From nuclei to stars
Introduction to nuclear astrophysics

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Outline

0. General introduction

Lecture 1: Introduction to nuclear astrophysics

Lecture 2: Nucleosynthesis processes in the Universe

Lecture 3: Cross-sections and thermonuclear reaction rates

Lecture 4: Experimental approaches in nuclear astrophysics

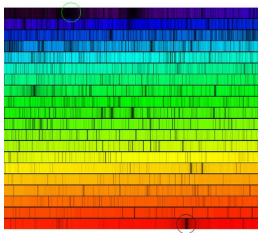
0. General introduction

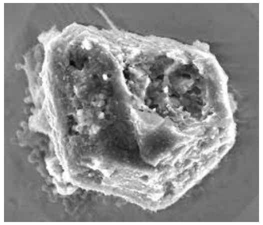
Nuclear astrophysics is a field which addresses some of the most compelling questions in nature

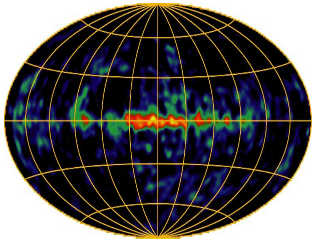
- What is the origin of the chemical elements in the Universe?
- How do stars form and evolve? What is their fate?
- What is the energy source powering stars?
- Which nucleosynthesis processes are responsible of the observed solar abundances?

An interdisciplinary field

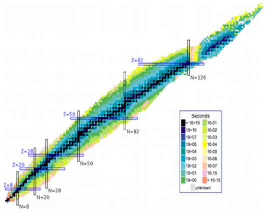
Observations
(astronomy and meteorites)

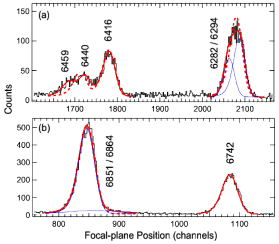


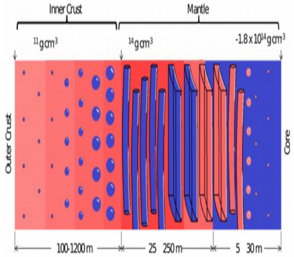




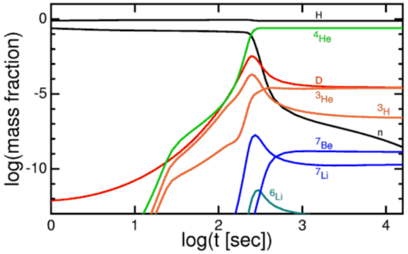
Nuclear physics
(properties of nuclei)

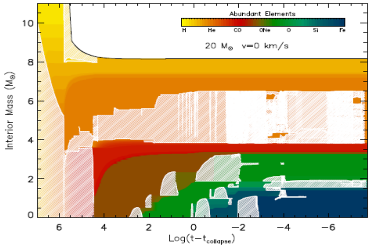






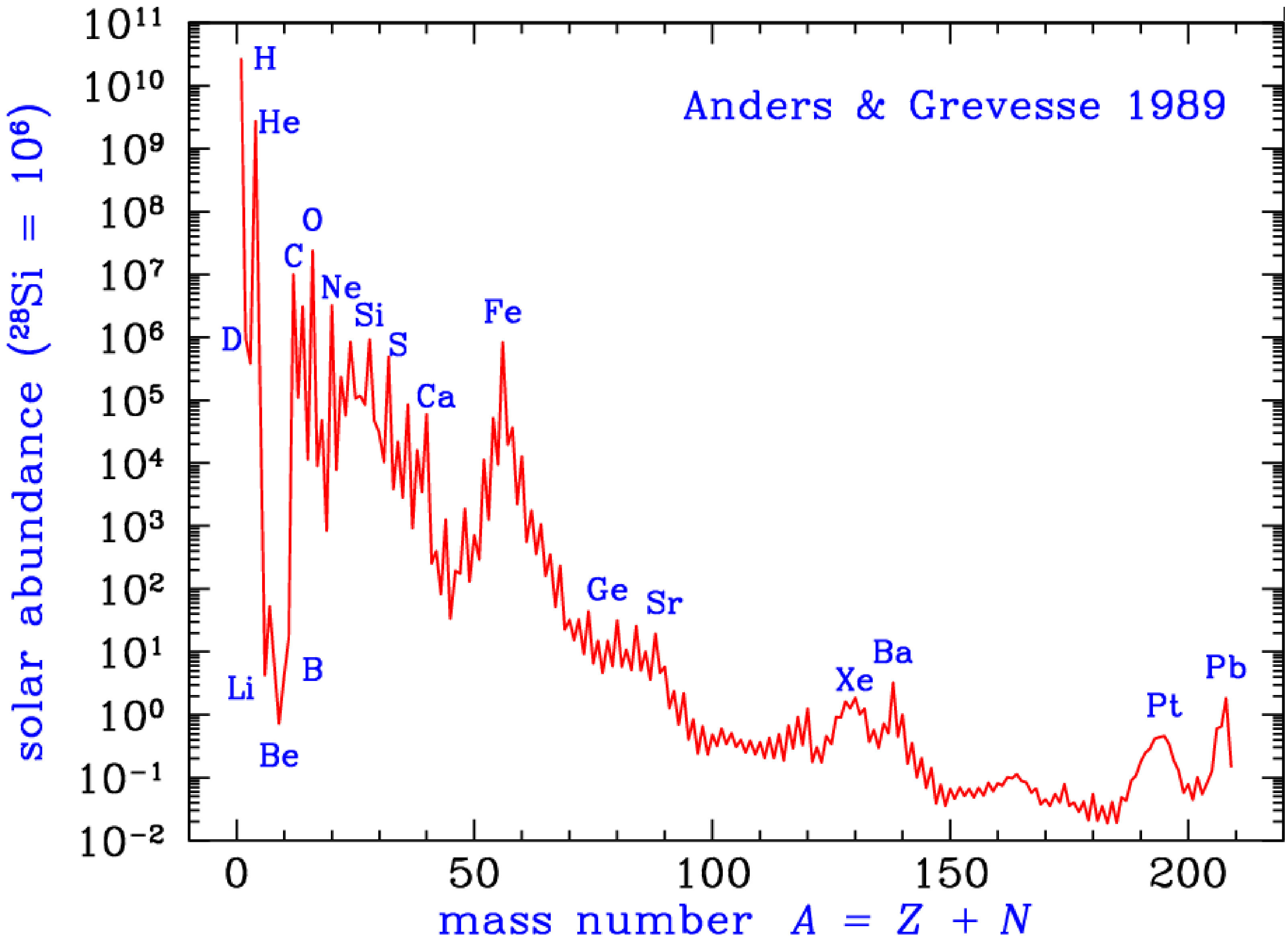
Astrophysic modeling
(Big-Bang, stars, ...)





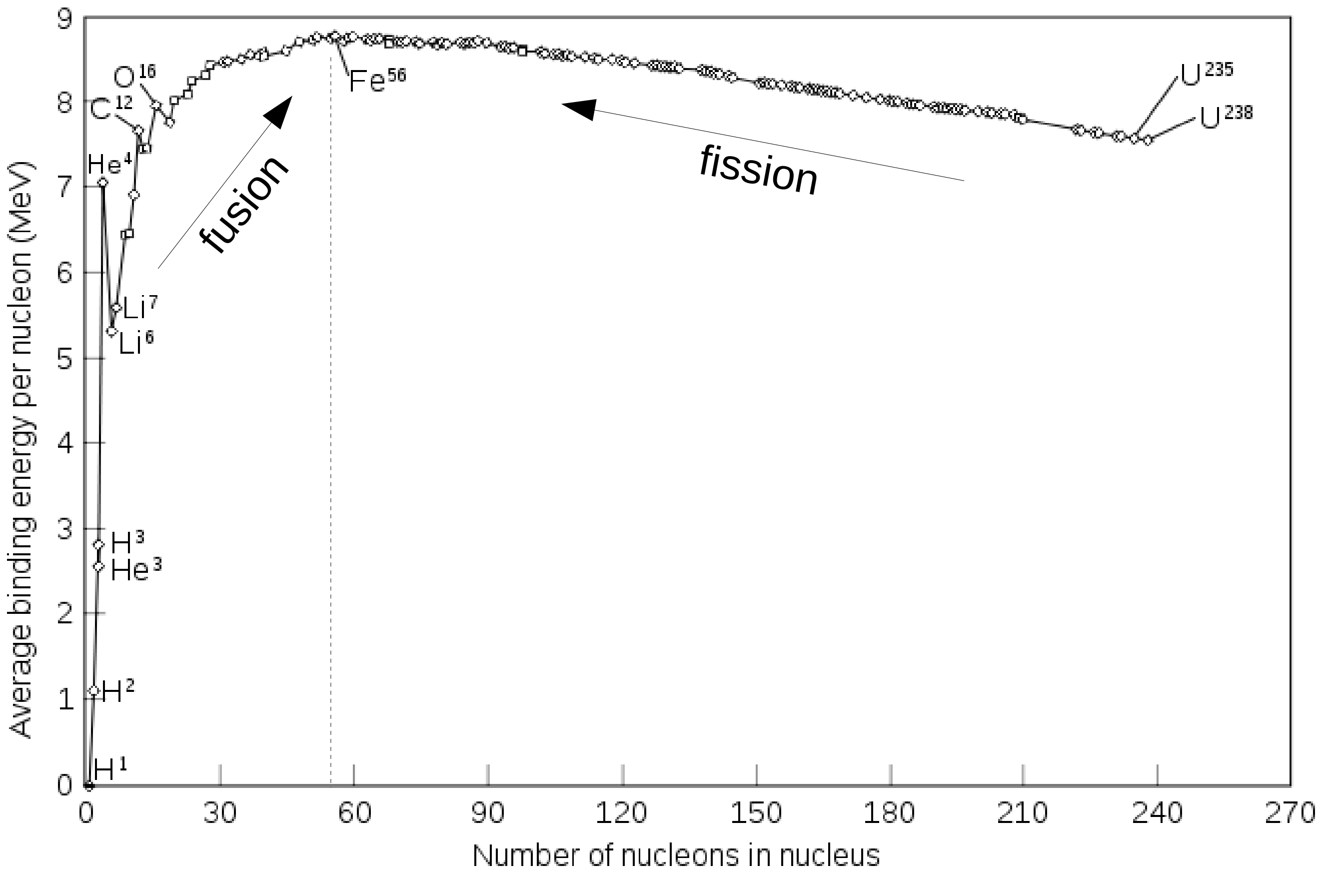
The solar abundance curve

The Rosetta stone in nuclear astrophysics



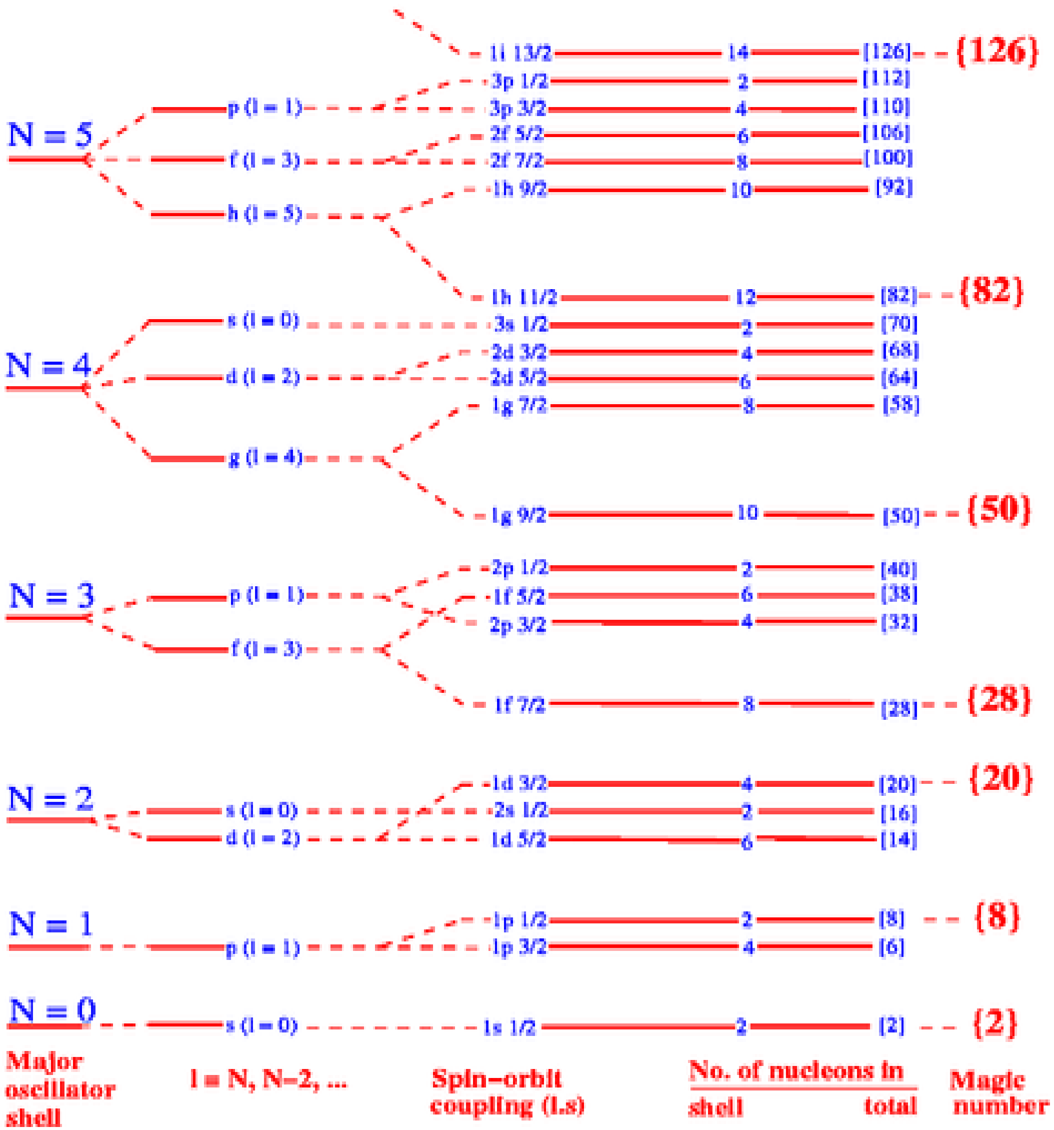
The binding energy per nucleon

$$\Delta E/A = -\left[M_{nucleus} - ZM_p - (A - Z)M_n\right] c^2/A$$



- The light elements Li, Be, B are relatively fragile
- The "α-nuclei" (*A* is multiple of 4) are particularly stable
- *ΔE/A* is maximum (8.8 MeV) near ^{56}Fe → "iron peak"

The nuclear shell-model



- Nuclear stability is related to shell closure and pairing

- *Z*, *N* odd or even
  → oscillation in the abundance curve

- Nuclei with *Z* or *N* equal to a magic number
  → abundances peak

- Double magicity *Z* = 82 and *N* = 126
  → ^{208}Pb peak

Outline

0. General introduction

Lecture 1: Introduction to nuclear astrophysics

1. Stellar astronomy
2. The cycle of matter and chemical evolution of the Galaxy
3. The solar or "cosmic" abundances
4. Birth of stars
5. Stellar structure

Lecture 2: Nucleosynthesis processes in the Universe

Lecture 3: Cross-sections and thermonuclear reaction rates

Lecture 4: Experimental approaches in nuclear astrophysics

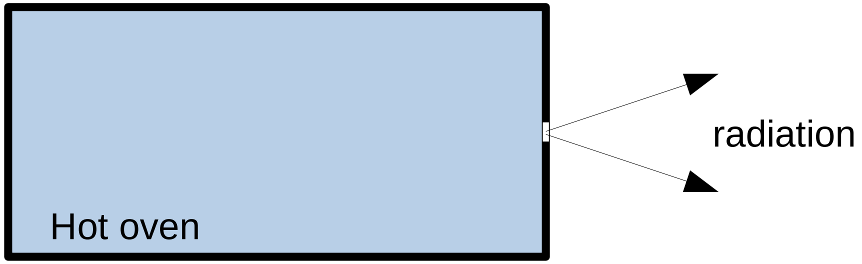
1. Stellar astronomy

Buldge of the Milky Way – Hubble Space Telescope – Wide Field Camera 3

Black body radiation

- Idealized physical body that absorbs all incident electromagnetic radiation and which is at thermodynamic equilibrium

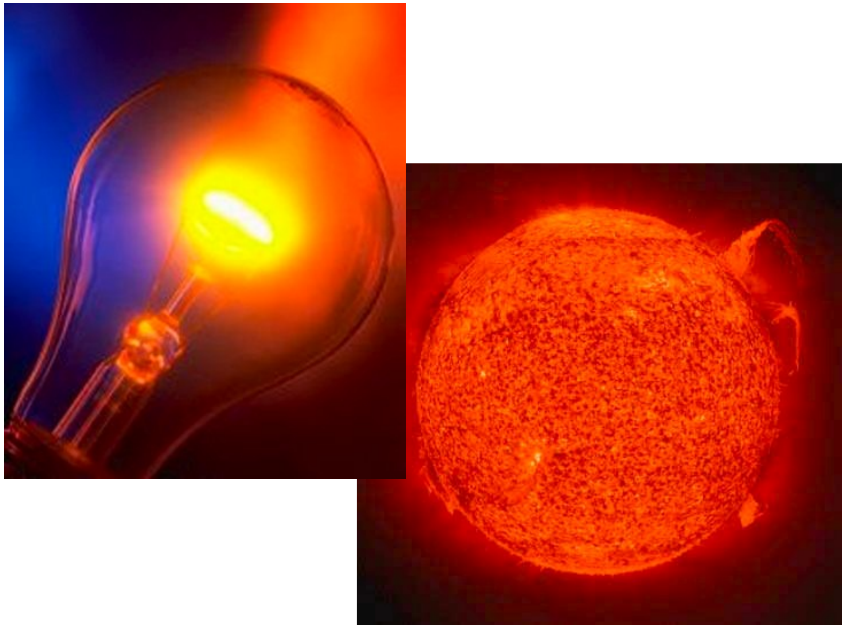
→ hot furnace with a small hole which does not disturb thermal equilibrium inside



- Surface brightness (erg cm^{-2} s^{-1} sr^{-1} Hz^{-1} or erg cm^{-2} s^{-1} sr^{-1} cm^{-1}) given by Plank's law

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

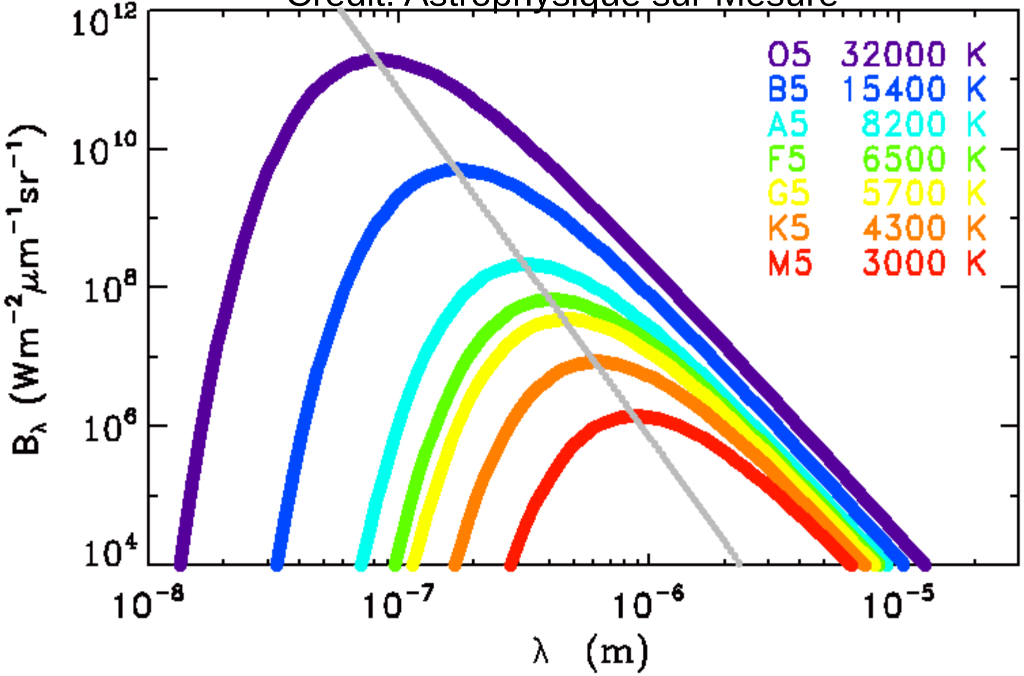
h is the Planck constant, *c* the speed of light, *k* the Boltzmann constant and *T* the black body temperature



