Master NPAC:
An introduction to the theory of nuclear reactions

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IPN Orsay

prepared with inputs of D. Lacroix

Lecture 1 : Generalities from classical to quantum scattering

THE SCOPE OF THE PRESENT COURSE

General aspects

- Generalities from classical to quantum scattering
- Two-body quantum scattering: phase-shifts, resonances…
- Formal theory of scattering to optical potential

Nuclear Physics

- Inelastic channels, Fusion
- Direct reactions: continuum effect, Break-up, knock-out

Research topics

- Microscopic approaches to reactions: TDHF
- Few-body approaches to reactions

AN INCOMPLETE LIST OF REFERENCES (TEXTBOOKS)

- Quantum Collision Theory C. J. Joachain, ISBN 978-0-444-86773-5, North-Holland, 1975.
- Scattering Theory, J. R. Taylor, 978-0-486-45013-1, Dover publications, 1983.
- Introduction to Nuclear Reactions, C. A. Bertulani and P. Danielewicz,978-0-750-30932-5, Taylor & Francis, 2003.
- Nuclear Reactions for Astrophysics, I. J. Thompson and F. M. Nunes, 978-0-524-85635-5, Cambridge University press, 2009.
- Nuclear Reactions, H.P. gen. Schieck, 978-3-642-53985-5, Springer, 2014.
- Introduction to Nuclear Reactions, G.R. Satchler, 0-333-25907-6, Macmillan Press, 1980.

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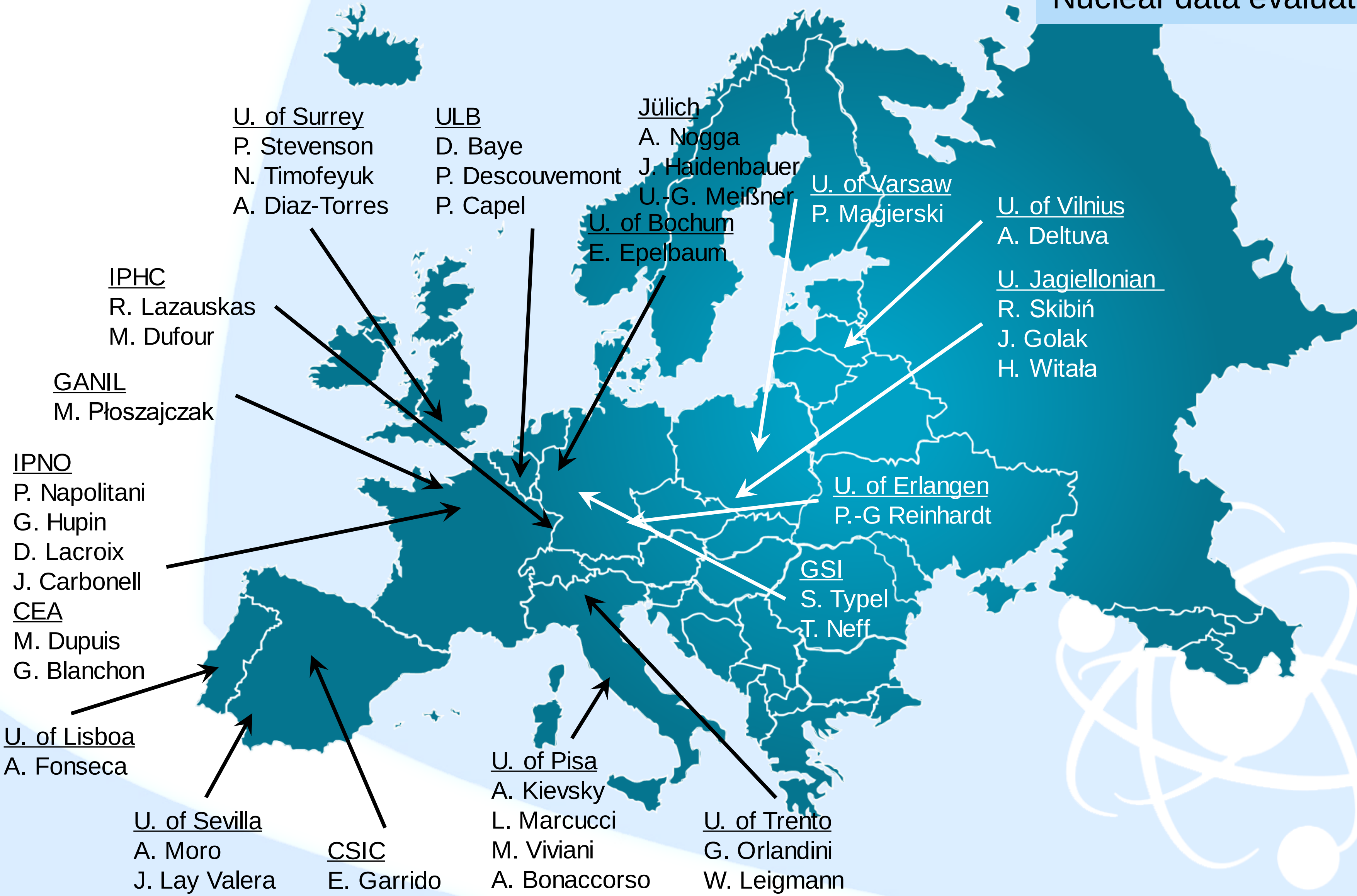
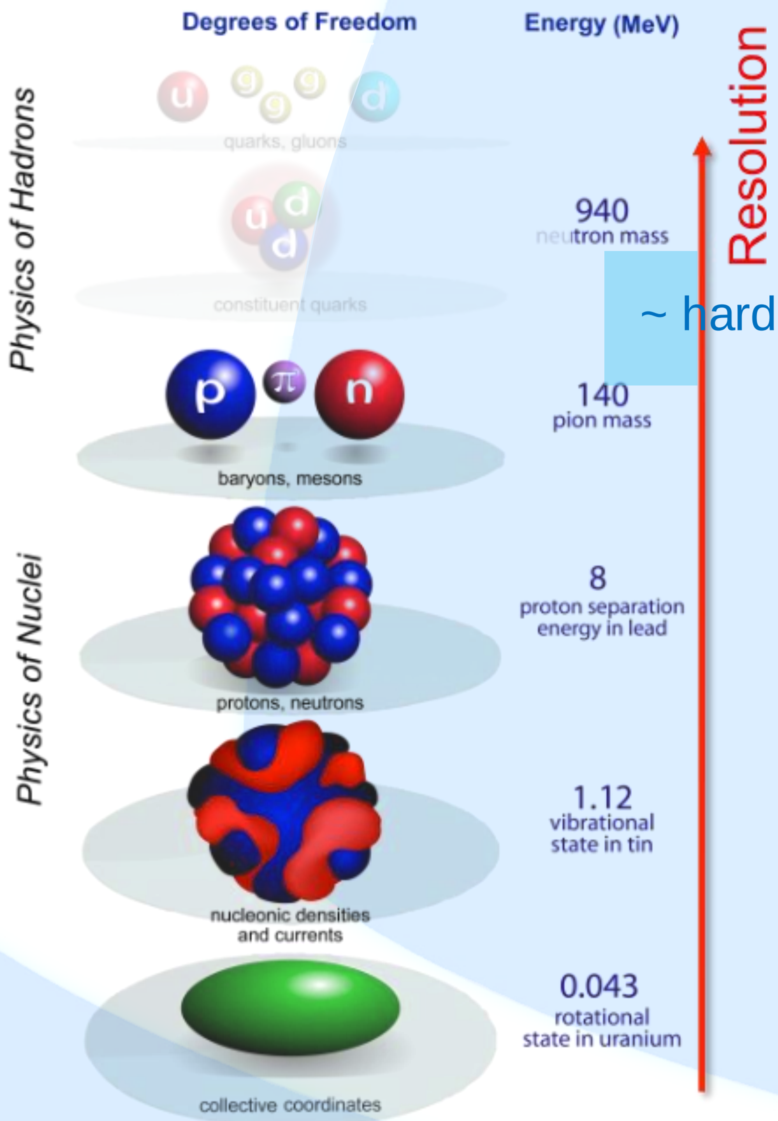
Laboratoire de Physique des 2 Infinis

AN INCOMPLETE LIST OF PRACTITIONERS IN EUROPE

Missing:
Nuclear data evaluators

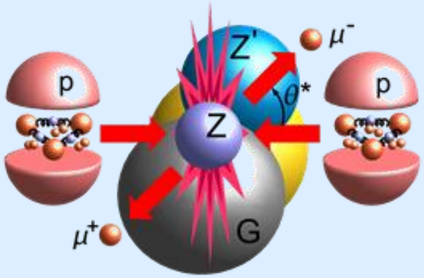


LOW ENERGY NUCLEAR PHYSICS

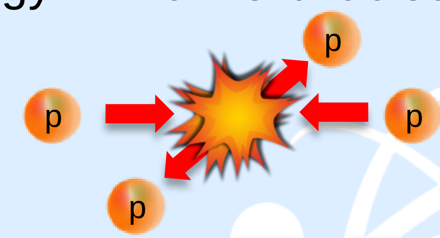


Here we consider that there is <u>not</u> enough energy to excite <u>internal degrees of freedom</u> of nucleons

High energy picture: from CMS (CERN)



Beam energy < 140 MeV/nucleon (π threshold)



The low energy nuclear physicist viewpoint

- NN-interaction is a fundamental force
- Nucleons are considered as elementary (point-like) particles

From D. Furnstahl, Nucl. Phys. B Proc. Suppl. (2012).

