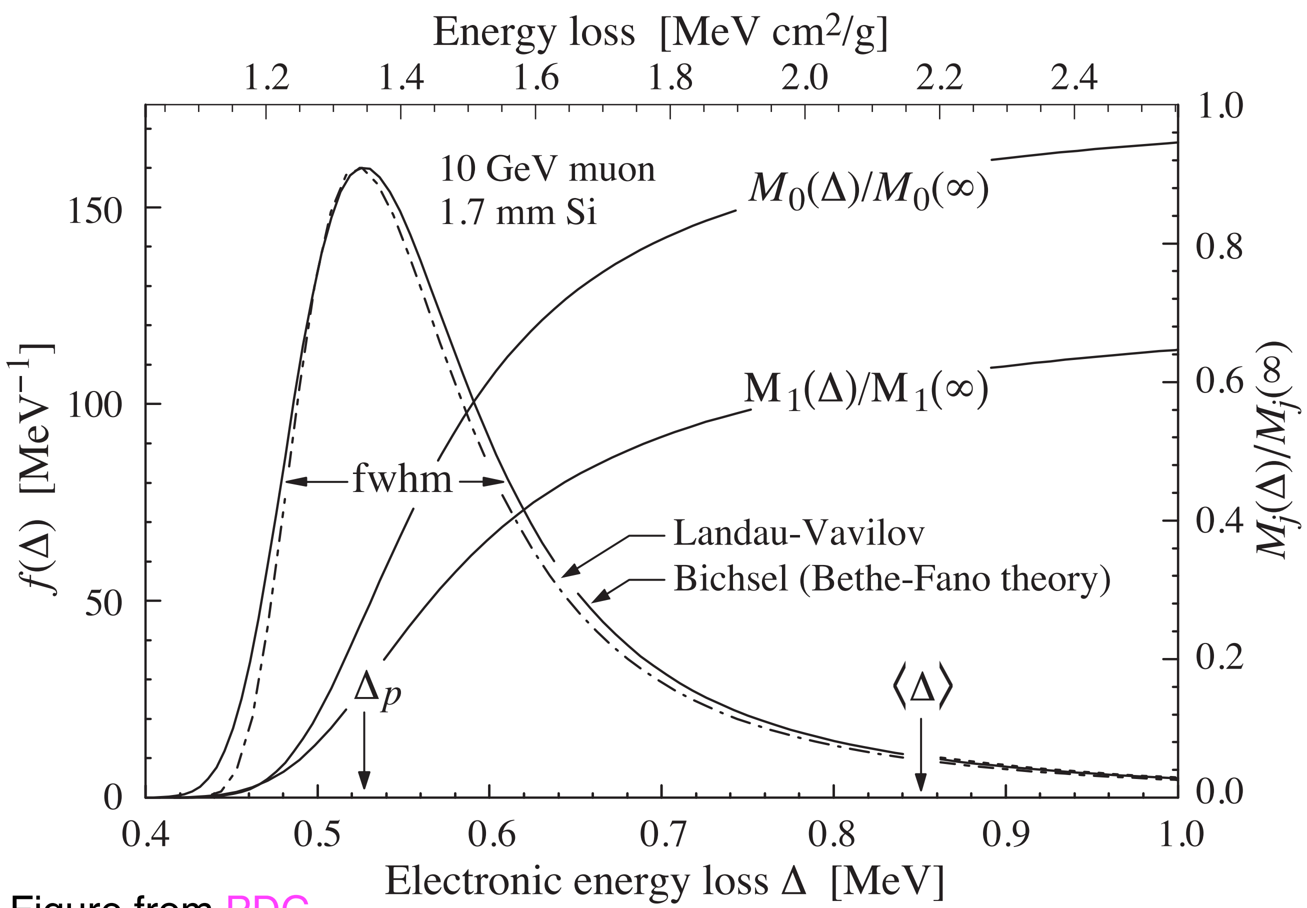
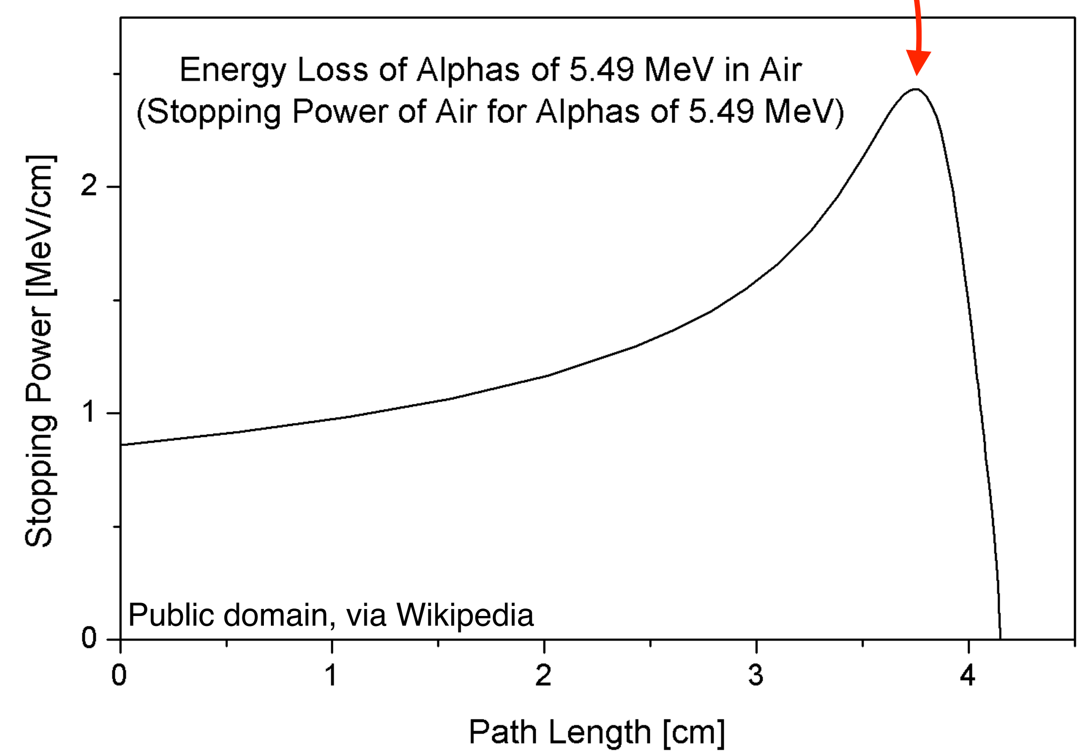