The colour of stars

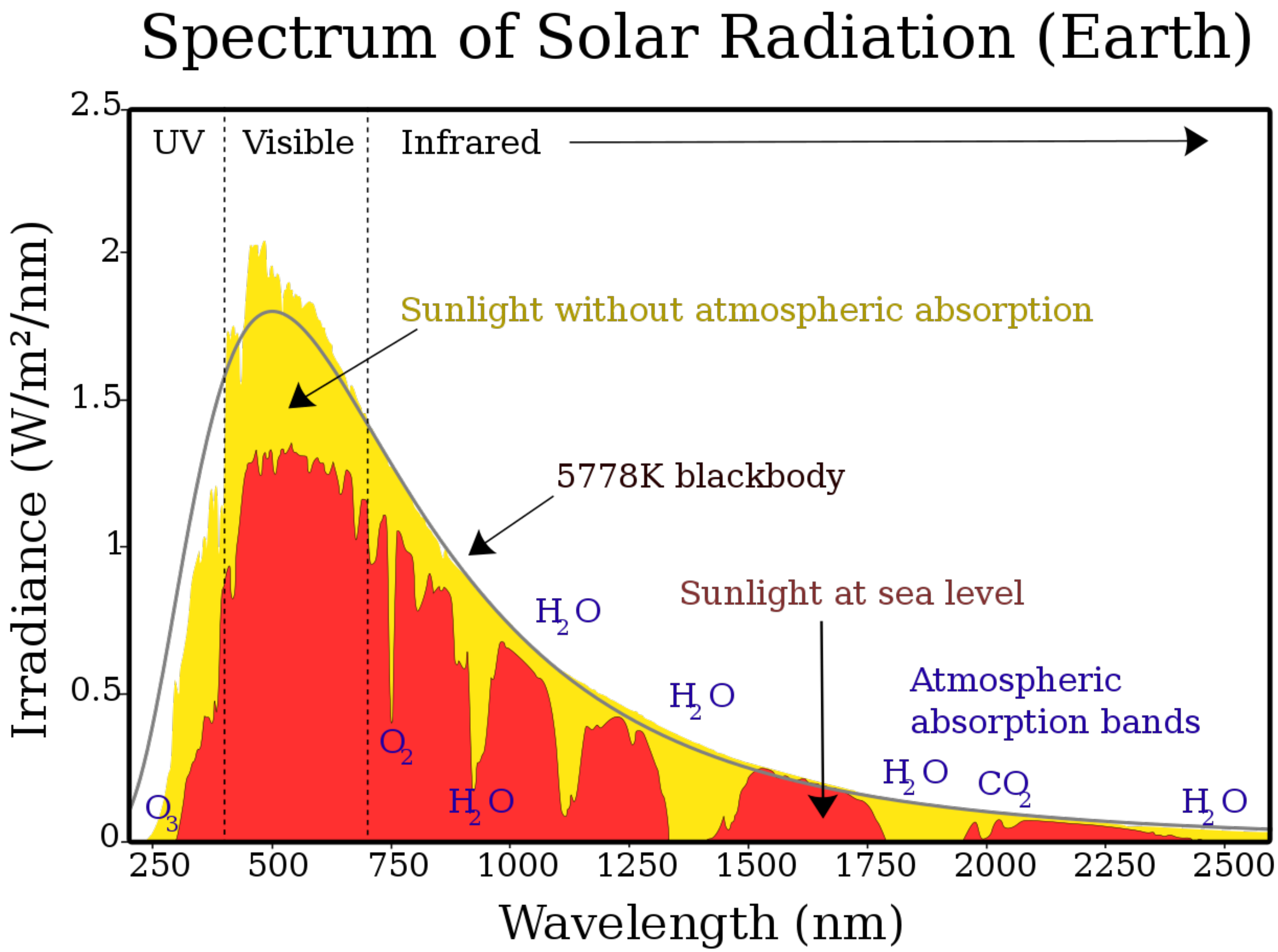
- The spectrum of a star is very similar to that of a black body
- Wien's law: $\lambda_{max} T = 0.29 \text{ K cm}$
- Stefan-Boltzmann law: $L = 4\pi R^2 \sigma_s T_{eff}^4$
  - σ_s = 5.67 10^{-5} erg cm^{-2} s^{-1} K^{-4} is the Stefan-Boltzmann constant
  - T_{eff} is the effective temperature of a star with radius R and luminosity L (≡ temperature of black body having same radiated power per unit area)

Credit: Astrophysique sur Mesure



- In practice, T_{eff} is generally estimated from the colour index B-V which is the brightness ratio λ_B~4350 Å and λ_V~5550 Å
- Johnson photometric system UBV = Ultraviolet Blue Visible

Are stars good black bodies?

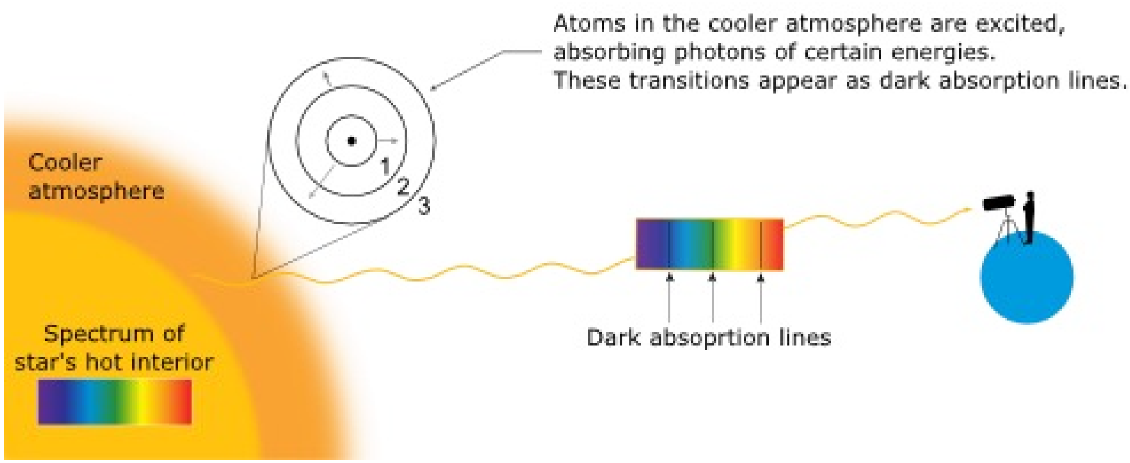


Deviations from black body emission

- Absorption and emission lines
- Contribution of several thermal components (photosphere, corona...)

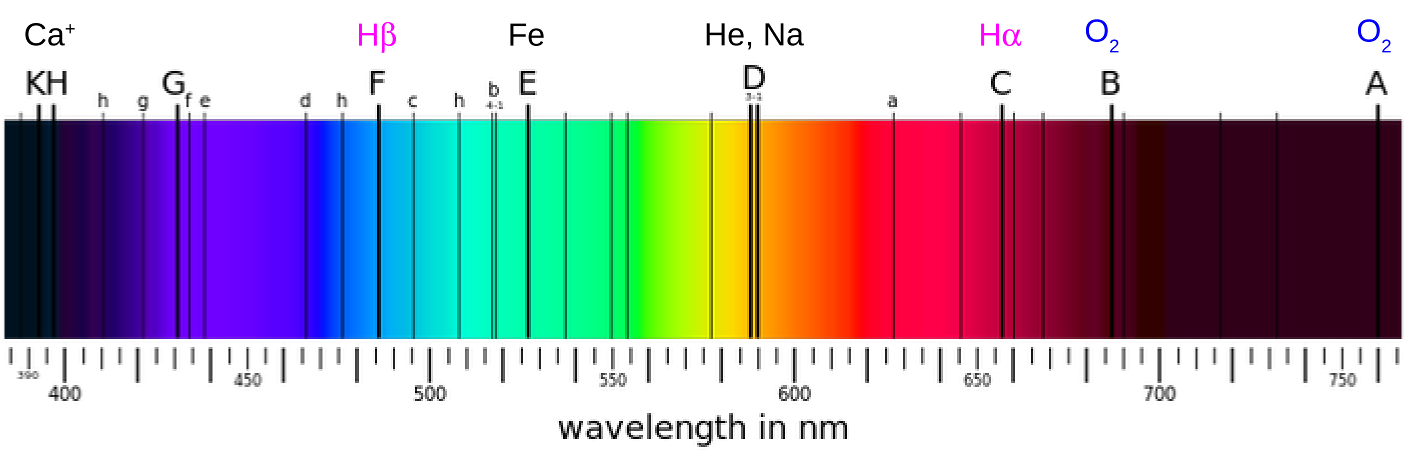
Stellar spectra

Absorption spectra from stars



- Each element absorbs light at characteristics frequencies
- Information on:
  - Chemical composition
  - Surface temperature
  - Ionization degree
  - Gas pressure and density
  - ….

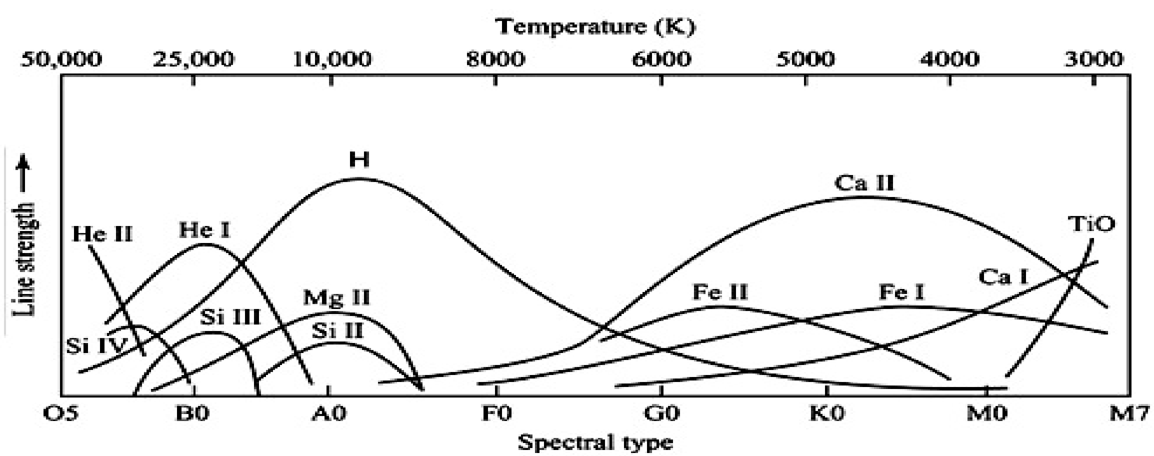
Solar spectrum



Spectral classification

The Harvard classification of stars (“Oh, Be A Fine Girl/Guy, Kiss Me”)

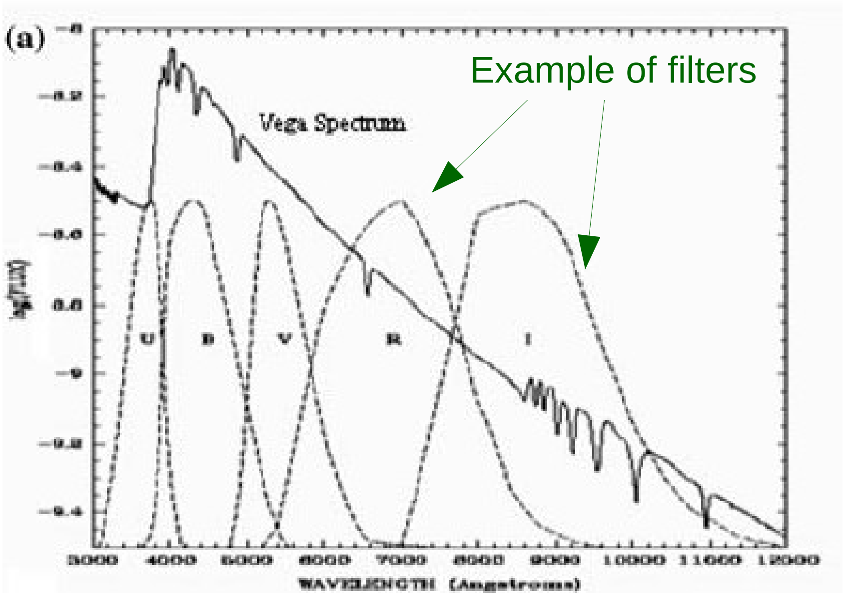
Class	T_{eff}	Colour	Absorption lines
O	> 25000 K	blue	Helium, nitrogen, carbon & oxygen
B	10000 – 25000 K	blue – white	Neutral helium, moderate hydrogen
A	7500 – 10000 K	white	Strong hydrogen
F	6000 – 7500 K	yellow – white	Metals: Fe, Ti, Ca, Sr, Mg
G	5000 – 6000 K	yellow **(sun)**	Calcium, helium, hydrogen, metals
K	3500 – 5000 K	yellow – orange	Metals
M	< 3500 K	red	Metals and titanium oxide



The apparent magnitude

- The magnitude of a celestial body is a measure of its brightness (*F*) according to a logarithmic scale (adapted from human visual perception)

- Apparent magnitude: $m_X(obj) - m_X(Vega) = -2.5\log_{10}\left(\frac{F_X(obj)}{F_X(Vega)}\right)$



X = U, B, V, ...

- Vega (A0, 2nd brighest star in the northern hemisphere) was chosen as the zero point

- Now m_x(Vega) = +0.03 !

- Inverted scale: apparent magnitude of +1 means 2.5 times less luminous than Vega

The absolute magnitude

- The interesting physical quantity is the luminosity:

$$L_X = F_X \times 4\pi D^2$$

where *D* is the distance to the object

- To compare the luminosity of different objects they are placed at a common distance of 10 pc (1 pc = 3.26 ly)

- The absolute magnitude *M* of an object is its apparent magnitude if it were at a distance of 10 pc

$$m - M = -2.5 \log_{10}\left(\frac{L_X}{4\pi D_{pc}^2} \times \frac{4\pi 10^2}{L_X}\right) = 5 \log_{10} D_{pc} - 5$$

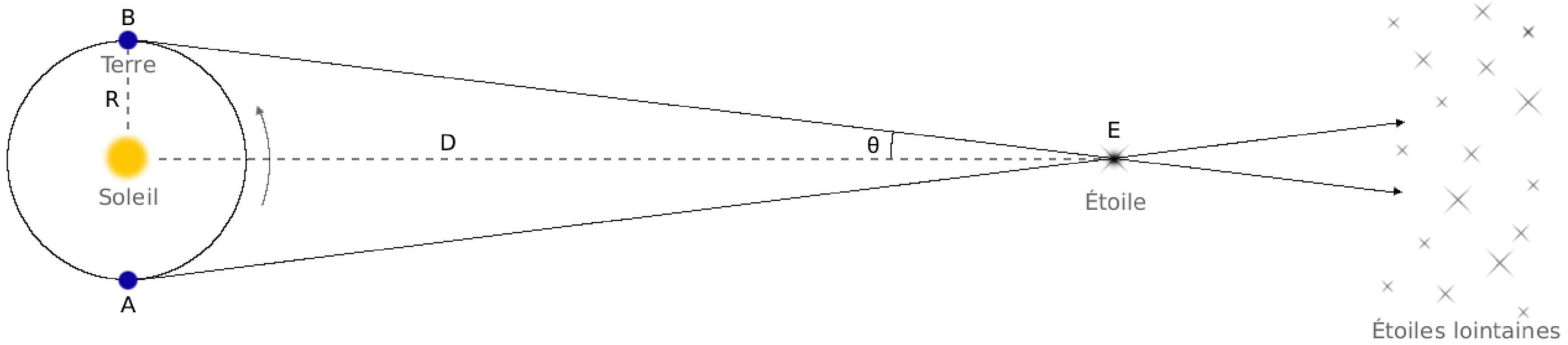
where D_{pc} is the distance in pc and *m – M* is the distance modulus

Some typical magnitudes

Object	m	M
Sun	-26.8	+4.8
Full moon	-12	invisible
Venus	-4	invisible
Betelgeuse (supergiant star)	+0.5	-5.6
The faintest star visible with naked eye	+6	
Andromeda galaxy	+3.4	-20.7
Quasar in the distant Universe	+28	-30

Distance measurement – parallax

- The method of annual parallax (p) is the only one to give a direct measurement of stellar distance



$D = R/\theta$ (θ in radian), and R is the mean Earth-Sun distance (= 1 AU)

$D_{pc} = 1/p$ (p in arcseconds ≡ 1/3600 degree)

- 1 parsec: distance from which R is 1” (= 3.26 ly)

- Photographic plates: $p \sim 0.01''$
  HIPPARCOS ESA satellite (1989): $p \sim 0.001''$
  GAIA ESA satellite (2013): $p \sim 10^{-6}\,''$
  → D ~ Mpc (!) [Milky Way ~ 25 kpc]

The Hertzprung-Russell (HR) diagram

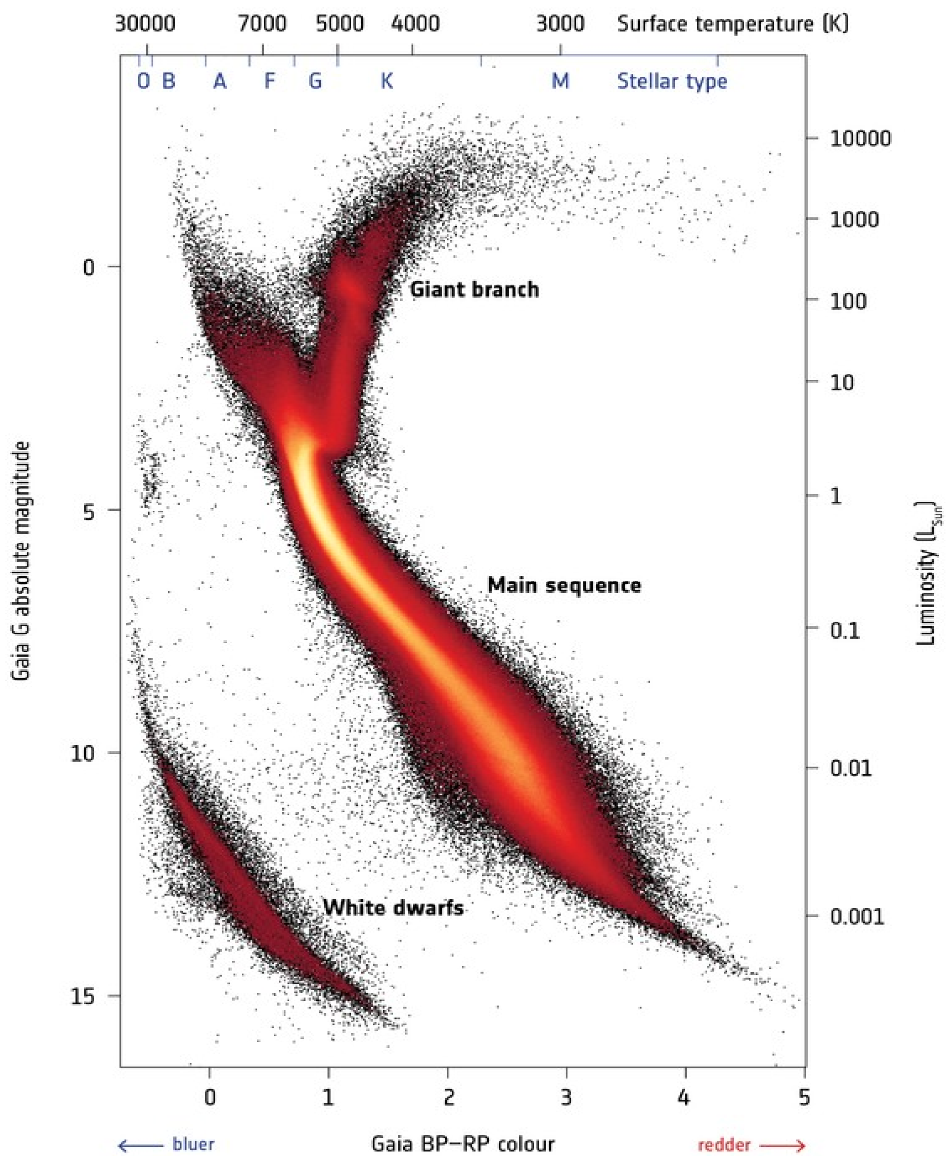
- Diagram found in 1911–1913 named after the two astronomers Hertzprung and Russell