SCHEMATIC VIEW OF A NUCLEAR REACTION

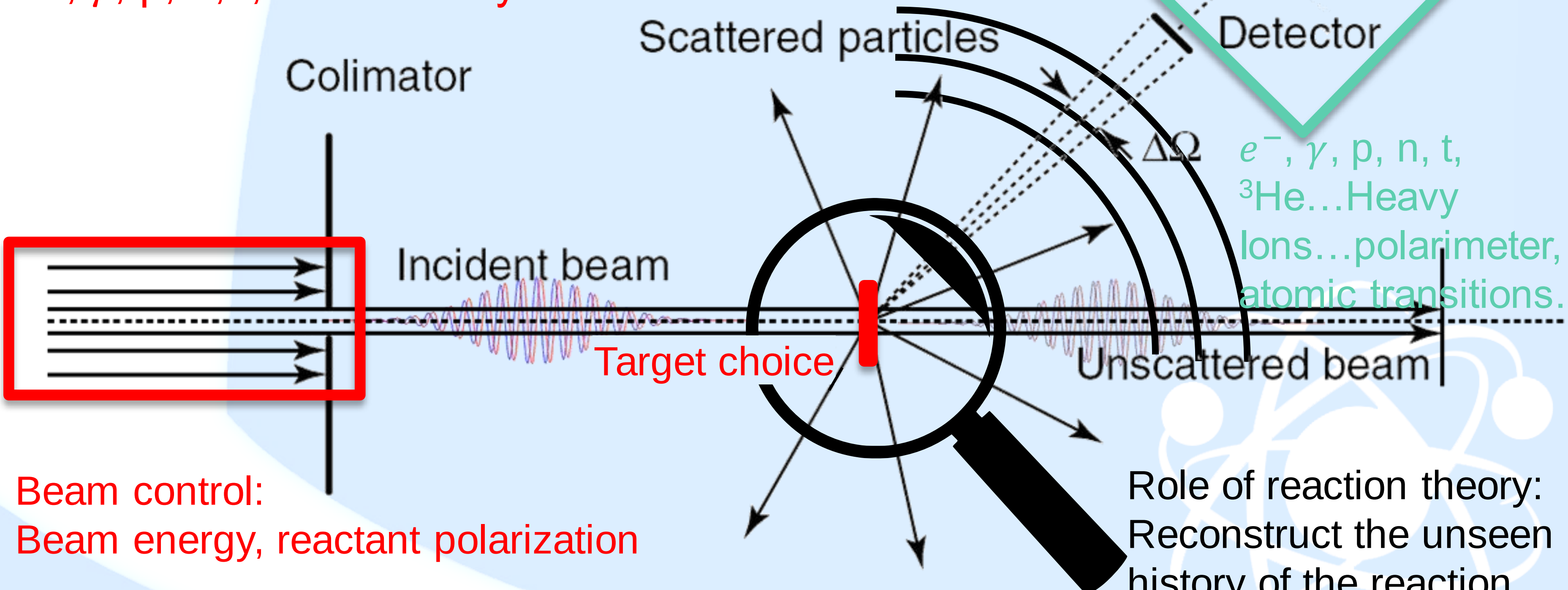
Typical experiment:

Expt. parameters

Theory

Detection

e^-, γ, p, n, t, ^{3}He…Heavy Ions…



Target choice

e^-, γ, p, n, t, ^{3}He…Heavy Ions…polarimeter, atomic transitions…

Beam control:
Beam energy, reactant polarization

Role of reaction theory:
Reconstruct the unseen history of the reaction

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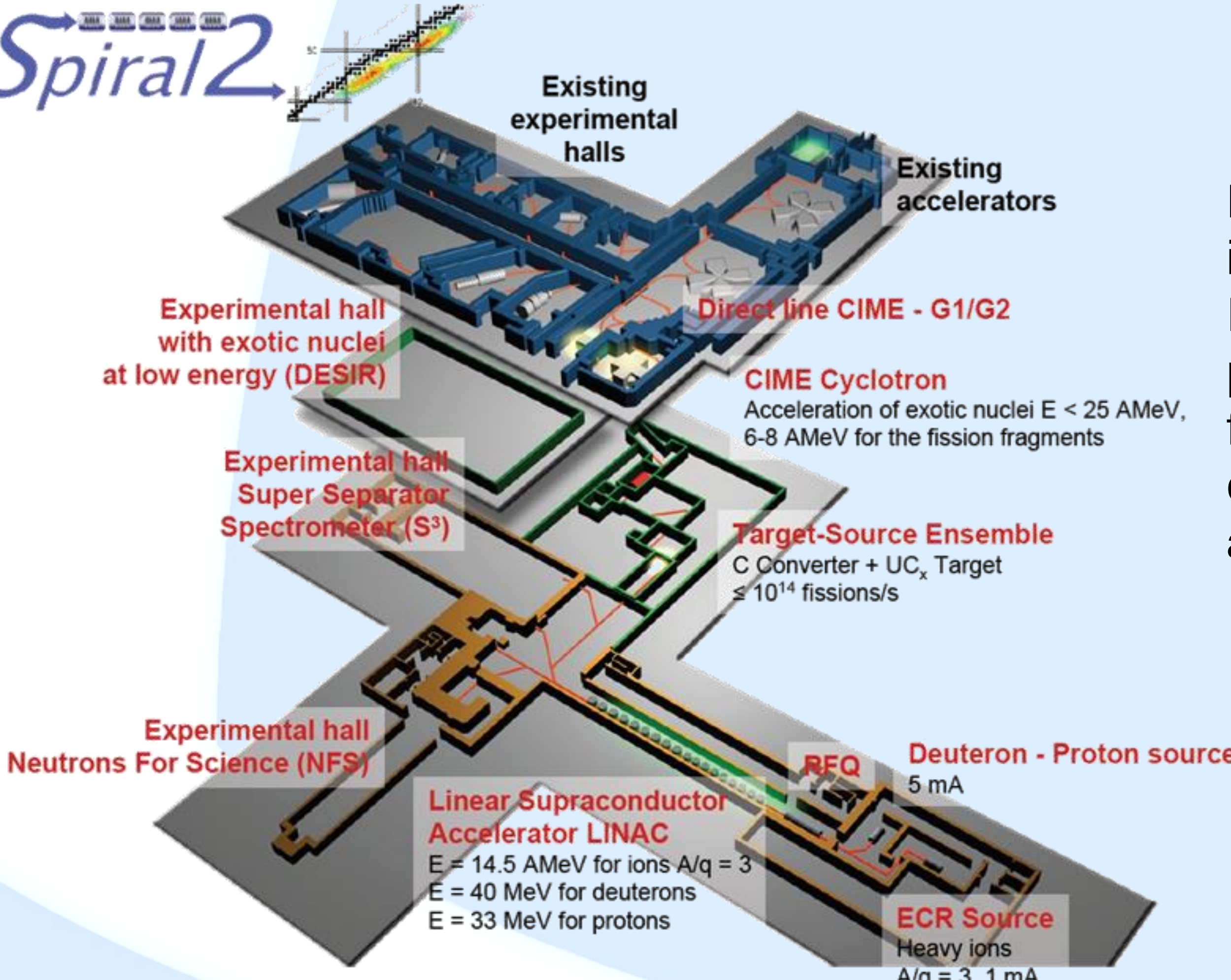
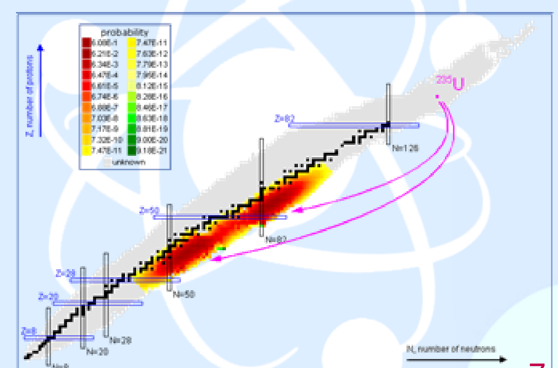
NECESSARY PREMISES: PARTICLE ACCELERATOR

GANIL spiral2
laboratoire commun CEA/DSM CNRS/IN2P3



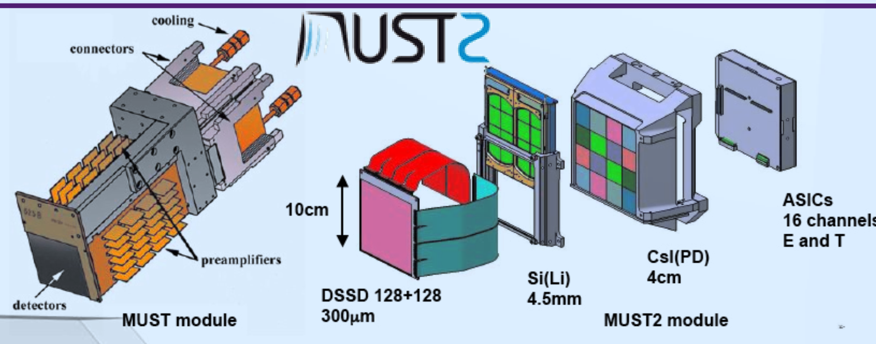
Nowadays, scientists are interested in Radioactive Ion Beams (RIB) to probe the physics far from the stability/ of extreme isospin/ of astrophysical interest/ …

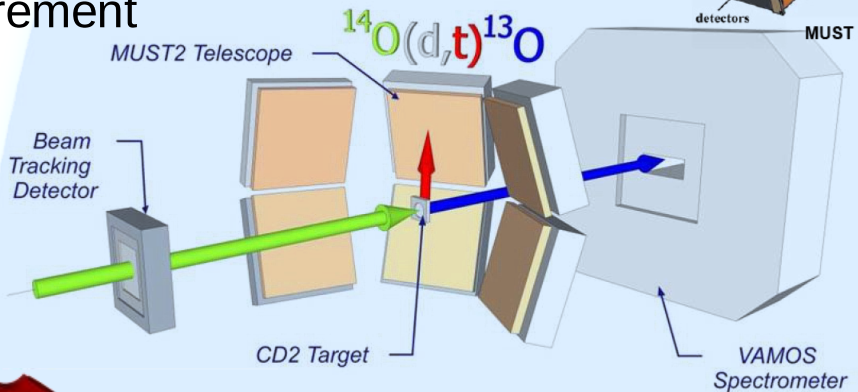


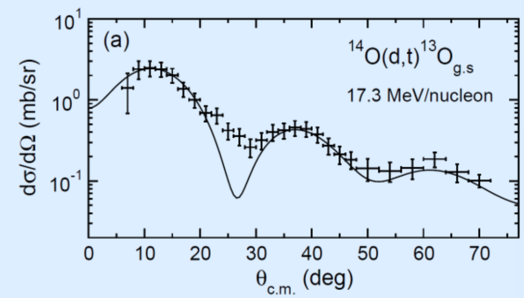
REALISTIC VIEW OF TODAY'S NUCLEAR REACTIONS

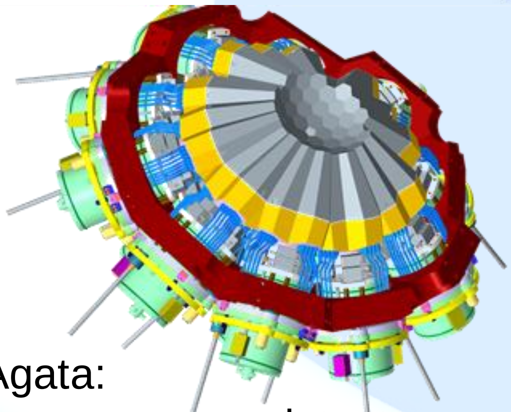
An example of experiment

- Precisely measure the ejectile
- Particle ID with good energy and angles
- γ measurement

