Irène Joliot-Curie
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Master NPAC:
An introduction to the theory of nuclear reactions

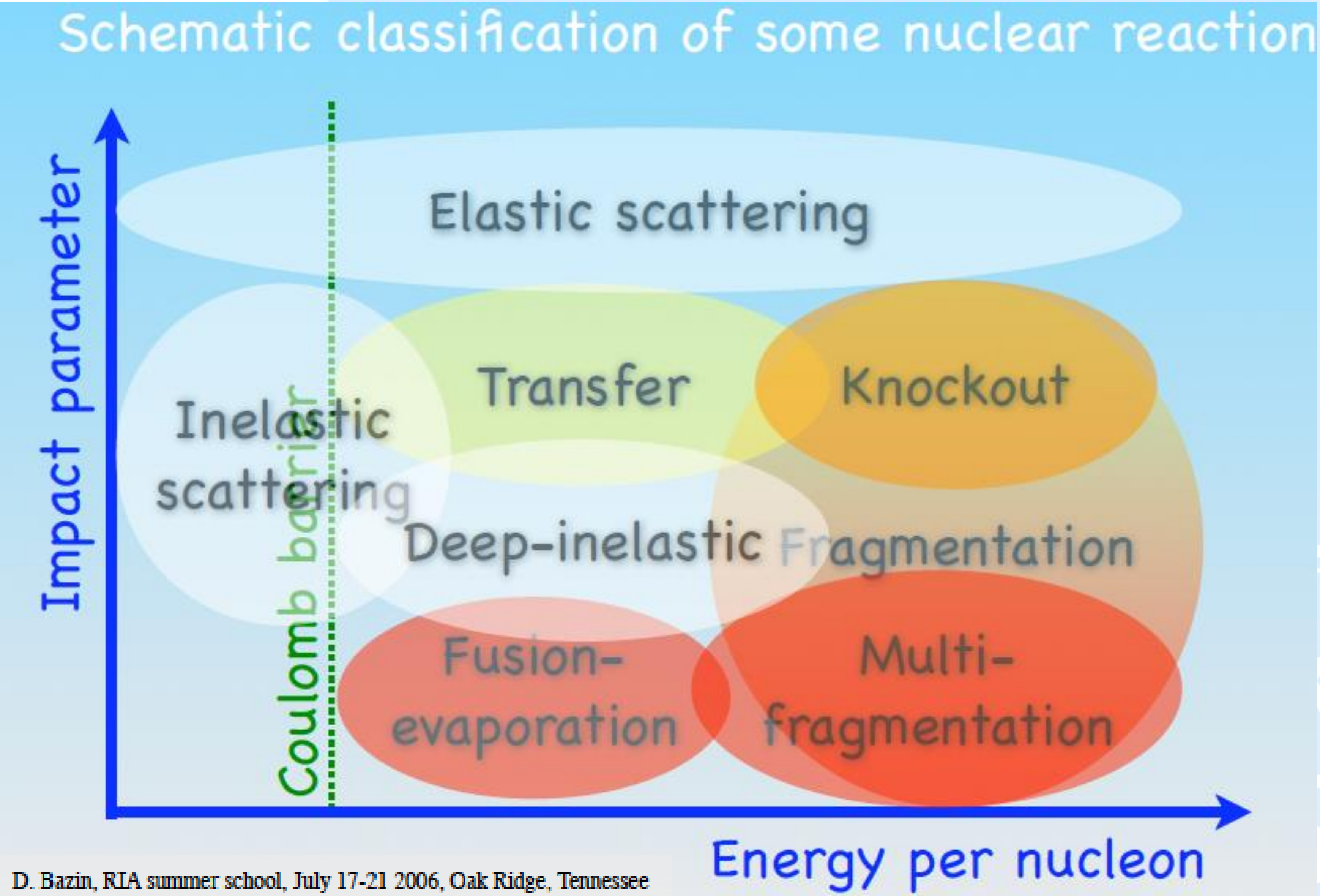
Guillaume Hupin

IPN Orsay

prepared with inputs of D. Lacroix

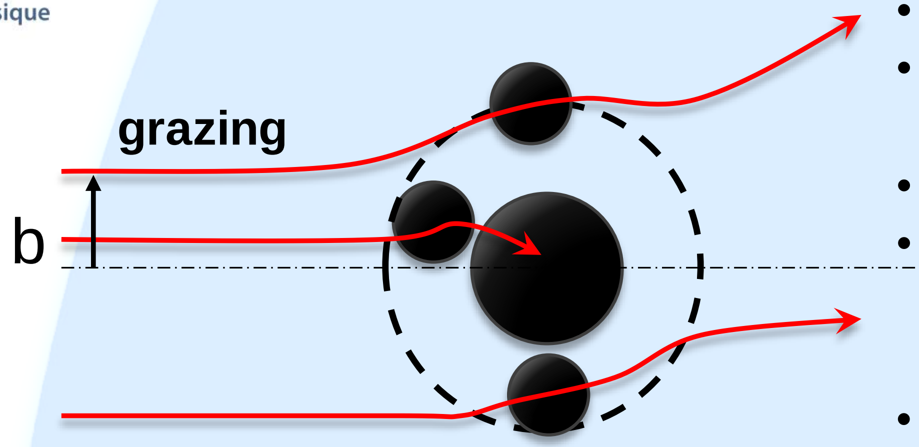
Lecture 4: Direct nuclear reactions Coulomb excitation, transfer

SCHEMATIC CLASSIFICATION INCLUDING HIGHER ENERGY
E/A FROM FEW MEV TO 200 MEV



D. Bazin, RIA summer school, July 17-21 2006, Oak Ridge, Tennessee

DIRECT REACTIONS
INTRODUCTION



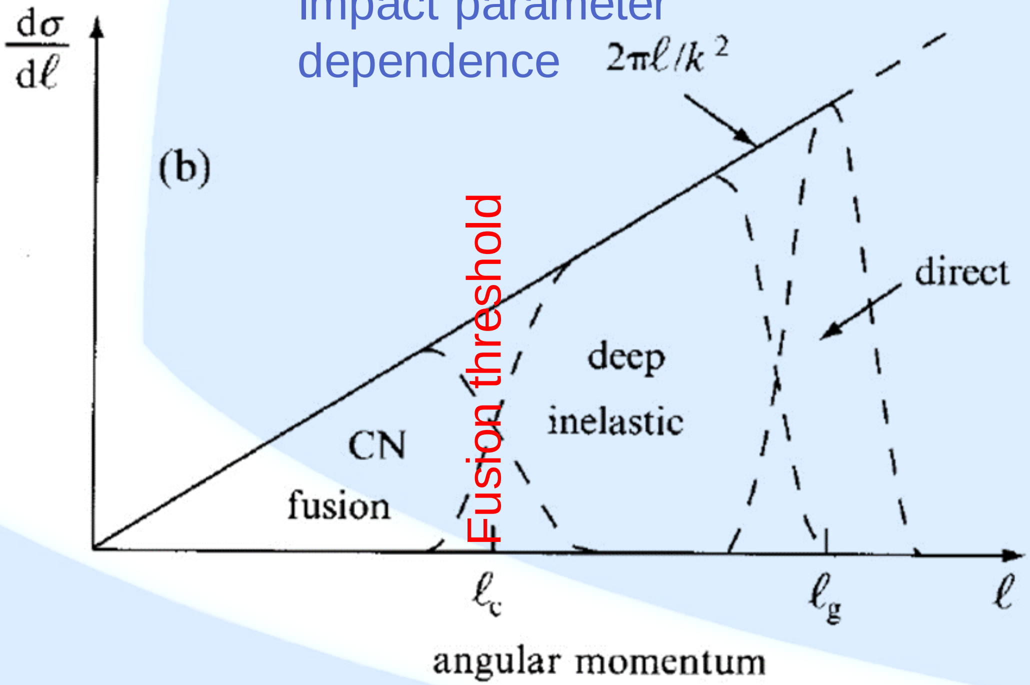
- Direct reaction
- Elastic and quasi-elastic

- Fusion
- Large amplitude Collective motion

- Multi-nucleon transfer
- Deep inelastic

Adapted from : W. Nörenberg and H.A. Weidenmüller, "Introduction to Heavy-Ion theory", Springer-Verlag 1981.

Impact parameter dependence



From J.S. Lilley, "Nuclear Physics"

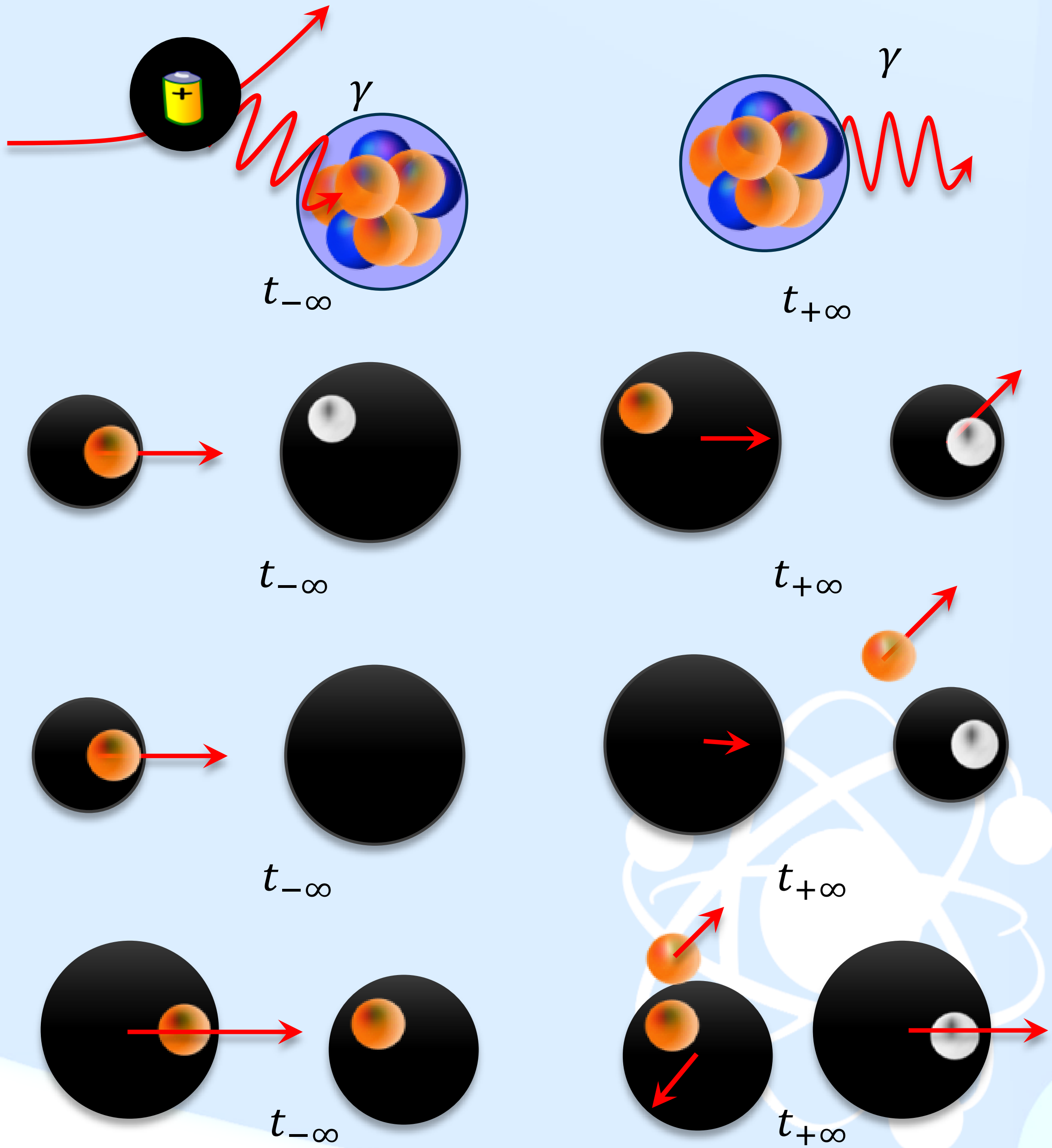
DIRECT REACTIONS
INTRODUCTION

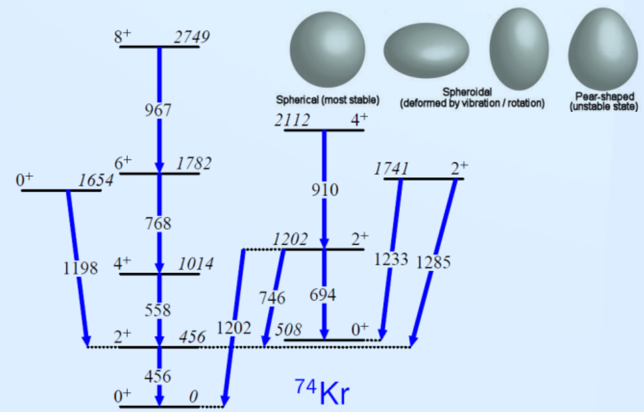
Coulomb excitation

Transfer reaction

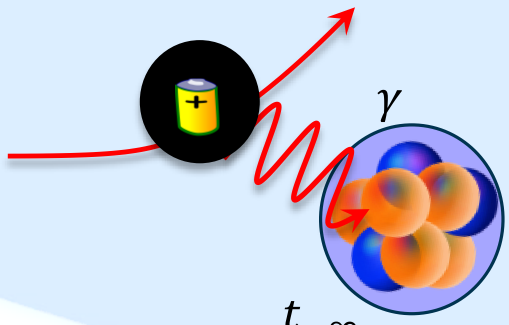
Break-up

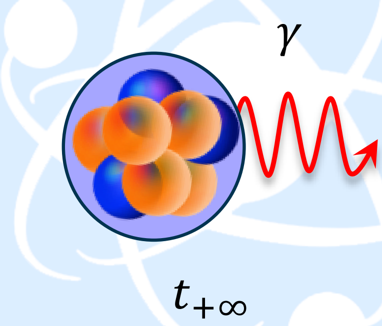
Knock-out





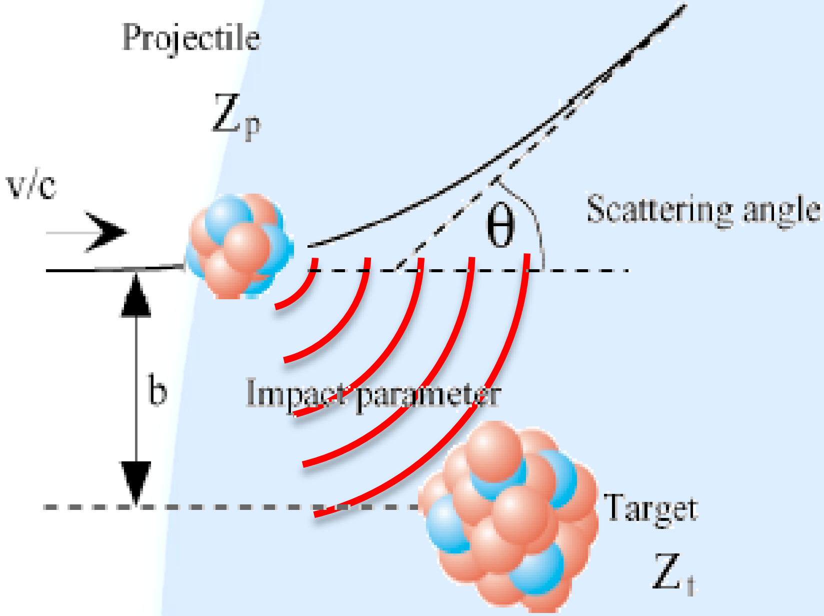
Coulomb Excitation (COULEX)

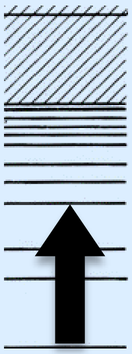




COULOMB EXCITATIONS

A. Gade et al., Phys. Rev. C 68, 014302 (2003).





Complete Hamiltonian

$$H = H_0 + V$$

Internal excitations

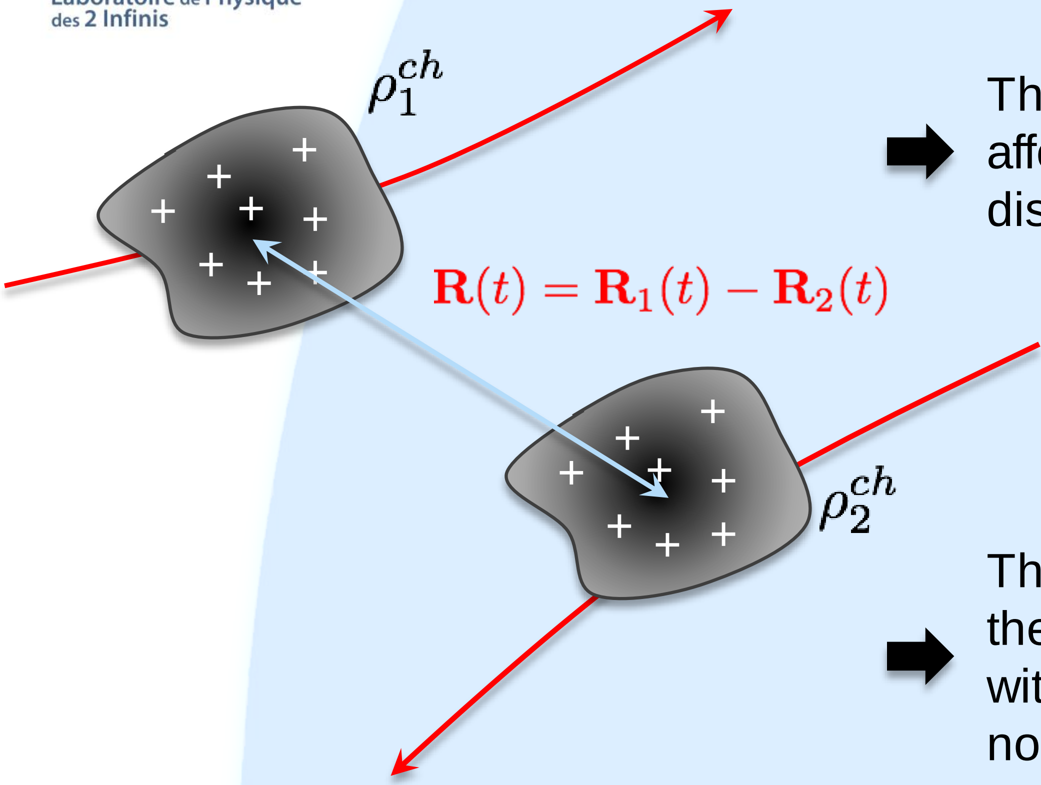
$$H_0 \psi_n = E_n \psi_n$$

- Typical Coupled-Channel problem that could be solved by looking to the asymptotic properties of the solutions:

$$(H - E)\Psi = 0$$

- In most cases a time-dependent perturbative treatment is enough

COULOMB EXCITATIONS



- The short range nuclear part does not affect the nuclei trajectory at large distance
- The relative motion is usually treated in the *Born-Oppenheimer* approximation with a simple classical relativistic or non-relativistic trajectory