Figure from PDG

How far can particles travel?

- Bethe-Block tells us mean energy loss, $\langle \mathrm{d}E/\mathrm{d}x \rangle$.
- Fluctuations in energy loss => calculating distribution of how far a particle travels in material (its range, R) is not easy*.
- But if you assume energy loss is continuous, you can integrate formula for certain cases:
  - Low-energy hadrons (R < interaction length, see later)
  - Muons not dominated by radiation (below few hundred GeV)
- As particles slow down, dE/dx increases => energy loss spikes at the end of particle's path (Bragg peak).
- Important application in cancer treatment: proton & hadron therapy (energy delivery is concentrated at the target).



* There's a tail in the range distribution from particles that travel further than average. Jargon: this is called "straggling".

Electrons & positrons

- So far we've discussed Coulomb scattering for particles heavy relative to atomic electron targets, i.e. nearly everything.
- … but what about electrons & positrons? Two complications:
  1) Some of the assumptions behind the Bohr/Bethe-Bloch calculations are broken (e.g. that incoming particle isn't deflected because it's heavy; also implicit assumption that the particles are non-identical breaks for electron-electron scattering)
  2) We only considered energy loss from Coulomb scattering on atomic electrons. For $e^{\pm}$, energy loss from radiation becomes important even at moderate energies.
- To fix these, we need to
  1) Correct the Bethe-Bloch formula for the case of $e^{\pm}$
  2) Include another term for radiation energy loss

$$\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{total}} = \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{Bethe-Bloch}} + \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{radiation}}$$

Electrons & positrons: Coulomb term

Bethe-Bloch for heavy particles:

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi z^2 r_e^2 m_e c^2}{\beta^2}\frac{Z}{A}N_A\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

Bethe-Bloch corrected for $e^{\pm}$ (with $z^2 = 1$):