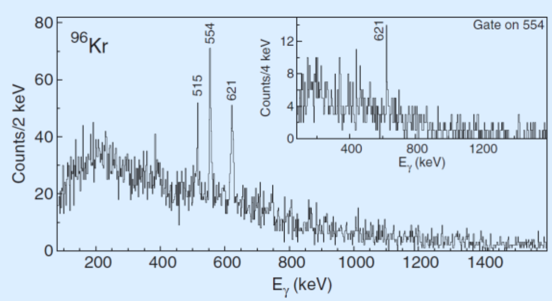


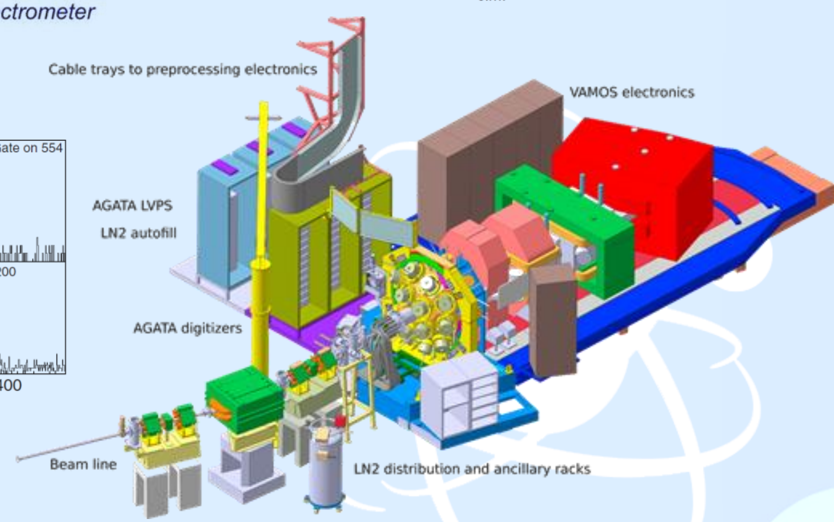






Agata:
4π gamma-ray detector
FWHM 6keV @ 1 MeV
ε~43%





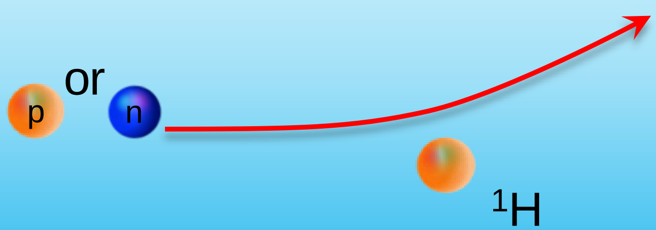
VAMOS spectrometer

INFORMATION INFER FROM REACTION OBSERVABLES

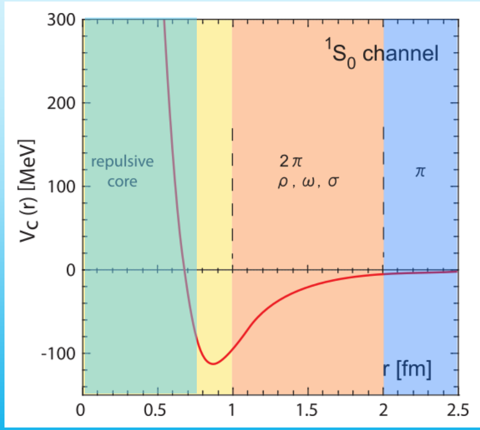
Why do we need so many beams/detectors ?

Reactions observables depends on the wavelength of the projectile and how it interacts with target

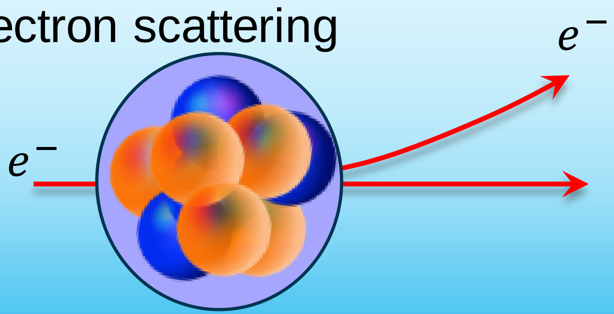
Nucleon-nucleon collisions



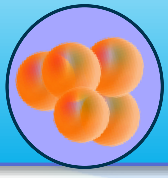
NN interactions

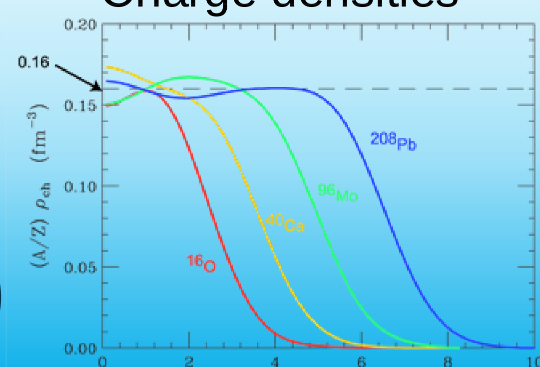


Electron scattering



Charge densities

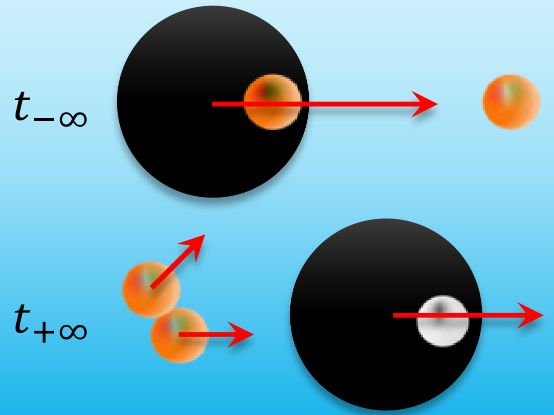




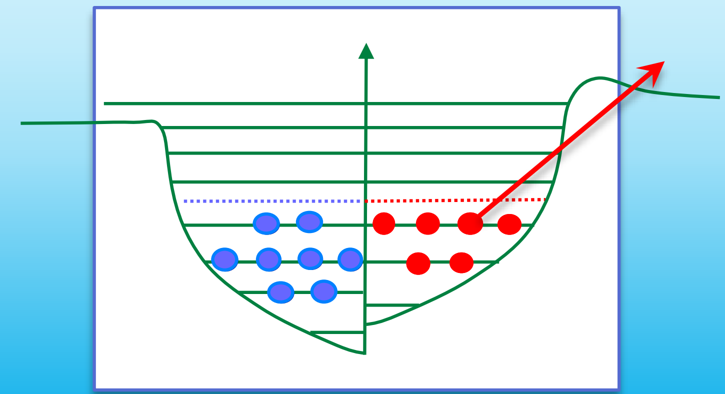
INFORMATION INFER FROM REACTION OBSERVABLES

Some reactions are designed to study nuclear structure

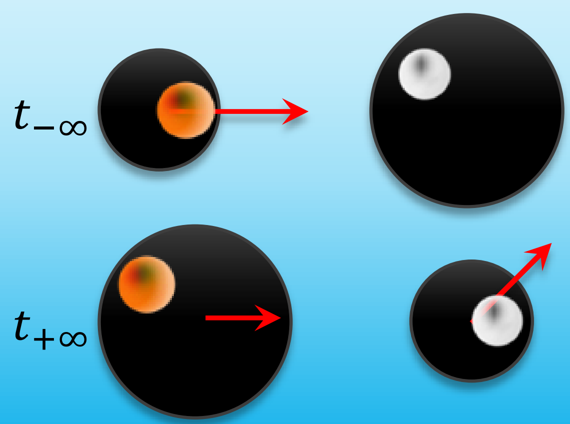
Knockout reactions (high energy)



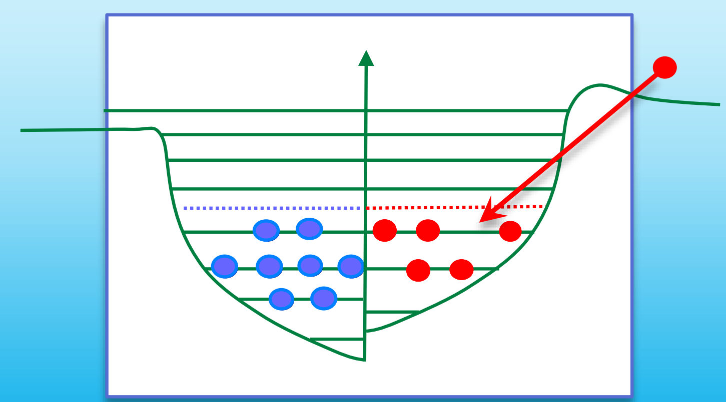
Spectroscopic information



Transfer reactions (low energy)



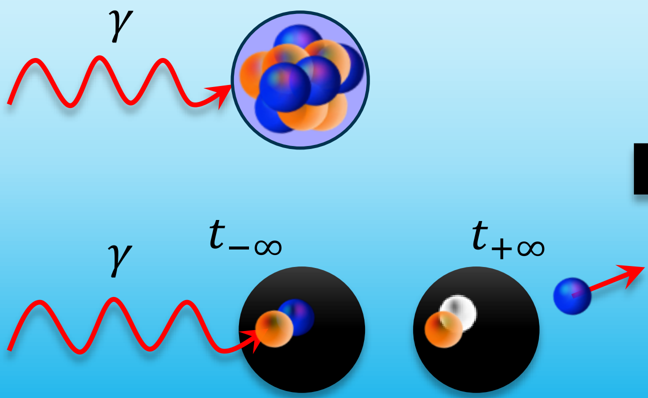
Spectroscopic information



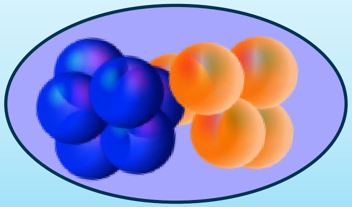
INFORMATION INFER FROM REACTION OBSERVABLES