- Luminosity classes
  - I. Supergiants
  - II. & III. Giants
  - IV. Subgiants
  - V. Main sequence (MS)
  - VI. White dwarfs (WD)

- $L = 4\pi R^2 \sigma_s T^4$
  for a same temperature, the lower the luminosity, the smaller the star radius

- GAIA, 2nd data release
  > 4 million stars within 5 kly from Sun

→ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM



HR diagram is a key tool to understand star population and evolution

Star clusters

Open cluster

- M45 – The Pleiades

  ~ 500 stars (some are hot)

  age: 4×10^{7} years (young)

  distance: 120 pc (close)

Globular cluster

- M13 – Hercules

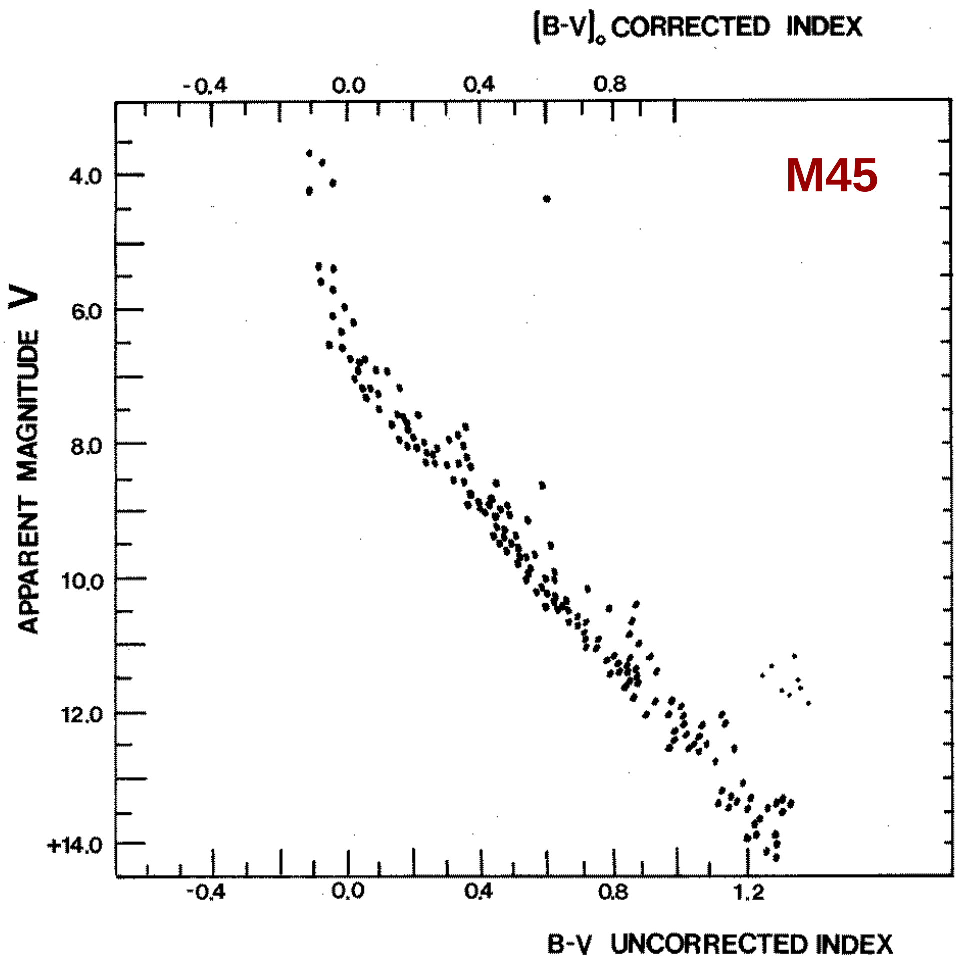
  ~ 300 000 stars (!)

  age: 11.6×10^{9} years (old)

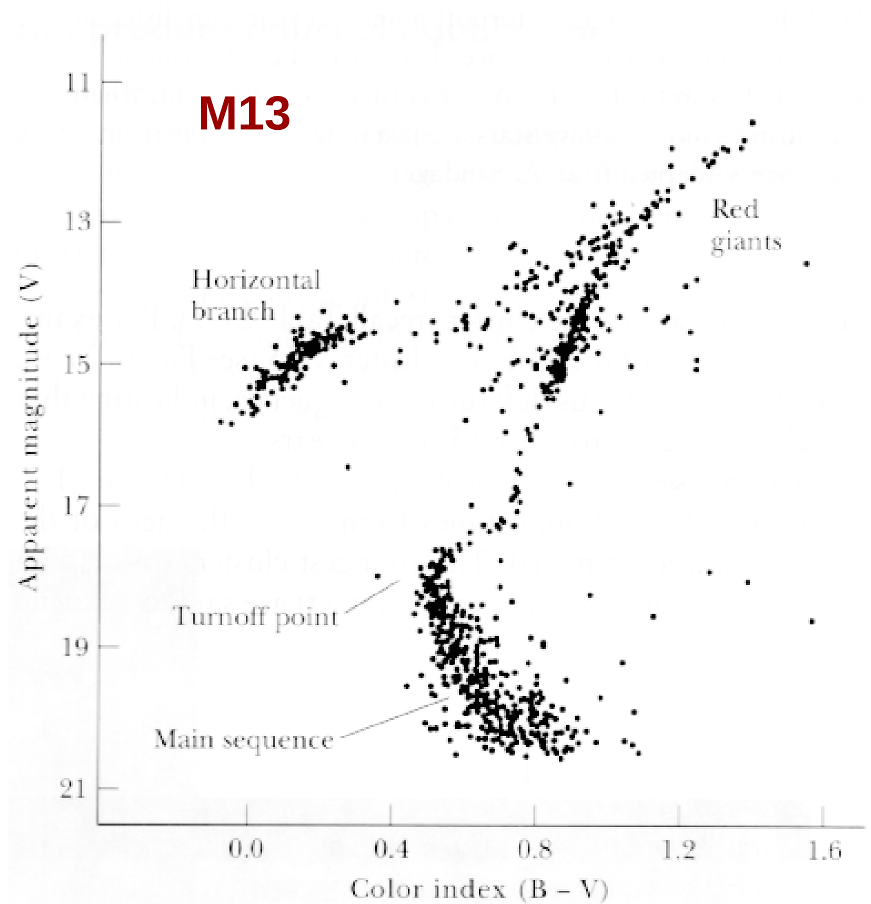
  distance: 6.8 kpc (far)

The HR diagram of star clusters

Open cluster



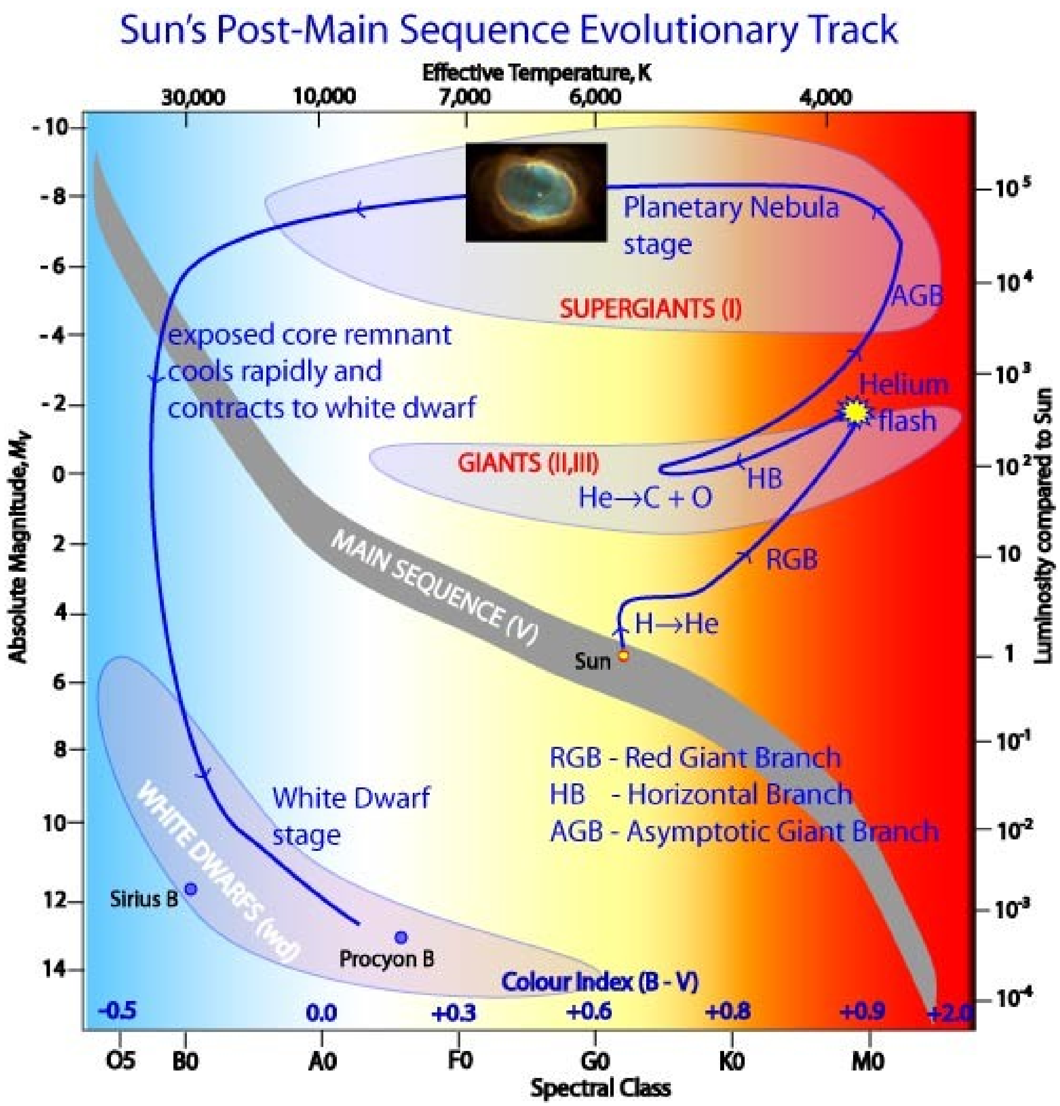
Globular cluster

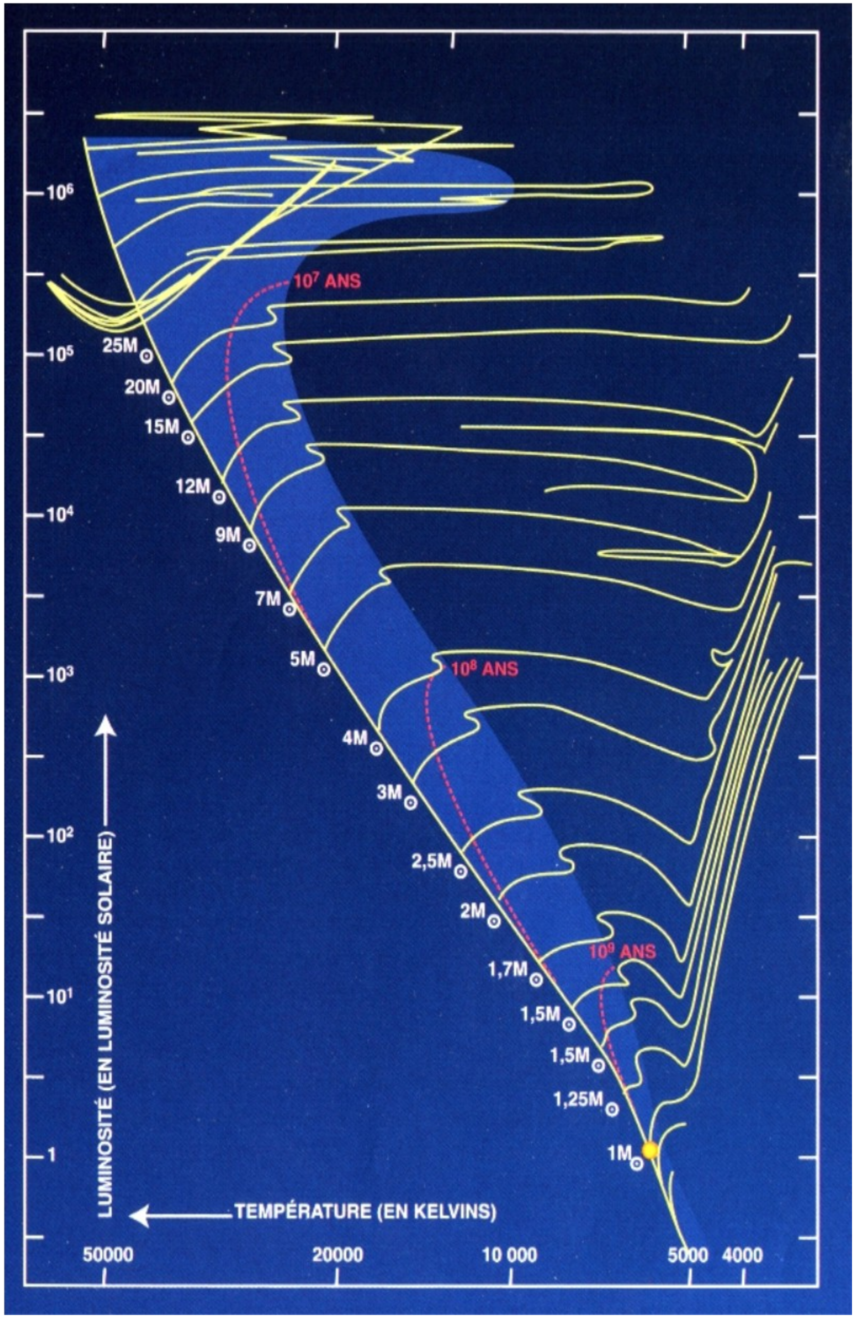


- Most of the stars are in the main sequence for the Pleiades
- Much more complex HR diagram in case of M13

HR diagram is a key tool to understand star population and evolution

Theoretical HR diagrams





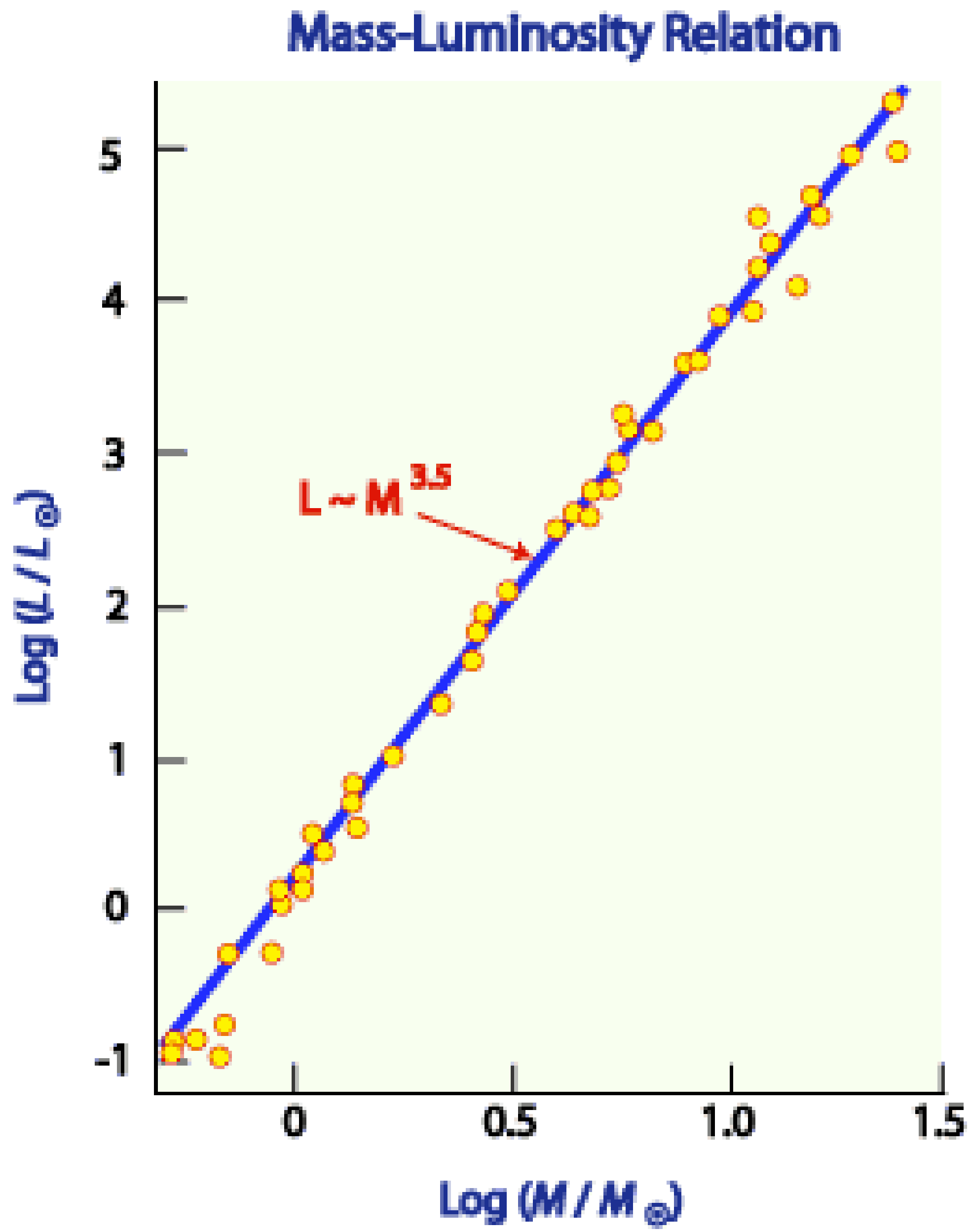
- The cluster age can be found from the position of the turn-off phase

The mass-luminosity relation...

