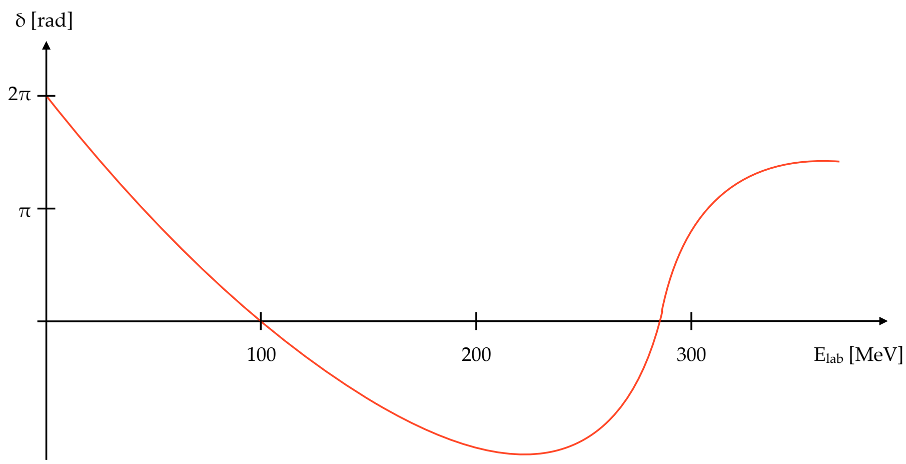
From nuclei to stars
Theoretical course

NPAC 2021-2022

Final exam 9/2/2022

1. Define the *nuclear many-body problem* and explain what are the main difficulties associated to its solution.

2. Given the phase shift shown below, what can you tell about the corresponding potential?



3. Prove the anticommutation relation between fermionic annihilation operators a_α and a_β.

   Hint: Evaluate the action of the anticommutator on a generic Slater determinant $|\mu\nu\dots\rangle$.

4. Hartree-Fock method

   a. What are the main ideas behind the Hartree-Fock approximation?

   [*Bonus* (the answer can be schematic):

   b. How does the Hartree-Fock energy E_{HF} change in presence of a three-body interaction?

   c. How does the Hartree-Fock one-body Hamiltonian h_{HF} change in presence of a three-body interaction?]

5. Compute the expectation value

$$\langle \Phi_{\mathrm{HF}} | H | \Phi_i^a \rangle \,, \tag{1}$$

   where $|\Phi_{\mathrm{HF}}\rangle$ is the Hartree-Fock Slater determinant and $|\Phi_i^a\rangle \equiv a_a^+ a_i |\Phi_{\mathrm{HF}}\rangle$ a determinant built from $|\Phi_{\mathrm{HF}}\rangle$ via a one particle-one hole excitation. The Hamiltonian is given as a sum of a one- and an antisymmetrised two-body operator

$$H = \sum_{\mu\nu} t_{\mu\nu}\, a_\mu^+ a_\nu + \frac{1}{4} \sum_{\mu\nu\lambda\epsilon} \bar{v}_{\mu\nu\lambda\epsilon}\, a_\mu^+ a_\nu^+ a_\epsilon a_\lambda \,. \tag{2}$$

   Hint: Make use of Wick's theorem and of the one-body density matrix defined in the course. To simplify the expression, use the fact that the density matrix is automatically zero for certain combinations of indices.