Others give access to specific mode of excitations

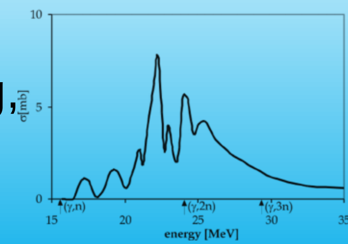
Photonuclear reactions



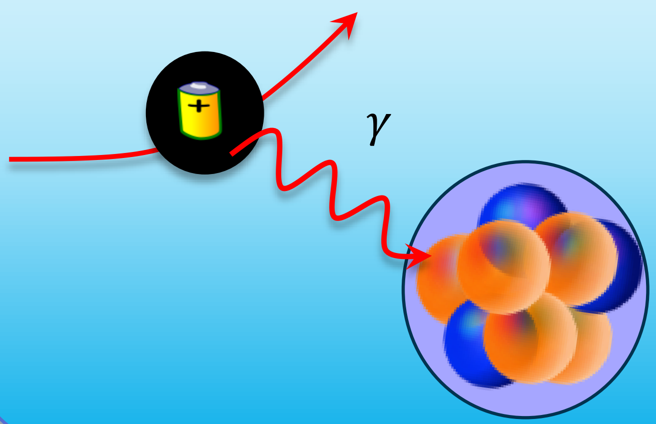
GDR excitation



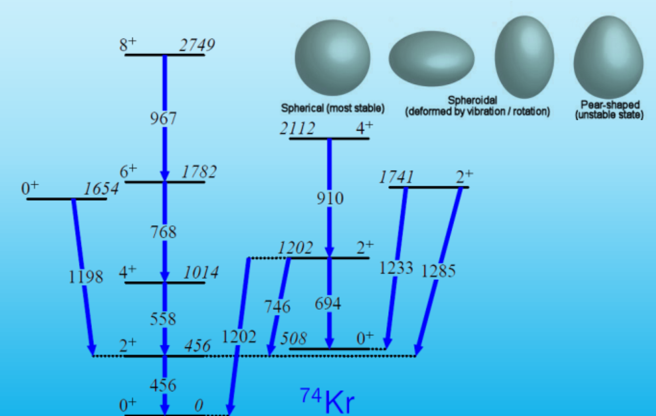
Also Bremsstrahlung, radiative capture



Coulomb excitation



Collective motion

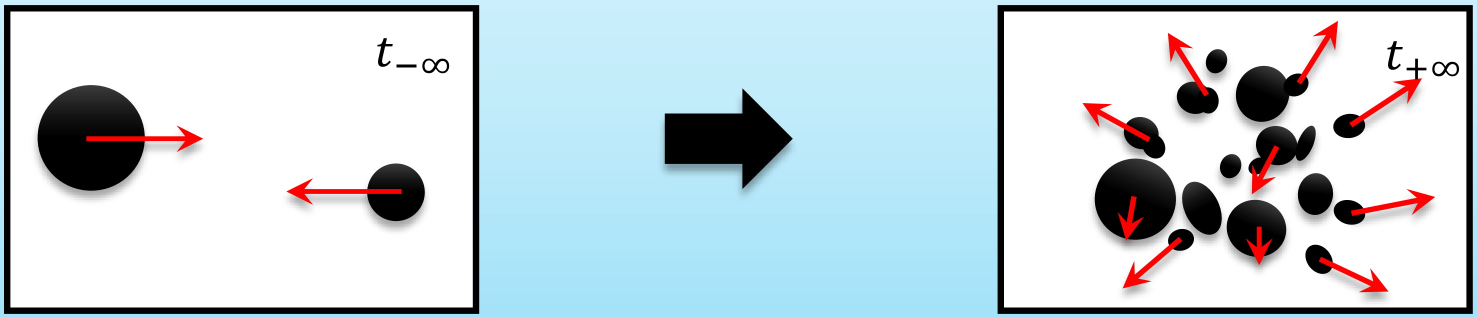


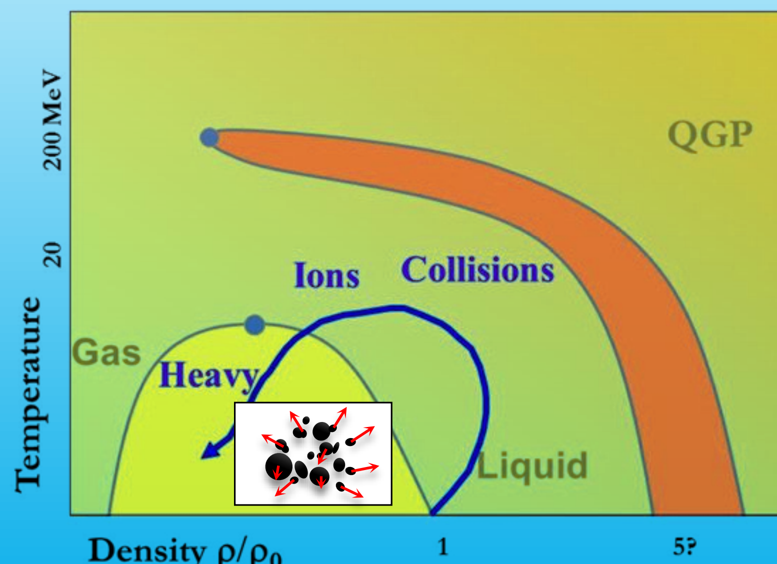
INFORMATION INFER FROM REACTION OBSERVABLES

Out of equilibrium properties (away from nuclear density + cold)

Reactions at Fermi energy
(30 to 150 MeV/A)

Fragmentation

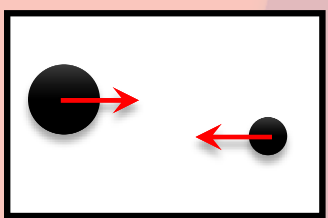


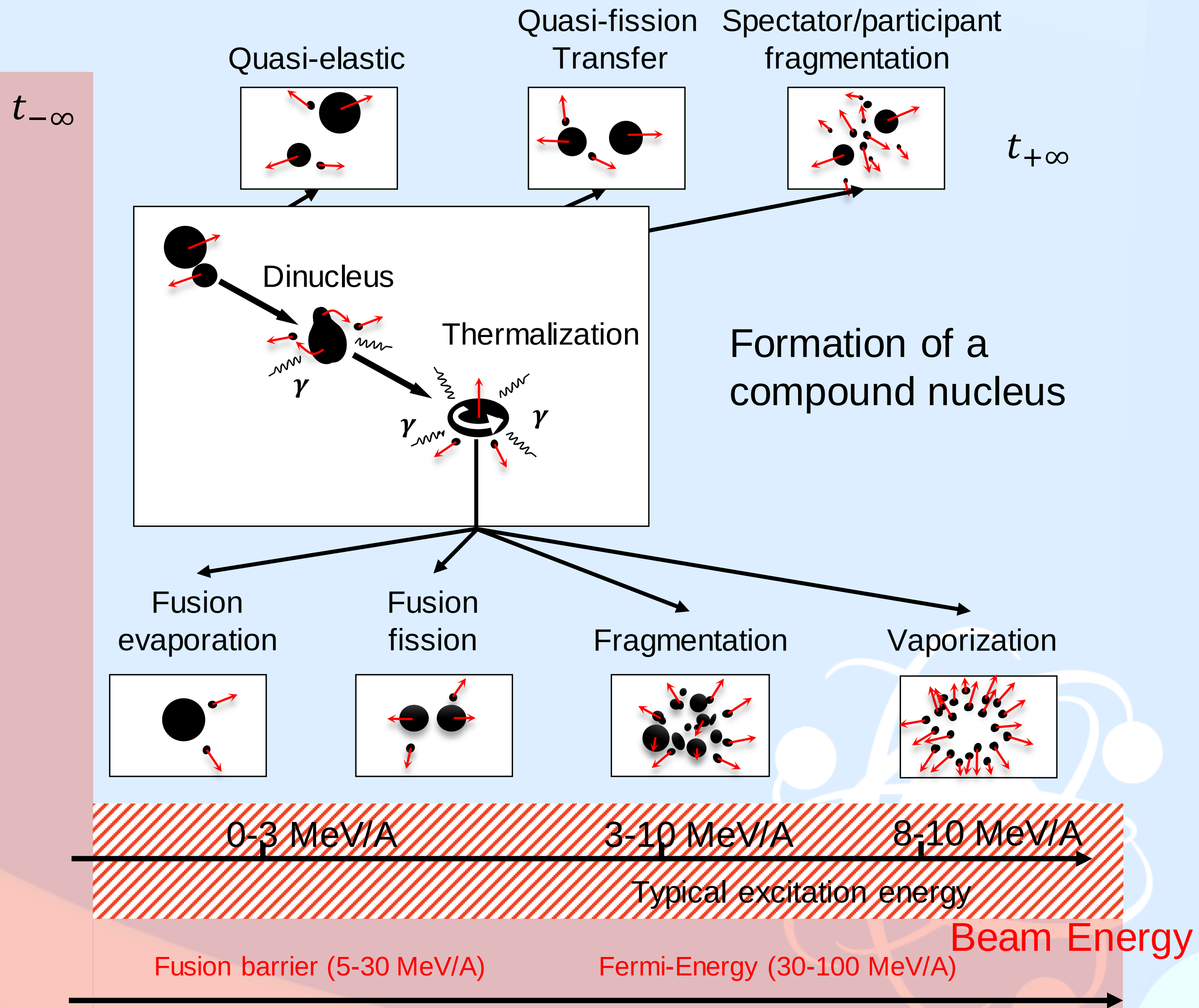


RICHNESS AND COMPLEXITY OF NUCLEAR REACTIONS

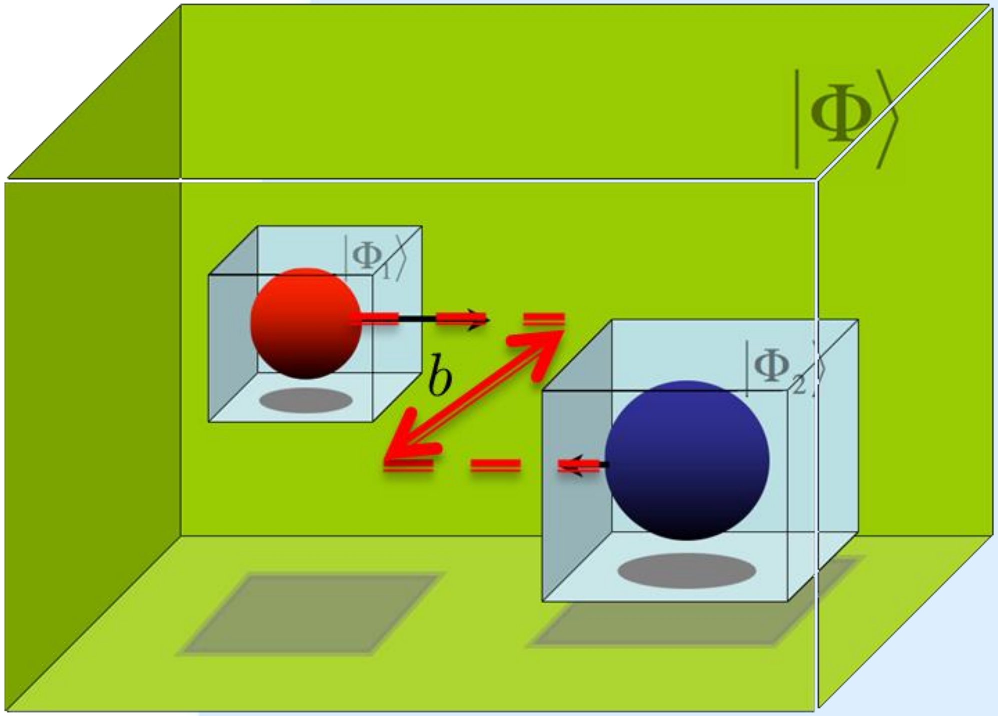
We control

Incoming channel





REMINDER: CONTROLLABLE INPUTS OF A COLLISION



Mass/Charge:

- Projectile (N_p, Z_p)
- Target (N_t, Z_t)

Beam Energy

- $E_b/A, E_p$

Spin moments

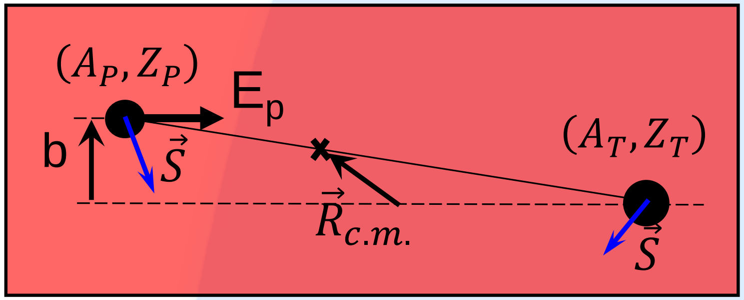
- Projectile $(p_z, p_{zz} \ldots)$
- Target $(q_z, q_{zz} \ldots)$