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{2\pi r_e^2 m_e c^2}{\beta^2}\frac{Z}{A}N_A\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_ec^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

where $\tau \equiv T/(m_ec^2)$ for kinetic energy T, and:

for electrons, $F(\tau) = 1 - \beta^2 + f_e(\tau)$

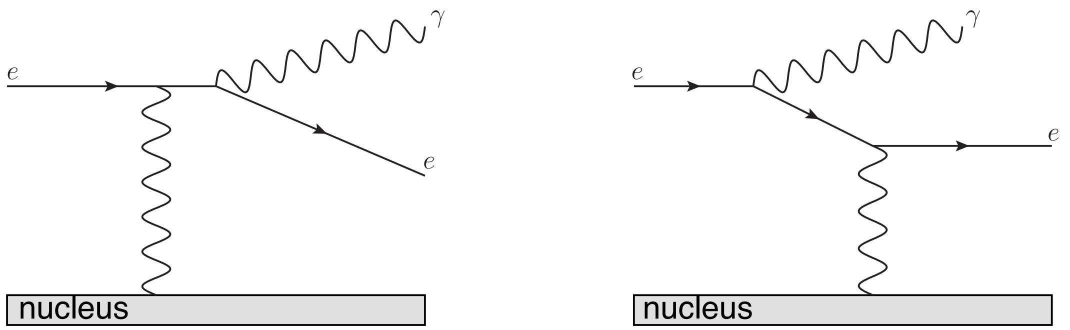
for positrons, $F(\tau) = 2\ln 2 - \frac{23}{12}\beta^2 + \beta^2 f_p(\tau)$

(see Leo or other texts for the full functional form; again, you don't need to memorise the full expressions)

[Formulae from Leo]

Electrons & positrons: Bremsstrahlung

- Bremsstrahlung (braking radiation): a charged particle scatters in the electric field of a nucleus and emits a photon.



- Can **only** happen in matter, not in vacuum.

- Cross-section scales like $\sigma \propto 1/M^2$ for incident particle mass M
  - This is why brem is much more severe for $e^{\pm}$, e.g. $(m_\pi/m_e)^2 \sim 10^5$

- E_γ can be up to full energy of $e^{\pm}$, but skews to lower energy. High-energy brem can have a big impact on particle's path, final energy.

- Mean energy loss is difficult to calculate exactly (depends on atomic parameters, particularly screening) but **at first order** for particles of energy E and mass M in material (A,Z) it goes like:

$$-\frac{1}{\rho}\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{rad}} \propto \frac{N_A}{A}\frac{E\,Z^2}{M^2}$$

[Complete formulae are long and messy. If you need to calculate a value, see PDG review or a textbook.]

Electrons & positrons: Bremsstrahlung

- Compare energy loss from Coulomb inelastic term vs radiation for relativistic electrons/positrons* at first order:

$$-\frac{1}{\rho}\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{Coulomb}} \propto \frac{Z}{\beta^2} \qquad -\frac{1}{\rho}\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{rad}} \propto \frac{N_A}{A}\frac{E Z^2}{M^2}$$