The time-dependent perturbation that induces internal transitions can be written as

$$V(t) = \int d^3r_1 d^3r_2 \frac{\rho_1^{ch}(\mathbf{r}_1 - \mathbf{R}_1)\rho_2^{ch}(\mathbf{r}_2 - \mathbf{R}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} - \frac{Z_1 Z_2 e^2}{R(t)}$$

with $\mathbf{R}(t) = \mathbf{R}_1(t) - \mathbf{R}_2(t)$

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Starting point: $H = H_0 + V(t)$ $H_0\psi_n = E_n\psi_n$

Time-dependent solution

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H(t)\Psi(t)$$

$$\Psi(t) = \sum_n a_n(t)\psi_n e^{-iE_nt/\hbar}$$

Coupled channel time-dependent equations

$$\dot{a}_k(t) = -\frac{i}{\hbar}\sum_n a_n(t)\; V_{kn}(t)\; e^{i\frac{E_k-E_n}{\hbar}t}$$

with $V_{kn}(t) = \langle\Psi_k|V(t)|\Psi_n\rangle$

This situation is equivalent to the scattering problem (cf. lecture 2 and 3)

$$\Psi(t) = a_i(t)\psi_i e^{-iE_it/\hbar} + \sum_f a_f(t)\psi_f e^{-iE_ft/\hbar}$$

Entrance — Exit

Asymptotic conditions

$t \to -\infty$ — Reaction time T — $t \to +\infty$

$a_i = 1$

$a_f = 0$

$|a_f(+\infty)|^2$ Probability to populate the final state "f" given the initial state "i" (transition rate)

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$t \to -\infty$ Reaction time T $t \to +\infty$

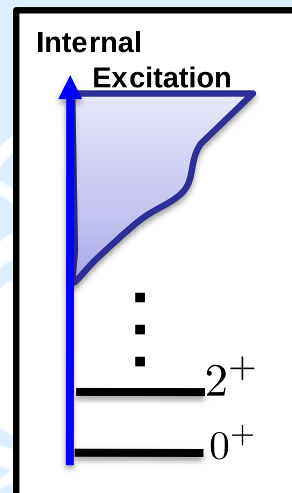
$$a_i = 1$$
$$a_f = 0$$

$|a_f(+\infty)|^2$ Probability to populate the final state "f" given the initial state "i" (transition rate)

$$\dot{a}_k(t) = -\frac{i}{\hbar}\sum_n a_n(t)\; V_{kn}(t)\; e^{i\frac{E_k - E_n}{\hbar}t}$$

$$\Rightarrow \quad a_k = -\frac{i}{\hbar}\int_0^T V_{kn}(t)e^{i(E_k - E_n)t/\hbar}\,dt$$

$$\rho(E) = \frac{dN}{dE}$$



First order Perturbation theory (Born approximation)

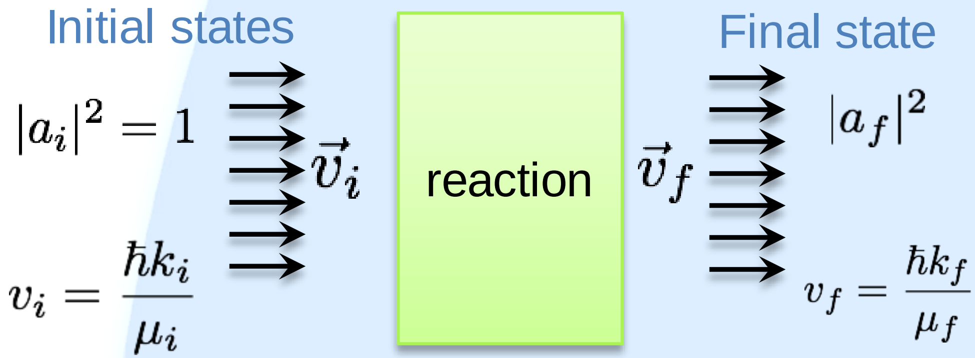
$$P(i \to f) = \frac{1}{\hbar^2}\int_0^T \left|\langle\Psi_f|V(s)|\Psi_i\rangle e^{-i(E_f - E_i)s}\right|^2 ds$$

Transition to a set of states with density ρ

$$dP(i \to f) = \frac{1}{\hbar^2}\int_0^T \left|\langle\Psi_f|V(s)|\Psi_i\rangle e^{-i(E_f - E_i)s}\right|^2 ds\rho(E_f)dE_f$$

COULOMB EXCITATIONS
LINK TO FERMI GOLDEN RULE

From time-dependent to time independent picture



Cross section = number of particles scattered per unit time into a given final state normalized to the incident flux

$$\sigma_{i \to f} = \frac{\Gamma_{i \to f}}{v_i |a_i|^2}$$

with

$$\Gamma_{i \to f}(E_f) = \frac{1}{T} dP(i \to f)$$

Link with the Fermi Golden Rule

If V and ρ varies slowly compared to the typical interaction time

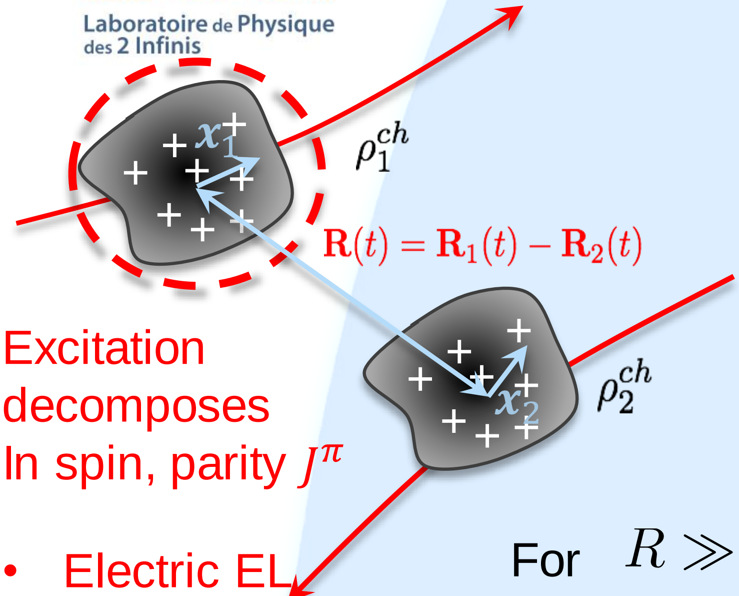
$$\Rightarrow \quad \Gamma_{i \to f} \simeq \frac{2\pi}{\hbar} |V_{if}|^2 \rho(E_f)$$

For a isotropic distribution of final momentum

$$\sigma_{i \to f} = \frac{\mu_f \mu_i}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} |V_{fi}|^2$$

(see C. Bertulani, arXiv 0908.430)

APPLICATION TO COULOMB EXCITATIONS
EXCITATION CHANNELS



Excitation decomposes In spin, parity J^π

- Electric EL
- Magnetic ML

Time-dependent perturbation

$$V(t) = \int d^3x_1 d^3x_2 \frac{\rho_1^{ch}(\mathbf{x}_1)\rho_2^{ch}(\mathbf{x}_2)}{|\mathbf{R}(t) + \mathbf{x}_1 - \mathbf{x}_2|} - \frac{Z_1 Z_2 e^2}{R(t)}$$

$(\mathbf{x}_1, \mathbf{x}_2)$ Intrinsic coordinates

Multipole decomposition of the perturbation

$$\frac{1}{|\mathbf{r} - \mathbf{s}|} = \sum_{LK} \frac{4\pi}{2L+1} \frac{r_<^L}{r_>^{L+1}} Y_{LK}^*(\Omega_s) Y_{LK}(\Omega)$$

For $R \gg x_{1/2}$

$$V(t) = \int d^3x_1 d^3x_2 \rho_1^{ch}(\mathbf{x}_1)\rho_2^{ch}(\mathbf{x}_2) \sum_{L>0,M} \frac{4\pi}{2L+1} \frac{x_{12}^L}{R^{L+1}(t)} Y_{LM}(\mathbf{x}_{12}) Y_{LM}^*(\mathbf{R})$$

Suppose that we focus on excitation in nucleus 1 induced by the Coulomb field of the second nucleus: $\rho_2^{ch}(\mathbf{x}_2) \approx Z_2 e \delta(\mathbf{x}_2)$

$$\Rightarrow V(t) = Z_2 e \sum_{L>0,M} \frac{4\pi}{2L+1} \frac{1}{R^{L+1}(t)} Y_{LM}^*(\mathbf{R}(t)) \int \rho_1^{ch}(\mathbf{r}) r^{L+1} Y_{LM}(\mathbf{r}) d^3r$$

APPLICATION TO COULOMB EXCITATIONS
EXCITATION CHANNELS

$$V(t) = Z_2 e \sum_{L>0,M} \frac{4\pi}{2L+1}\frac{1}{R^{L+1}(t)} Y^*_{LM}(\mathbf{R}(t)) \int \rho_1^{ch}(\mathbf{r}) r^{L+1} Y_{LM}(\mathbf{r}) d^3 r$$

Contains all the information on the nuclei trajectories

Defines the operator that will induces a specific excitation. Independent of the trajectory

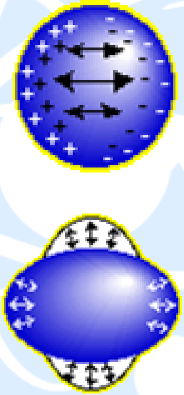
Schematic expression

$$V(t) = Z_2 e \sum_{L>0M} \frac{4\pi}{2L+1} F_{LM}(t) \mathcal{M}(EL, M)$$

Electric Multipole moments independent on the perturbation type

Example

- monopole $\mathcal{M}(E0,0) = \frac{1}{\sqrt{4\pi}} \int d^3 r\, \rho^{\text{ch}}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}} Z e$
- dipole $\mathcal{M}(E1,0) = \sqrt{\frac{3}{4\pi}} \int d^3 r\, \mathbf{r} \rho^{\text{ch}}(\mathbf{r}) = \sqrt{\frac{3}{4\pi}} d$
- quadrupole $\mathcal{M}(E2,0) = \frac{1}{4}\sqrt{\frac{5}{\pi}} \int d^3 r\, (2z^2 - x^2 - y^2) \rho^{\text{ch}}(\mathbf{r})$



APPLICATION TO COULOMB EXCITATIONS
PERTURBATION THEORY

$$V(t) = Z_2 e \sum_{L>0M} \frac{4\pi}{2L+1} F_{LM}(t)\mathcal{M}(EL,M) \qquad \text{with} \qquad \Phi(t) \sim \chi(\mathbf{R}(t)) \sum_i a_{fi}(t)\Phi_f$$

Transition probability
$$a_{fi} = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt \langle \chi_{\mathbf{R}(t)} \Phi_f | V(t) | \chi_{\mathbf{R}(t)} \Phi_i \rangle e^{i\omega t}$$

$$\hbar\omega = E_f - E_i$$

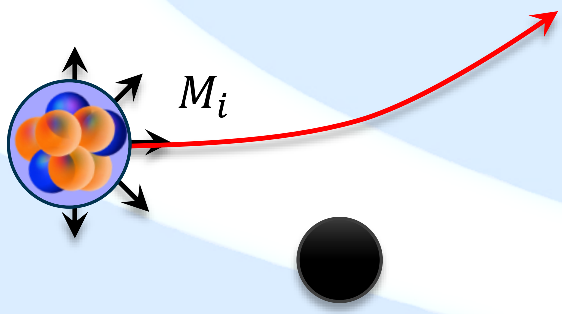
Reaction part

$$I_{LM}(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{R^{L+1}(t)} Y_{LM}(\mathbf{R}(t)) e^{i\omega t}$$

Excitation part

$$\langle \Phi_f | \mathcal{M}(EL,M) | \Phi_i \rangle$$

For unpolarized beam (all $\mathcal{M}$ contributes and should be summed up for final J and average for initial)



$$w_{fi} = \frac{1}{2J_i+1} \sum_{M_f M_i} |a_{fi}|^2$$

APPLICATION TO COULOMB EXCITATIONS
PERTURBATION THEORY

Average transition probability

$$w_{fi} = \left(\frac{4\pi Z_2 e}{\hbar}\right)^2 \sum_{L>0} \frac{B(\mathrm{EL}; i \to f)}{(2L+1)^3} \sum_M |I_{LM}(\omega_{fi})|^2$$

Reduced transition probability

$$B(\mathrm{EL}; i \to f) = \sum_{M_i M_f} \left|\mathcal{M}(\mathrm{EL}, M)_{fi}\right|^2$$

Differential cross-section:

$$\frac{d\sigma_{fi}(\mathrm{L})}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} \times \frac{B(\mathrm{EL}; i \to f)}{(2L+1)^3} \sum_M |I_{LM}(\omega_{fi})|^2$$

Reaction only independent on the channel and *J*

Intrinsic structure only

Both reaction and structure

$$\hbar\omega = E_f - E_i$$

with

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} = \frac{(e^2 Z_1 Z_2)^2}{16E^2} \frac{1}{\sin^4 \frac{\theta}{2}}$$

(For magnetic excitation see C. Bertulani, arXiv 0908.430)

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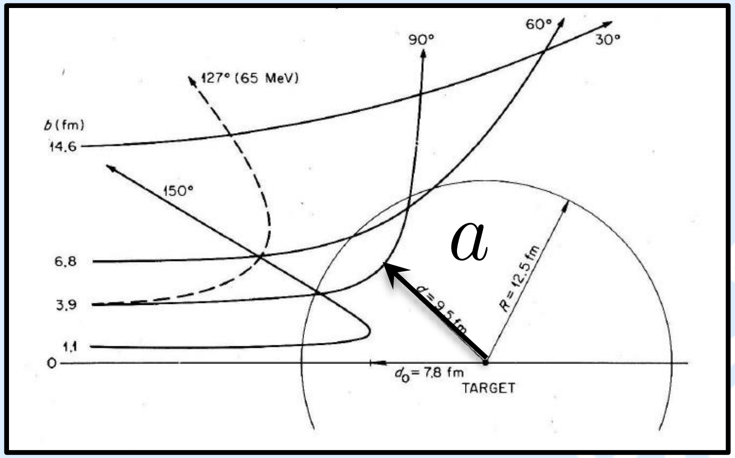
COULOMB EXCITATIONS

$$\frac{d\sigma_{fi}(\mathrm{L})}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} \times \frac{B(\mathrm{EL}; i \to f)}{(2L+1)^3} \sum_M |I_{LM}(\omega_{fi})|^2$$

$$f_L(\omega) = \sum_M |I_{LM}(\omega_{fi})|^2$$ is a rather complex quantity

with $$I_{LM}(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{R^{L+1}(t)} Y_{LM}(\mathbf{R}(t)) e^{i\omega t}$$



Collision time $\tau_{\mathrm{col}} \simeq \frac{a}{v}$

Period of the transition $\tau_{fi} \simeq \frac{1}{\omega_{fi}}$ ➡ Adiabatic parameter $\zeta = \frac{\tau_{\mathrm{col}}}{\tau_{fi}} \propto \frac{\Delta E_{fi}}{E}$

COULOMB EXCITATIONS

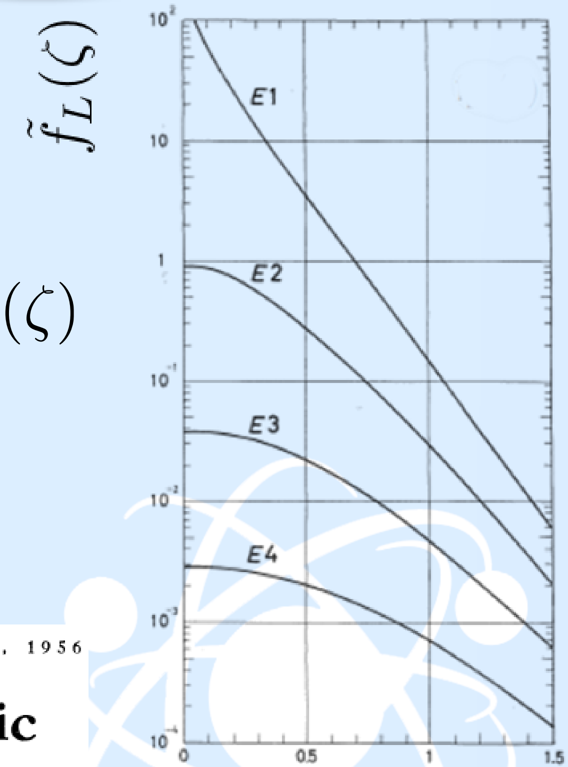
$$\frac{d\sigma_{fi}(\mathrm{L})}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} \times \frac{B(\mathrm{EL}; i \to f)}{(2L+1)^3} \sum_M |I_{LM}(\omega_{fi})|^2$$

$f_L(\omega) = \sum_M |I_{LM}(\omega_{fi})|^2$ is a rather complex quantity

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} \propto a^2$$

$$f_L(\omega) \propto \frac{1}{a^{2L}} \tilde{f}(\zeta)$$

$$\Rightarrow \sigma(\mathrm{EL}) \propto \frac{B(\mathrm{EL})}{a^{2L-2}} \tilde{f}_L(\zeta)$$



$$\zeta \propto \frac{\Delta E_{fi}}{E}$$

More phenomenology in

REVIEWS OF MODERN PHYSICS VOLUME 28, NUMBER 4, OCTOBER, 1956

Study of Nuclear Structure by Electromagnetic Excitation with Accelerated Ions

K. ALDER, A. BOHR, T. HUUS, B. MOTTELSON, AND A. WINTHER

CERN Theoretical Study Division and Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