How the probe interacts

- Strong nuclear
- Electromagnetic
- Weak

CLASSIFICATION OF REACTIONS

Entrance channel



Exit channel

????
Inclusive/exclusive
Strongly detectors dependent

No energy transfer

$$A + a \rightarrow A + a$$ Elastic scattering $A(a,a)A$

$$A + \vec{a} \rightarrow \vec{A} + a$$ Spin transfer $A(\vec{a},a)\vec{A}$

Energy transfer

$$A + a \rightarrow B + b$$ Transfer reaction $A(a,b)B$

$$A + a \rightarrow A + a + \gamma$$ Nuclear Bremsstrahlung

$$A + a \rightarrow B^* + b^* + \cdots$$ Inelastic scattering
Deeply inelastic…

$$A + a \rightarrow C^* \rightarrow \text{decay}$$ Compound nucleus

X^* Means that X is not in its ground state

CONSERVATION LAWS

- Total charge:

$$\sum_i Z_i = \sum_f Z_f$$

- Total Baryonic number; i.e. when $E < E_{\pi\ \mathrm{treshold}}$ protons and neutron are equilibrated:

$$\begin{cases} \sum_i Z_i = \sum_f Z_f \\ \sum_i N_i = \sum_f N_f \end{cases}$$

- Total energy
- Total linear momentum

Important kinematics relations through, which one related quantities to initial parameter

- Total angular momentum
- Total parity
- Total isospin

$$J_i^{\pi_i} T_i = J_f^{\pi_f} T_f$$

where

$$\left[[A(I_A, \pi_A, T_A), a(, I_a \pi_a, T_a)]^S \psi(r)\right]^{J_i^{\pi_i} T_i}$$

$$\left[[B(I_B, \pi_B, T_B), b(I_b \pi_b, T_b)]^S \tilde{\psi}(r)\right]^{J_f^{\pi_f} T_f}$$

GOALS OF REACTION THEORY: FILL THE BLACK BOX

Challenge of nuclear reaction theory

- Understand the reaction mechanism to get information on nuclei
- Explain the diversity of nuclear phenomena
- Give a unified description of these phenomena

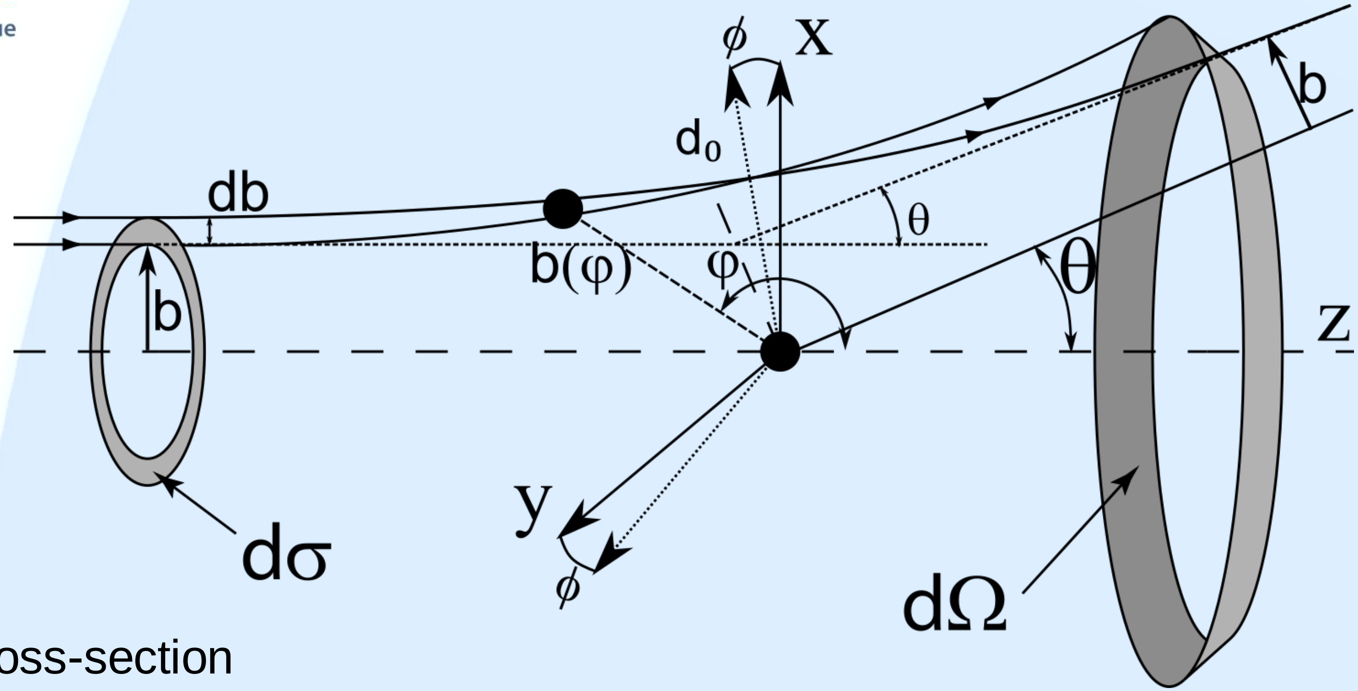
Challenge of this lecture

- Gives some hint on how these (some) phenomena are described
- Gives you some starting point/minimal background on nuclear reactions models
- Gives you overview of hot topics in the field

Scattering in Classical Mechanics

As an introduction

DEFINITIONS OF CROSS-SECTION AND OTHER QUANTITIES



Differential cross-section

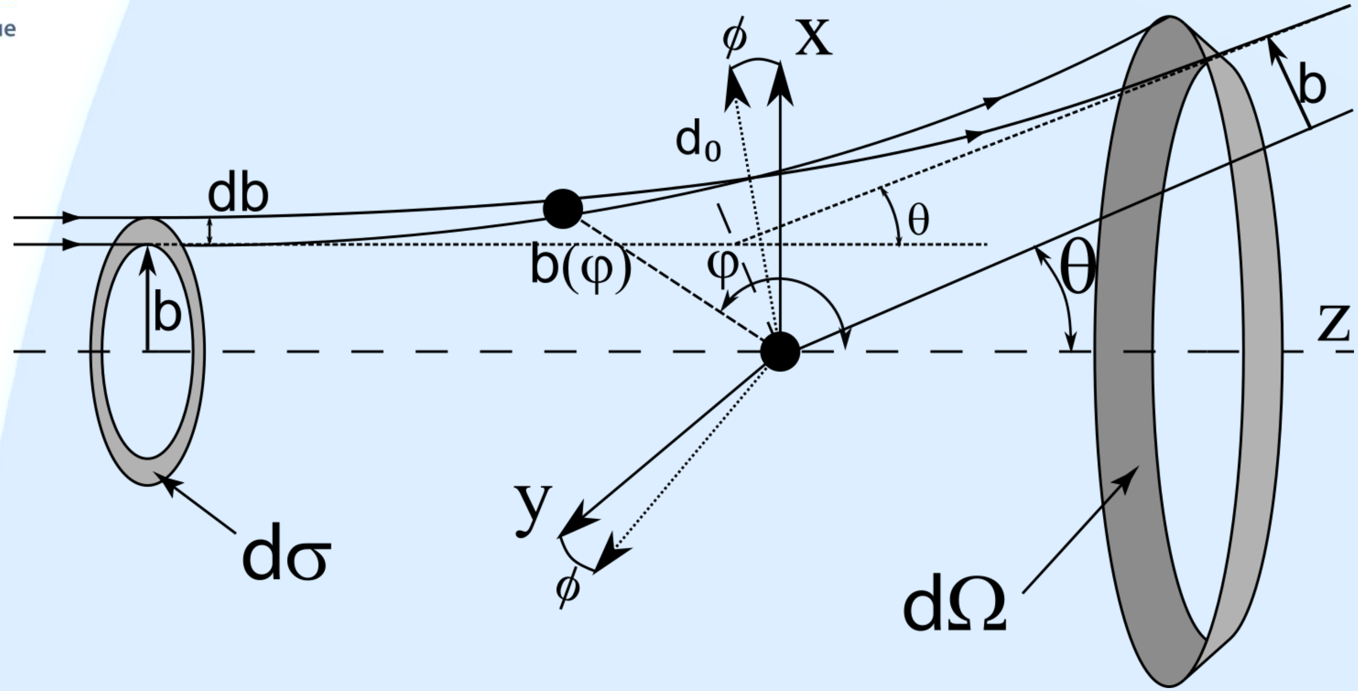
$$\frac{d\sigma_{\text{channel}}(\theta,\varphi)}{d\Omega} = \frac{\text{(Number of scattered particle per unit time)}(\theta,\varphi)}{\text{(Incident flux) (Scattering centers, n)(unit of solid angle)}}$$

$$\xrightarrow{n=1} \frac{j_f(\theta,\varphi)}{j_i}$$

Total cross-section

$$\sigma_{\text{tot}} = \int \sum_{\text{c}} \frac{d\sigma_{\text{c}}(\theta,\varphi)}{d\Omega}\, d\Omega$$

DEFINITIONS OF CROSS-SECTION AND OTHER QUANTITIES



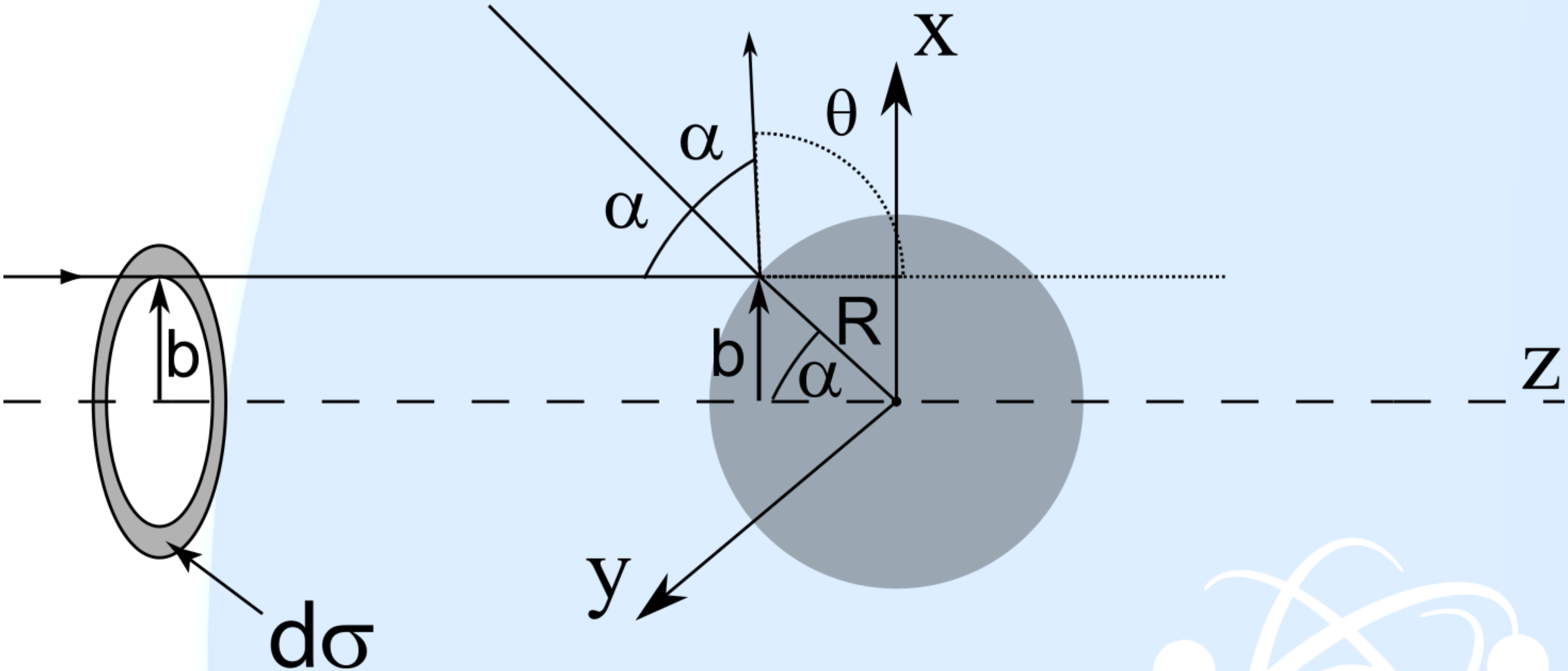
In classical mechanic, equations are deterministic i.e. the trajectory depends on b (or equivalently $b(\pi)$).
So number of scattered particle per unit time:

$$j_i \pi[(b+db)^2 - b^2] \sim j_i 2\pi b db$$

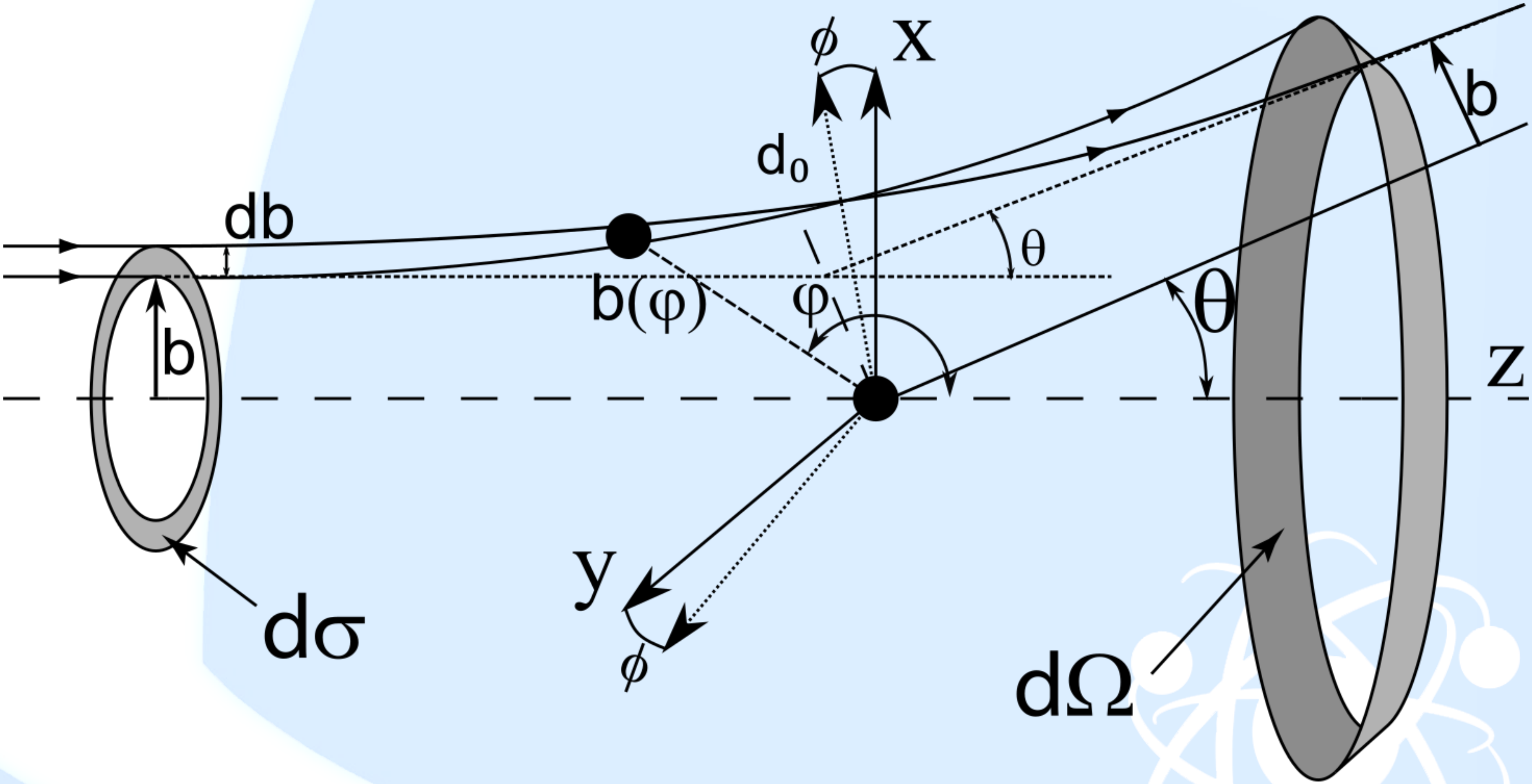
The differential cross-section is:

$$\frac{d\sigma_c(\theta,\varphi)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

HARD SPHERE SCATTERING



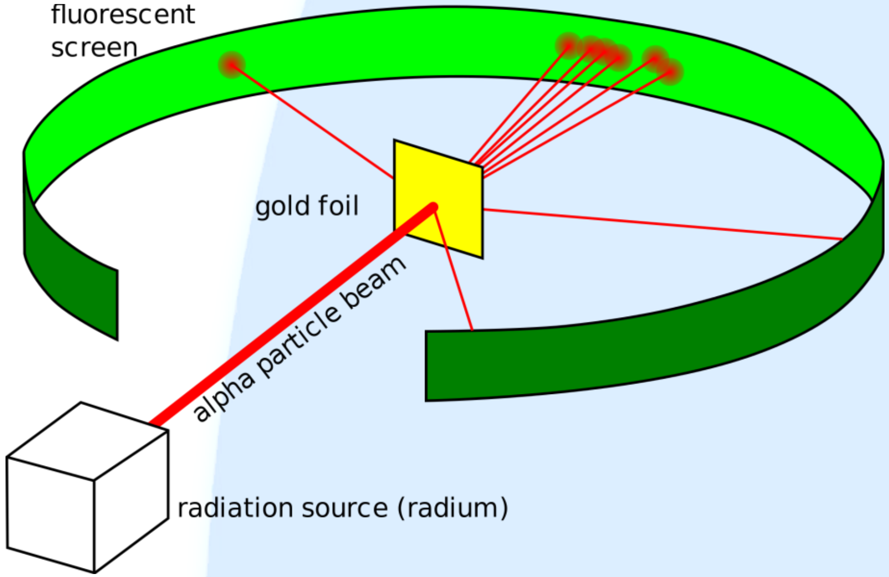
CENTRAL FORCE SCATTERING (COULOMB)

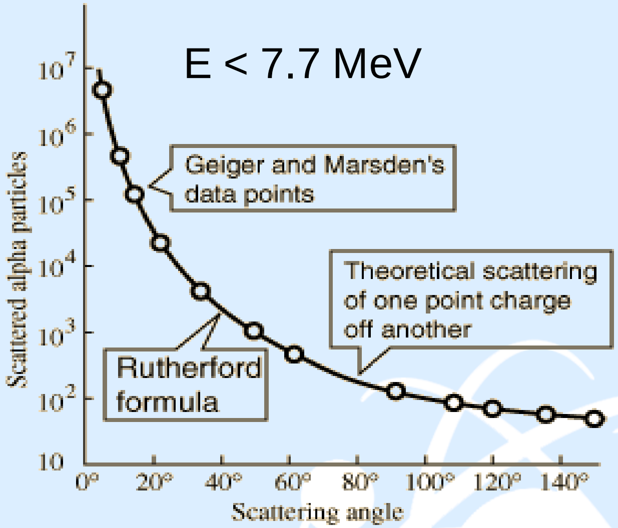


RUTHERFORD SCATTERING: EXPERIMENTAL PROOF AND VALIDATION

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 m v_0^2}\right)^2 \csc^4\left(\frac{\theta}{2}\right)$$

Geiger-Marsden experiment (1908-1913)