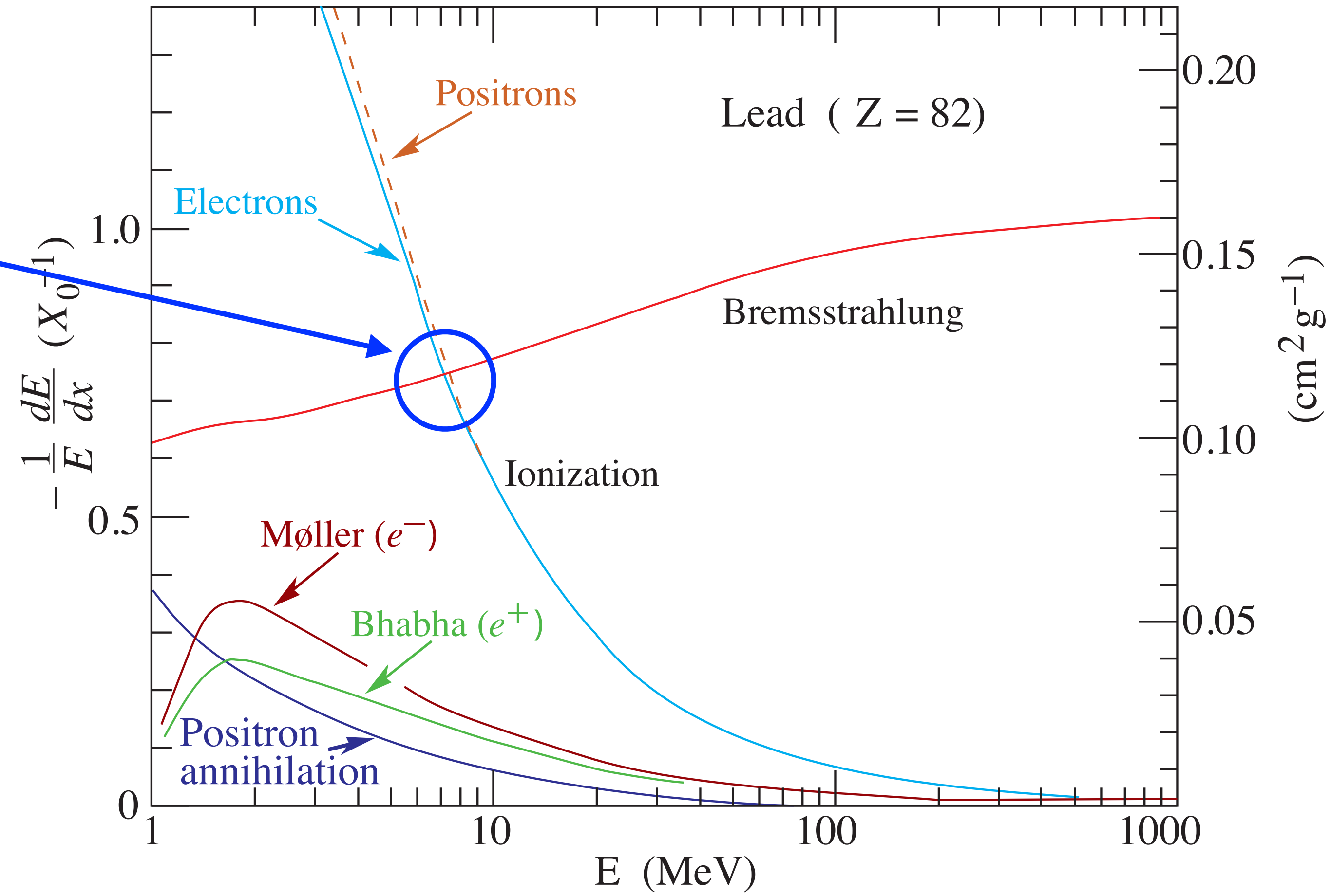
- Cross-over point is known as the critical energy, E_c

  $E < E_c$: Coulomb dominates

  $E > E_c$: Radiation dominates

- Approximately, for $e^{\pm}$,

$$E_c \approx \frac{800\ \text{MeV}}{Z+1.2} \sim 10^1 \text{ to } 10^2\ \text{MeV}$$



Caution: this is not the only definition of the critical energy. They're all similar within a factor of order 1, but you will see slightly different numbers. Will come back to this when discussing EM showers. You should know how it's defined, and the rough magnitude.

* For $e^{\pm}$, Coulomb term goes like $1/\beta^2$ in the relevant energy range. (For heavier particles, the log term is important but dE/dx is still rising a lot slower than linearly with E, so general argument still holds.)

Radiation length

- Radiation length X_0 is defined as the distance over which electron/positron energy is reduced by a factor of 1/e **due to brem radiation losses only.** This works because:

$$-\frac{1}{\rho}\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{rad}} \propto \left(\frac{N_A Z^2}{AM^2}\right)E \;\Rightarrow\; \frac{\mathrm{d}E}{\mathrm{d}x} \propto -E \;\Rightarrow\; E(x) = E_0\, e^{-x/X_0}$$

- Note connection between X_0 and energy loss:

$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{E}{X_0}$, so using formula from previous slide: $X_0 \propto \frac{A}{\rho Z^2}$ at first order

- Empirical formulae exist (Sec 34.4 of PDG) but for most materials you'd look up the value in the PDG or other data table.