COULOMB EXCITATIONS

$$\frac{d\sigma_{fi}(\mathrm{L})}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} \times \frac{B(\mathrm{EL}; i \to f)}{(2L+1)^3} \sum_M |I_{LM}(\omega_{fi})|^2$$

- Describe the excitation process from an initial to a final state of the nucleus

- For a given multipolarity, it can be obtained by applying a time-dependent field to the nucleus

$$V(t) = Z_2 e \sum_{L>0M} \frac{4\pi}{2L+1} F_{LM}(t) \mathcal{M}(EL, M)$$

In the weak coupling regime= linear response theory to a set of Time-dependent field

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \left(\hat{H} + \varepsilon f(t) \hat{F}\right) |\Psi(t)\rangle$$

Examples: Monopole Quadrupole Dipole

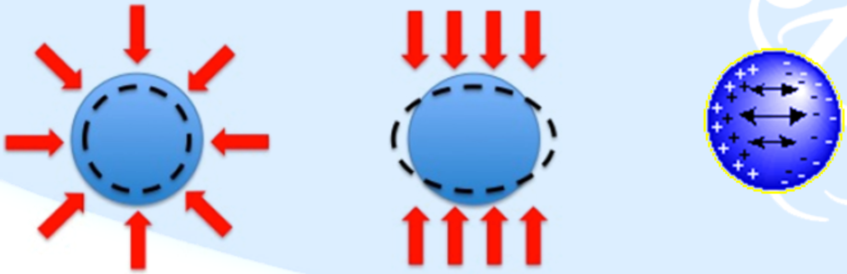
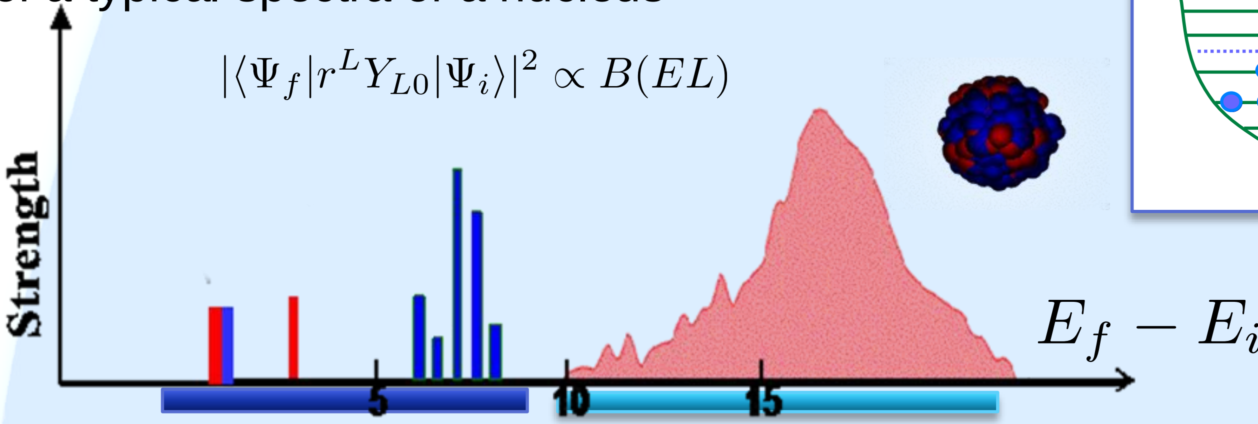
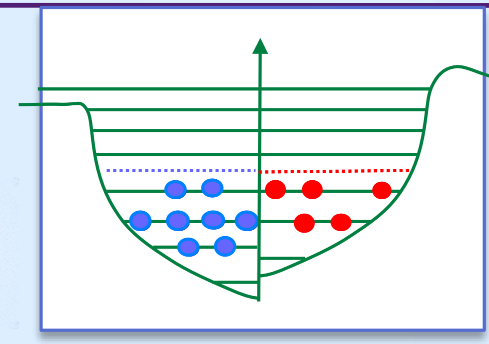


Illustration of a typical spectra of a nucleus

$$|\langle\Psi_f|r^L Y_{L0}|\Psi_i\rangle|^2 \propto B(EL)$$

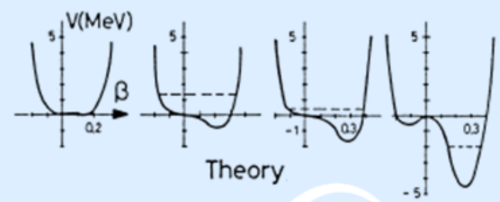


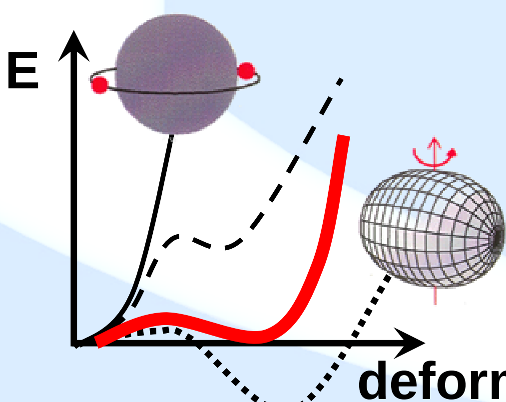


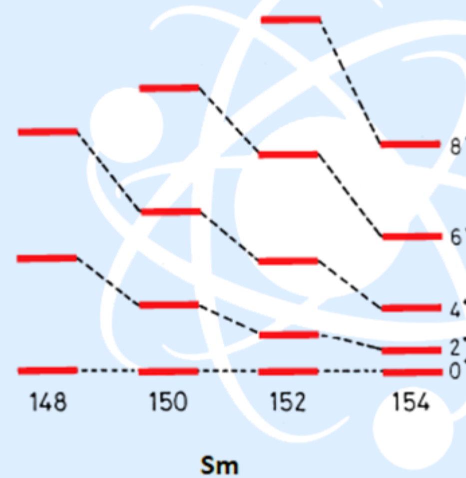
Low lying excitation (collective and non-collective)

Sensitive to “magicity”, shell evolution…

Give information on shape coexistence

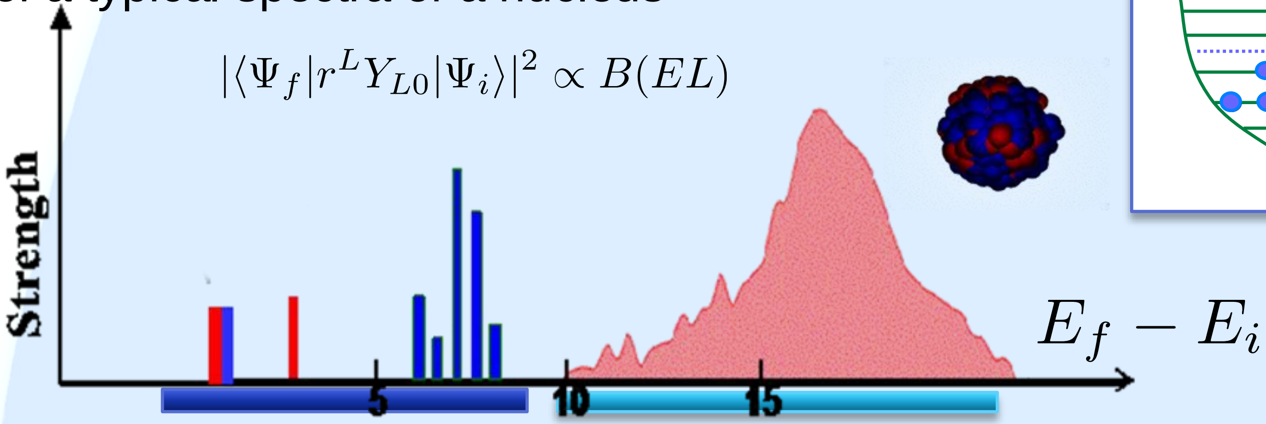


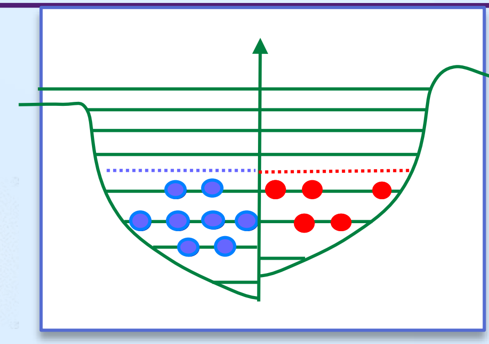




COULOMB EXCITATIONS

Illustration of a typical spectra of a nucleus

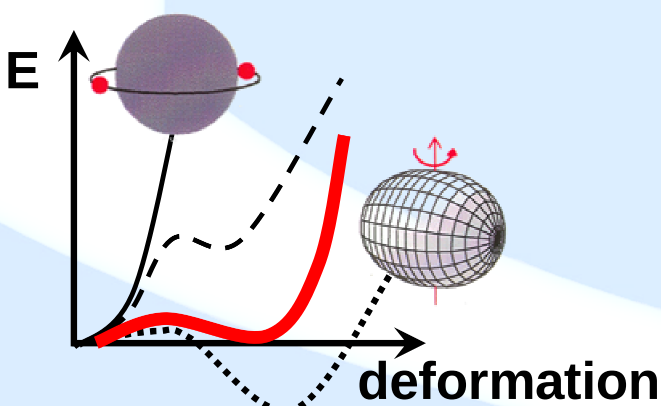




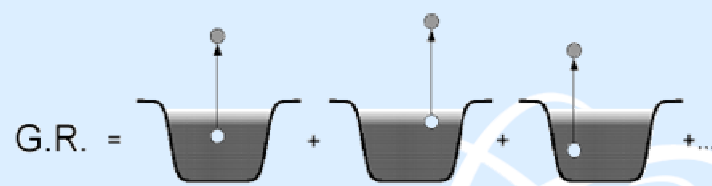
Low lying excitation (collective and non-collective)

Sensitive to “magicity”, shell evolution…

Give information on shape coexistence



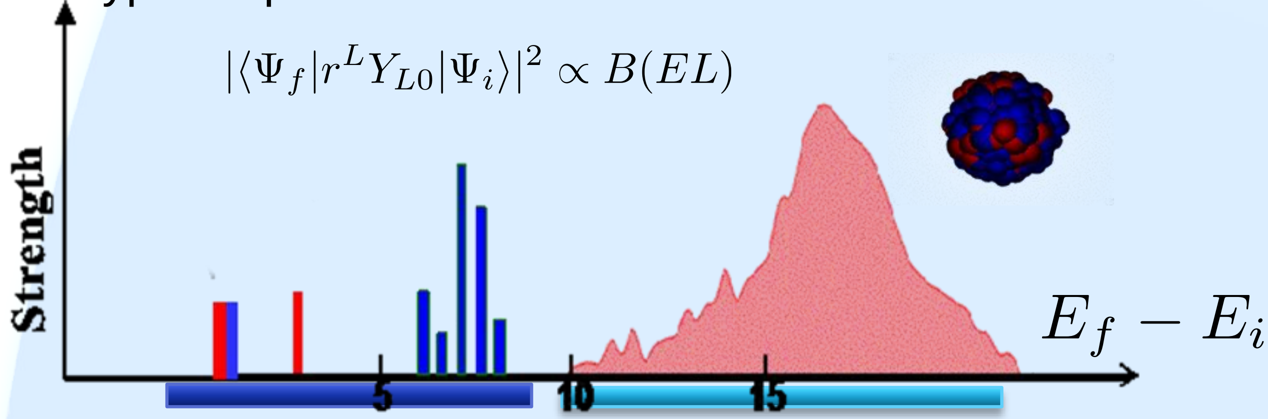
High lying excitation (highly collective)

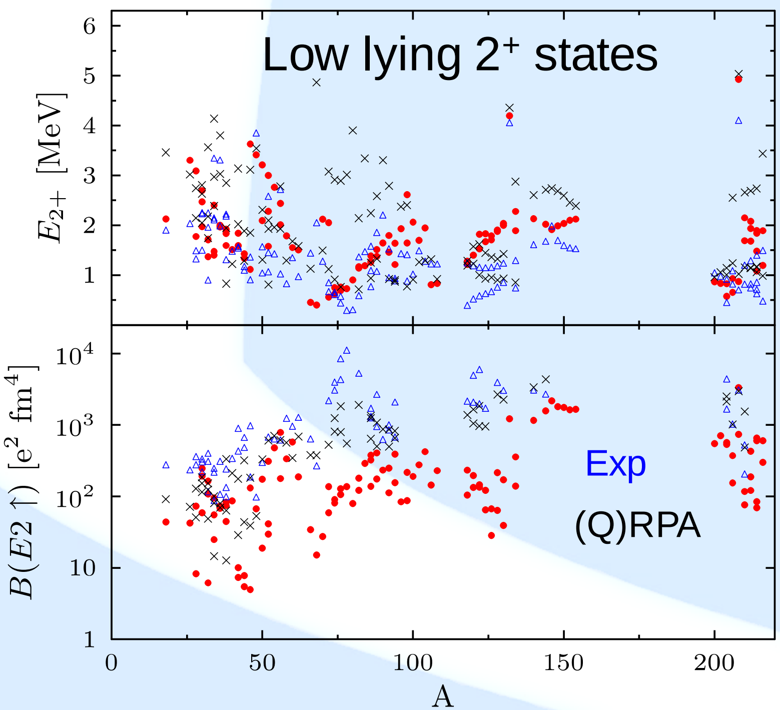


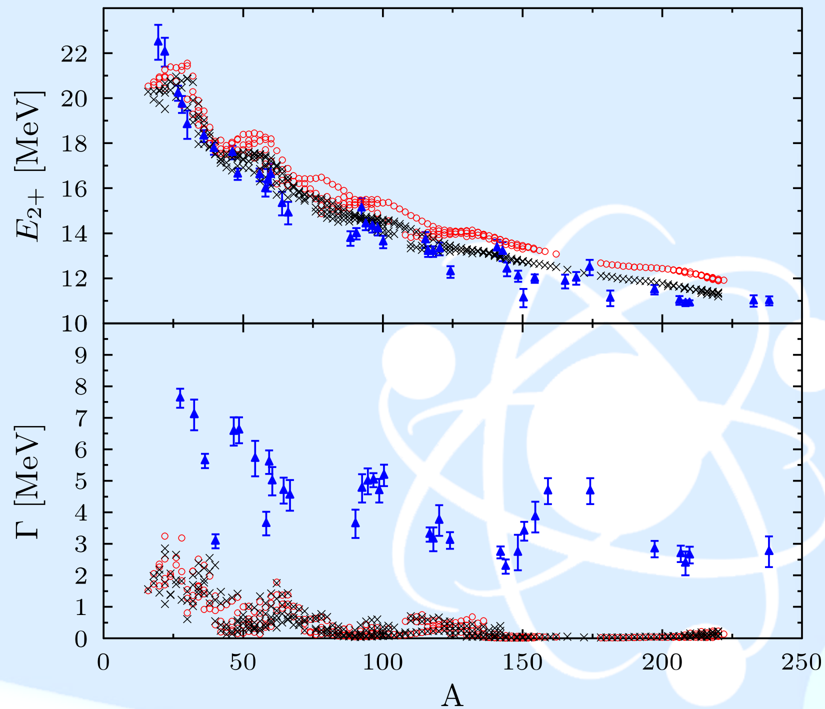
Sensitive to magicity, shell evolution, …

Sensitive to internal correlations

Illustration of a typical spectra of a nucleus







COULOMB EXCITATIONS: ILLUSTRATION OF CROSS-SECTION

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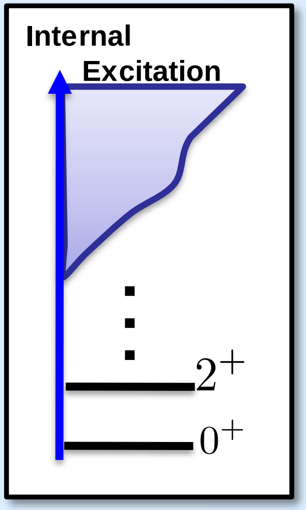
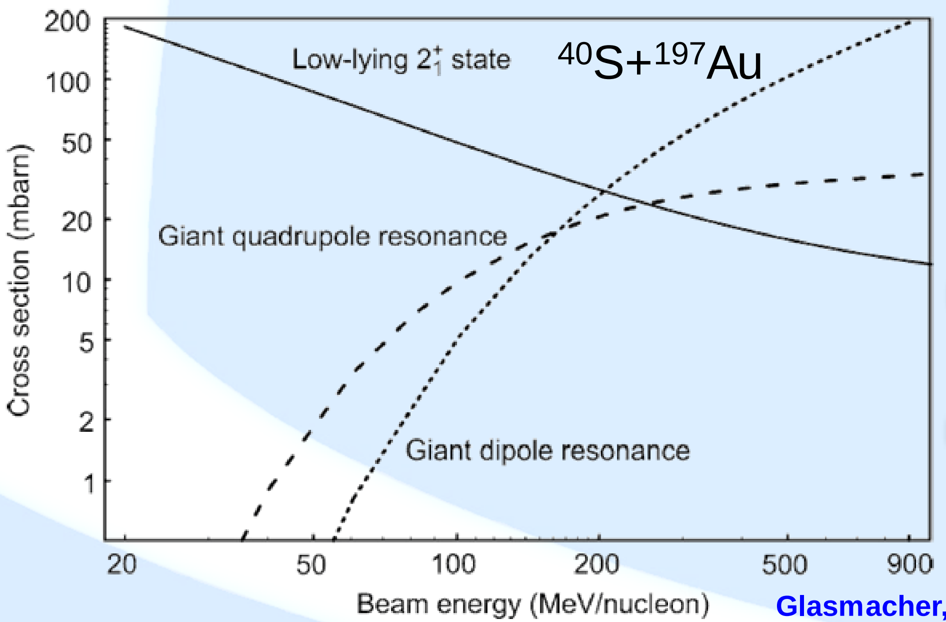
$$\rho(E) = \frac{dN}{dE}$$

$$\frac{d\sigma_{fi}(\mathrm{L})}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth}} \times \frac{B(\mathrm{EL}; i \rightarrow f)}{(2L+1)^3} \sum_M |I_{LM}(\omega_{fi})|^2$$



Total differential cross-section $\frac{d\sigma(L)}{d\Omega} = \sum_f \int \frac{d\sigma_{fi}(L)}{d\Omega} \rho_f(E) dE$

Total cross-section $\sigma(L, E) = \int d\Omega \frac{d\sigma(L)}{d\Omega}$



Glasmacher, Nucl. Phys. A 693, 90 (2001).