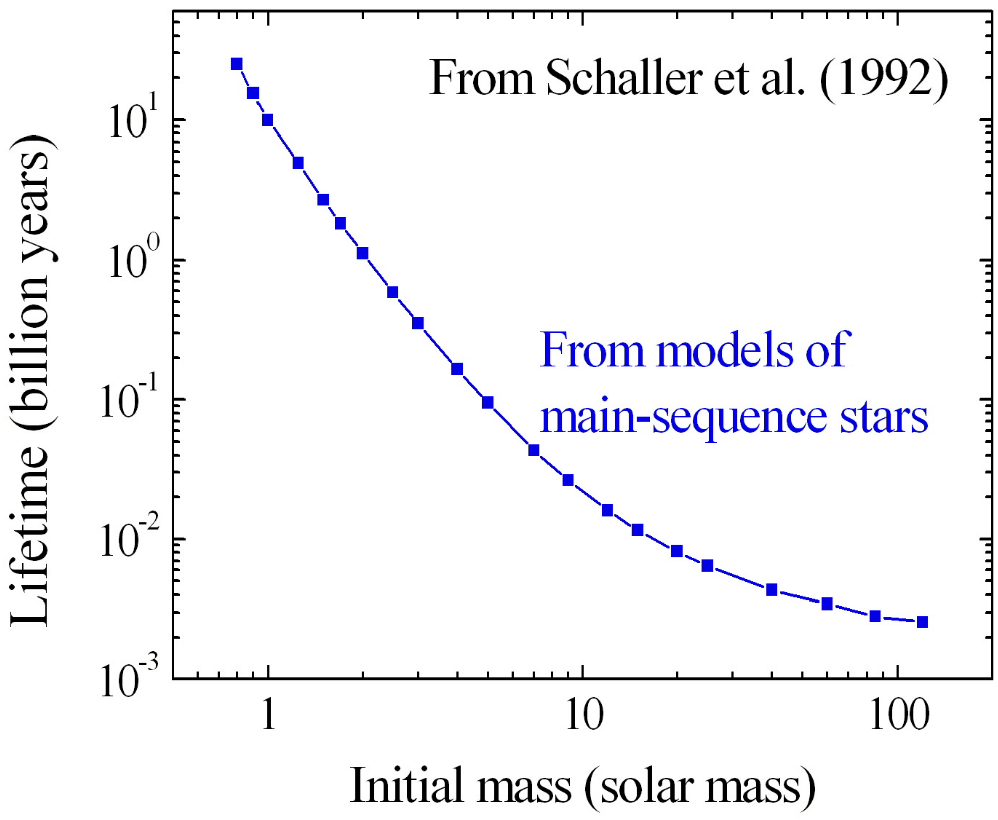
- ….of stars in the main sequence (not good for, e.g., white dwarfs and giants)

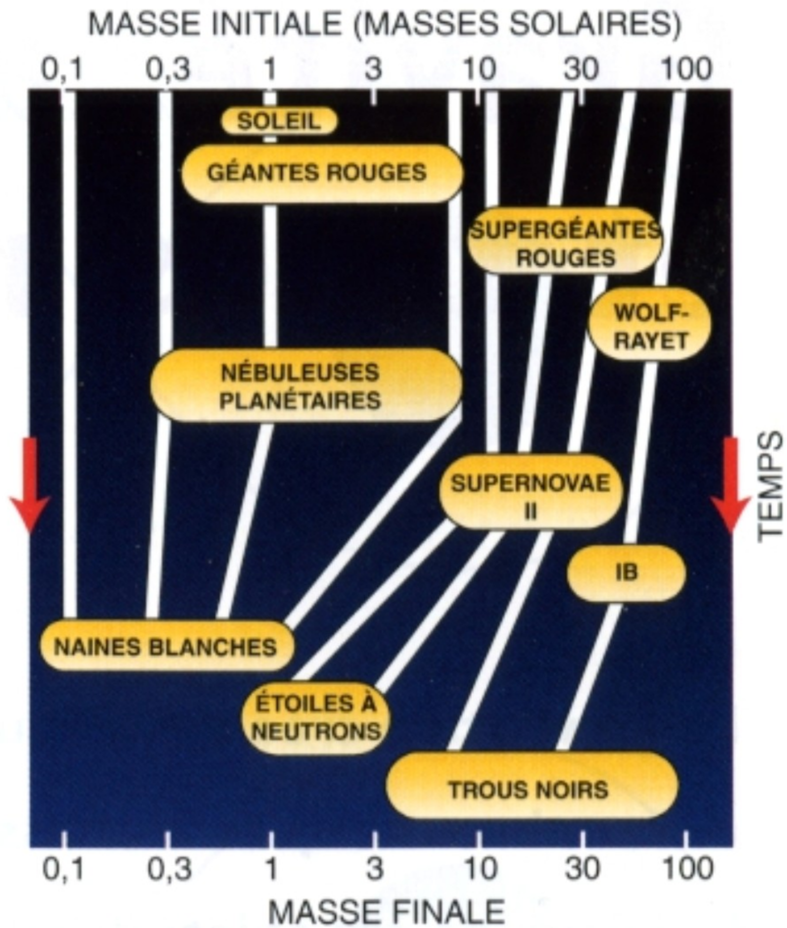
- Based on observations of relatively nearby eclipsing binaries (binary star systems where the orbit plane is along our line of sight)

$\Rightarrow L\ \mu\ M^{\nu}$ with $\nu \sim 3 - 4$



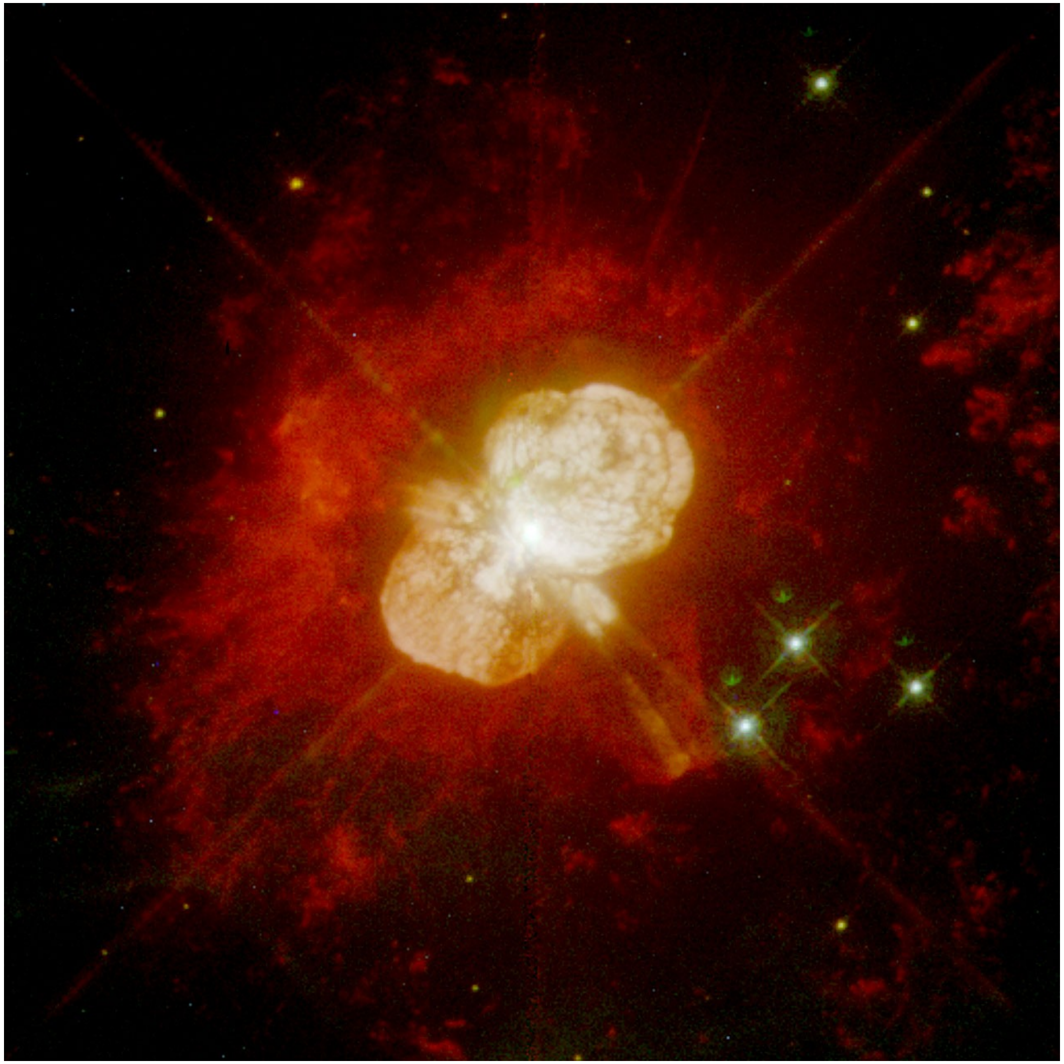
The mass and fate of stars





The initial mass of stars fixes their lifetime and ultimate fate (white dwarfs, neutron stars, black holes)

2. The cycle of matter and chemical evolution of the galaxy

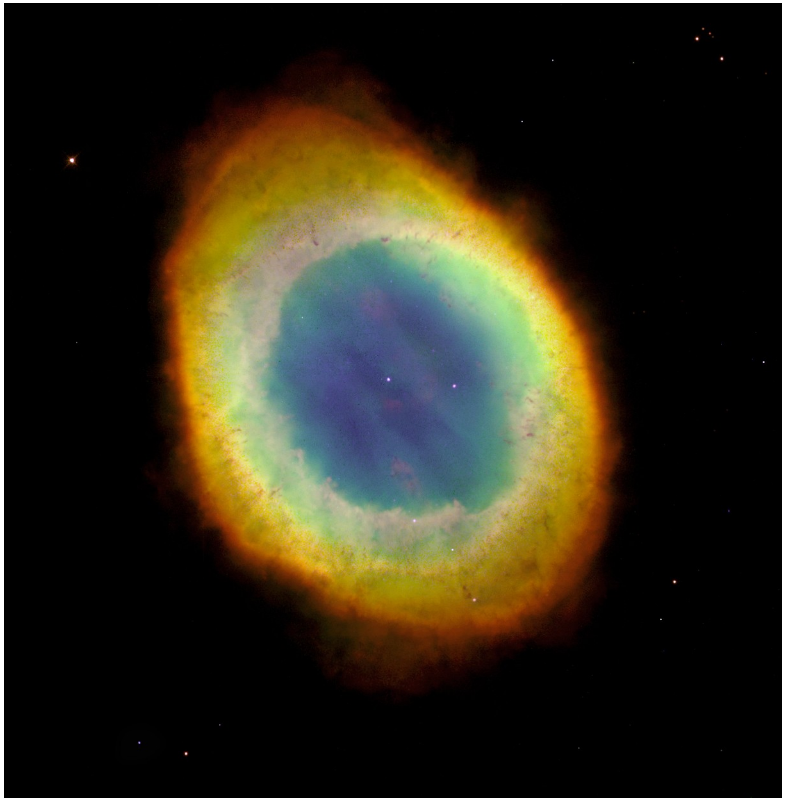


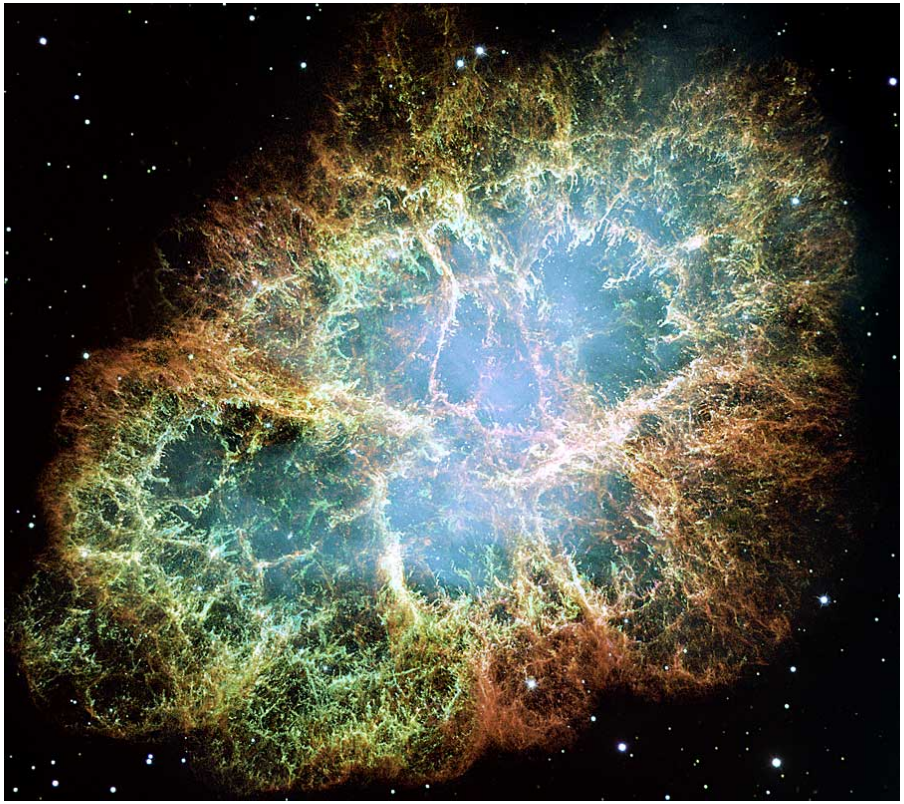
Eta Carinae – Hubble Space Telescope – WFPC-2

Eta Carinae (η Car A)

- 100 – 150 $M_\odot$
- 2.6 kpc
- Variable blue hypergiant
- Huge explosion 150 years ago (still here)

The “death” of stars

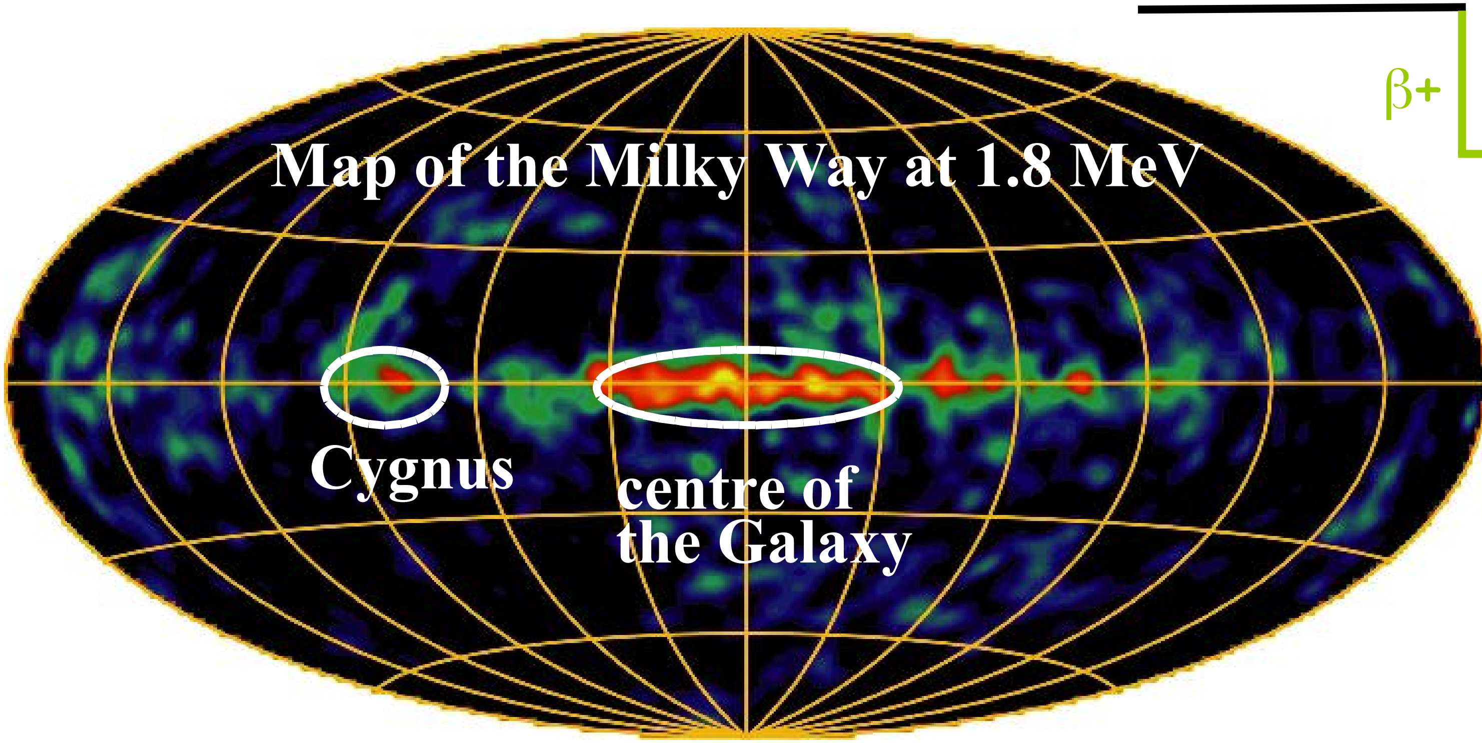


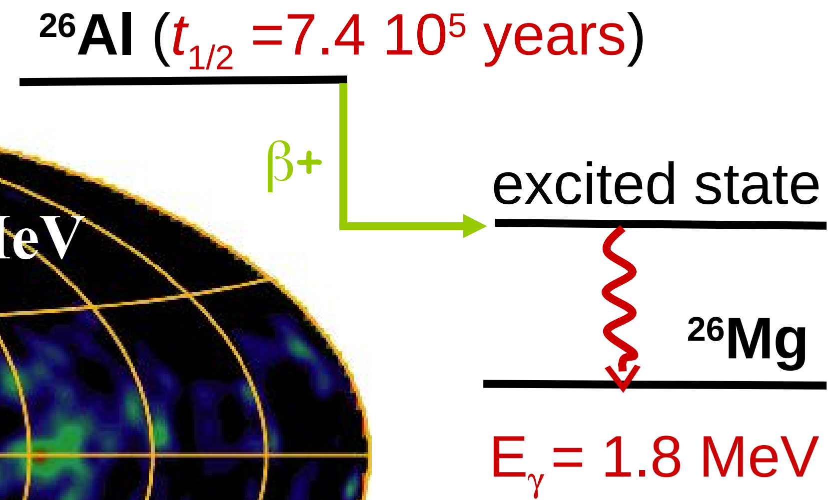


- The ring nebula (M57) in the Lyra constellation is a “planetary” nebula
  (look for the white dwarf)

- The crab nebula (M1) in the Taurus constellation is a supernova remnant
  (look for the neutron star)