Caution: often an implicit factor of ρ (e.g. for Pb, X_0 is 0.56cm or 6.37 g cm^{-2})

- A typical EM calorimeter for a particle physics experiment has a depth of 20-30 X_0, i.e. enough to fully contain most showers.

Examples: [PDG, Leo]

W:	0.35 cm	PbO:	1.27 cm	Water:	36 cm
Pb:	0.56 cm	Fe:	1.76 cm	Polystyrene:	43 cm
$PbWO_4$:	0.89 cm	NaI:	2.59 cm	Air:	300m
(BGO):	1.12 cm	Al:	8.9 cm		

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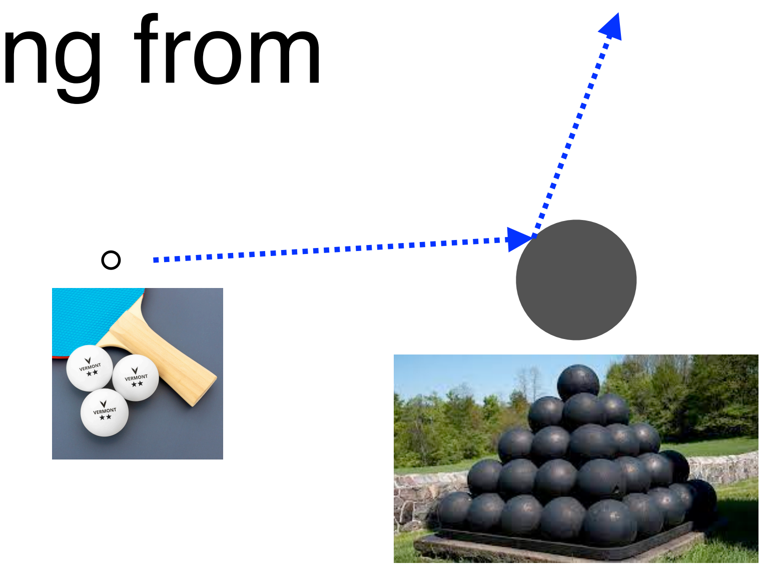
Aside: interaction length

- [Nuclear] interaction length λ_I is a loose analog of radiation length for strong/nuclear interactions of hadrons with nuclei.
- I'm not going to talk much about those here -- you'll hear more about them in the calorimeter lectures. (Just mentioning it here for completeness.)
- The interaction length represents the mean free path of a high-energy hadron between inelastic hadronic collisions.
- Applies to both charged and neutral hadrons.
- Interaction length can be much longer than radiation length, e.g. for lead $X_0(\mathrm{Pb}) \approx 0.56\,\mathrm{cm}$ but $\lambda_I(\mathrm{Pb}) \approx 17.6\,\mathrm{cm}$

Multiple scattering

- So far, we've talked about interactions with atomic electrons, and how they can cause energy loss (but minimal deflection).
- Particles can also undergo Coulomb scattering from nuclei. These collisions are ~ elastic but can deflect the particle.