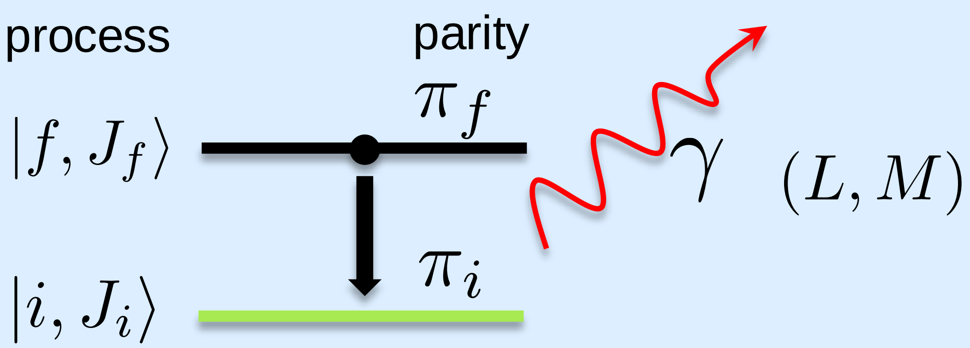
Note:
At high energy $\left(\frac{v}{c}\right) \nearrow$

➡ Relativistic treatment

➡ Wave-distortion is important (eikonal approximation)

EXPERIMENTAL OBSERVATION THROUGH GAMMA DECAY: SOME PRELIMINARY REMARKS

After the excitation process



Not all transitions can be seen (selection rules)

$$|J_f - J_i| \leq L \leq J_f + J_i$$

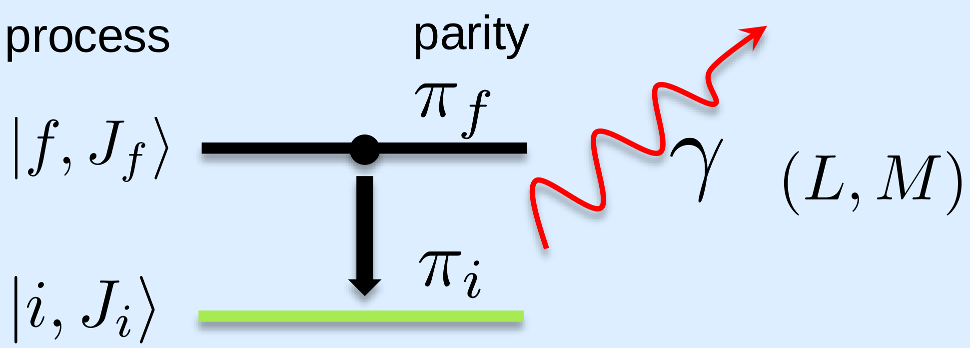
If $\pi_i = \pi_f$ Only possible for electric transition with $L = 2n$

Only possible for magnetic transition with $L = 2n + 1$

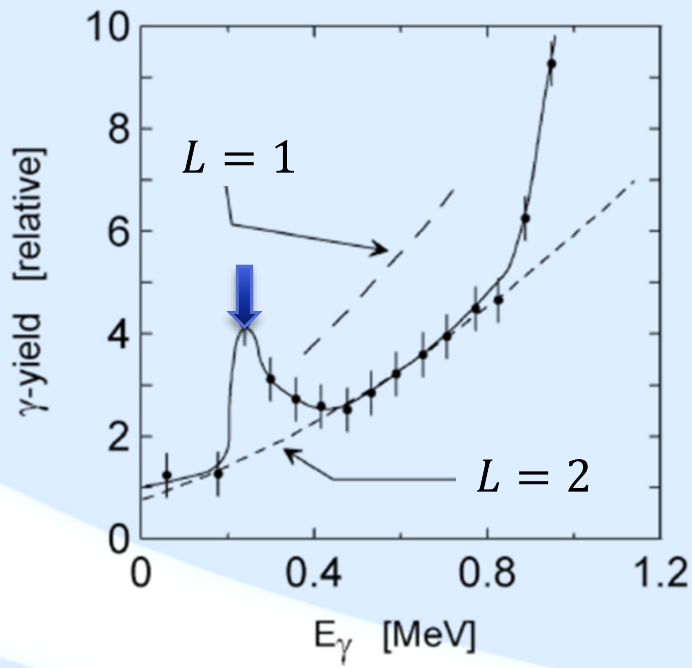
$\pi_i \neq \pi_f$ Just the opposite

EXPERIMENTAL OBSERVATION THROUGH GAMMA DECAY: SOME PRELIMINARY REMARKS

After the excitation process

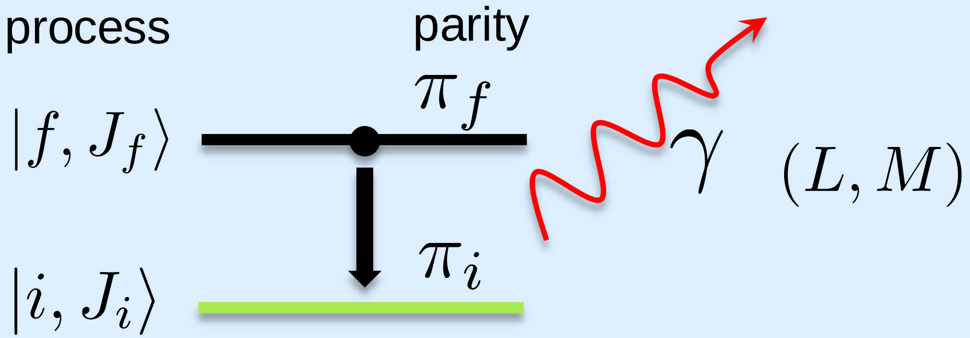


➡ Emitted γ have energetic and angular properties that depend on L



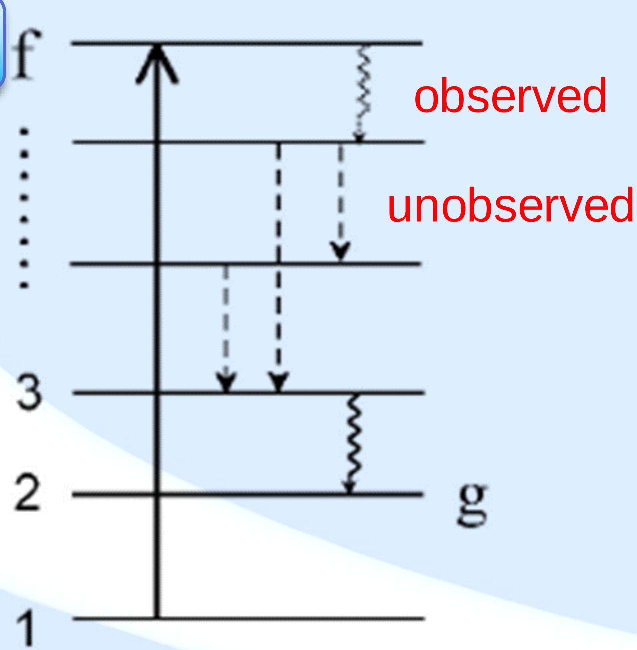
EXPERIMENTAL OBSERVATION THROUGH GAMMA DECAY: MORE REALISTIC CASE

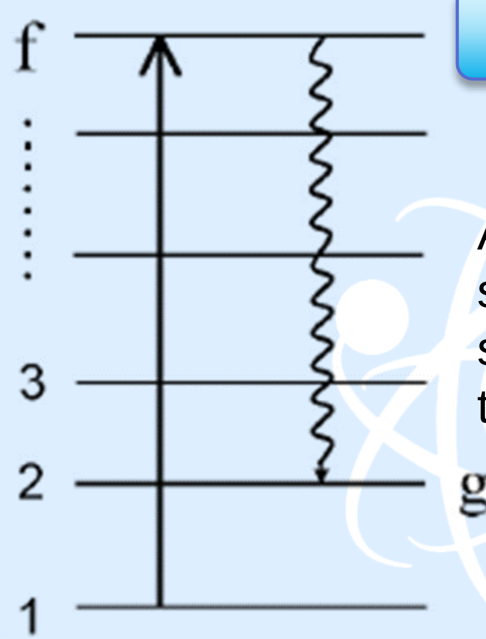
After the excitation process



Experimental spectra is a mixing of direct and multistep process (some of them unobserved because of selection rules)

multistep

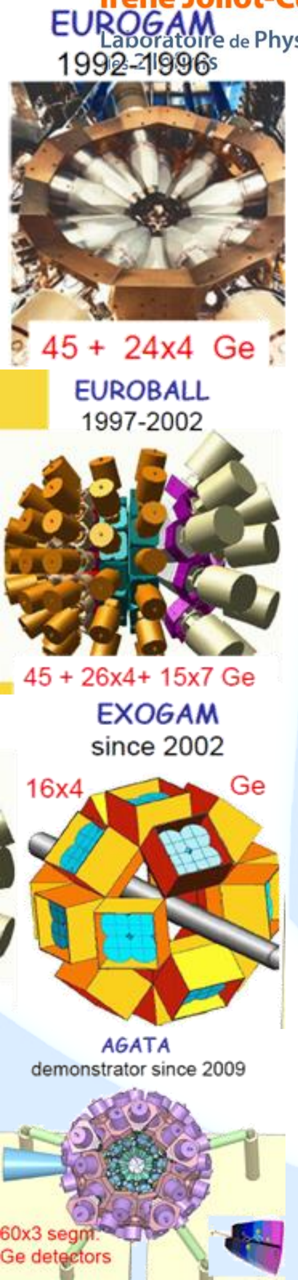




direct

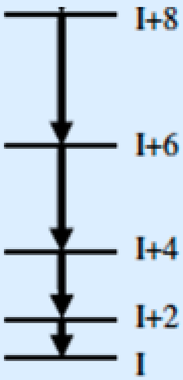
Assignment of spin parity is sometimes tricky

COULEX & SHAPE COEXISTENCE IN A NUTSHELL



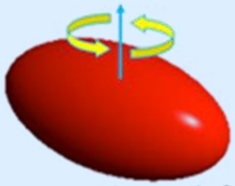
➡ Experiments requires high precision g detector

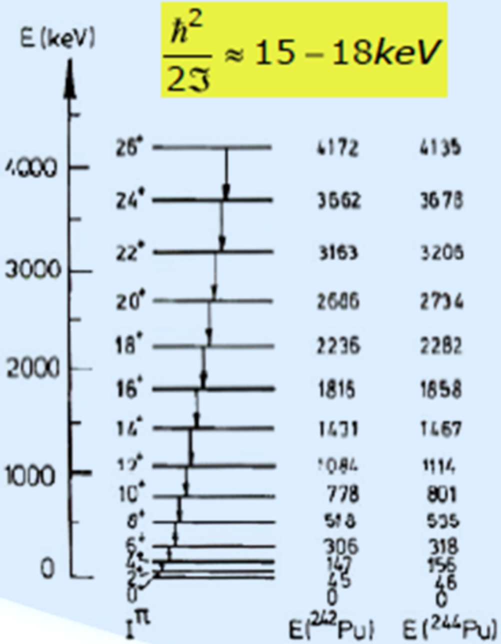
Physics case:
Probing the shape of nuclei

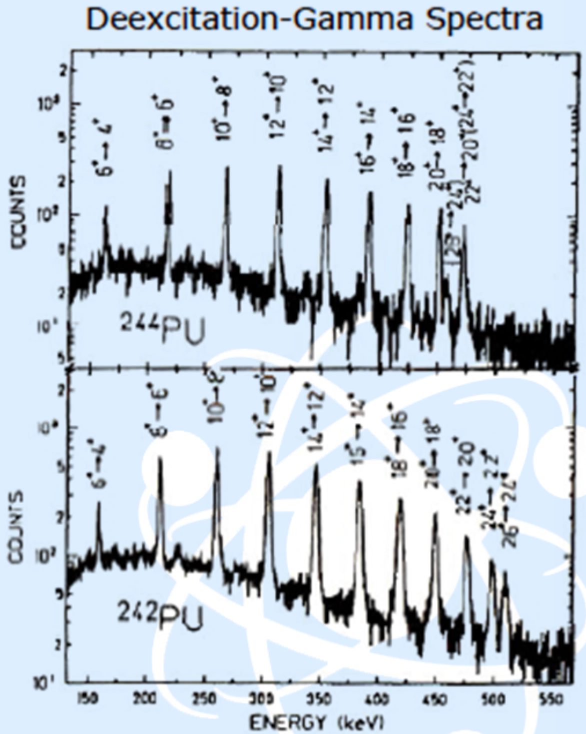


$$E_{\mathrm{rot}} = \frac{I(I+1)}{2J}\hbar^2$$

$$\mathcal{J} \rightarrow \beta_2$$

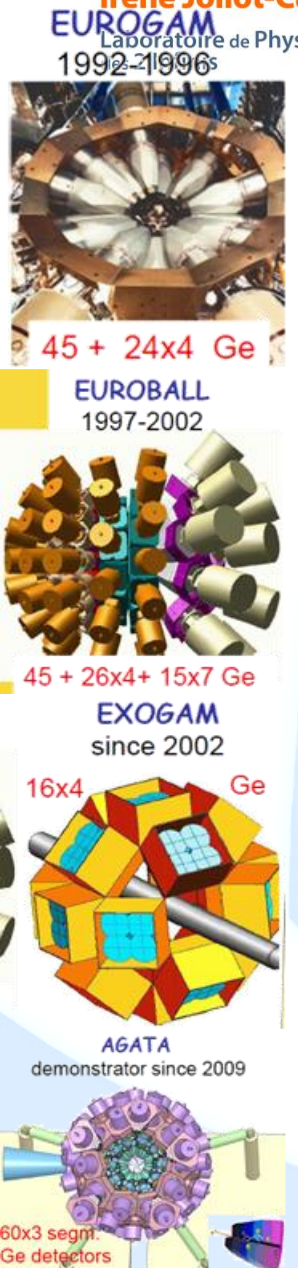




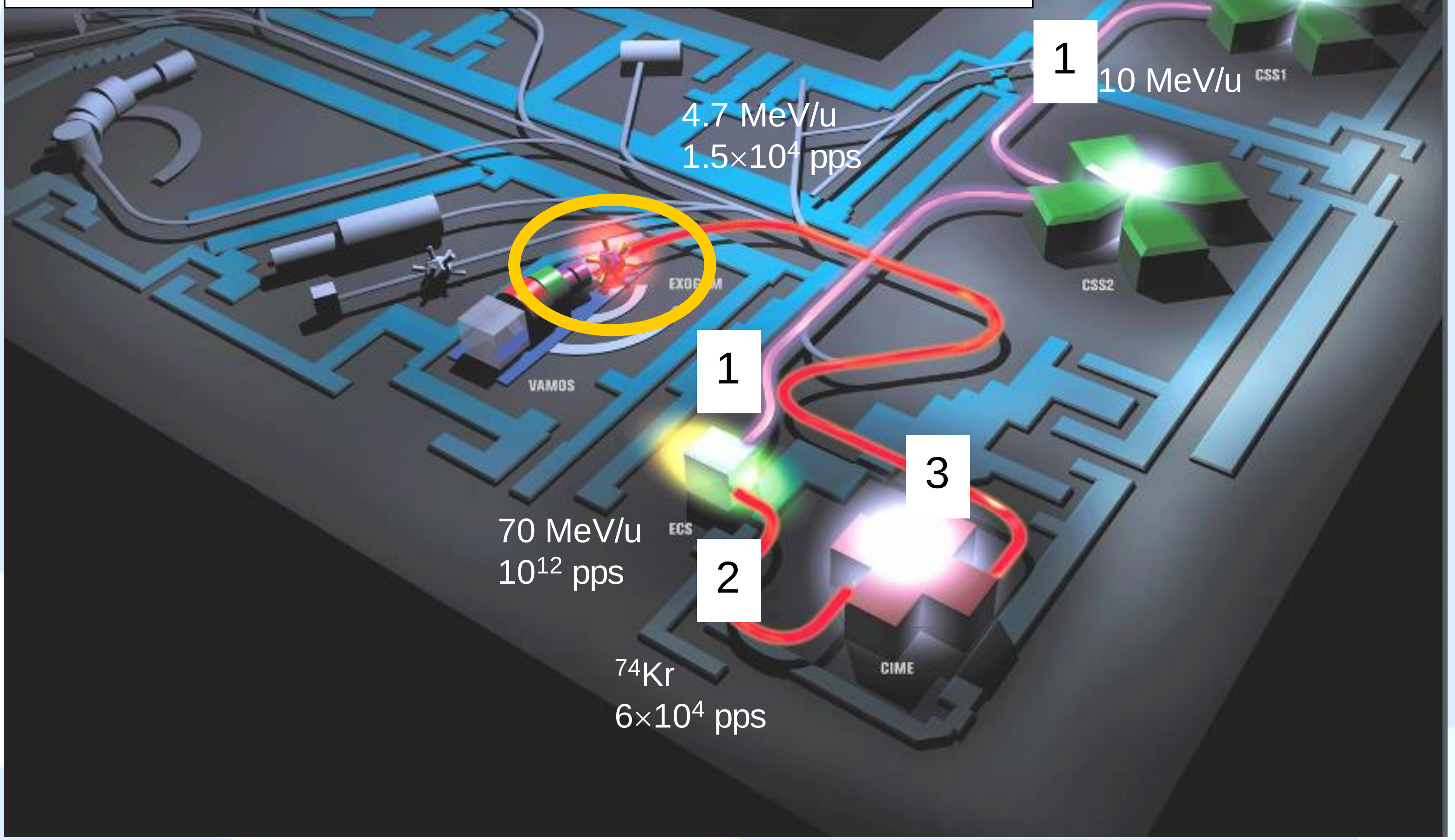


Wood et al.,Heyde

COULEX & SHAPE COEXISTENCE IN A NUTSHELL

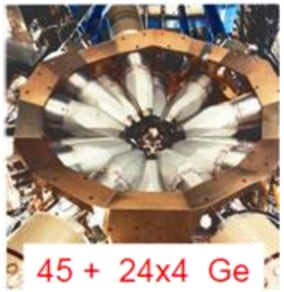


- The ^{74,76}Kr RIB are produced by fragmentation of a ^{78}Kr beam on a thick carbon target.
- Radioactive nuclei are extracted and ionized
- Post-accelaration of the RIB