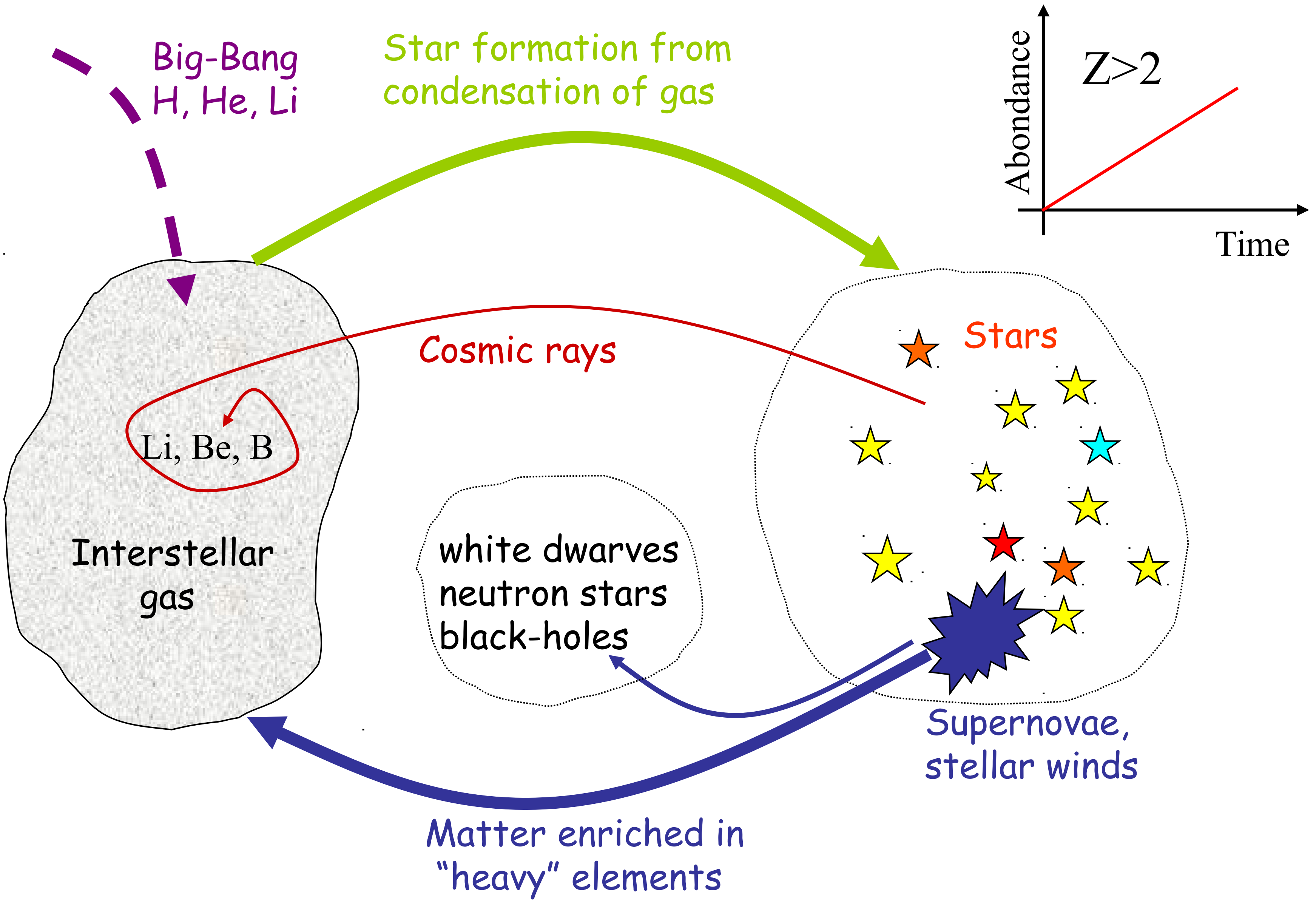
The radioactivity of the Galaxy



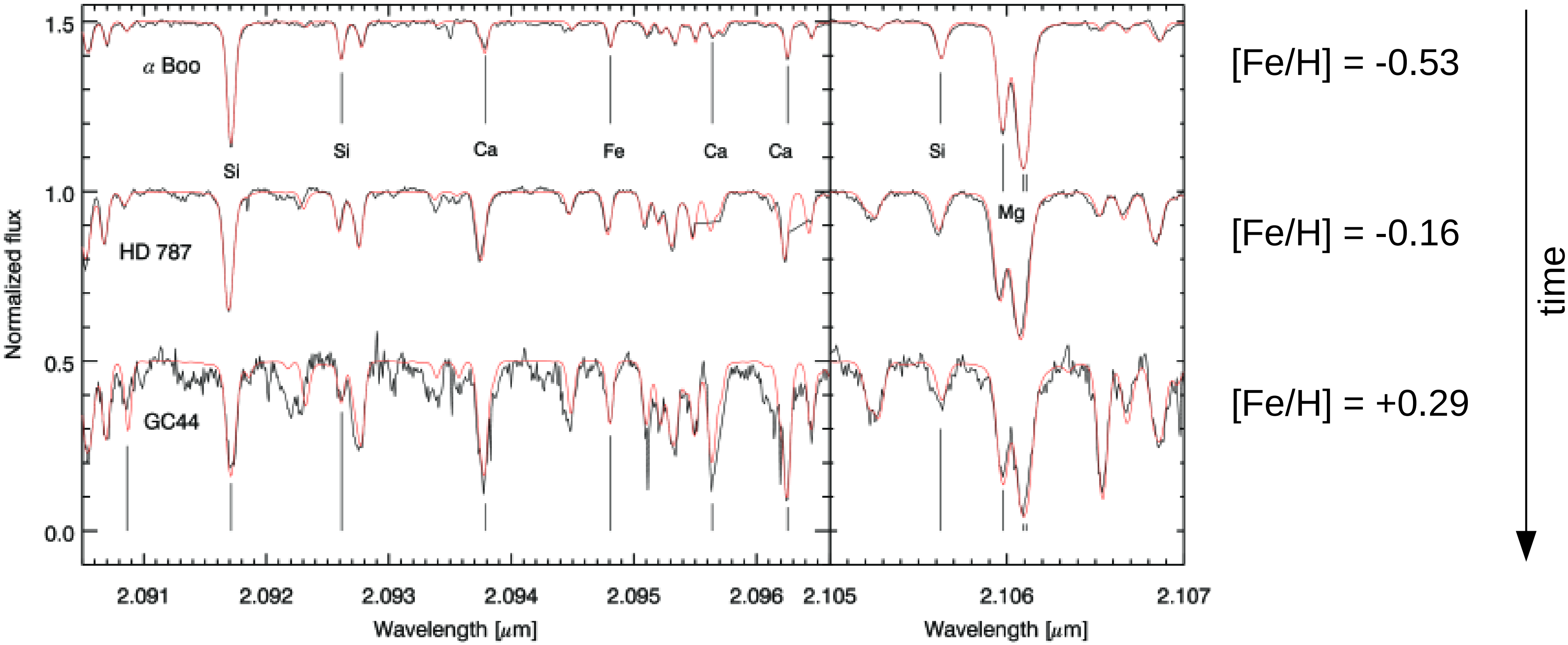


- Gamma-ray astronomy (MeV range) allows to observe in real time the Galactic enrichment in radioactive nuclei

The cycle of matter in the Galaxy



Stellar abundances



- HD 787 and GC44 are stars of population I
- α Boo (= Arcturus) is probably a star of population II (more metal poor)

The metallicity

- Metal (astronomy): every chemical element heavier than helium (Z>2)

- Metallicity:
  - Using mass fractions: $Z = 1 - X - Y$
    where X, Y and Z are mass fractions of H, He and metals, respectively
    → Sun (surface): $Z = 0.0134$, $X = 0.7381$, $Y = 0.2485$

  - Using chemical abundance ratios:

$$[Fe/H] = \log_{10}\left(\frac{n_{Fe}}{n_H}\right)_{star} - \log_{10}\left(\frac{n_{Fe}}{n_H}\right)_{sun}$$

    n_H and n_{Fe} are numbers of H and Fe per unit of volume (density)

  - The Fe abundance (n_{Fe}/n_H) is one of the most simple to measure in stellar spectra

- Examples
  - $[Fe/H]_{\odot} = 0$ (metallicity of the proto-solar cloud 4.6×10^9 years ago)
  - Stars of population II ("metal"-poor): [Fe/H] < -1 (1/10 of the solar metallicity

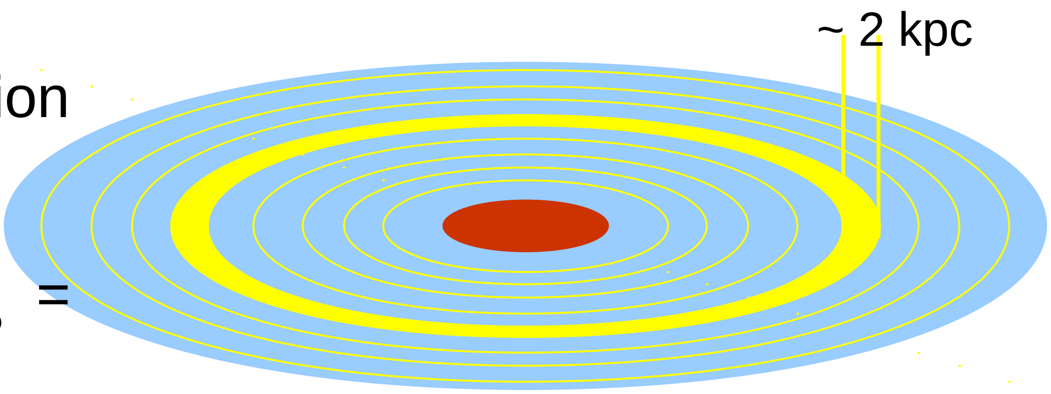
Models of Galactic Chemical Evolution

Goals:

- Compute time and spatial evolution of isotope abundance

Model:

- Independent radial annulus ($R_\odot$ = 8.5 kpc)



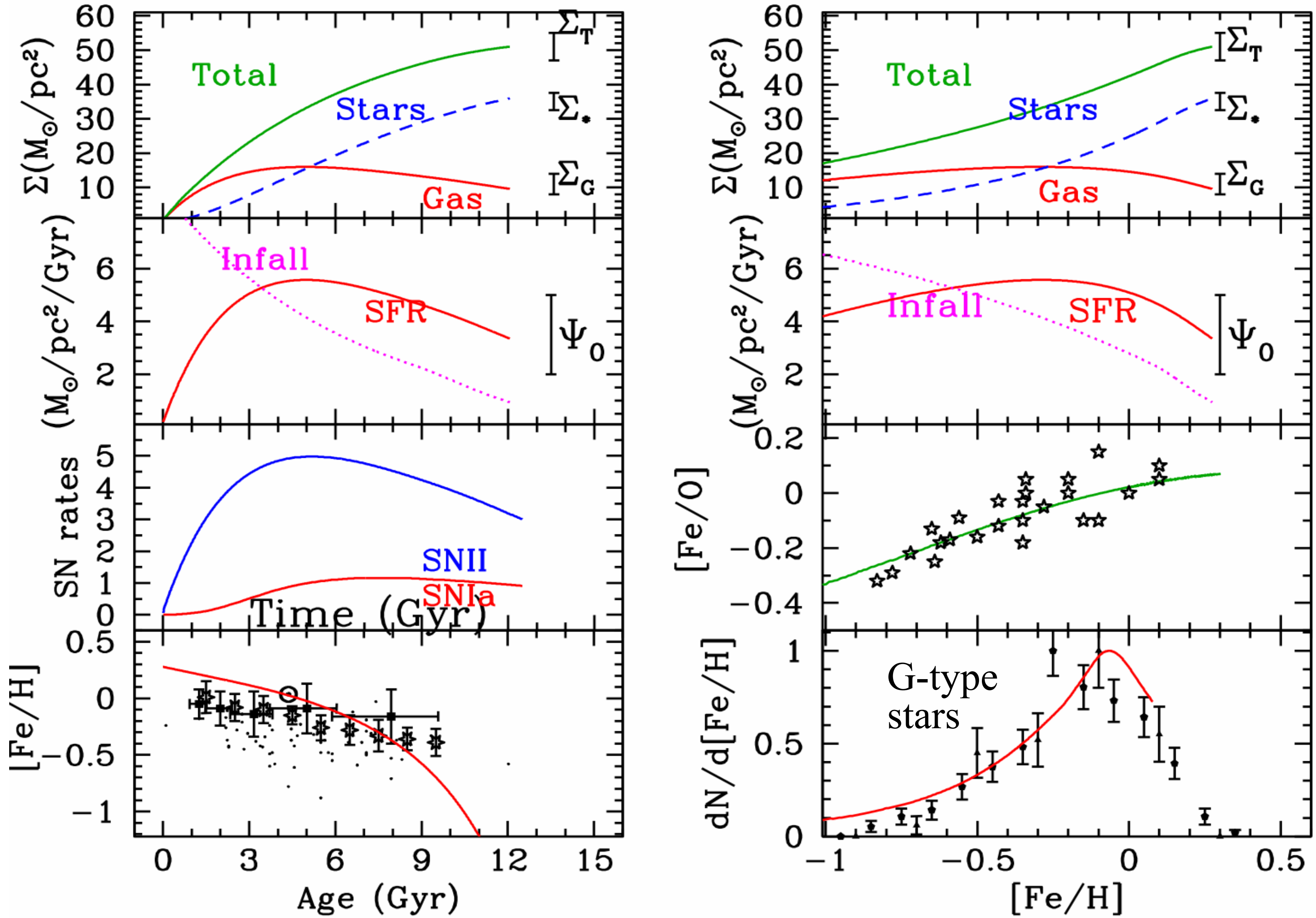
Key ingredients:

- Star Formation Rate (SFR) in the Galaxy ($M_\odot$ per year)
- Initial Mass Function (IMF) of the stars at the time of their formation
- Lifetime of the stars as a function of mass and metallicity
- Production yields of the isotopes in each star (nucleosynthesis)
- Stellar matter ejection rate (stellar winds, supernovae)
- Mixing with the interstellar gas (instantaneous or delayed)
- Interaction of the Galaxy with the intergalactic medium (gas infall, ejection by galactic wind, ...)

→ each of these ingredients is a research topic by itself

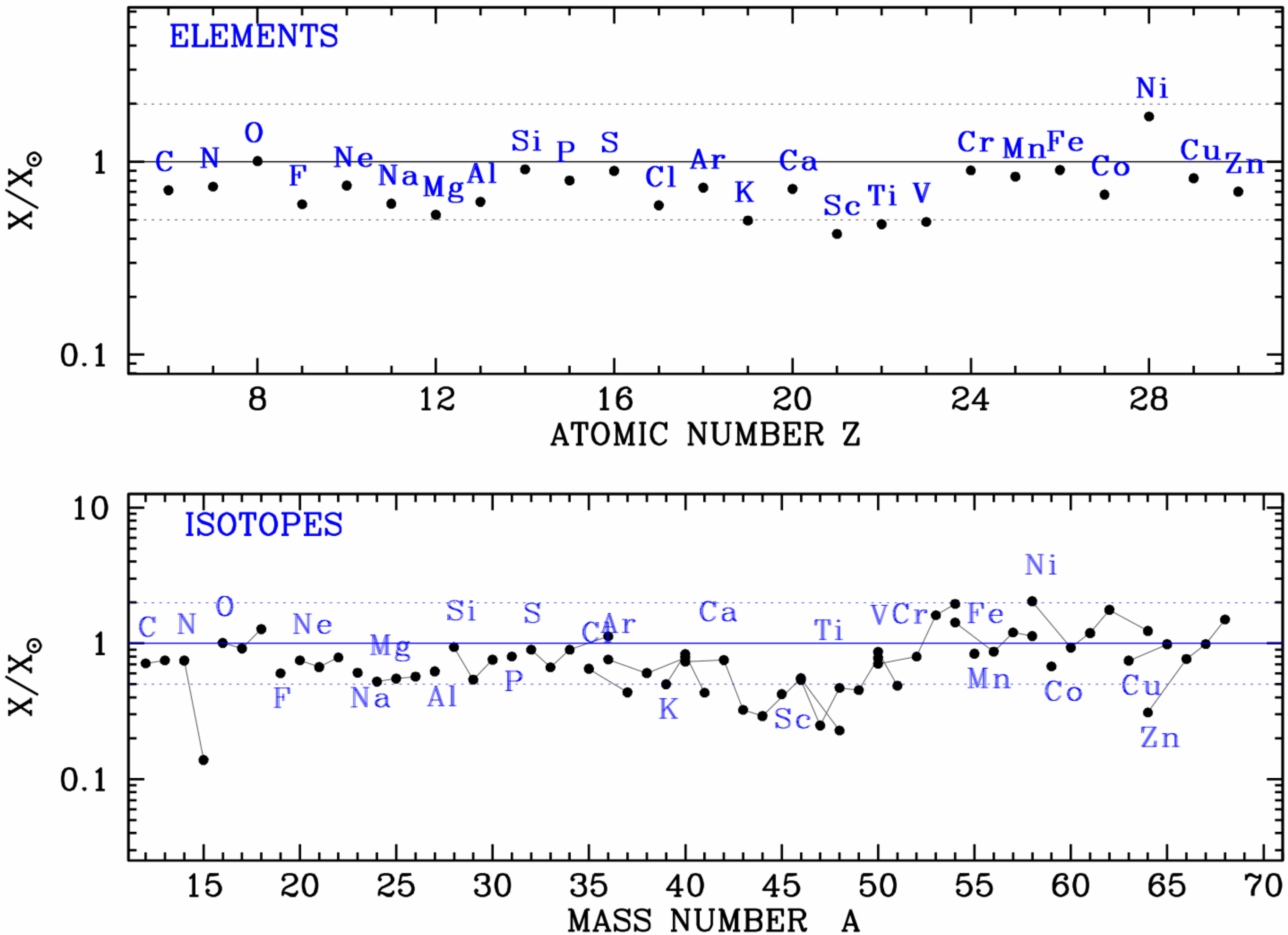
Galactic history in the solar neighbourhood

- Solar neighbourhood: region of volume < 0.5 kpc^3 around the sun



Prantzos (2008)

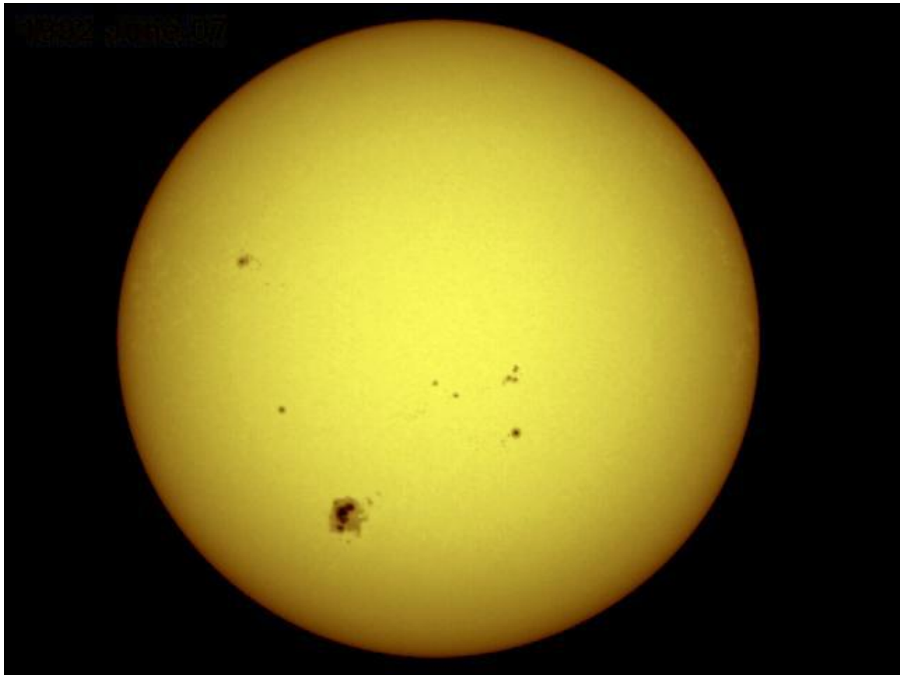
A model for the solar abundances



Prantzos (2008)

- The X_i (mass fraction) from ^{12}C to ^{68}Zn are reproduced within a factor of two (!)
- ^{15}N is most probably synthesized in classical novae

3. The solar or “cosmic” abundances



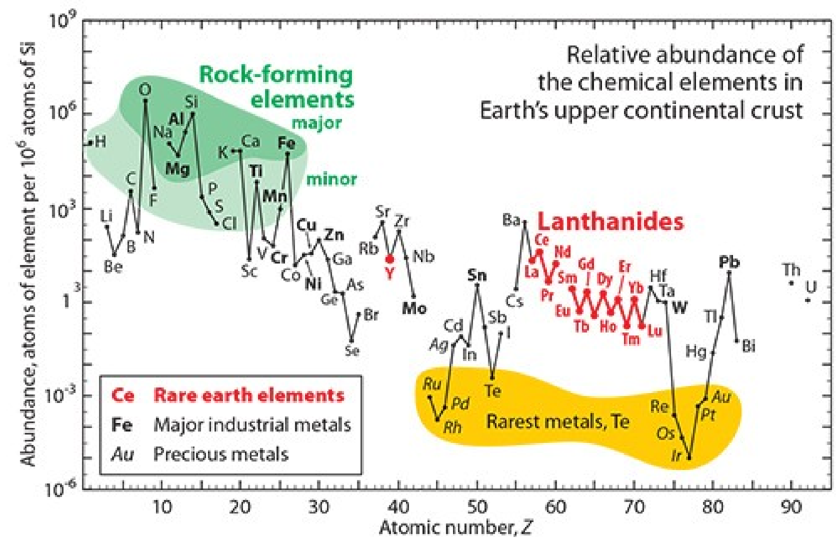
The solar photosphere (NASA)

Fragment of the Orgueil meteorite (France 1864), MNHN collection

How to determine solar abundances?