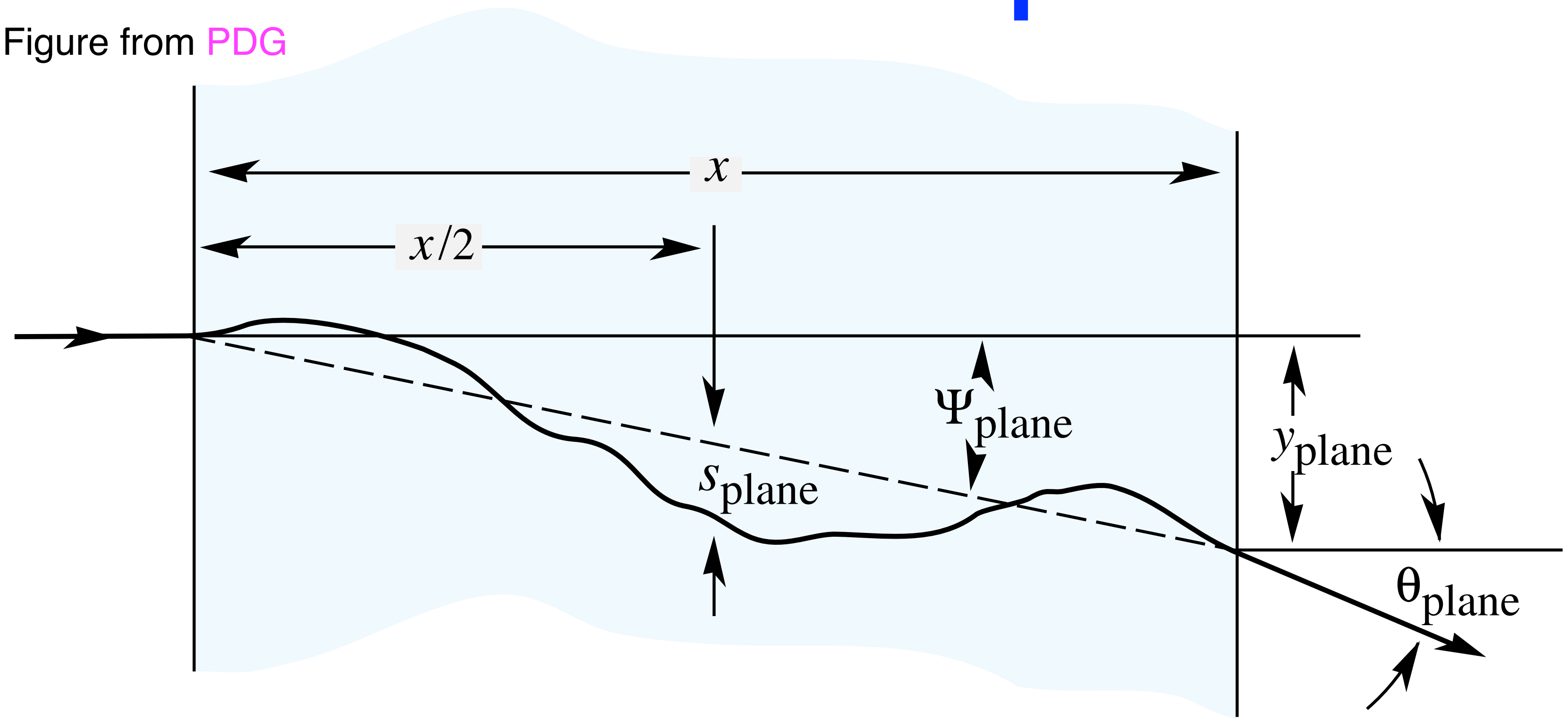
- What's the angular distribution of particles after passing through matter?
- For a single, individual nuclear scatter the angular distribution is well known [neglecting spin, screening]: Rutherford formula

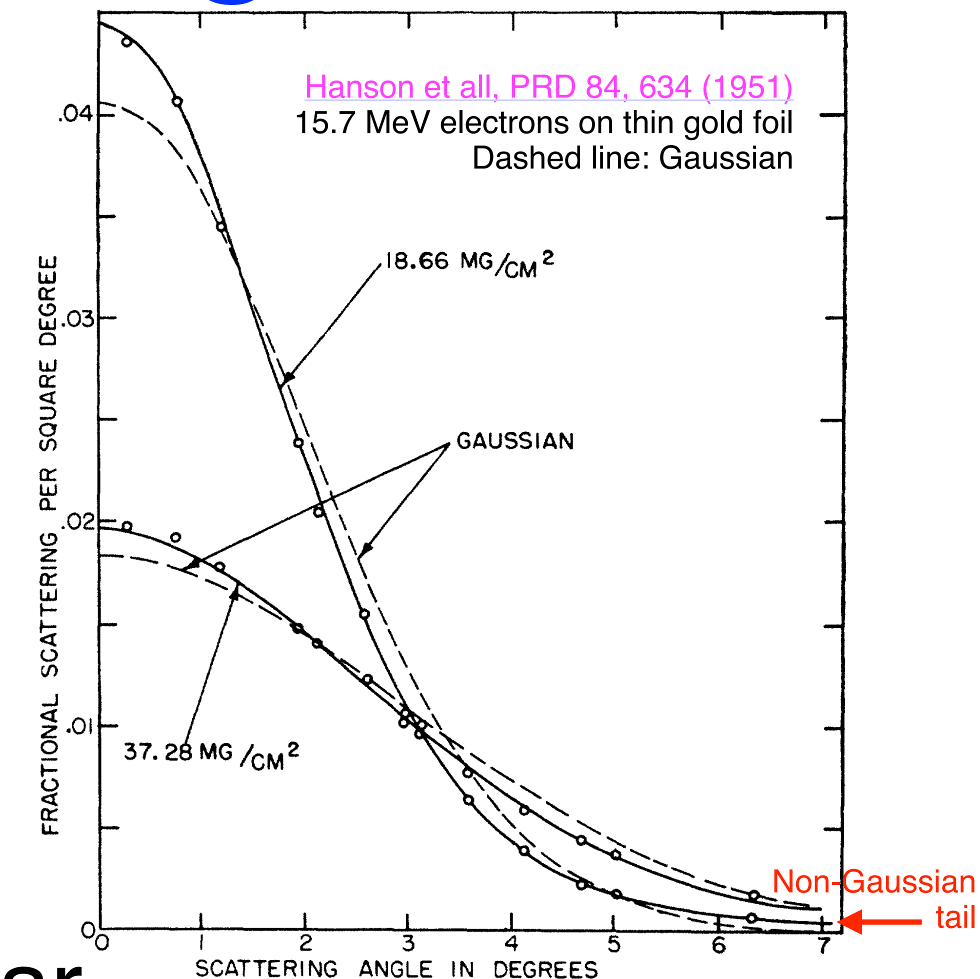
$$\frac{d\sigma}{d\Omega} = z_1^2 z_2^2 r_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{4\sin^4(\theta/2)} \propto \frac{1}{\sin^4(\theta/2)}$$