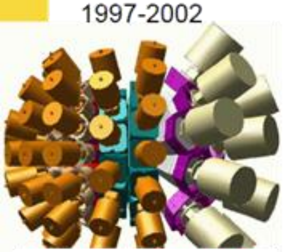
COULEX & SHAPE COEXISTENCE IN A NUTSHELL

EUROGAM
1992-1996



45 + 24x4 Ge

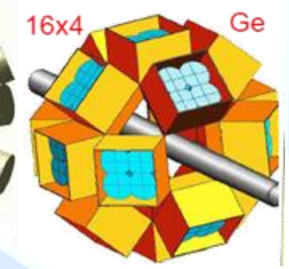
EUROBALL
1997-2002



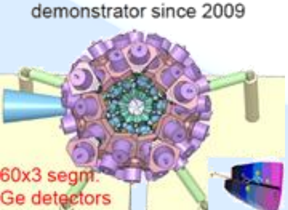
45 + 26x4+ 15x7 Ge

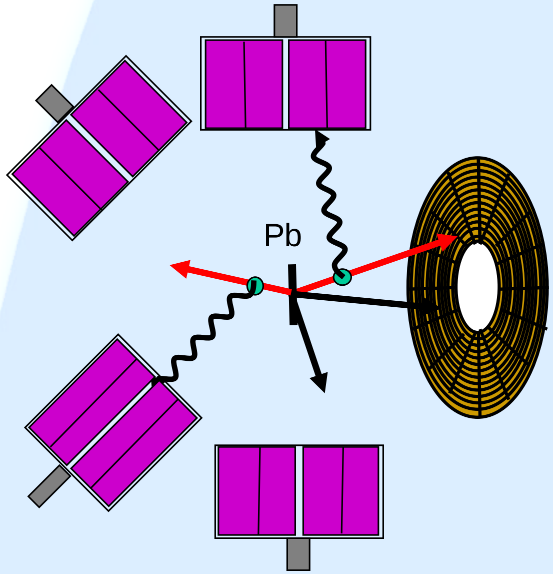
EXOGAM
since 2002

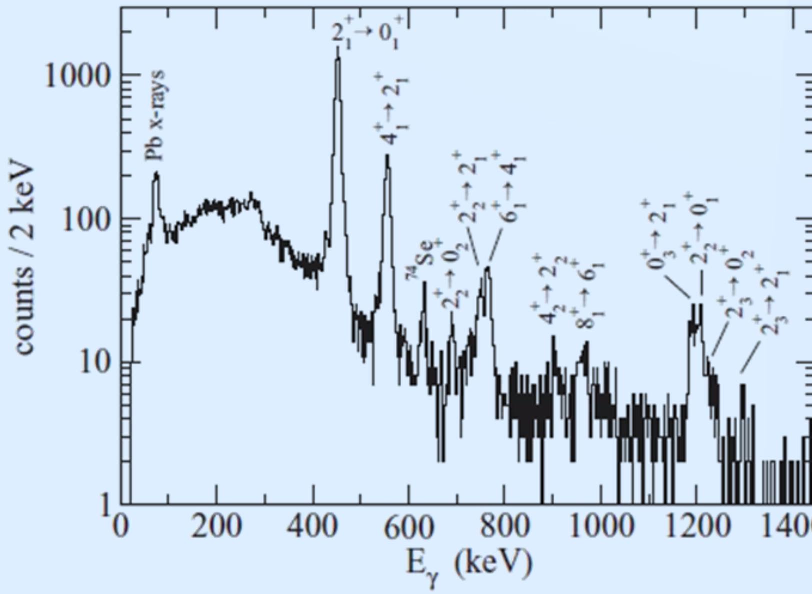
16x4 Ge



AGATA
demonstrator since 2009



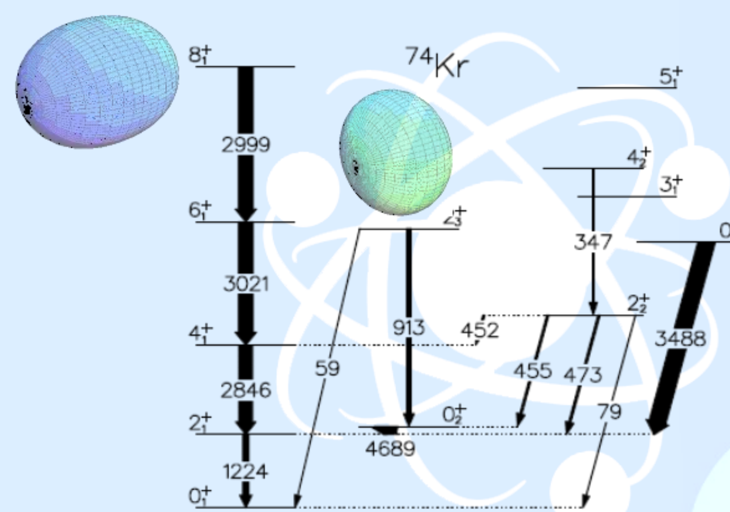




E. Clément et al. PRC 75, 054313 (2007)

γ detection

Particle detection



FEW WORDS ON EXPERIMENTS WITH COULEX

Physics case: Study of specific giant resonances (the Giant Dipole Resonance)

The GDR has high cross section at high beam energy and can suitably be studied with Coulomb excitation experiments

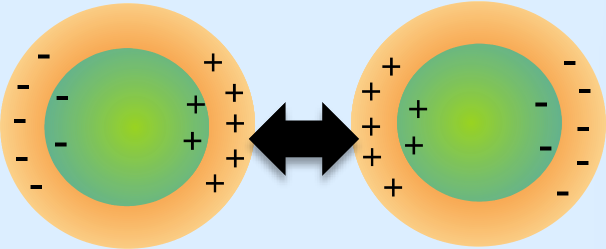
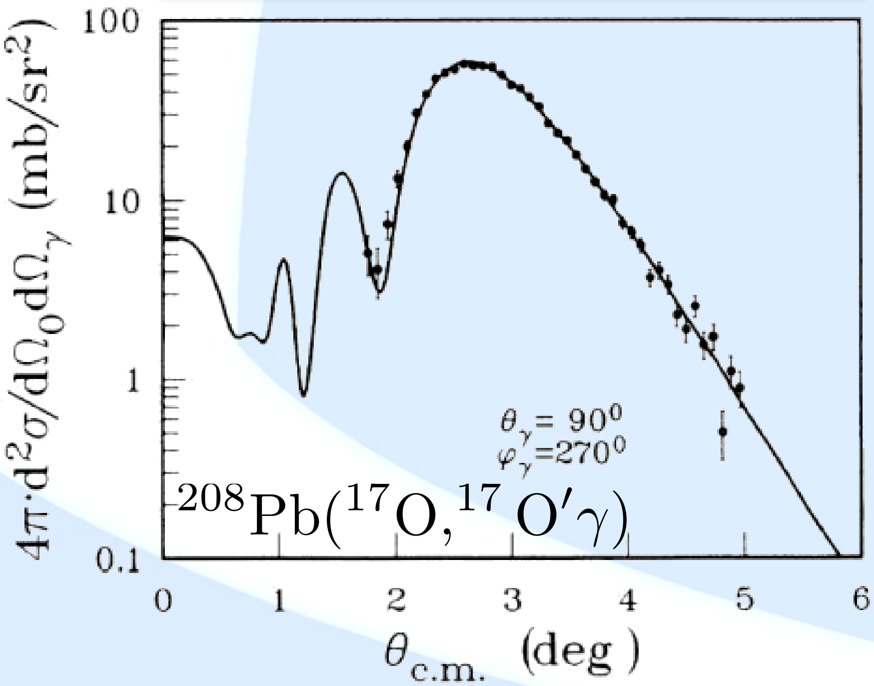


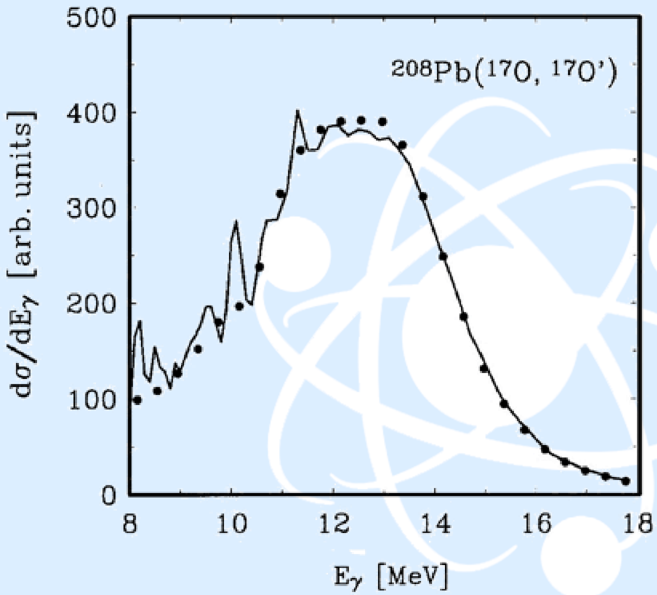
Illustration:

J. Beene et al., Phys. Rev. C 41, 920 (1990).

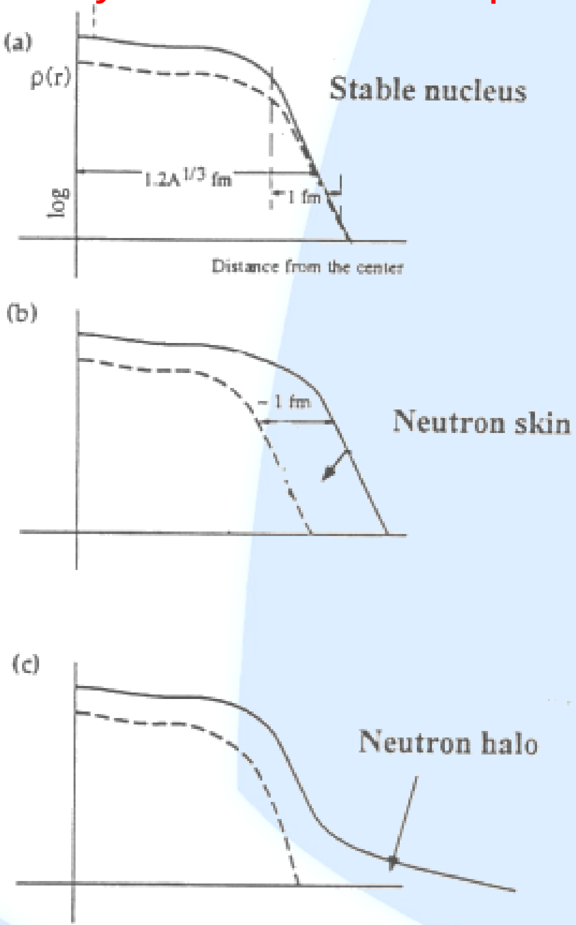
Differential cross-section

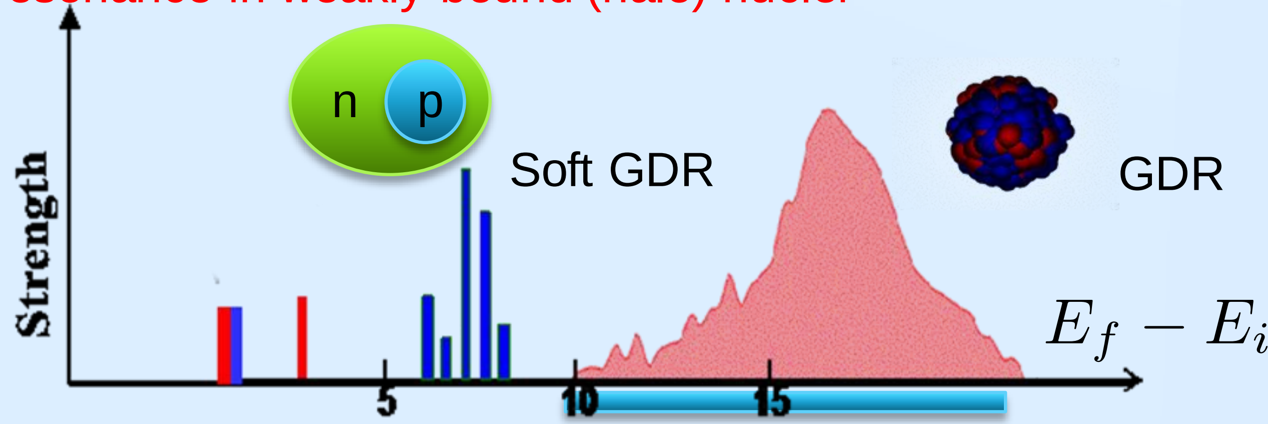


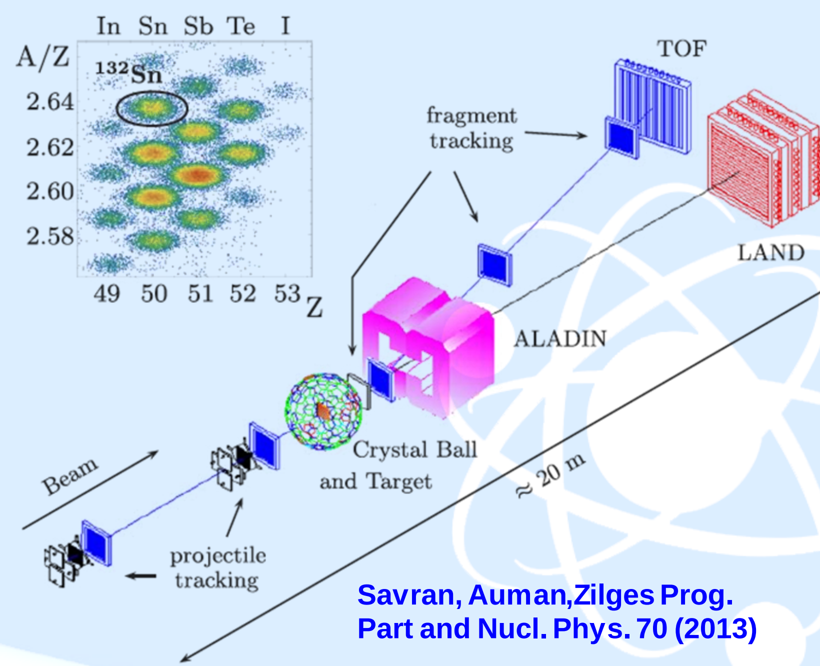
GDR (E1) response



Physics case: Soft Dipole resonance in weakly bound (halo) nuclei

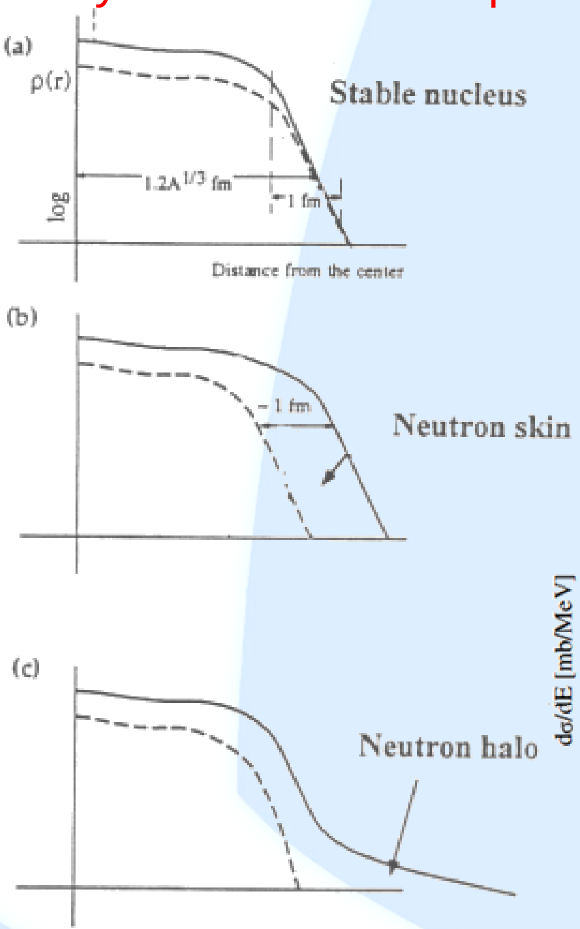


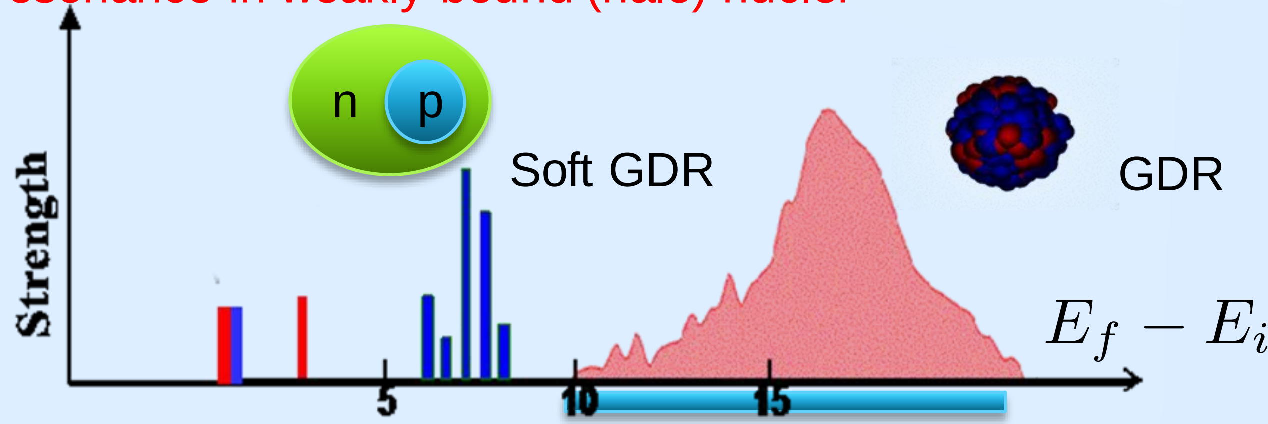


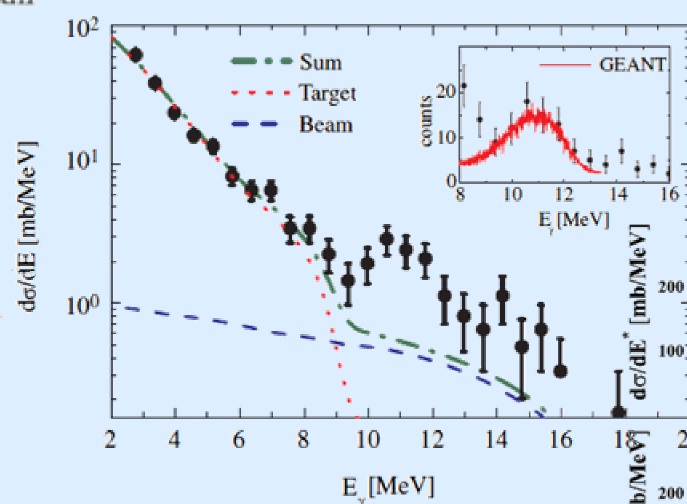


Savran, Auman,Zilges Prog. Part and Nucl. Phys. 70 (2013)