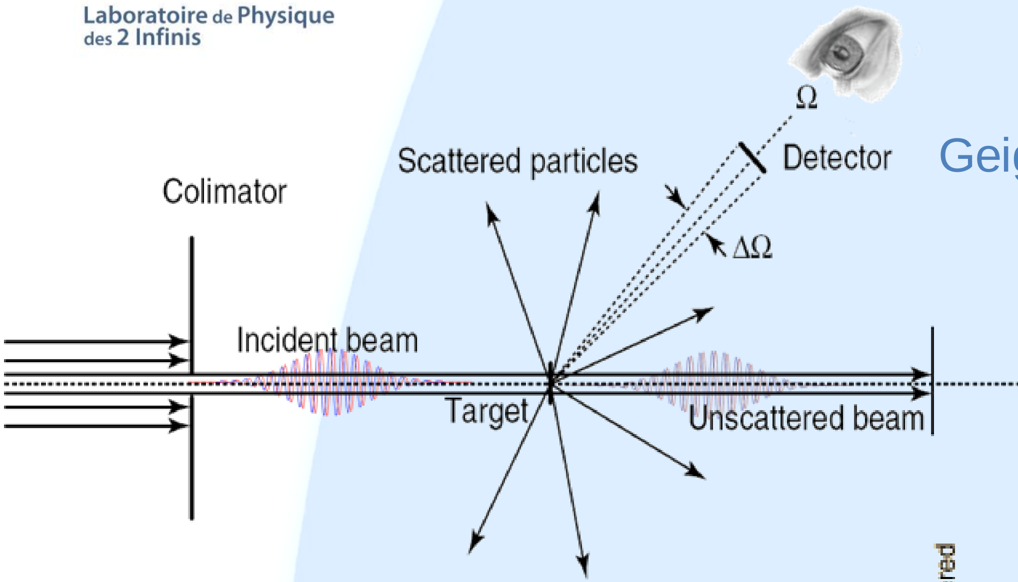




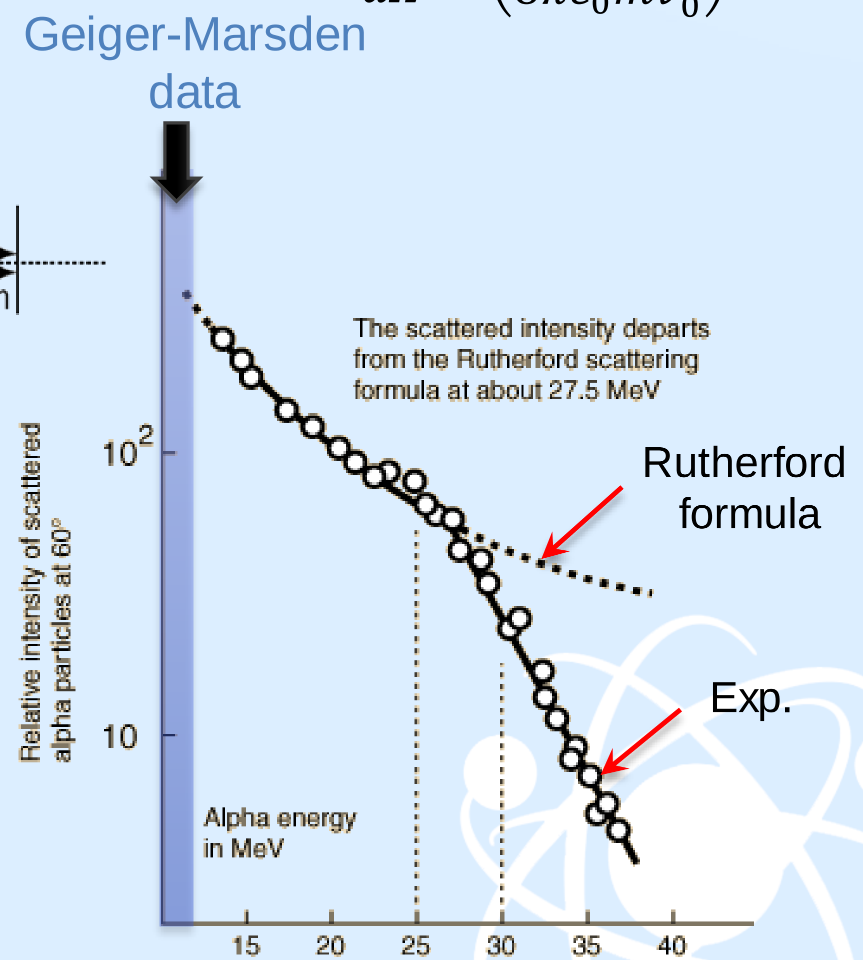
Data: http://hyperphysics.phy-astr.gsu.edu

DEPARTURE FROM RUTHERFORD

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 m v_0^2}\right)^2 \csc^4\left(\frac{\theta}{2}\right)$$

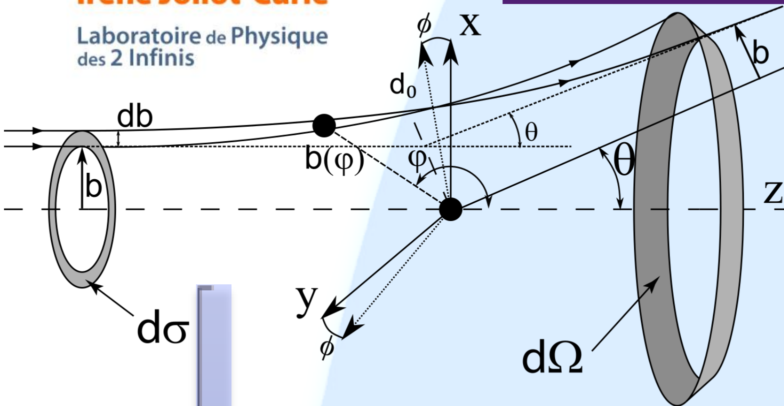


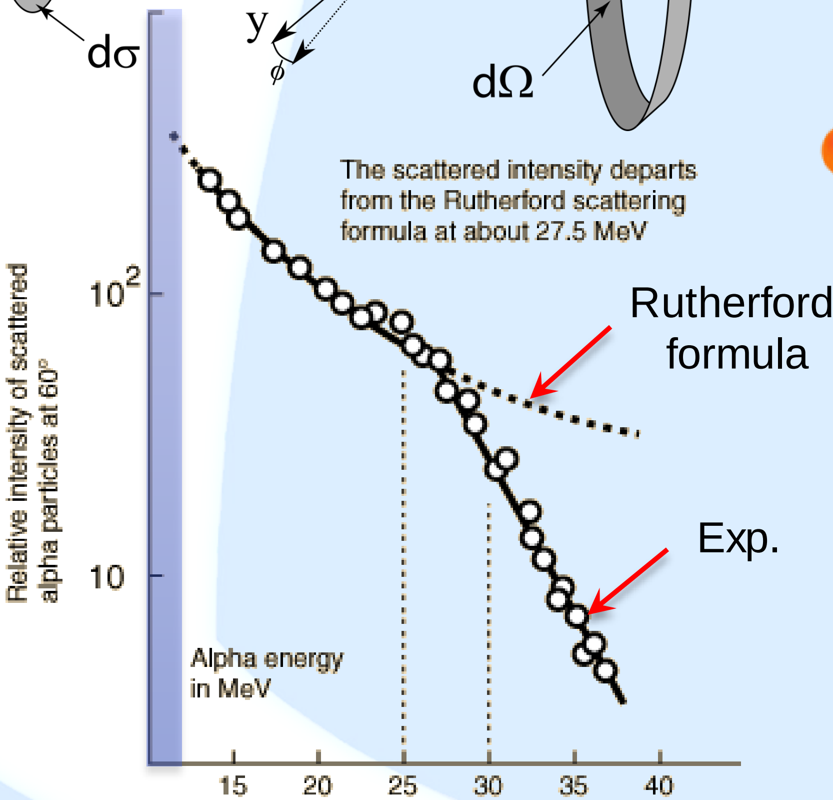
- Detector at a fixed θ angle
- Scan for various beam energy



Eisberg, R. M. and Porter, C. E., Rev. Mod. Phys. 33, 190 (1961)

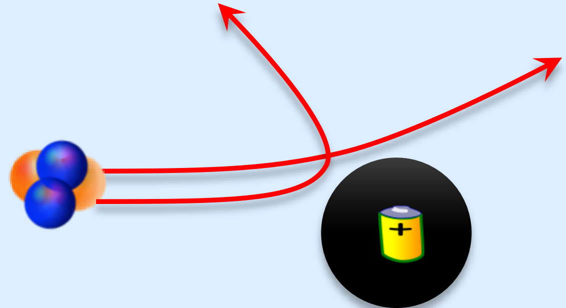
DEPARTURE FROM RUTHERFORD: INTERPRETATION

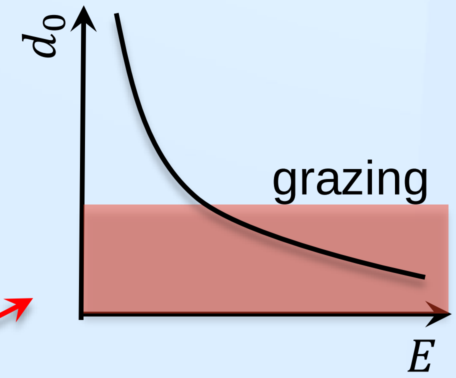




Eisberg, R. M. and Porter, C. E., Rev. Mod. Phys. 33, 190 (1961)

$$d_0 = \frac{e^2 Z_1 Z_2}{2\pi\varepsilon_0 2E}$$



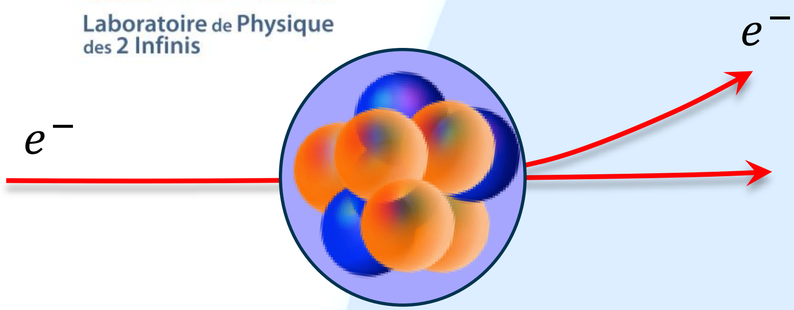


Alpha particle are not point like-particle

But this alone cannot explain the deviations from Rutherford

- Nucleons do interact also through the nuclear force.
- This interaction is a short-range interaction

ELECTRON SCATTERING TO PROBE NUCLEAR INTERIOR



See Z protons

Nucleons or α scattering on nuclei are sensitive to both the **nucleus spatial extension** and the **nuclear interaction**

Electron scattering is mostly **sensitive to charged matter** (proton) density

To probe nuclear properties with electrons, high energy electron beam are needed ($m_e \ll m_\alpha$)

⇒ $\beta = \frac{v}{c}$ is not small and relativistic effects are important

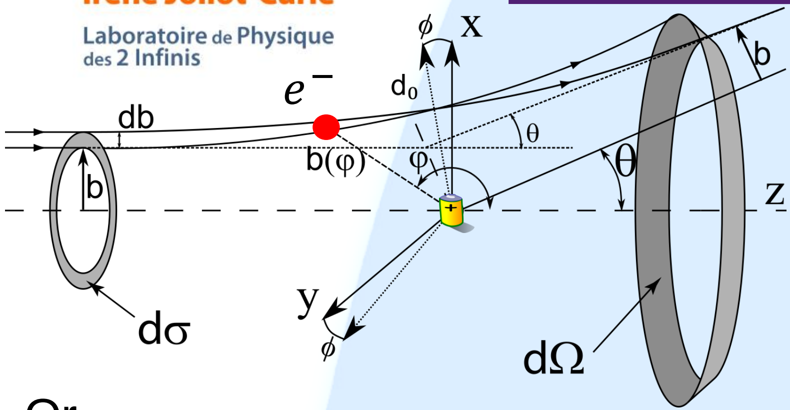
Rutherford with relativistic effects (assuming no nuclear spatial extension):

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth}} \left(1 - \beta^2 \sin^2\frac{\Theta}{2}\right)$$

valid for small *Z*

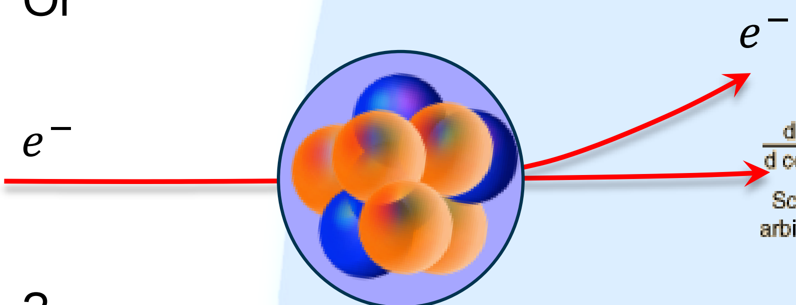
For $\beta = 1$ ⇒ Mott scattering formula $\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \simeq \left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth}} \cos^2\frac{\Theta}{2}$

ELECTRON SCATTERING TO PROBE NUCLEAR INTERIOR

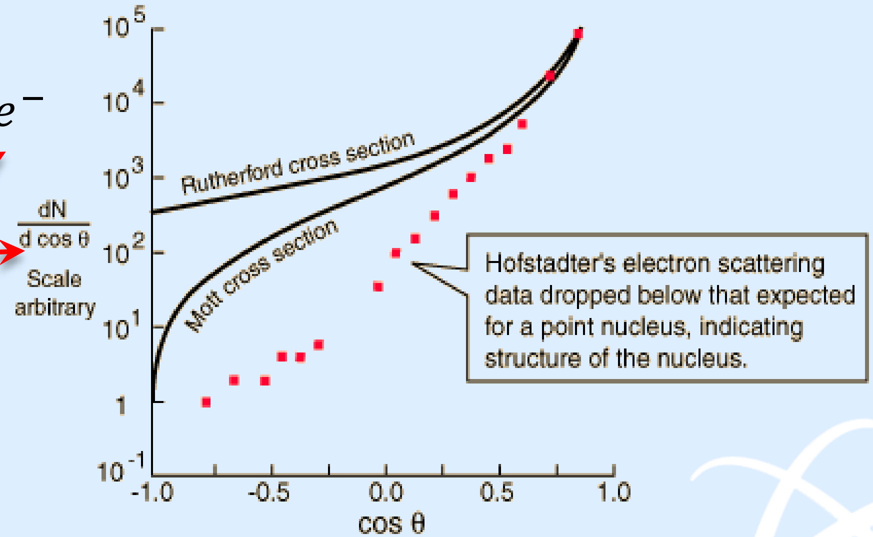


$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \simeq \left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth}} \cos^2\frac{\Theta}{2}$$

Or



?