- i.e. mostly small angles but occasional large-angle scatters.
- For very thin absorbers, this is enough. Tricky case: finite thickness where multiple scattering can occur.

Multiple scattering

Figure from PDG





- Finite width case is kind of a mess.
- For a thick absorber where many nuclear scatters expected, can model as Gaussian core (from many small-angle scatters) plus longer tails where angle is dominated by a single large-angle scatter.
- Track fitters need to allow for **correlated** fluctuations due to multiple scattering, e.g. using Kalman fitter.
  - Note that this is different from hit measurement resolution, i.e. independent random errors.

Summary

Charged particles

Light: electrons (and positrons)

- Bremsstrahlung dominates @ $E > 20$ MeV
- Inelastic scattering with atoms (ionization)
- Elastic scattering with nucleons
- Cherenkov radiation & transition radiation
- Nuclear reactions

Heavier: everything else*

- Inelastic scattering with atoms: Bethe&Block formula ($\sigma \sim 10^{-16}$ cm^2)
- Elastic scattering with nucleons
- Cherenkov radiation & transition radiation
- Nuclear reactions
- Bremsstrahlung

* Muons, pions, kaons, protons, ions, charged hyperons, ...

[end of lecture]

Example exam questions from previous years

Note: The exam format has changed over time, and the syllabus has also evolved somewhat. The point is not to give you the exact style or content of this year's questions, but to help you prepare.

Exam questions

From 2021:

Q1 (approx. 10–15 min)

LHCb has recently found 3σ evidence of tension with the Standard Model when comparing the rates of the two processes $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$. While the underlying physics of these two channels should be very similar in the SM, the behaviour of the final-state particles in the detector differs. (The typical momentum of the B^+ mesons is 10–100 GeV.)

[This part is as much about detector design in general as specifically on interactions of charged particles with matter.]

(a) List the key detector elements/subdetectors necessary to determine the following for the final-state particles:

(i) its momentum,

(ii) whether it is an electron, muon, or kaon.

[Treat (i) and (ii) separately. You do not need to propose particular technologies; it is sufficient to list subdetector/subsystem types.]

The momentum resolution of the e^+e^- system is significantly worse than that of the $\mu^+\mu^-$ system (when measuring both in the same LHCb subdetectors). This is due to the effect of an additional physical process that is negligible for muons but not for electrons.

(b) Identify this process.

[ditto]

(c) Suggest a way in which information from another subdetector might be used to help recover some or all of this degradation.

(d) When designing a future detector, what could you change in order to reduce this effect as much as possible? (i.e. what is the key quantity/parameter that determines the magnitude of the effect?)