Physics case: Soft Dipole resonance in weakly bound (halo) nuclei

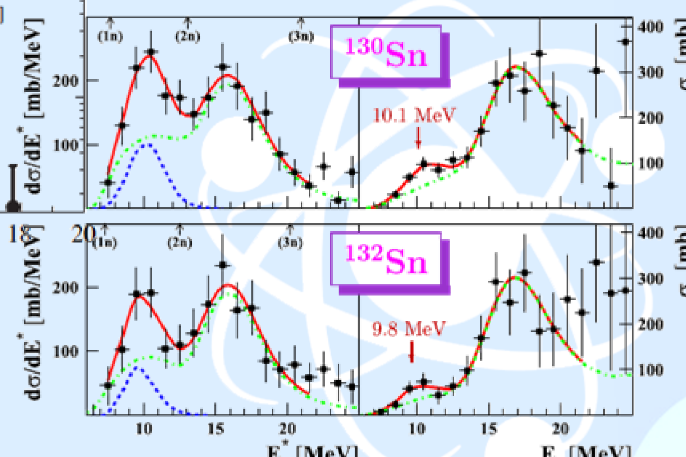






$^{68}\mathrm{Ni}[600\mathrm{MeV/A}] + ^{197}\mathrm{Au}$

$^{13x}\mathrm{Sn}[500\mathrm{MeV/A}] + ^{208}\mathrm{Pb}$



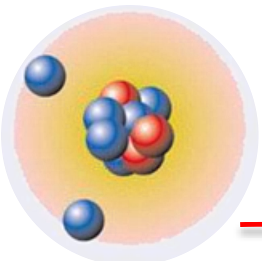
Savran, Auman,Zilges Prog. Part and Nucl. Phys. 70 (2013)

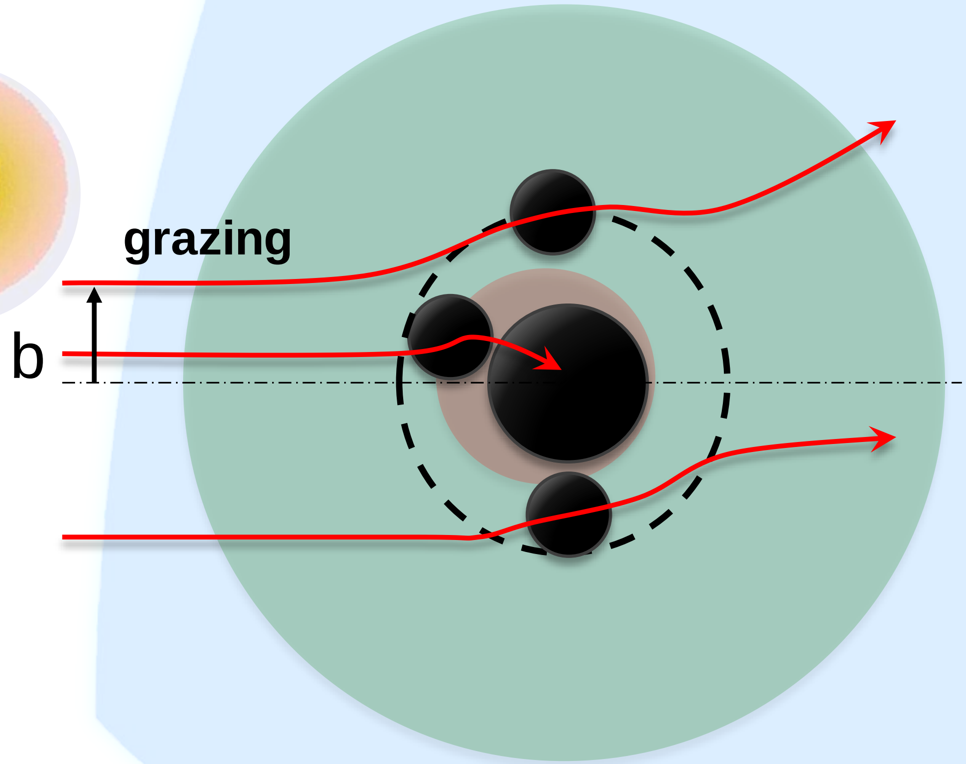
FEW WORDS ON EXPERIMENTS WITH COULEX

SOME REMARKS

Nuclear interaction range

Coulomb interaction range

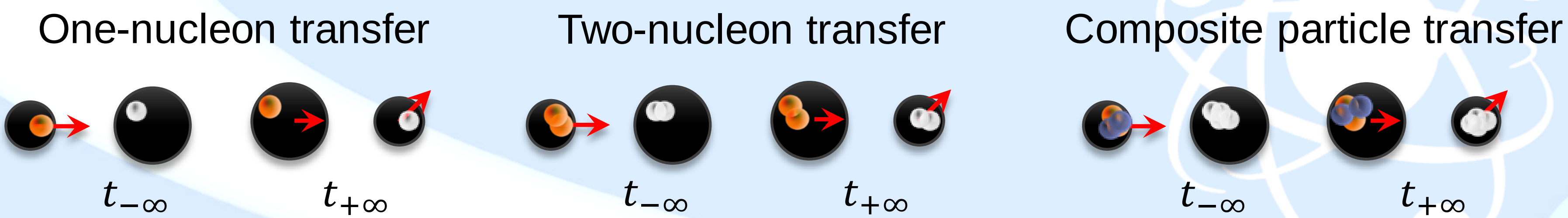




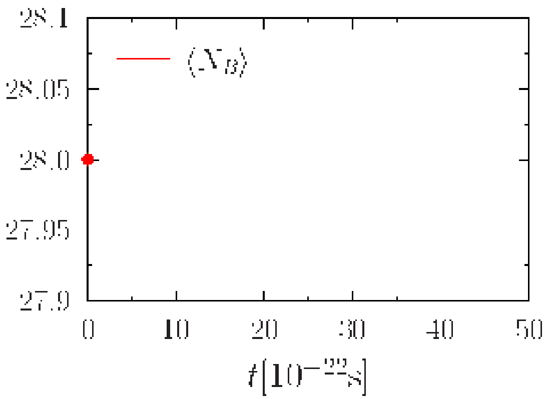
- In many cases, Coulomb and nuclear effects cannot be separated leading to interference
- This is even more true for very extended systems
- Most direct reactions are due to nuclear effects (transfer, knock-out, deep inelastic)
- Weakly bound systems are also easier to break (continuum effect)

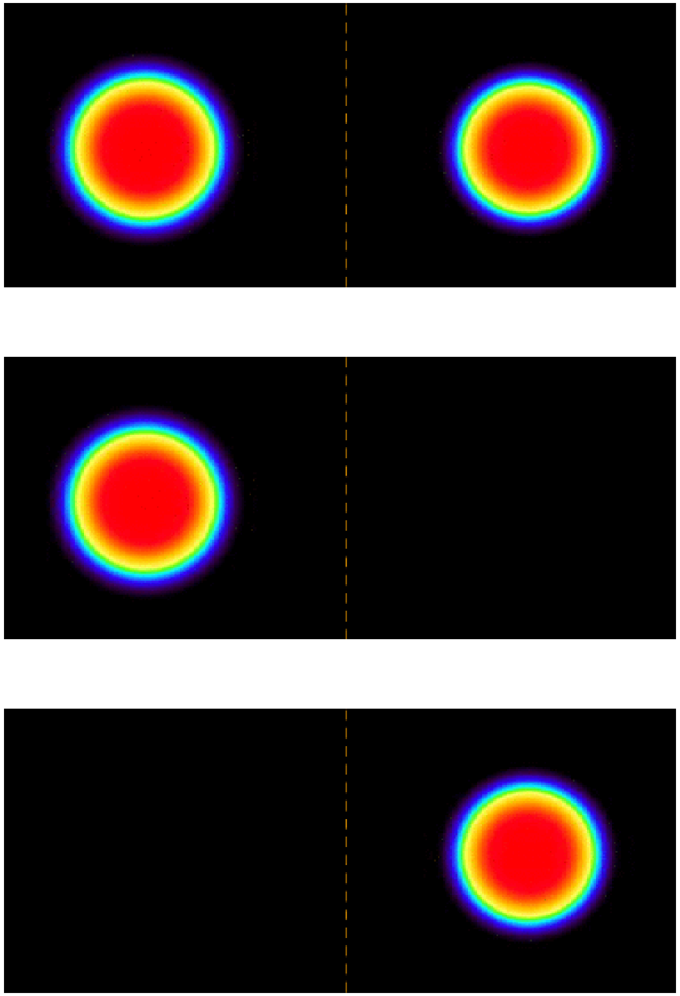
Transfer reactions

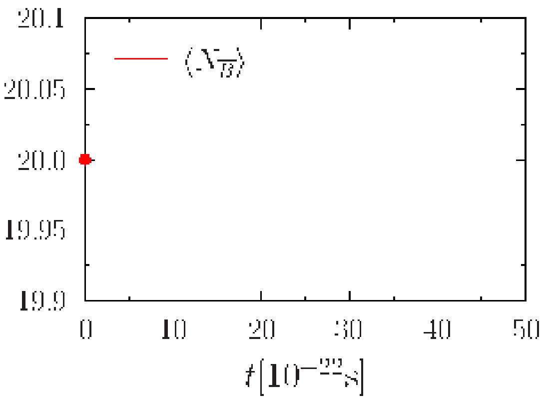
The many-facets of nuclear reactions



PHYSICAL PICTURE

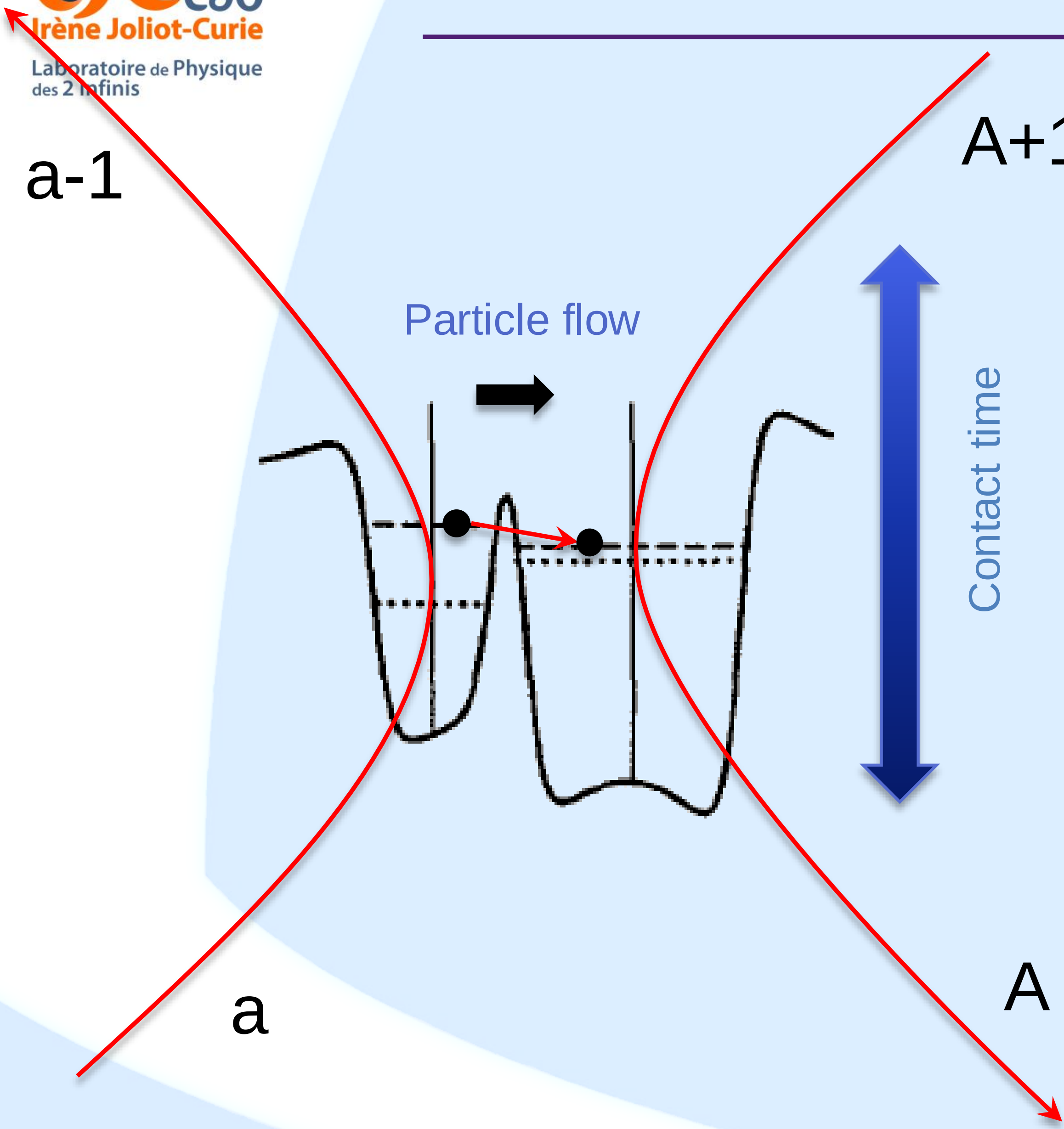






Central collisions

(Courtesy G. Scamps)



- The transfer will depend on the contact time

The contact should be enough to let the particle go from one nucleus to another (low energy process)

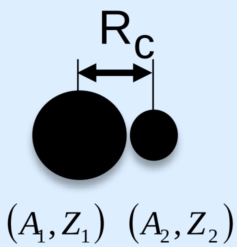
- The transfer depends on the initial and final single-particle states

(sensitivity to the internal structure of nuclei)

NUCLEON TRANSFER
PHENOMENOLOGICAL ASPECTS

The driving force to transfer

Suppose infinite contact time
(Di-nuclear picture)



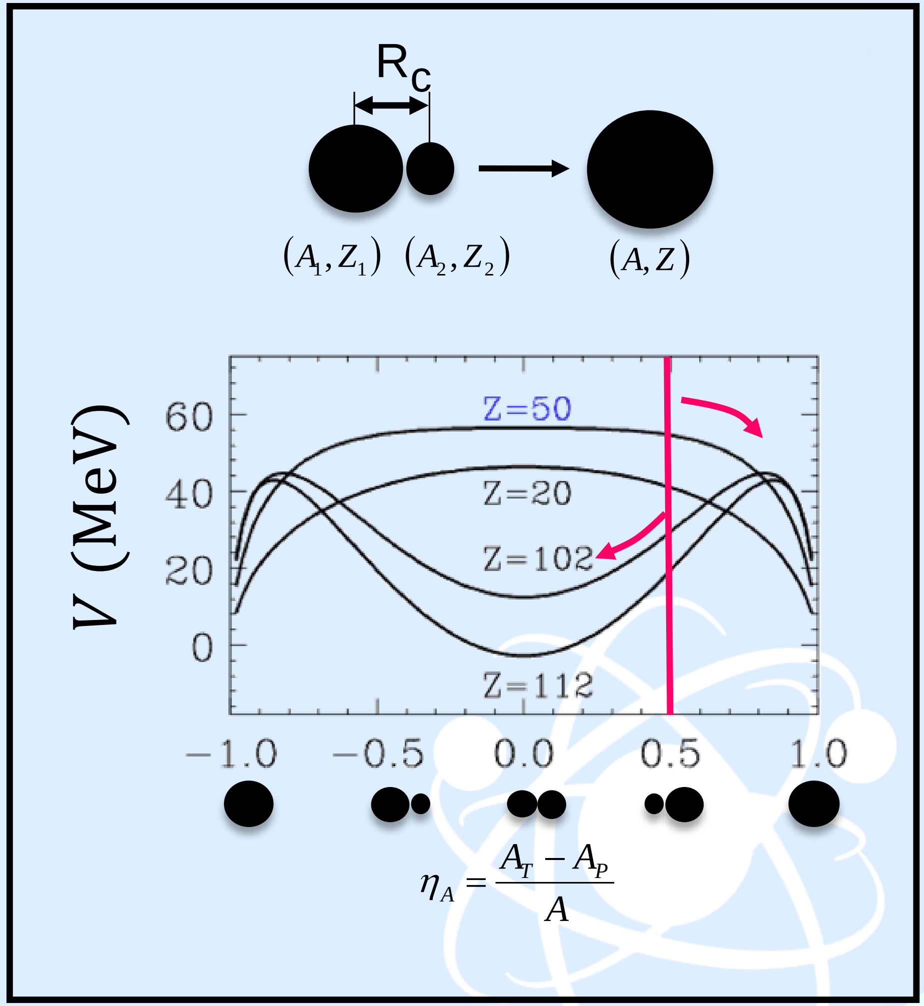
$$V(r) = \langle H \rangle_r - \langle H_1 \rangle_\infty - \langle H_2 \rangle_\infty$$

$$\Downarrow$$

$$V(A_T, Z_T, A_P, Z_P) = M_{LD}(A_T, Z_T) + M_{LD}(A_P, Z_P) - e^2 \frac{Z_T Z_P}{R_C} - M_{LD}(A, Z)$$

$$\Downarrow$$

$$\frac{dA_T}{dt} = F_{P \to T}$$



➡ Q-value dependence

The driving force to transfer

Suppose infinite contact time
(Di-nuclear picture)

