SOLUTIONS TO THE FINAL EXAM 2019-2020

1 Yukawa interaction

→ A similar problem was given in your mid-term exam.

2 Second quantisation

Let us first apply N to $|\psi\rangle$

$$N|\psi\rangle = \frac{1}{\sqrt{2}}\left[\sum_i a_i^\dagger a_i |\mu\nu\rangle + \sum_i a_i^\dagger a_i |\mu\alpha\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left(2|\mu\nu\rangle + 2|\mu\alpha\rangle\right) = 2|\psi\rangle$$

⇒ $|\psi\rangle$ is indeed an eigenstate of N with eigenvalue 2

Now the individual components

- $$a^\dagger_\mu a_\mu |\psi\rangle = \frac{1}{\sqrt{2}}\left[a^\dagger_\mu a_\mu |\mu\nu\rangle + a^\dagger_\mu a_\mu |\mu\alpha\rangle \right]$$

$$= \frac{1}{\sqrt{2}}\left[|\mu\nu\rangle + |\mu\alpha\rangle \right] = |\psi\rangle$$

$\Rightarrow$ $|\psi\rangle$ is an eigenstate of the component $i=\mu$

- $$a^\dagger_\nu a_\nu |\psi\rangle = \frac{1}{\sqrt{2}}\left[a^\dagger_\nu a_\nu |\mu\nu\rangle + \underbrace{a^\dagger_\nu a_\nu |\mu\alpha}_{=0} \right]$$

$$= \frac{1}{\sqrt{2}} |\mu\nu\rangle \neq |\psi\rangle$$

$\Rightarrow$ $|\psi\rangle$ is <u>not</u> an eigenstate of the component $i=\nu$

· same for the component $i = \alpha$

3 Wick's theorem

a) $a_\alpha |\Phi_0\rangle = 0 \qquad \forall \alpha$

b) Applying Wick's theorem

$$a_\alpha a_\beta a^\dagger_\gamma a^\dagger_\delta = :a_\alpha a_\beta a^\dagger_\gamma a^\dagger_\delta:$$

+ terms with one contraction and one normal product

$$- \overline{a_\alpha a^\dagger_\gamma}\; \overline{a_\beta a^\dagger_\delta} + \overline{a_\alpha a^\dagger_\delta}\; \overline{a_\beta a^\dagger_\gamma}$$

the term with two anomalous contractions vanishes because $|\Phi_0\rangle$ is a Slater

Taking the expectation value w.r.t. $|\phi_0\rangle$, only fully-contracted terms survive

$$\langle \phi_0 | a_\alpha a_\beta a^\dagger_\gamma a^\dagger_\delta | \phi_0 \rangle = - \overline{a_\alpha a^\dagger_\gamma}\, \overline{a_\beta a^\dagger_\delta} + \overline{a_\alpha a^\dagger_\delta}\, \overline{a_\beta a^\dagger_\gamma}$$

$$= - \left(\delta_{\alpha\gamma} - \rho_{\alpha\gamma} \right) \left(\delta_{\beta\delta} - \rho_{\beta\delta} \right)$$

$$+ \left(\delta_{\alpha\delta} - \rho_{\alpha\delta} \right) \left(\delta_{\beta\gamma} - \rho_{\beta\gamma} \right)$$

see definitions in chapter IV page 5

c) Applying Wick's theorem

$$a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta a_\lambda a^\dagger_\mu = :a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta a_\lambda a^\dagger_\mu:$$

+ terms with 1 contraction and 2 normal products

+ terms with 2 contractions and 1 normal products

$$- \overline{a^\dagger_\alpha a_\gamma}\ \overline{a^\dagger_\beta a_\delta}\ \overline{a_\lambda a^\dagger_\mu}$$

$$+ \overline{a^\dagger_\alpha a_\gamma}\ \overline{a^\dagger_\beta a_\lambda}\ \overline{a_\delta a^\dagger_\mu}$$

$$+ \overline{a^\dagger_\alpha a_\delta}\ \overline{a^\dagger_\beta a_\gamma}\ \overline{a_\lambda a^\dagger_\mu}$$

$$- \overline{a^\dagger_\alpha a_\delta}\ \overline{a^\dagger_\beta a_\lambda}\ \overline{a_\gamma a^\dagger_\mu}$$

$$- \overline{a^\dagger_\alpha a_\lambda}\; \overline{a^\dagger_\beta a_\gamma}\; \overline{a_\delta a^\dagger_\mu}$$

$$+ \overline{a^\dagger_\alpha a_\lambda}\; \overline{a^\dagger_\beta a_\delta}\; \overline{a_\gamma a^\dagger_\mu}$$

similarly to point b), all anomalous contractions are zero

Again, taking the expectation value w.r.t. $|\Phi_0\rangle$ only the 6 fully-contracted terms survive

$$\langle \Phi_0 | a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta a_\lambda a^\dagger_\mu | \Phi_0 \rangle = -\rho_{\gamma\alpha}\rho_{\delta\beta}(\delta_{\lambda\mu} - \rho_{\lambda\mu})$$

$$+ \rho_{\gamma\alpha}\rho_{\lambda\beta}(\delta_{\delta\mu} - \rho_{\delta\mu}) + \rho_{\delta\alpha}\rho_{\gamma\beta}(\delta_{\lambda\mu} - \rho_{\lambda\mu})$$

$$- \rho_{\delta\alpha}\rho_{\lambda\beta}(\delta_{\gamma\mu} - \rho_{\gamma\mu}) - \rho_{\lambda\alpha}\rho_{\gamma\beta}(\delta_{\delta\mu} - \rho_{\delta\mu}) + \rho_{\lambda\alpha}\rho_{\delta\beta}(\delta_{\gamma\mu} - \rho_{\gamma\mu})$$

4 Hartree-Fock

(I think you can find all this in the notes of chapter V)