- Earth material (crust)
  Problem: chemical fractionation strongly modifies the local composition compared to pre-solar nebula

  Example: quartz is dominantly SiO_2, which is not the composition of the solar system, e.g. N(O) = 16 x N(Si)



→ Main source for isotopic composition of elements since chemistry is governed by the number of electrons/protons (not the neutrons)

- Meteorites
  Some categories of meteorite formed from material that never experienced high pressure or temperatures, and therefore were never fractionated
  → direct sampling of the pre-solar nebula

- Photosphere
  Sun is formed directly from pre-solar nebula material and its outer layers (largely unmodified) create spectral features

Gifts from Heaven

Meteorite find in Atacama desert

Up to ~ 200 meteorites / km^2, mean age 710 kyr, max age 2.5 Myr ! (^{36}**Cl**)

Classification of meteorites

- Many different types of meteorites
- Classification relies on chemical composition, mineral properties, ….

Group	Subgroup	Composition	Frequency	Origin
Stony	Chondrites	Fe & Mg silicates	86 %	Primitive asteroids & comets
	Achondrites		8.4 %	surface
Mixed (stony-iron)		Metallic Fe + Fe/Mg silicates	1.1 %	Mantle/core
Iron		Metallic Fe	4.5 %	core

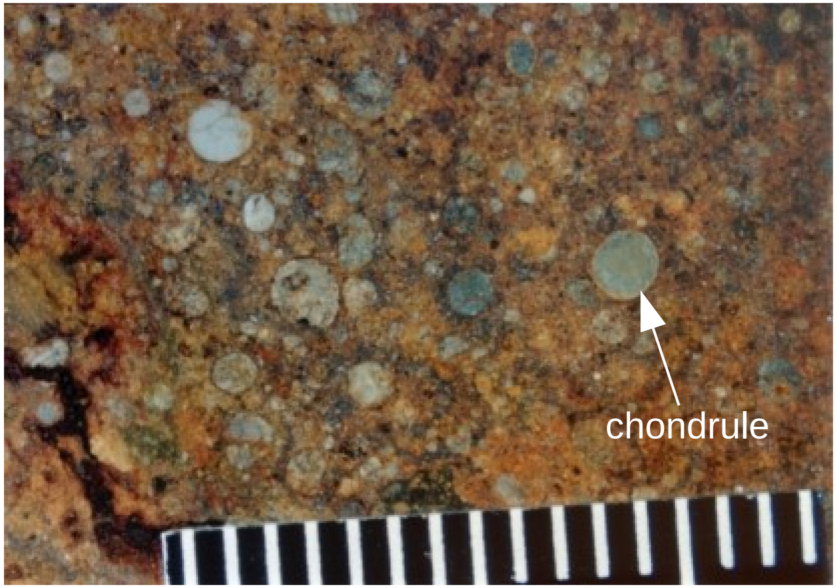
Differentiated bodies

- Not all meteorites provide representative solar abundances (most of them are differentiated or have undergone gas-solid fractionation)
- Chondrites are primitive meteorites that underwent little modification after their formation

Chondrites

- Chondrites have chondrules which are small 0.1 – 1 mm size spherical inclusions in matrix
- Chondrites have formed very early in the presolar nebula and remained largely unchanged since then
- Different types: ordinary (79.9 %), enstatite (1.6 %) and carbonaceous (4.3 %)

Grassland chondrite



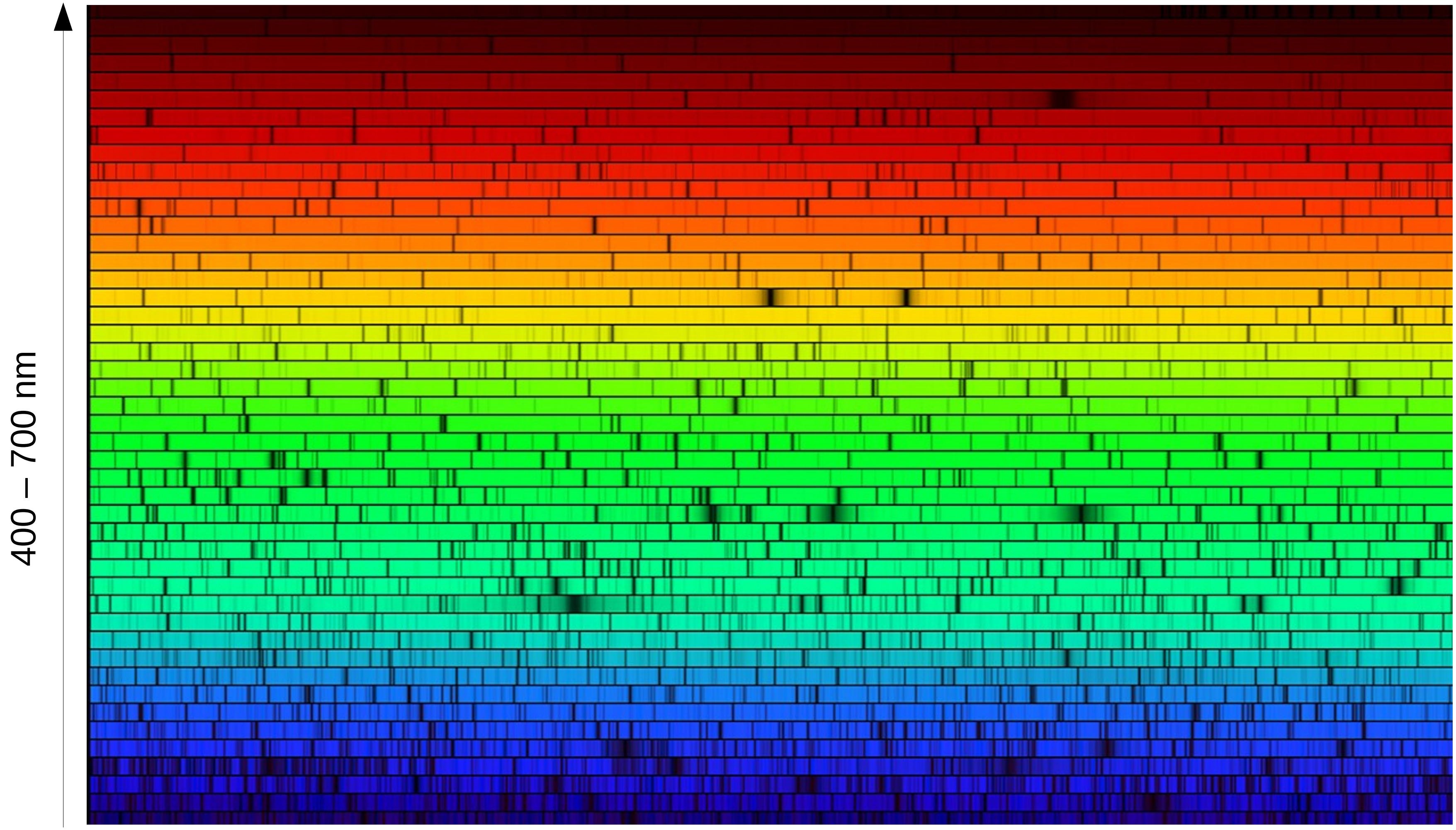
Pieced of Orgueil meteorite

- Carbonaceous chondrites have different properties (very little heating)
- CI are considered to be the least altered meteorites

  → named after Ivuna meteorite (Tanzania, 1938, 705 g)
- Only 5 known meteorites contain CIs chondrites (Alais, Ivuna, Orgueil, Revelstoke, Tonk)

Solar spectrum

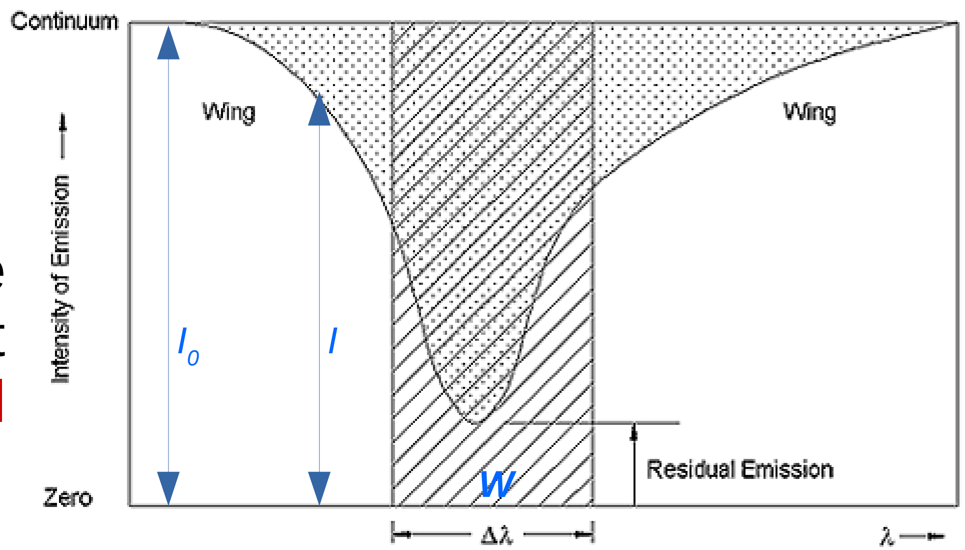
Absorption spectra provide the majority of data because the largest number of elements can be observed, and because they are well understood (good models available)



N. A. Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF

From spectral lines to abundances

- Each absorption line originates from a specific atomic transition in a specific atom/ion
  - Wavelength → atomic species
  - Intensity → abundance



- The equivalent width (*W*) describes the width a rectangular spectral line must have in order to have the same total absorption line as the actual line

- Simple absorption in an atmosphere layer of thickness Δ*x*

$$I = I_0 e^{-n\sigma\Delta x}$$

where *I* is the flux, I_0 the continuum flux, σ the absorption cross-section, and *n* the number density of absorbing atoms

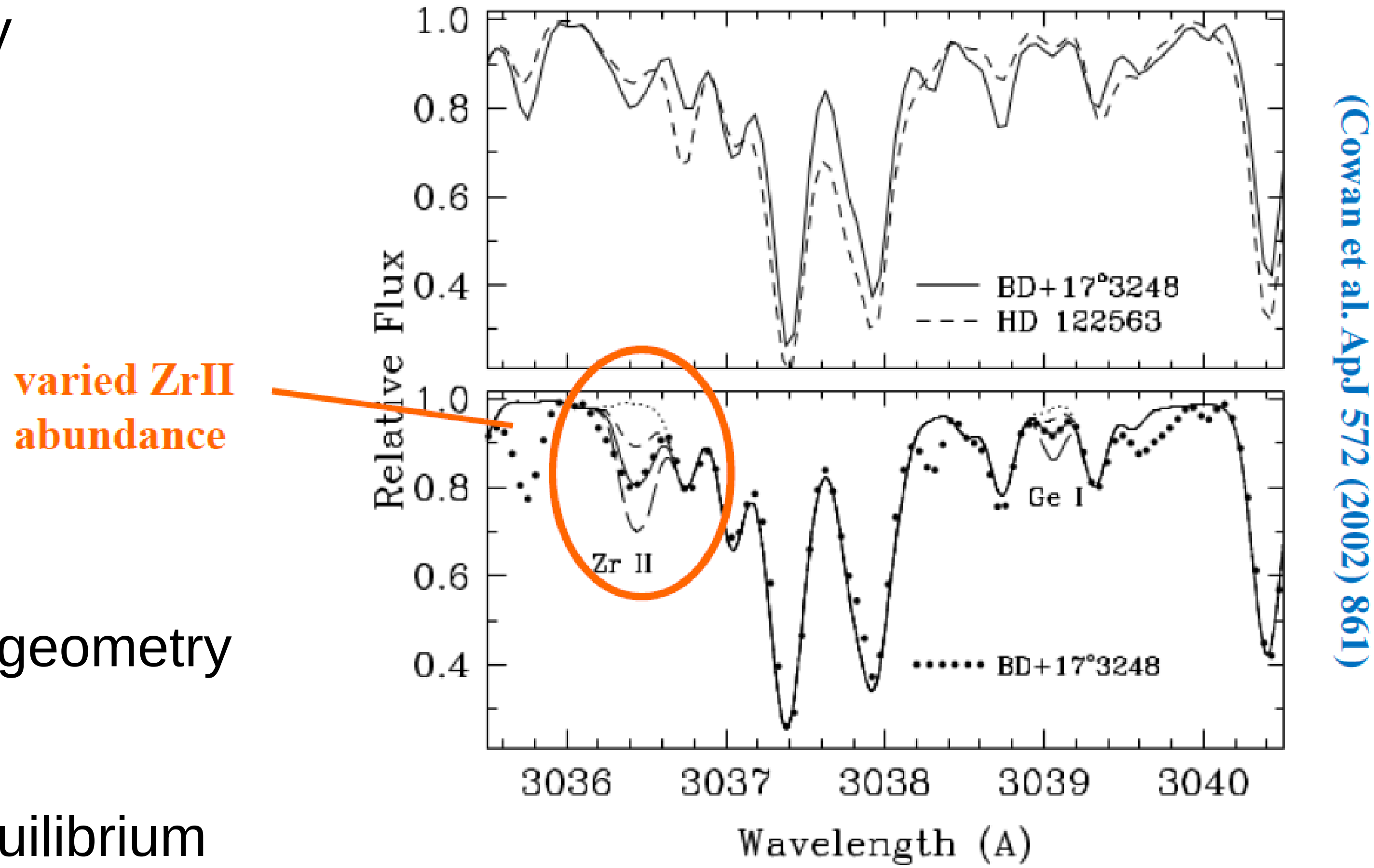
→ if σ is known, one can determine the abundances

- Determination of cross-section is not easy! Oscillator strength (em transition probability between atomic levels), line width (lifetime) depends on natural width, frequency of collisions (*P*), Doppler broadening (*T*) ….

Spectrum synthesis

A good stellar atmosphere model is needed

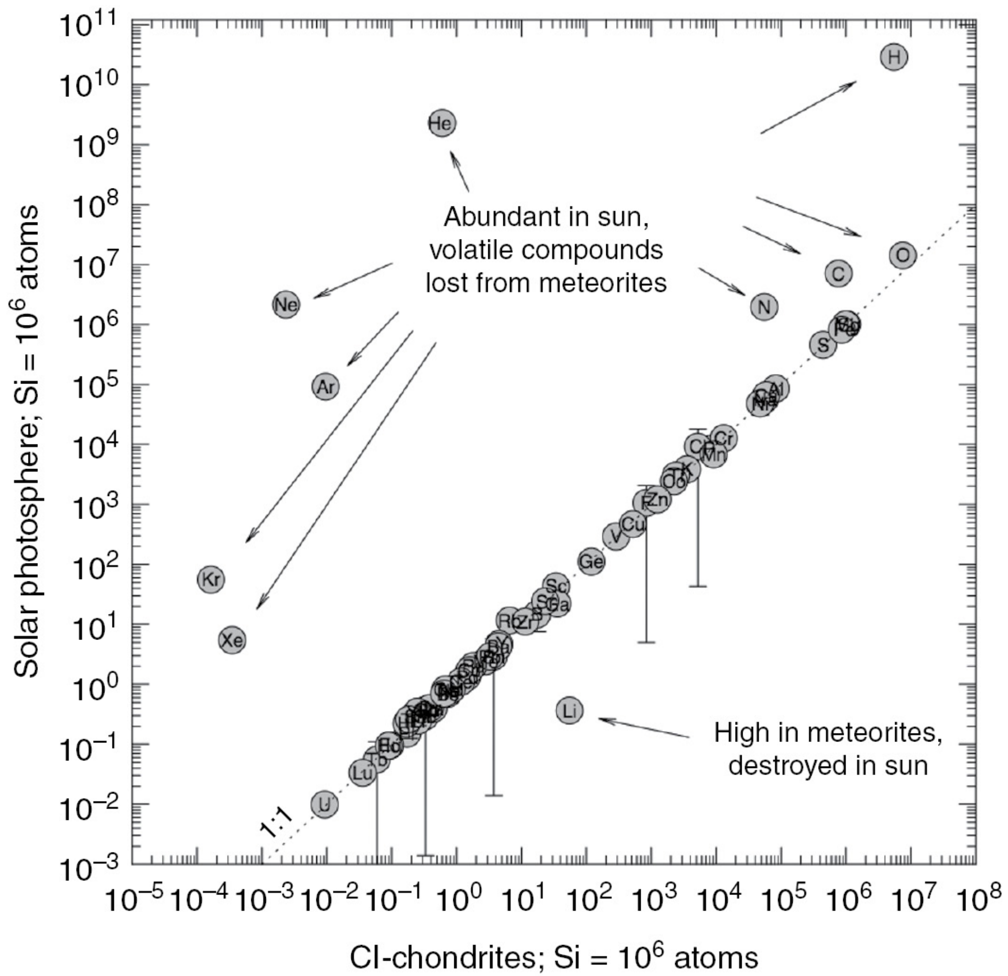
- Effective temperature
- Surface gravity
- “metallicity”



Assumptions

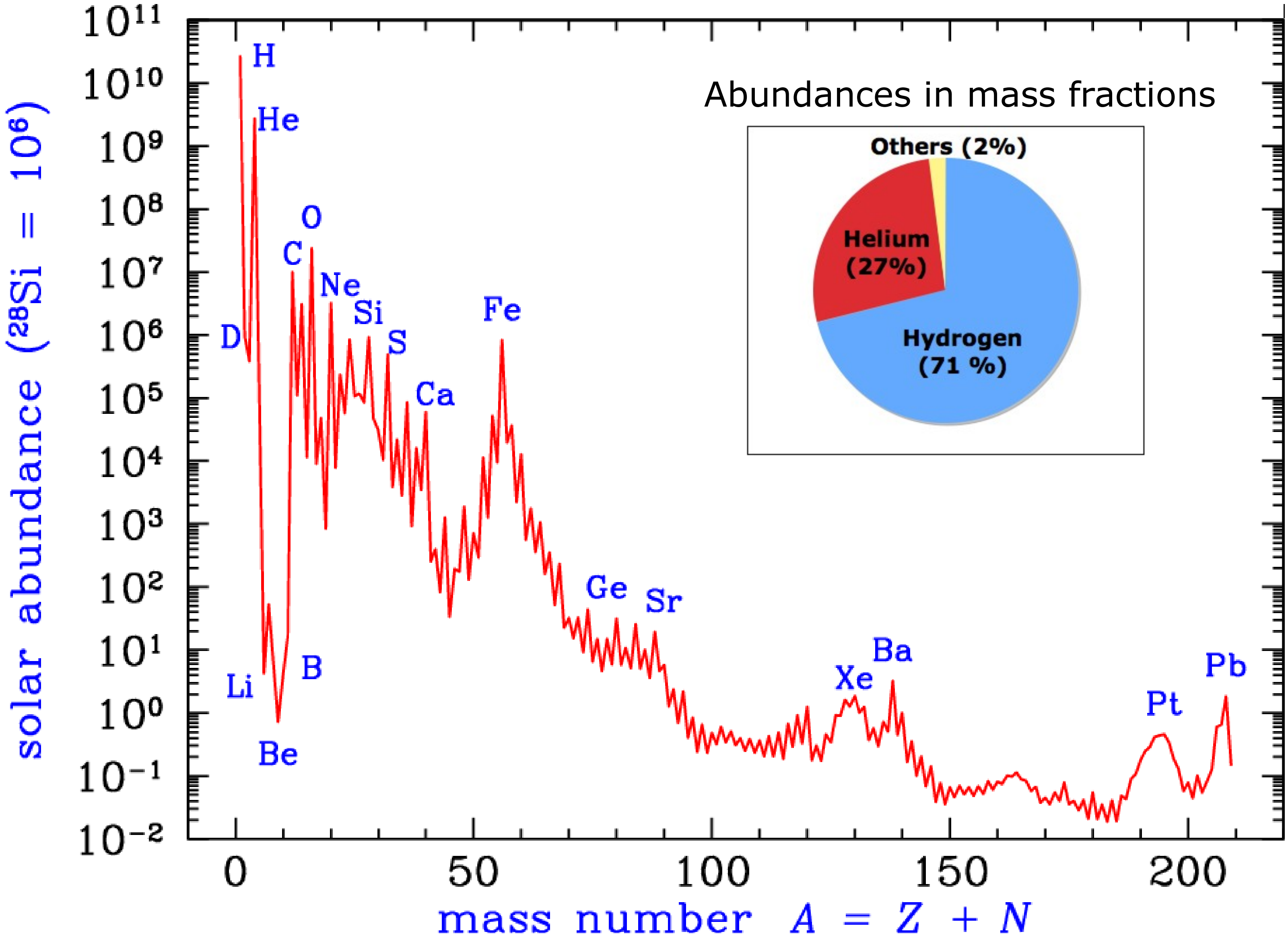
- Plane-parallel geometry
- Homogeneity
- Stationarity
- Hydrostatic equilibrium
- Radiative equilibrium
- Local Thermodynamic Equilibrium (LTE)