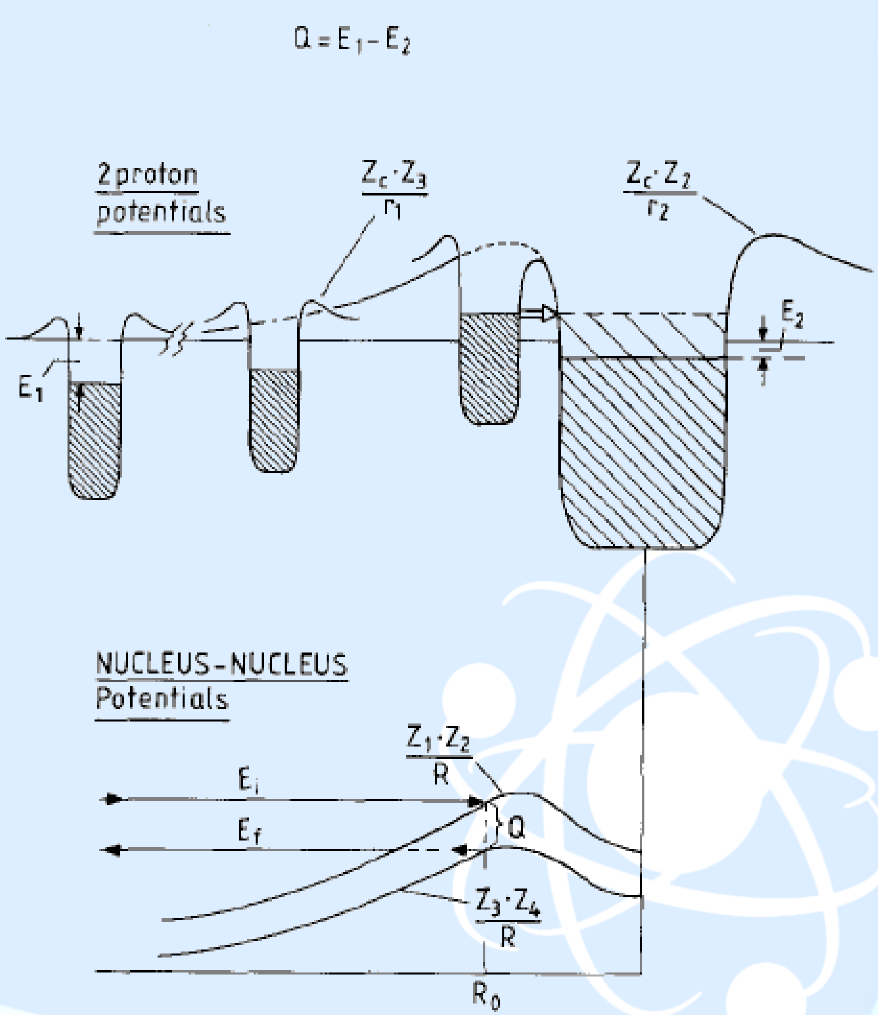
$$V(r)=\langle H\rangle_r-\langle H_1\rangle_\infty-\langle H_2\rangle_\infty$$

$$V(A_T,Z_T,A_P,Z_P)=M_{LD}(A_T,Z_T)+M_{LD}(A_P,Z_P)-e^2\frac{Z_TZ_P}{R_C}-M_{LD}(A,Z)$$

$$\frac{dA_T}{dt}=F_{P\to T}$$

➡ Q-value dependence

Schematic view of the process



NUCLEON TRANSFER
PHENOMENOLOGICAL ASPECTS

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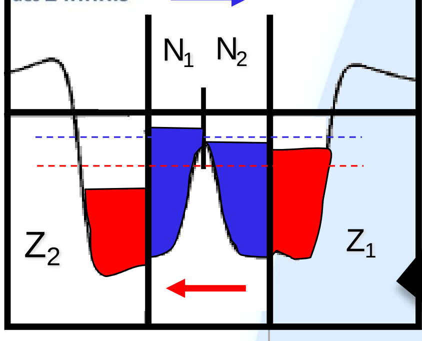
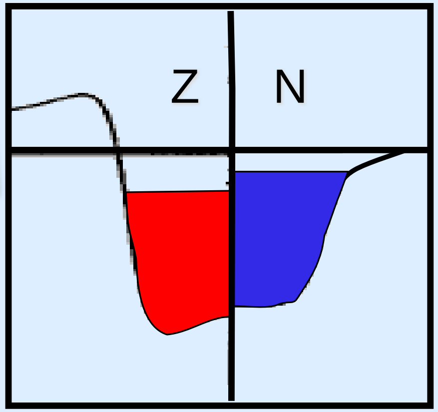
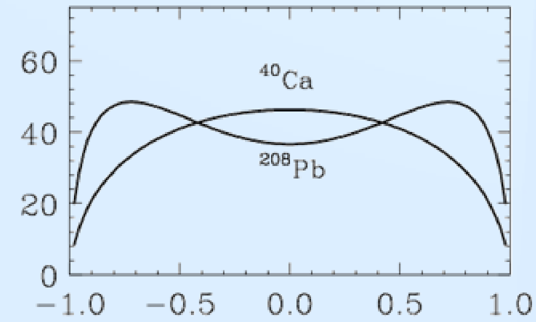


Two-fluid dynamics picture





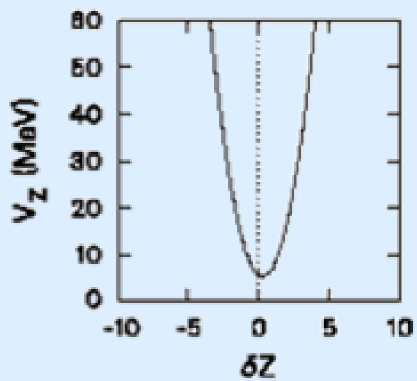
$$\eta_A = \frac{A_1 - A_2}{A_{tot}}$$

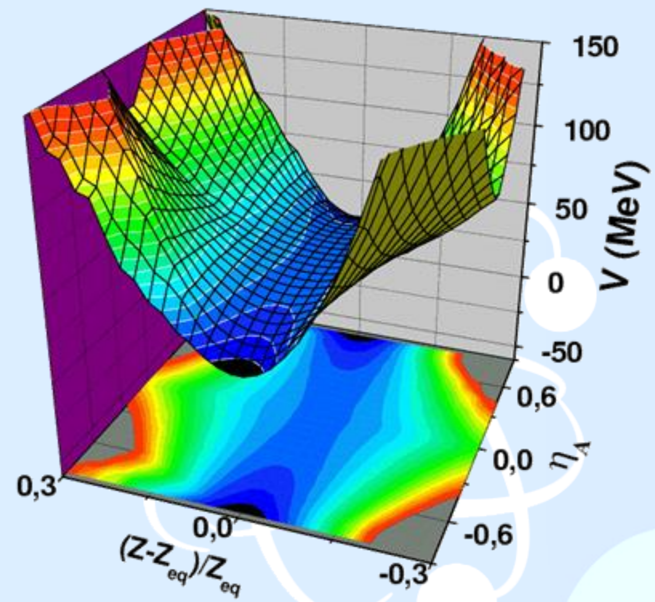
N/Z equilibration is much faster

Linked to the equilibration of chemical potential

Ultimately linked to the symmetry energy of the masse formula

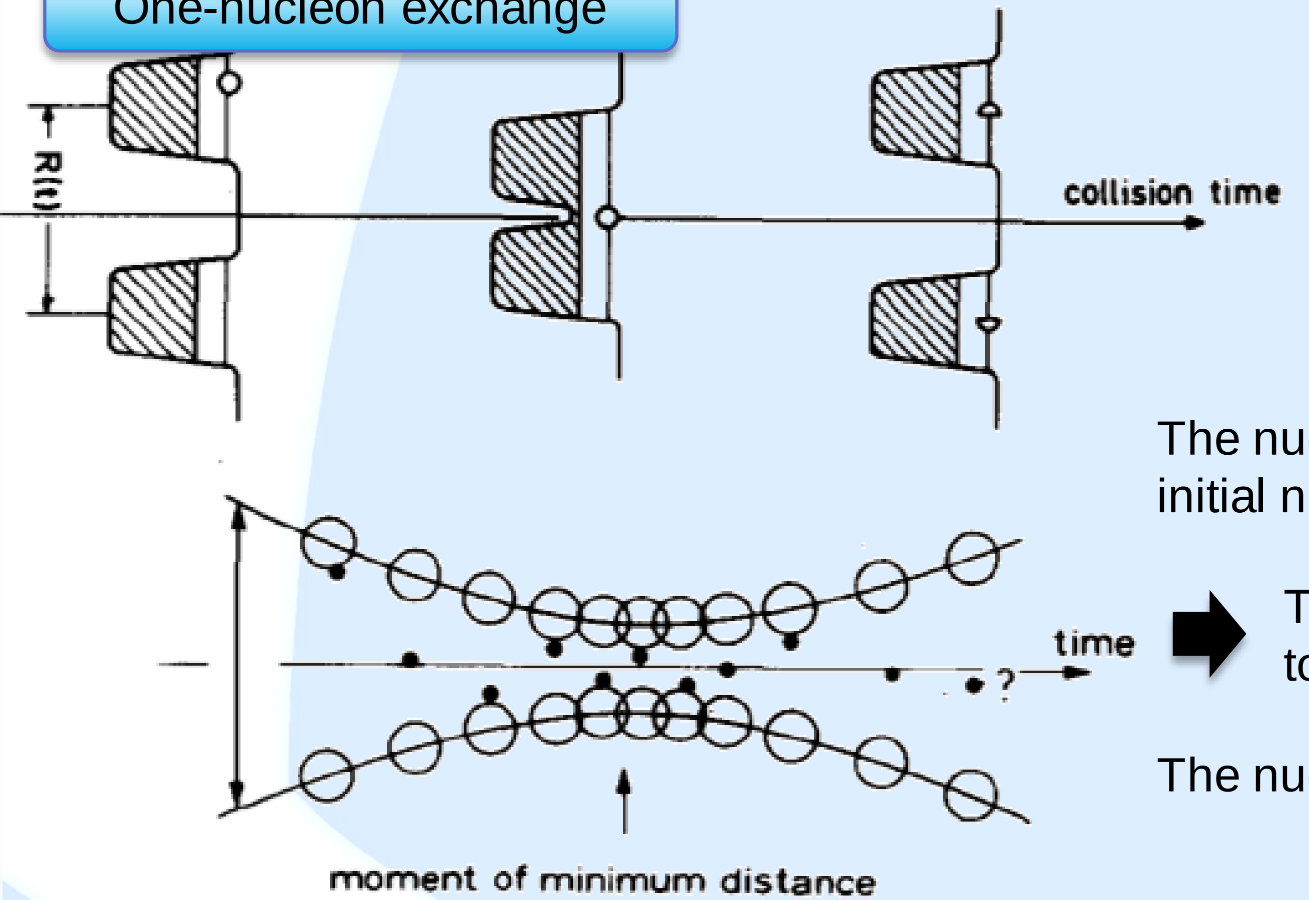
$$\tau_{N/Z} << \tau_{\left(\frac{A_1 - A_2}{A}\right)}$$





Lacroix, Ecole Joliot-Curie 2002.

One-nucleon exchange



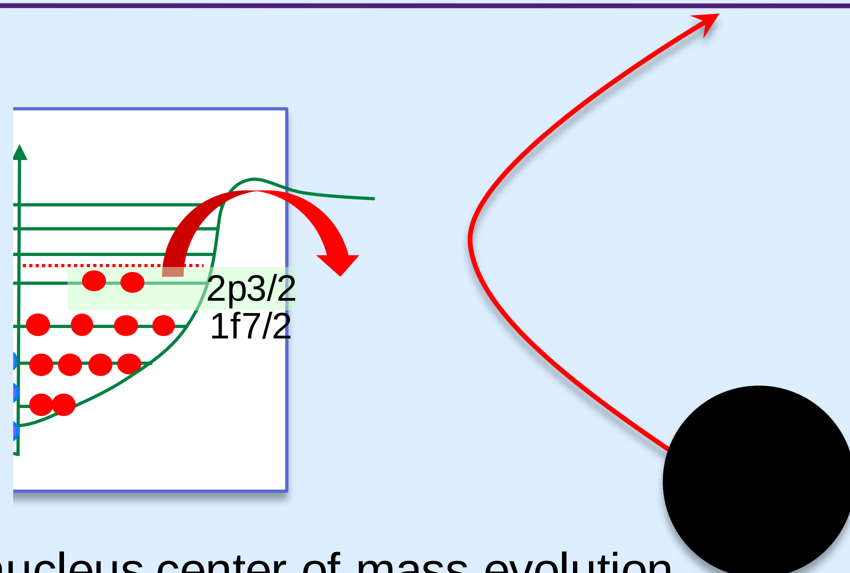
The nucleon can stay in the initial nucleus

➡ The nucleon can participate to other open channels

The nucleon can be transmitted

von Oerzen, Bohlen, Phys. Rep. 19 (1975)

Single-particle scenario



Hypothesis:

- Treat classically the nucleus-nucleus center of mass evolution
- Consider independently the transfer from each initial state
- Consider that the particle transfer does not affect the nuclei potentials

$$i\hbar\partial_t|\Phi_\alpha(t)\rangle = \left\{ \frac{\mathbf{p}^2}{2m} + V_P(\vec{\mathbf{r}},t) + V_T(\vec{\mathbf{r}},t) \right\} |\Phi_\alpha(t)\rangle$$

Wood-Saxon potentials

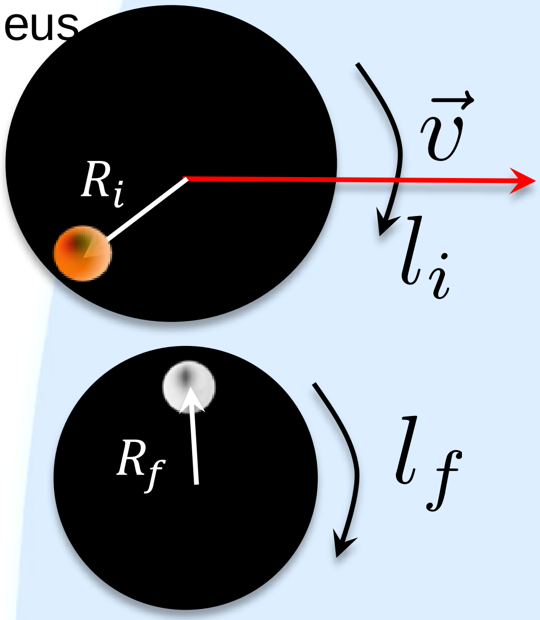
$$V_{P/T}(\vec{r},t) = \frac{V_0}{1+\exp\{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_{T/P}(t)|/a\}}$$

➡ Equivalent to solve a three-dimensional wave equation

Brink, Phys. Lett B 40 (1972)

Semi-classical selection rules

Initial nucleus



Conservation of parallel momentum before and after the transfer:

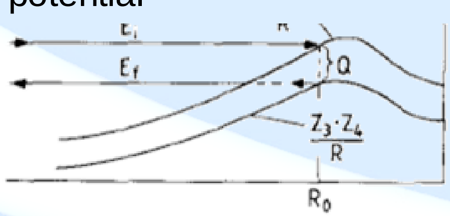
$$mv - \frac{\hbar l_i}{R_i} = \frac{\hbar l_f}{R_f}$$

(angular momentum matching condition)

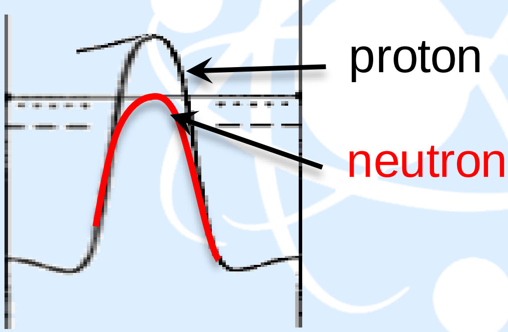
Other conditions might be derived from the conservation of perpendicular angular momentum

Proton-neutron difference in the exchange

In the nucleus-nucleus potential



In single-particle potential



SOME PHENOMENOLOGICAL ASPECTS: INFLUENCE OF CORRELATION, SPECTROSCOPIC FACTORS

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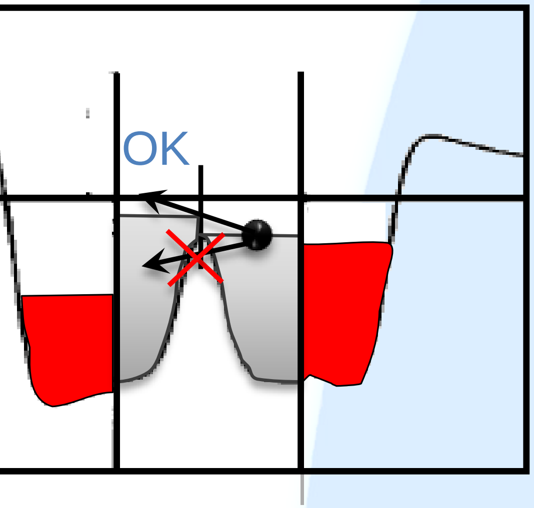
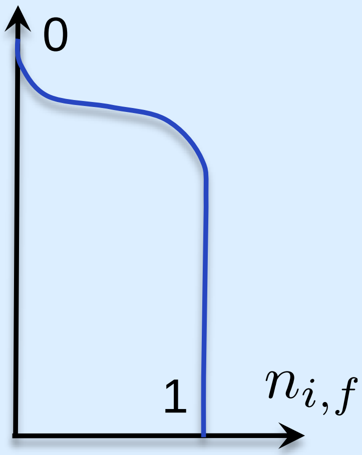
Nucleons are fermions (subject to Pauli exclusion principle)





