# From nuclei to stars
# Theoretical course

**NPAC 2020-2021**

**Mid-term exam 5/11/2020**

1. What is a Slater determinant and which properties of $N$-fermion states can be deduced from the fact that it can be expressed as a matrix determinant?

2. For each of the five possible operator types in the nucleon-nucleon interaction

   a) $\mathbb{1}$

   b) $\vec{L}\cdot\vec{S}$

   c) $S_{12}^{(r)} \equiv 3(\vec{\sigma}_1\cdot\bar{r})(\vec{\sigma}_2\cdot\bar{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$, with $\bar{r} \equiv \dfrac{\vec{r}}{|\vec{r}|}$

   d) $S_{12}^{(p)} \equiv 3(\vec{\sigma}_1\cdot\bar{p})(\vec{\sigma}_2\cdot\bar{p}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$, with $\bar{p} \equiv \dfrac{\vec{p}}{|\vec{p}|}$

   e) $Q_{12} \equiv \dfrac{1}{2}\left[(\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L}) + (\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\right]$

   determine whether the operator commutes (i) with $\vec{S}^2$ and (ii) with $\vec{L}^2$, i.e. whether S and L are conserved by each of them.

   *Hint:* For the tensor operators, use the relation $6(\vec{S}\cdot\bar{x})^2 - 2\vec{S}^2 = S_{12}^{(x)}$, where $x = r, p$.

   *Bonus:* Prove the above relation (at some point you have to use the formula $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot\vec{a}\times\vec{b}$).

3. One between $S$ and $L$ is not conserved by some of the operators listed above. Which experimental observation confirms this result?

4. Nuclear interaction

   a. What is Yukawa's model of nucleon-nucleon interaction and what is the logic behind it?

   b. In which way one-boson-exchange (OBE) potentials improve on Yukawa's idea?

   c. In which way models from effective field theory improve on OBE potentials?

5. What is the tetraneutron, why is it interesting to study it and why is it difficult to study it?

6. Let $a_\uparrow$ and $a_\downarrow$ be annihilation operators for states with $\sigma = +1/2$ and $\sigma = -1/2$ respectively, where $\sigma$ is the eigenvalue of the third component of the spin operator, $s_z$ (let us set $\hbar = 1$).

   a. Given the operators

   $$O_1 \equiv \frac{1}{2}[a_\uparrow^+ a_\downarrow + a_\downarrow^+ a_\uparrow]$$

   and

   $$O_2 \equiv \frac{1}{2i}[a_\uparrow^+ a_\downarrow - a_\downarrow^+ a_\uparrow]$$

   compute their commutator.

   b. Defining $[O_1, O_2] \equiv iO_3$, which physical operator does $O_3$ represent?

# FROM NUCLEI TO STARS



M2 NPAC 26 / 11 / 2020 V. Somà

## MID TERM EXAM

1) - form a (complete orthonormal) basis of $\mathcal{H}_N^F$

- exchange of two lines $\rightarrow$ - sign
- two equal lines $\rightarrow$ 0 Pauli

2) a) trivial

b) $\vec{L} \cdot \vec{S} = \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$ $\Rightarrow$ commutes with both $\vec{L}^2$ and $\vec{S}^2$

e) $Q_{12}$ → function of $\vec{L}\cdot\vec{S}$, $(\vec{L}\cdot\vec{S})^2$, $\vec{L}^2$

⇒ commutes with both

c, d) $G(\vec{S}\cdot\bar{x})^2 - 2\vec{S}^2$ ⇒ commutes with $\vec{S}^2$

↘ does not commute with $\vec{L}^2$ ← use $\vec{L} = \vec{r} \times \vec{p}$

⇒ $[\tilde{V}, \vec{L}^2] \neq 0$ ⇒ $\vec{L}$ is not conserved!

3) From the magnetic dipole moment of the deuteron (consistent only with a combination of $L=0$ and $L=2$)

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4) a) pion exchange, finite range

b) exchange of other bosons → short range

c) - resolution scale → more efficient

- <u>E</u>FT → more systematic

6) $[O_1, O_2] = i O_3$ with $O_3 = \frac{1}{2}\left(a^\dagger_\uparrow a_\uparrow - a^\dagger_\downarrow a_\downarrow\right)$

→ spin z-component

$\pm\frac{1}{2}\langle i|j\rangle$

$$S_z = \sum_{ij} \langle i | S_z | j \rangle a^\dagger_i a_j = \frac{1}{2} a^\dagger_\uparrow a_\uparrow - \frac{1}{2} a^\dagger_\downarrow a_\downarrow$$

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$$\{a^\dagger_\alpha, a_\beta\} = \delta_{\alpha\beta} \qquad \alpha, \beta = \uparrow, \downarrow$$

$$\{a^\dagger_\uparrow, a_\uparrow\} = \delta_{\uparrow\uparrow} = 1 \quad \Rightarrow \quad a^\dagger_\uparrow a_\uparrow = 1 - a_\uparrow a^\dagger_\uparrow$$

$$\{a^\dagger_\uparrow, a_\downarrow\} = \delta_{\uparrow\downarrow} = 0 \quad \Rightarrow \quad a^\dagger_\uparrow a_\downarrow = - a_\downarrow a^\dagger_\uparrow$$