Hofstadter, R., et al., Phys. Rev. 92, 978 (1953).

Nuclear systems cannot be treated as point-like particles and have finite extension

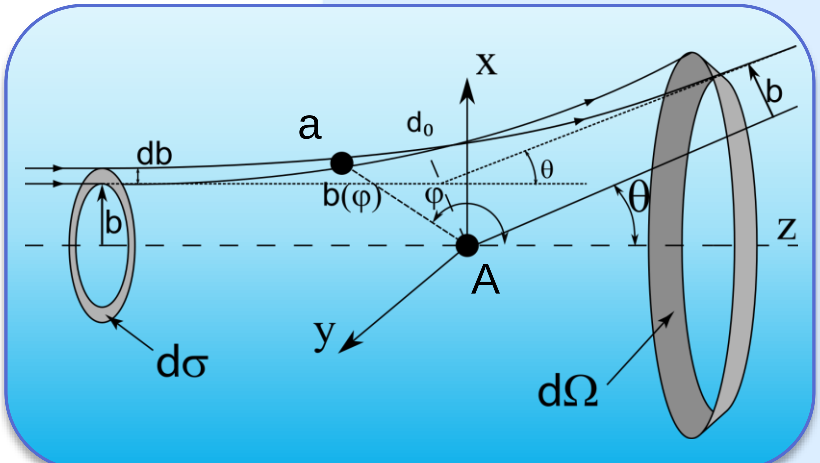
Quantum collisions

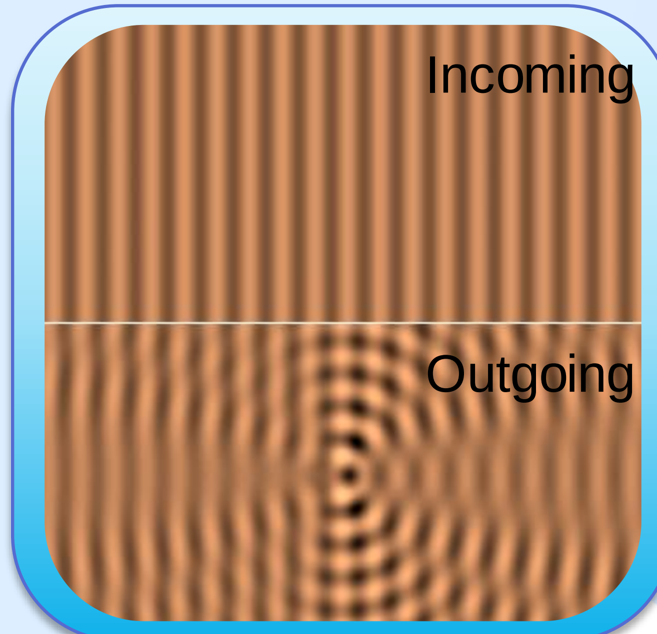
What ? Why for nuclei ?

BINARY ELASTIC COLLISIONS
THE (SEMI-)CLASSICAL VS QUANTUM PICTURE

We assume that the "entities" interact through a central potential $V(|\vec{r}_A - \vec{r}_a|)$

Classical picture | **Quantum picture**

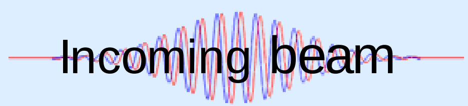




Incoming beam

$$E_B = \frac{P_a^2}{2M_a}$$

Trajectories are deterministic



Incoming beam

$$E_B = \frac{\hbar^2 k^2}{2M_a}$$ plane wave $$|\phi_i\rangle \xrightarrow{r\to\infty} e^{i\vec{k}.\vec{r}}$$

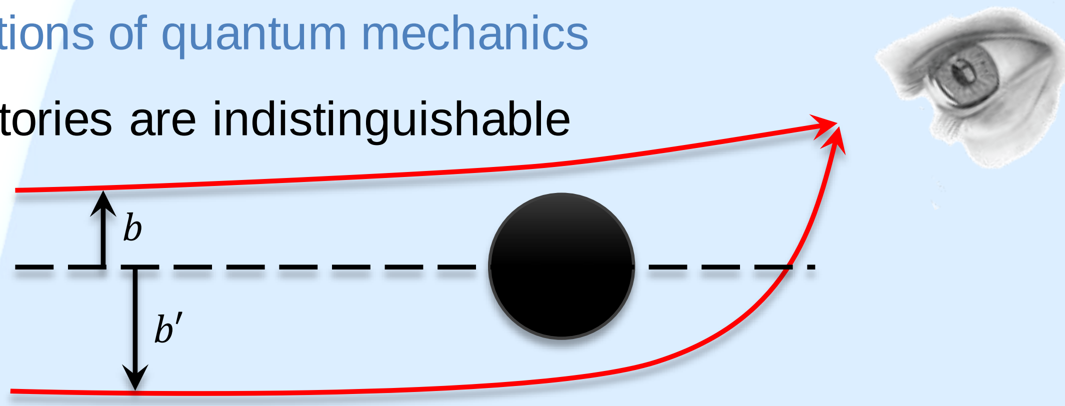
Outgoing spherical wave

$$|\phi_f\rangle \xrightarrow{r\to\infty} f(\Theta,\varphi)\frac{e^{ikr}}{r}$$

BINARY ELASTIC COLLISIONS
THE (SEMI-)CLASSICAL VS QUANTUM PICTURE

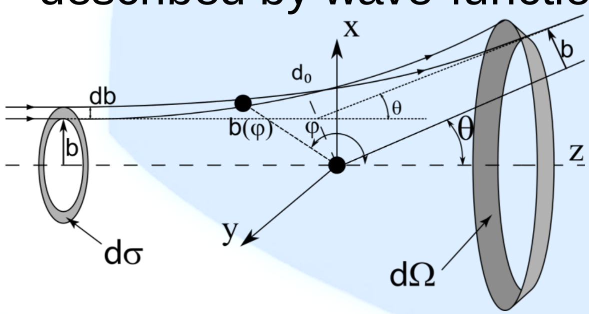
Few implications of quantum mechanics

Trajectories are indistinguishable



- Impact parameter is not defined in the quantum world

Particle-Target motion must be described by wave functions



- Quantum interference phenomenon

- Interfering trajectories can be constructive or destructive.

We start from

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \frac{j_f(\theta,\varphi)}{j_i}$$

and

$$|\psi^+\rangle = |\phi_i\rangle + |\phi_f\rangle$$

$$\psi_k^+(\vec{r}) \xrightarrow{r\to\infty} A\left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

Quantum current is defined as

$$\vec{J} = \frac{\hbar}{2\mu i}\left((\psi_k^+)^*\nabla\psi_k^+ - \psi_k^+\nabla(\psi_k^+)^*\right)$$

$$\vec{J} = \vec{j}_i + \vec{j}_f + \vec{j}_{\text{int}}$$

$$\begin{cases} j_f(\theta,\varphi) = \vec{J}\cdot\hat{r}\, r^{-2} \\ j_i = \vec{J}\cdot\hat{z} \end{cases}$$

$$\begin{cases} j_f(\theta,\varphi) = v|A|^2|f(\Theta,\varphi)|^2 \\ j_i = v|A|^2 \\ j_{\text{int}} = 0 \text{ for } \theta \neq 0 \end{cases}$$

$^1/_r$ leading contributions

We find the cross section to be:

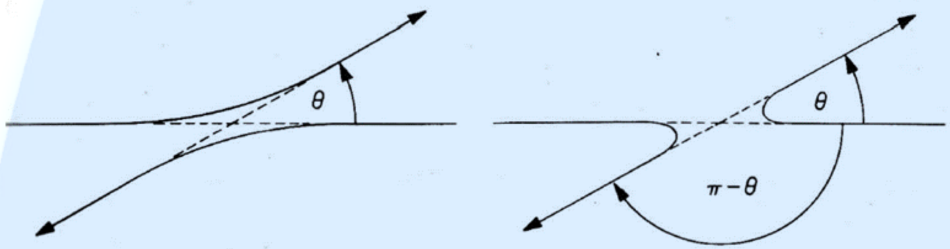
$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\Theta,\varphi)|^2$$

gives the optical theorem

ILLUSTRATION OF QUANTUM EFFECTS : SCATTERING OF IDENTICAL PARTICLES

Since nucleons are fermions

One cannot distinguish the two cases i.e. wf is antisymetric



This leads to

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\Theta,\varphi) + f(\pi - \Theta,\varphi)|^2$$

causes interferences !

Note that the same happens with Coulomb

Ex: scattering of 12C + 12C @ 5 MeV

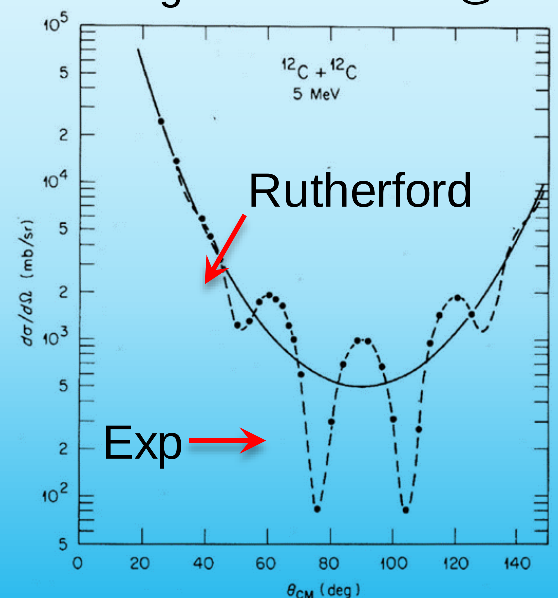


Illustration from G. R. Satchler "Introduction to direct reactions"