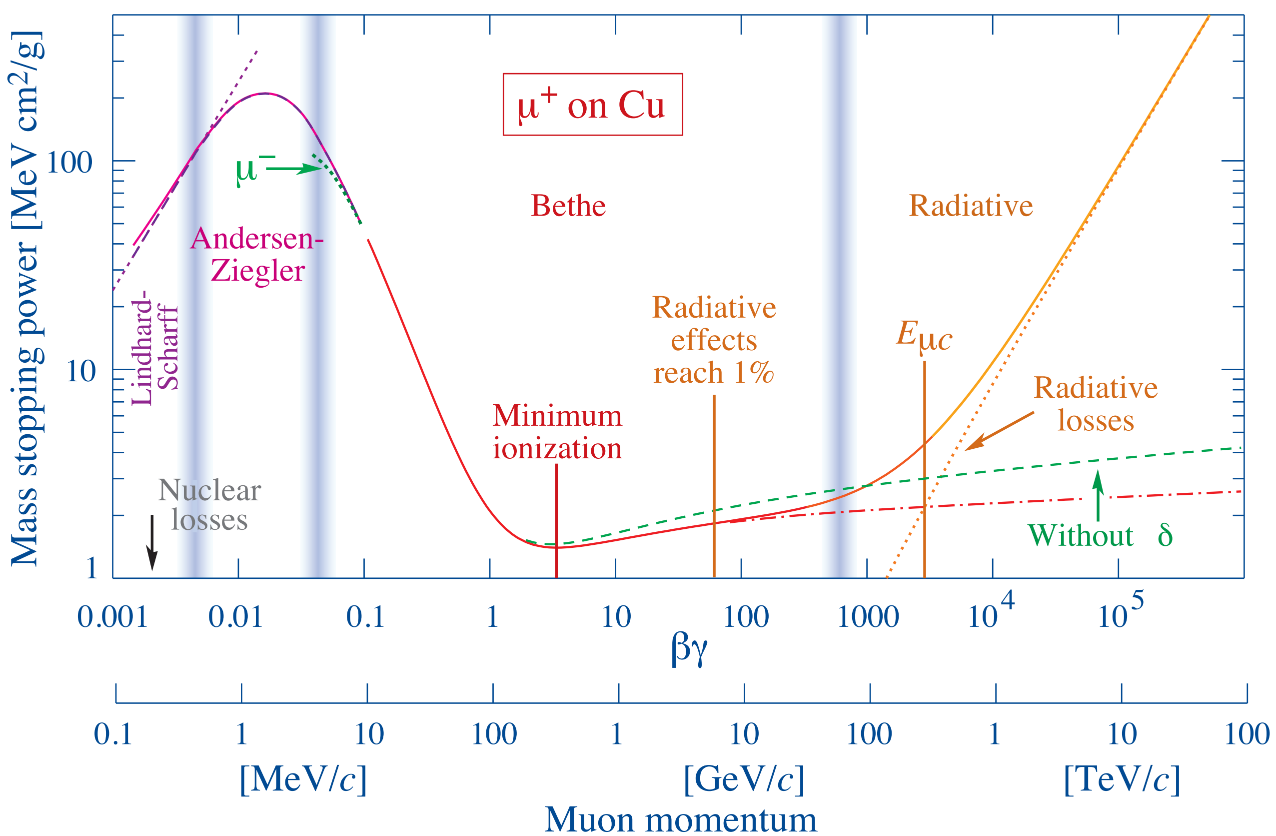
Exam questions

From 2021:

Q2 (approx. 15 min)

Consider the plot of mass stopping power below, as well as the following constants:

- Density of copper: $\rho = 8.96\,\mathrm{g\,cm^{-3}}$
- Radiation length of copper: $X_0 = 12.86\,\mathrm{g\,cm^{-2}}$
- Atomic number of copper $Z = 29$, and mean mass number $A \approx 63.5$
- Critical energy of copper: $E_c = 24.8\,\mathrm{MeV}$



(a) Estimate the thickness of copper required to stop a 10 GeV beam of muons.

(b) Estimate the thickness of copper required to stop a 10 GeV beam of electrons.

[Justify your answers. Approximate estimates, good to within a factor of two or so, are fine. It is enough to find the thickness that will stop the majority of the beam; you do not need to consider stragglers.]