Photospheric vs meteoritic abundances



- Chondrites CI and solar photosphere have extremely similar composition over at least 9 orders of magnitude
- Chondrites CI condensed from a gas having the same chemical composition as the Sun

The solar abundance curve



4. Birth of stars

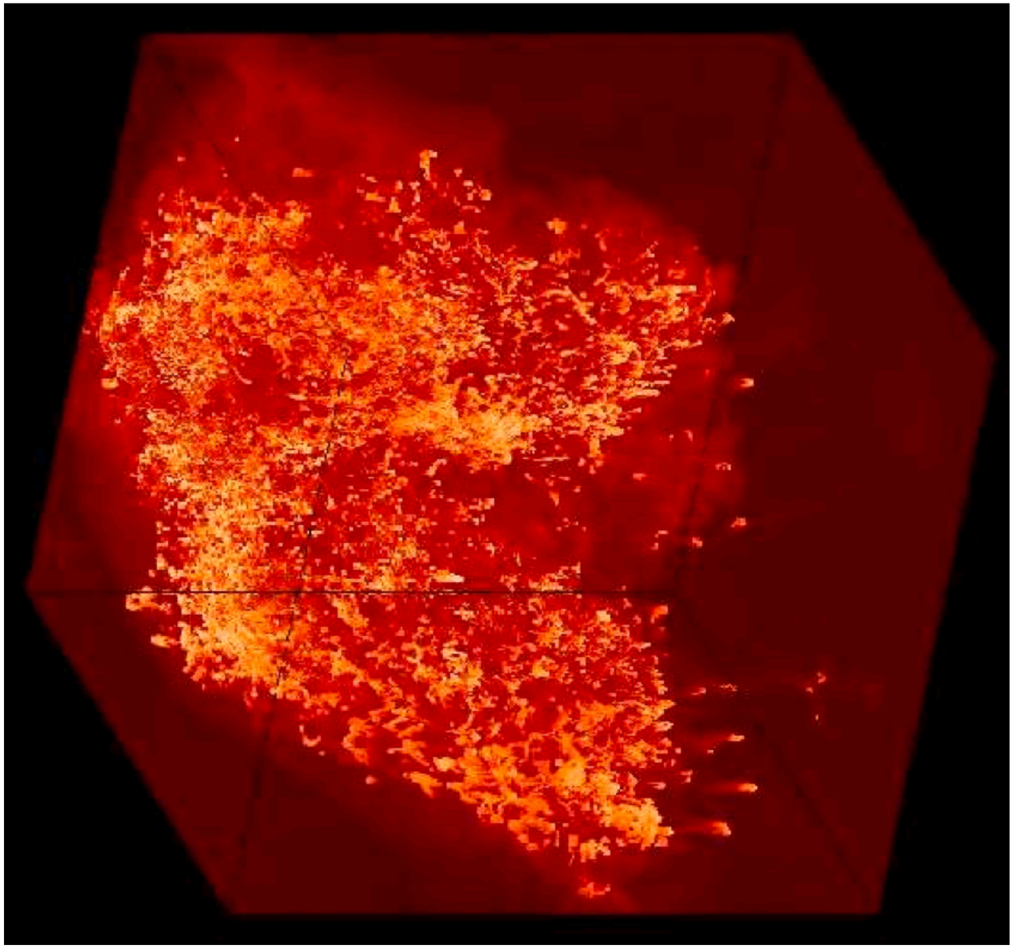
The "Pillars of Creation" within the Eagle nebula (M16), Hubble (2014)

MHD simulations of star formation

- Basic principle: gravitational contraction of a molecular (H_2) gas nebula which becomes unstable

- But it depends on the turbulence generated by the winds from massive stars and the shock waves from supernovae, the interstellar magnetic field, the cosmic rays, …
  → Magneto Hydro Dynamic (MHD) simulations

- Gravitational collapse can be spontaneous or triggered by external influence

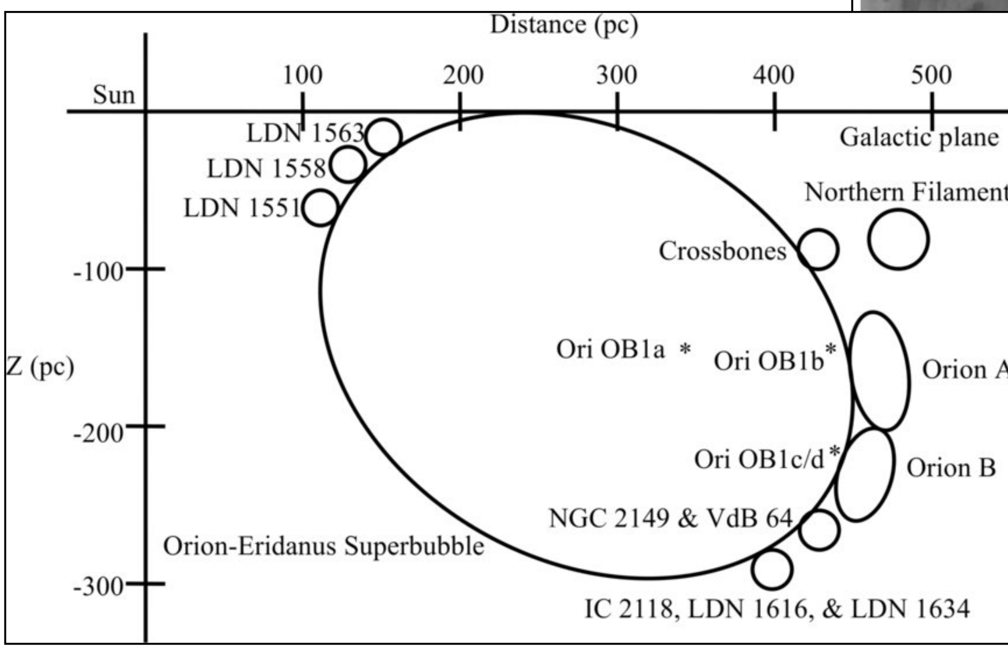


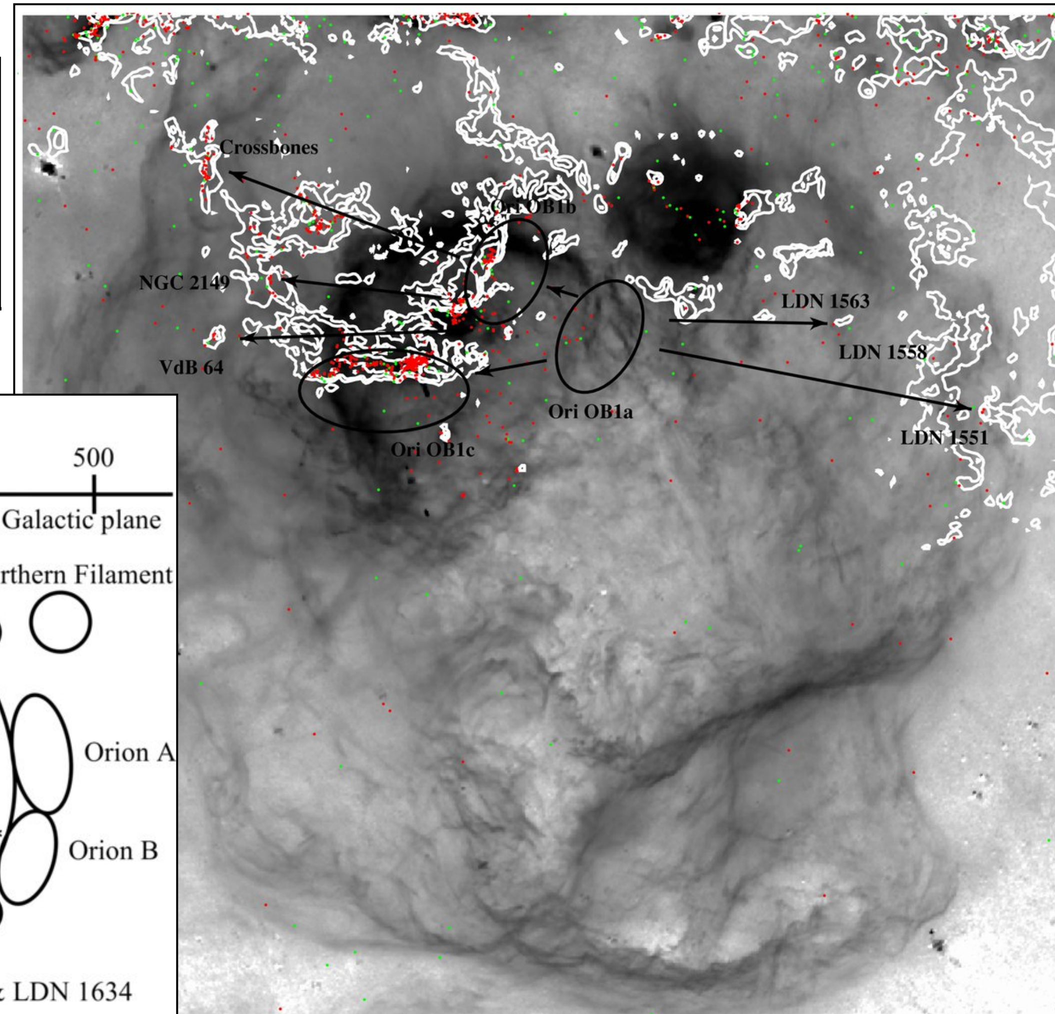
Density distribution of interstellar gas
Audit & Hennebelle (2010)

The role of massive stars

- Massive stars (winds, supernovae) trigger the birth of new generation of stars

Triggered star formation on the border of the Orion-Eradinus superbubble
(Lee & Chen 2009)





- Gray scale → Hα → ionized hot gas
- Contours → ^{12}CO line → cold gas

The virial theorem

- Fundamental theorem describing the properties of auto-gravitating systems at hydrostatic equilibrium (e.g. stars)

$$\Omega = -2K$$

- Gravitational potential energy *Ω* of a spherical cloud of mass *M* and radius *R*:

$$\Omega = -\int_0^R \frac{Gm(r)}{r} \times 4\pi r^2 \rho(r) dr$$

$$\Omega = -\lambda \frac{GM^2}{R}$$

where λ ~ 1 is a factor which depends on the mass density distribution *ρ(r)*
→ λ = 3/5 for a homogeneous sphere

- Kinetic energy of a perfect gas of temperature *T* where μm_H is the mean mass per particle

$$K = \frac{3}{2} \frac{MkT}{\mu m_H}$$

The Jeans mass

- The Jeans mass is the minimum mass a cloud must have if gravity is to overwhelm pressure and initiate collapse
- Equating *2K* and *-Ω*, one can write $3\frac{MkT}{\mu m_H} = \lambda \frac{GM^2}{R}$,

  and introducing the mean density number *n*, such as $M = n\mu m_H \times \frac{4}{3}\pi R^3$,

  we get the critical Jeans mass:

$$M_J = \left(\frac{1}{\mu m_H}\right)^2 \left(\frac{3kT}{\lambda G}\right)^{3/2} \left(\frac{4}{3}n\pi\right)^{-1/2}$$

- Stability criterion: an isolated, spherical and isothermal cloud is unstable if its mass is greater than M_J
- In the molecular gas of the interstellar medium, the mean molecular weight is $\mu \approx 2.4$, and we get:

$$M_J \approx 24\lambda^{-3/2} \left(\frac{T}{10K}\right)^{3/2} \left(\frac{n}{10^2 cm^{-3}}\right)^{-1/2} M_\odot$$

Stars are born in clusters

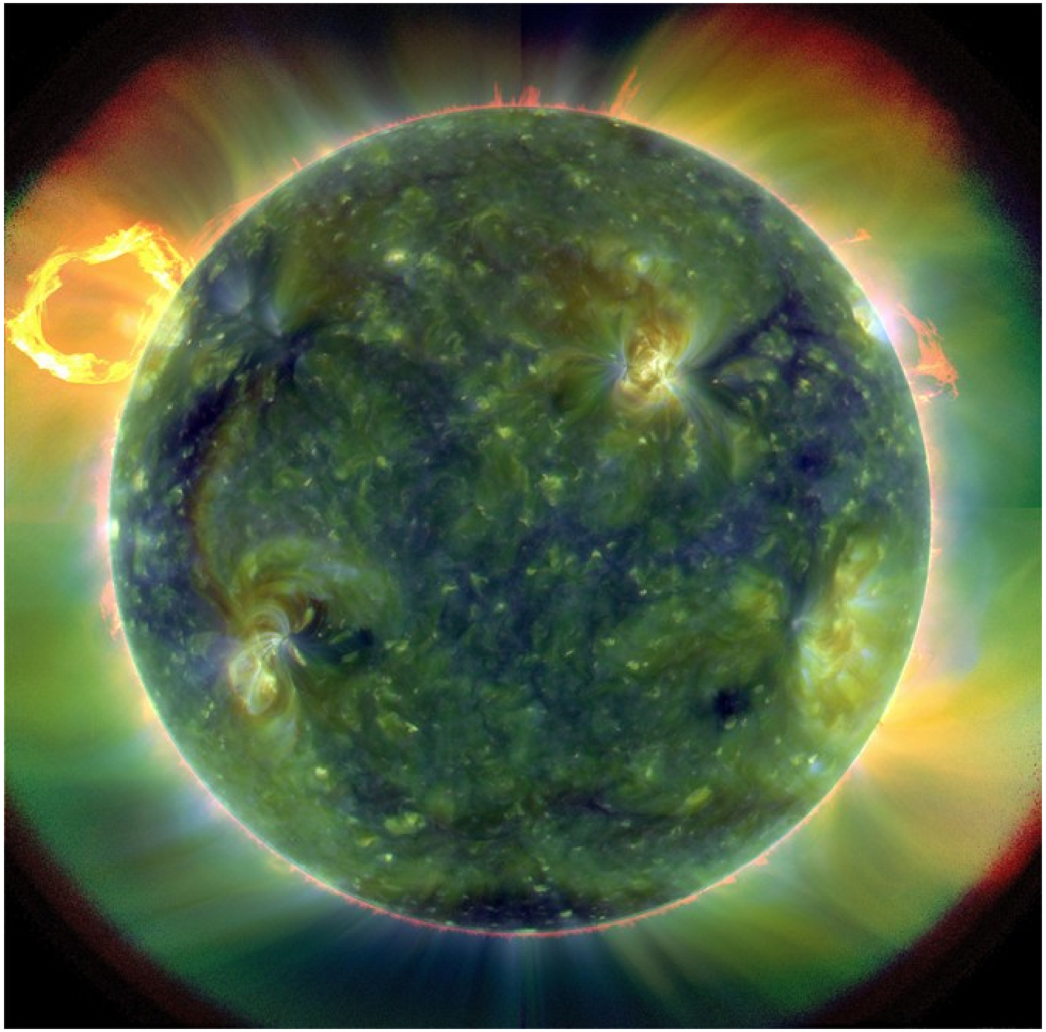
- During the contraction of a cloud, the central density increases but T ~ constant if radiative cooling is efficient
  → M_J ($\propto n^{-1/2}$) decreases → smaller and smaller regions of the cloud become unstable → the cloud fragments → star cluster

- About 10^{10} years ago, T was typically ~10^4 K → globular clusters (e.g. M13) formed from clouds of mass $M_J \sim 10^6\ M_\odot$

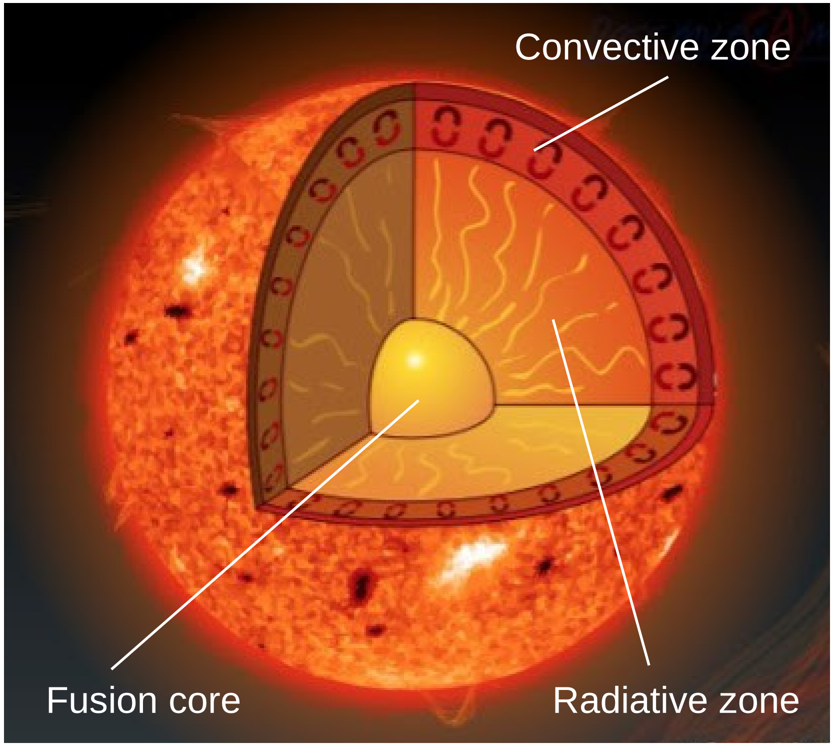
M45 – The Pleiades

M13 – Hercules

5. The internal structure of stars



The sun in extreme ultraviolet (Solar Dynamics Observatory, March 30, 2010)



Equations of stellar struture

For an isolated, static, spherically symmetric star, four basic laws/equations are needed to describe their internal structure

- **Mass conservation**
- **Hydrostatic equilibrium** (momentum conservation)
  → at each radius, forces due to pressure differences balance gravity
- **Conservation of thermal energy**
  → at each radius, the change in the energy flux equals the local rate of energy release
- **Thermal energy transport**
  → relation between the energy flux and the local gradient of temperature

These basic equations are supplemented by :

- **Equation of state** (pressure of a gas as a function of its temperature and density)
- **Opacities** (how transparent the star is to radiation)
- **Nuclear energy generation rate**

Mass conservation

- Let consider a thin shell at a distance *r* from the center of the star
- Let define M_r as the mass contained inside the sphere of radius *r*

- Conservation of mass implies that:

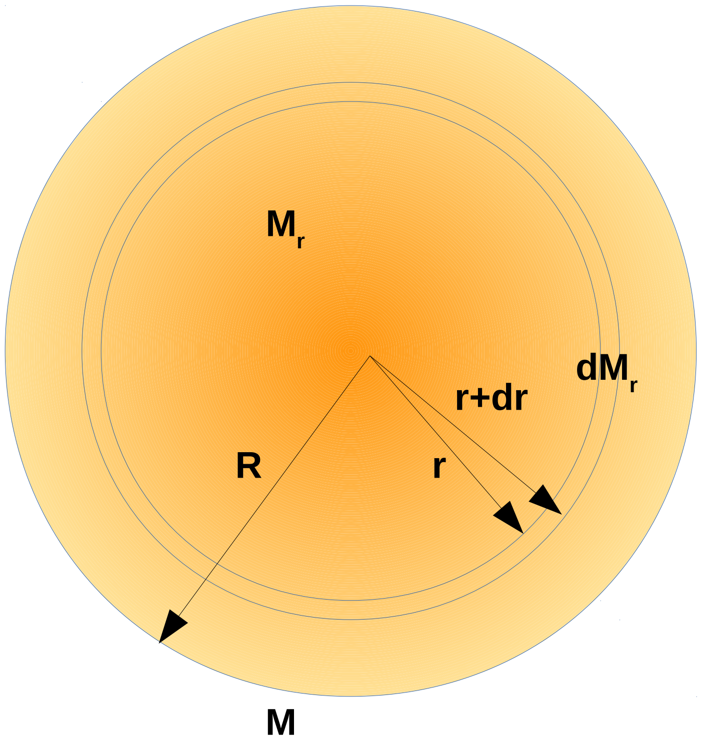
$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$$

1st stellar structure equation

where *ρ(r)* is the density as a function of the radius

- Total mass of the star is given by

$$M = \int_0^R 4\pi r^2 \rho(r) dr$$



Hydrostatic equilibrium

- Hydrostatic equilibrium: balance between gravity and internal pressure
- Pressure (net force due to difference in pressure between upper and lower faces of a cylinder)

$$F_P = P(r)dS - P(r+dr)dS = -\frac{dP}{dr}drdS$$

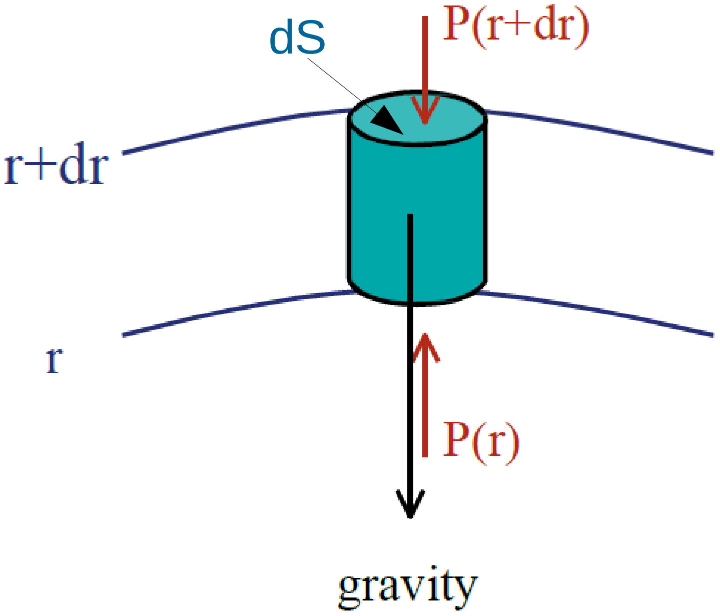
- Gravity:

$$F_g = -\frac{GM_r \times [\rho(r)drdS]}{r^2}$$

- Momentum conservation: $F_P + F_g = 0$

$$\boxed{\frac{dP}{dr} = -\frac{GM_r\rho(r)}{r^2}}$$

2nd stellar structure equation



- Mass coordinate M_r is often preferred

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4}$$

(using mass conservation)

Alternate form of hydrostatic equilibrium equation

Energy generation in stars

How much energy does the Sun need to generate in order to shine as it is?

→ Sun radiating steadily $L_\odot = 4\times10^{26}$ J.s^{-1} over ~1 Gyr (geological records) has lost ~1.2×10^{43} J, corresponding to a converted mass of 10^{26} kg (0.01% of mass of Sun)

Four possible sources of energy

- Cooling or contraction
  → either Sun would have been much hotter in the past, or contracting slowly same approach (recall Virial theorem: $\Omega = -2K$)
  → time during which the total release of gravitational potential energy would have supported the luminosity of the sun (thermal time scale):
  $$t_{th} = -\frac{\Omega}{L} = \lambda \frac{GM_\odot^2}{L_\odot R_\odot}$$
  → t_{th} ~ 3×10^7 yr → another energy source is needed!

- Chemical reactions
  → release ~5×10^{-10} of their rest mass energy << 10^{-4} needed!

- Nuclear reactions
  → nuclear timescale $t_{nucl} = \epsilon \times x M_\odot / L_\odot$, where ε~7 MeV/nucl is the energy obtained from the fusion of 4 ^1H into ^4He, x=0.1 is the mass fraction of the sun used as nuclear fuel → ~ 10^{10} years → main sequence

Conservation of thermal energy

- Luminosity *L(r)*
  → net power (erg.s^{-1}) leaving the sphere of radius *r*

- Energy production rate *ε(r)*
  → nuclear energy production rate per mass unit (erg.s^{-1}.g^{-1}) at a given density, temperature and chemical composition $\{X_i\}$ → $\varepsilon(\rho,T,\{X_i\})$

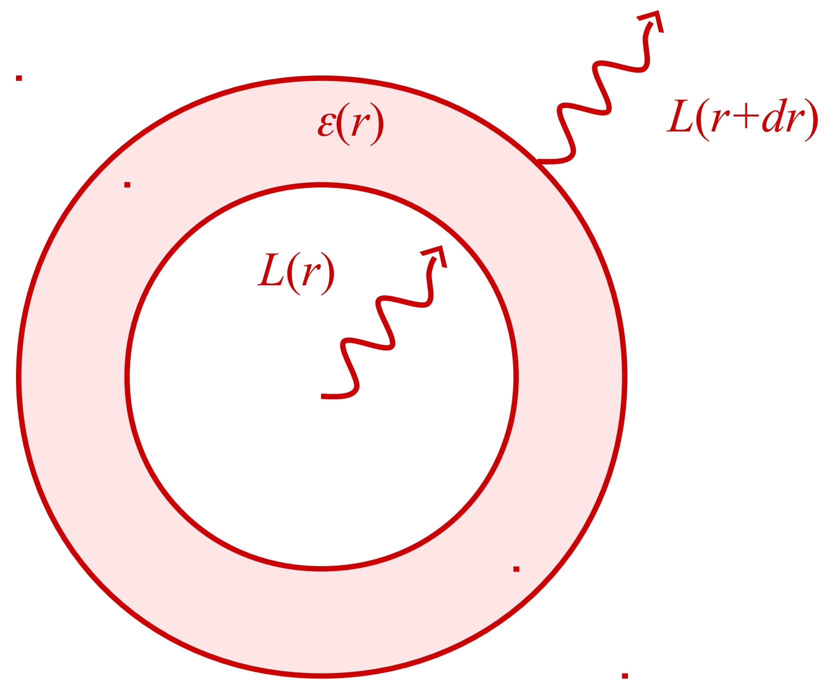
- Energy release in shell: $4\pi r^2\rho(r)\epsilon(r)dr$

- At thermal equilibrium:

$$L(r+dr) - L(r) = 4\pi r^2\rho(r)\epsilon(r)dr$$

$$\boxed{\frac{dL}{dr} = 4\pi r^2\rho(r)\epsilon(r)}$$

3rd stellar structure equation



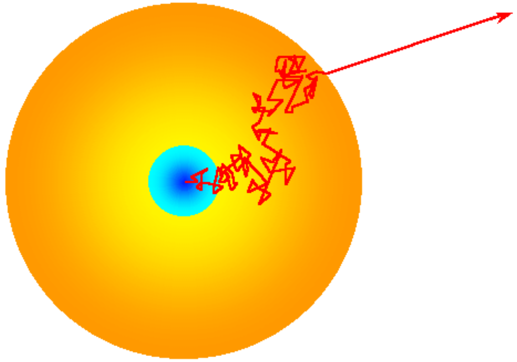
→ luminosity of the nuclear burning core

Thermal energy transport (1)

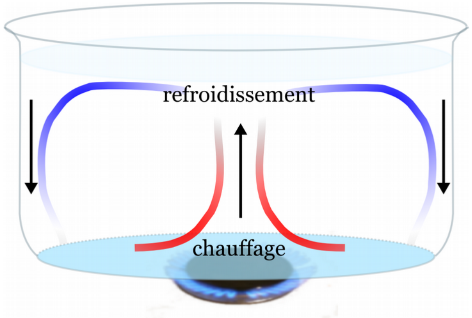
The energy transport processes determine the temperature gradient dT/dr inside the star

There are 3 ways to transport energy in stars:

- Radiation (energy is carried by photons)
  → photons produced by nuclear reactions and atomic transitions can (i) scatter with electrons and ions, and (ii) be absorbed and re-emitted many times before reaching the surface: random walk
  → $(dT/dr)_{rad}$ given by the opacity coefficients κ



- Convection (energy carried by bulk motions of gas)
  → convection if $-(dT/dr)_{ad} < -(dT/dr)_{rad}$
  (Schwarzschild criterion)
  1D treatment of stellar convection is uncertain



- Conduction (energy carried by particle motions)
  → only important in extremely dense medium (white dwarf, neutron star...)

Thermal energy transport (2)

- Radiation transport M. Schwartzschild, The Structure and Evolution of the Stars (Princeton; University Press, 1958)

$$\frac{dT}{dr} = -\frac{3L(r)\rho(r)\kappa(r)}{16\pi acr^2T^3}$$

4th stellar structure equation

κ is the opacity (a mass absoprtion coefficient) which depends on the gas composition

→ the photons emitted at high temperature *T* in the center of the star are continually emitted and reabsorbed, and gradually degraded to longer λ as they proceed outward. In case of the sun, they emerge from the surface as visible light.

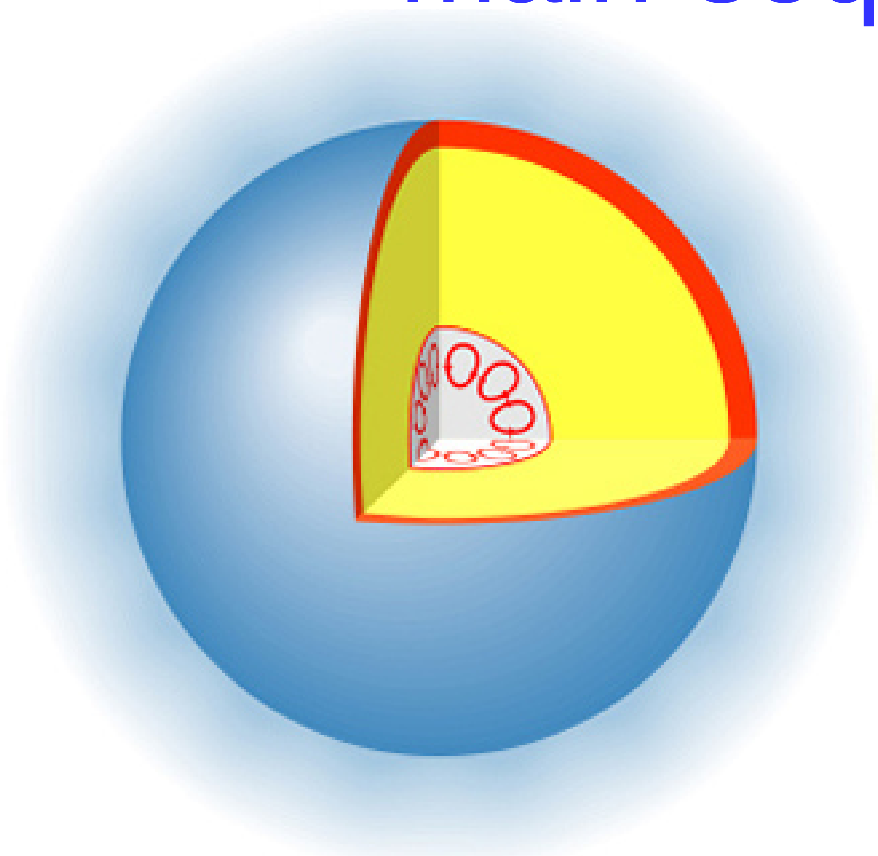
- Convection transport M. Harwitt, Astrophysical concepts (New-York; Wiley, 1973)

$$\frac{dT}{dr} = (1 - 1/\gamma)\frac{T(r)}{P(r)}\frac{dP(r)}{dr}$$

(adiabatic, mixing length theory)

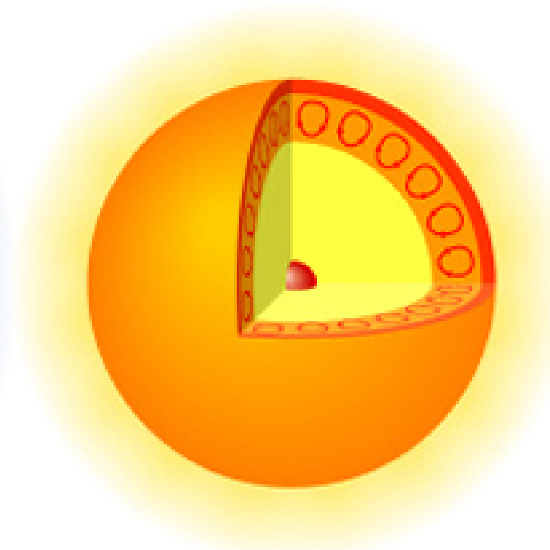
where the ratio of specific heats capacity γ = 5/3 for an ideal monoatomic gas

Convection & radiative zones in main-sequence stars



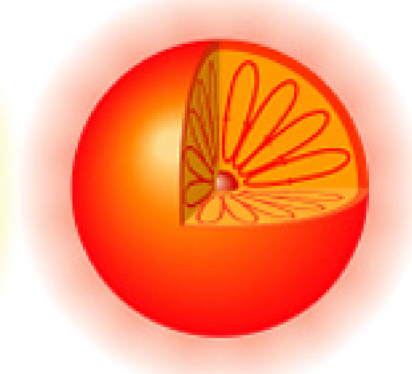
$M > 1.15\ M_\odot$

Radiative envelop
Convective core



$0.25\ M_\odot < M < 1.15\ M_\odot$

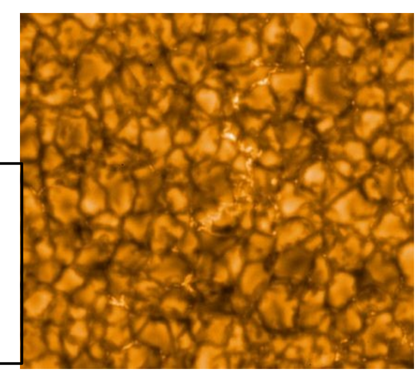
Convective envelop
Radiative core



$M < 0.25\ M_\odot$

Fully convective

Solar granulation
→ convective cells
(cell size ~ 100 km)



Equation of state (1)

- Total pressure: $P = P(\rho, T, X_i) = P_{gas} + P_{rad} = P_{ions} + P_{e^-} + P_{rad}$
- Pressure integral: $P = \frac{1}{3}\int_0^\infty vpn(p)dp$

where *v* is the particle velocity, *p* its momentum and *n(p)dp* is the number of particles per unit of volume with momenta within the interval *p* and *p+dp* [*vpn(p)dp* is a momentum flux]

- Radiative pressure → blackbody

$n_{rad}(p)$ = Planck's function → $$P_{rad} = aT^4/3$$

- Gas pressure → Maxwell-Boltzmann distribution (perfect gas)

$$n(p)dp = n\frac{4\pi p^2 dp}{(2\pi mkT)^{3/2}} e^{-\frac{p^2}{2mkT}} \longrightarrow P_{gas} = nkT = \frac{\rho}{\mu m_H}kT$$

At sun center (T = 16 MK, ρ = 150 g.cm^{-3}) → $P_{rad}/P_{gas} = 7\times10^{-4}$ (radiation pressure negligible!)

Equation of state (2)

- Degenerate electron gas → in the core of some stars the density is so high that quantum effects become important

  - Heisenberg uncertainty principle: $\Delta V \times \Delta^3 p \geq h^3$
    → if ρ increases – that is ΔV ($\propto \rho^{-1}$) decreases – until $\Delta^3 p > p_{th}$, the pressure becomes higher than that inferred from the temperature

  - In the limit of complete degeneracy, where all states of the phase space are occupied by 2 electrons of opposite spin (Pauli exclusion principle):

$$n_e(p) = \frac{2}{\Delta V} = \frac{2}{h^3}\Delta^3 p = \frac{8\pi p^2 dp}{h^3}$$ for $p < p_F$ (Fermi momentum)

$$\rightarrow P_{e^-} = k\rho^\eta$$ with η = 5/3 (non-relativistic) or 4/3 (relativistic)

P_{e-} does not depend anymore on the temperature (explosive situation!)

Summary

- **Structure equations:**

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$$ Mass conservation

$$\frac{dP}{dr} = -\frac{GM_r\rho(r)}{r^2}$$ Hydrostatic equilibrium

$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$ Thermal equilibrium

$$\left.\frac{dT(r)}{dr}\right|_{rad} = f(L(r), \kappa(r), T) \qquad \left.\frac{dT(r)}{dr}\right|_{conv} = f(P(r), T)$$ Energy transport

- **Equation of state:** $P = P(\rho, T, X_i)$
- **Nuclear energy production rate:** $\epsilon = \epsilon(\rho, T, X_i)$
- **Opacity coefficient:** $\kappa = \kappa(\rho, T, X_i)$

Back to the virial theorem

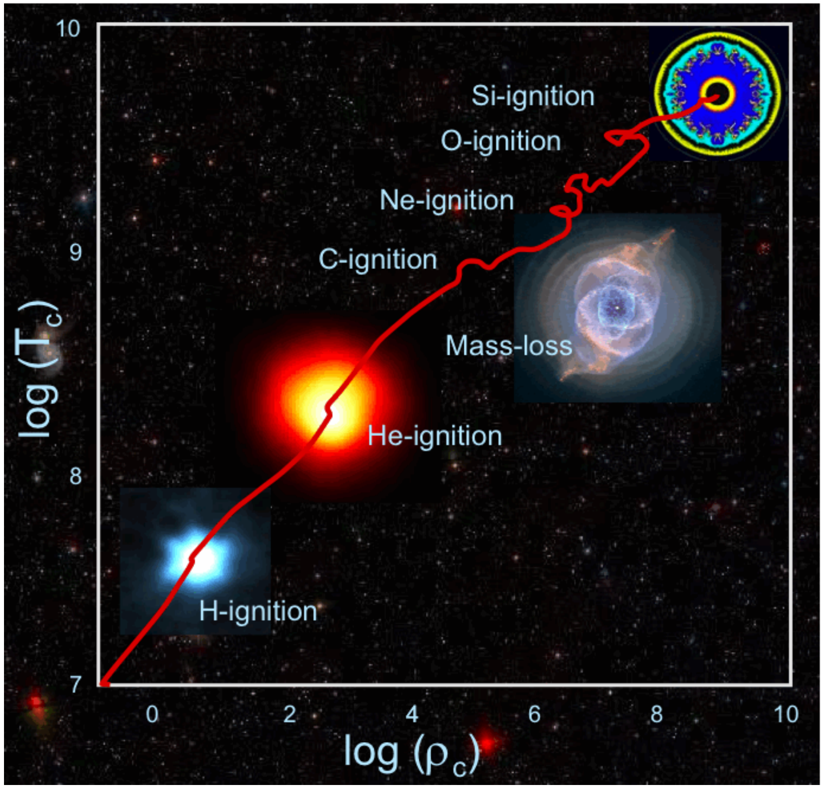
- Total energy of a star is $E = K + \Omega = \Omega/2 = -K$ (virial theorem)
- Because a star shines, *E* decreases with time

  → Ω decreases ($\Omega < 0$) → *R* decreases → the star contracts

  → *K* increases → the mean temperature of the star increases

- Half of the gravitational energy lost by the star turns into heat ($K = -\Omega/2$), the other half is radiated away ($E = \Omega/2$)

- The increase of the central temperature T_c allows the ignition of successive nuclear burning phases



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