→ Nucleon cannot be transferred to a level that is already occupied

$$P_{\rm tr} \simeq P_{\rm tr}^{\rm calc} n_i (1 - n_f)$$

n_i : probability to be initially occupied

$(1 - n_f)$: probability to be initially empty

For uncorrelated systems: $n_{i,f} = 0, 1$

With correlations: $n_{i,f} \neq 0, 1$

→ Transfer (as other spectroscopic tool) is sensitive to both single-particle structure and correlations

Spectroscopic factors

In practice people use a more general quantity called "Spectroscopic factor"

For a given state

$$U_i = \langle A+1 | a_i^\dagger | A \rangle$$

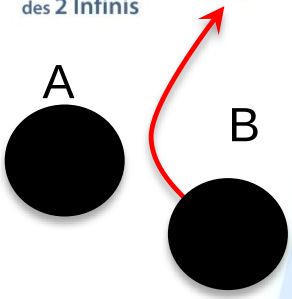
$$V_i = \langle A-1 | a_i | A \rangle$$

→

$$S_i^+ = |U_i|^2 \sim (1 - n_i)$$

$$S_i^- = |V_i|^2 \sim n_i$$

NUCLEON TRANSFER
LINK BETWEEN TDSE AND THE COUPLED CHANNEL APPROACH



In the frame of the emitter

$$i\hbar\frac{d}{dt}\Psi(\mathbf{r},t) \simeq \left\{\frac{\mathbf{p}^2}{2m} + V_A(\mathbf{r}) + \underline{V_B(\mathbf{r}-\mathbf{R}(t))}\right\}\Psi(\mathbf{r},t)$$

Time dependent perturbation

Introduce a complete set of states: $H_A|\Psi_a\rangle = E_a|\Phi_a\rangle$

$$\Psi(t) = \sum_a c_a(t)|\Psi_a\rangle$$

with $\Psi(-\infty) = \Psi_0$

$$c_a(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} \langle\Psi_a|V_B(\mathbf{r}-\mathbf{R}(s))|\Psi_0\rangle e^{i(E_a-E_0)dst/\hbar}\, ds$$

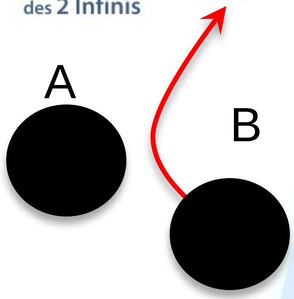
Problem: the Ψ_a basis is not so convenient for the asymptotic transfer

It would be better to develop on the moving single-particle Ψ_b basis $H_B|\Psi_b\rangle = E_b|\Phi_b\rangle$

This would be equivalent to the DWBA approach (for weak coupling)

$$c_{0\to b}(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} \langle\Psi_b(\mathbf{r}-\mathbf{R}(s))|V_B(\mathbf{r}-\mathbf{R}(s))|\Psi_0\rangle e^{i(E_b-E_0)dst/\hbar}\, ds$$

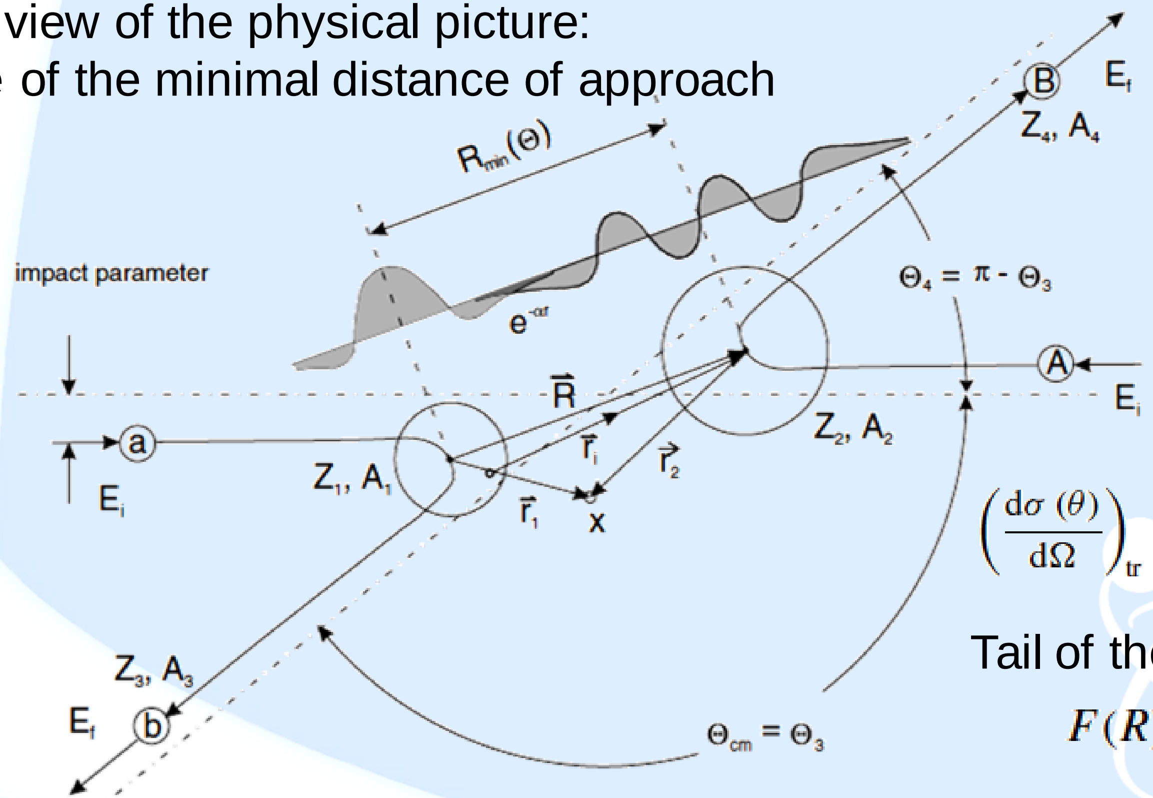
NUCLEON TRANSFER
SEMI-CLASSICAL APPROXIMATION



$$c_{0\to b}(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} \langle \Psi_b(\mathbf{r}-\mathbf{R}(s))|V_B(\mathbf{r}-\mathbf{R}(s))|\Psi_0\rangle e^{i(E_b-E_0)dst/\hbar}\, ds$$

➡ In most semi-classical approximation used today, the overlap is usually calculated using approximate expressions based on intuitive arguments.

Schematic view of the physical picture:
Importance of the minimal distance of approach



$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{tr} \approx |F(R_{min}(\theta))|^2 \left(\frac{d\sigma(\theta)}{d\Omega}\right)_{sc}$$

Tail of the transferred w.f.

$$F(R) = N e^{-\alpha R}/R$$

$\alpha = \sqrt{2\mu E_B/\hbar^2}$ $\quad E_B$ Binding energy

From: von Oertzen, Vitturi, Rep. Prog. Phys. 64 (2001)

NUCLEON TRANSFER

SEMI-CLASSICAL METHODS

A recent example: Corradi et al, Phys. Rev. C84 (2011)

- The transfer probability should account for the impact parameter and beam energy
- The transfer probability accounts for the initial and final quantum number plus the conservation of total angular momentum

$$(n_i, l_i, m_i) \rightarrow (n_f, l_f, m_f)$$

Example of more complete formula:

$$P_\beta(\ell) = P_{(a_1,a_1')}(\ell) = \sum_{m_1',m_1} |c_\beta(\ell)|^2$$

$$= U^2(a_1, I_A)V^2(a_1', I_a) \sum_{\lambda\mu} \frac{2j_1'+1}{2\lambda+1} \times \left|I_{\lambda\mu}^{(a_1,a_1')}(\ell)\right|^2$$

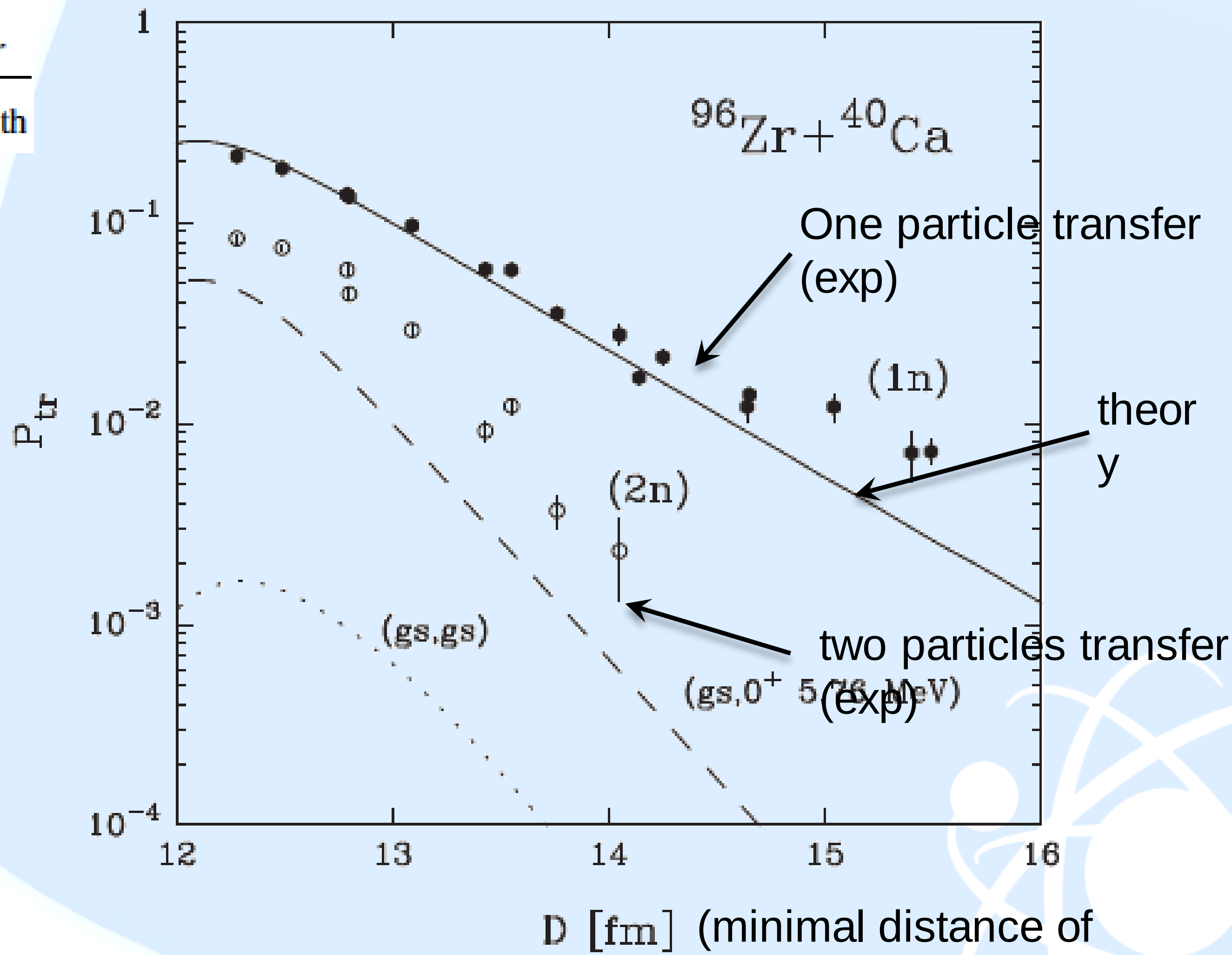
Angular momentum phase-space

Spectroscopic amplitude $a_1' \rightarrow a_1$

Contains all dynamical energetic and wave-function dependence

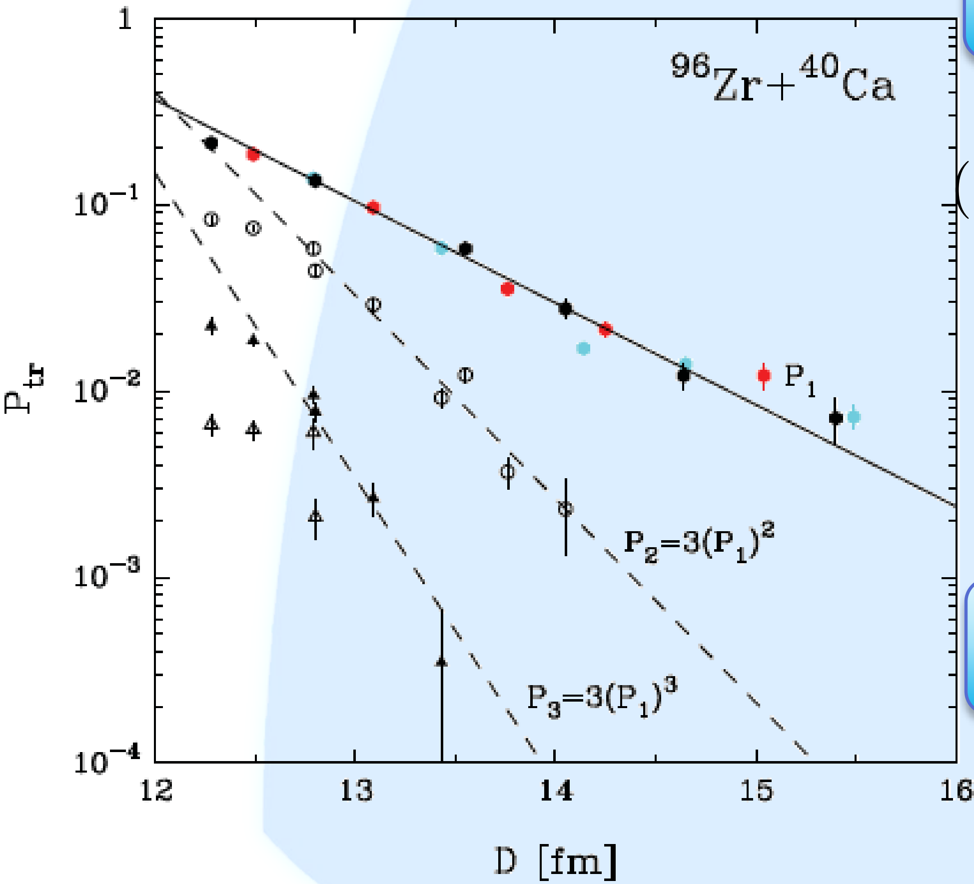
$$I_{\lambda\mu}^{(a_1,a_1')}(\ell) = \sqrt{\frac{2\pi a_{tr}}{\ddot{r}_o \hbar^2}} Y_{\lambda\mu}(\pi/2, 0) f_{\lambda 0}(D_\ell) \quad e^{-\frac{a_{tr}}{2\ddot{r}_0 \hbar^2}(\Delta E - Q_{opt} - \hbar\mu\dot{\phi}(0))^2}$$

$$P_{tr} = \frac{d\sigma_{tr}}{d\sigma_{\text{Ruth}}}$$



(minimal distance of approach)

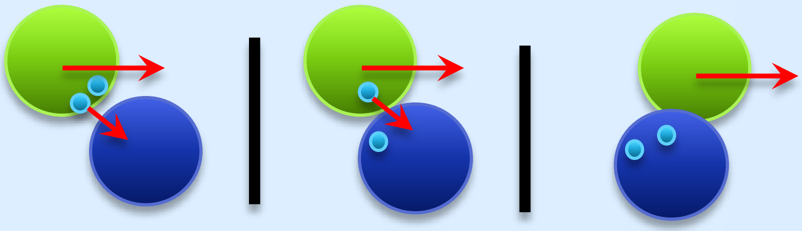
What about two-particle transfer?



Sequential (successive) transfer (two-step process)

$$a + A \longrightarrow (a-1) + (A+1)$$

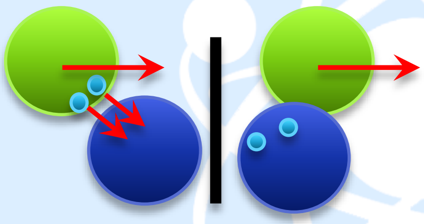
$$(a-1) + (A+1) \rightarrow (a-2) + (A+2)$$



$$P_2 \simeq (P_1)^2$$

Simultaneous transfer (one-step process)

$$a + A \longrightarrow (a-2) + (A+2)$$



$$P_2 \simeq (P_1)^2 + P_{Cor}$$

Note that the simple relation between probabilities are “naïve” (nucleons are Fermions)

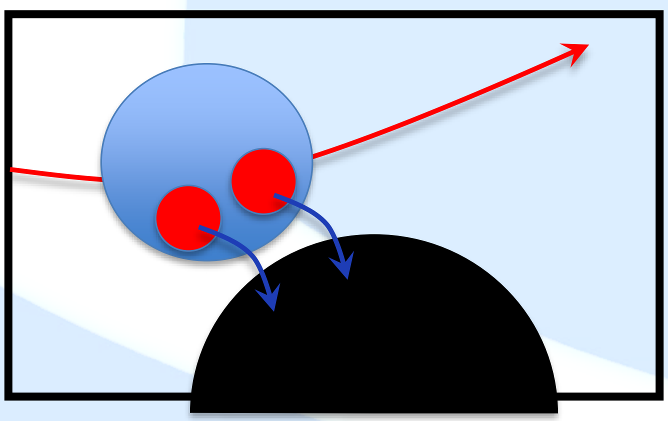
NUCLEON TRANSFER
SEMI-CLASSICAL METHODS: THEORETICAL CHALLENGE

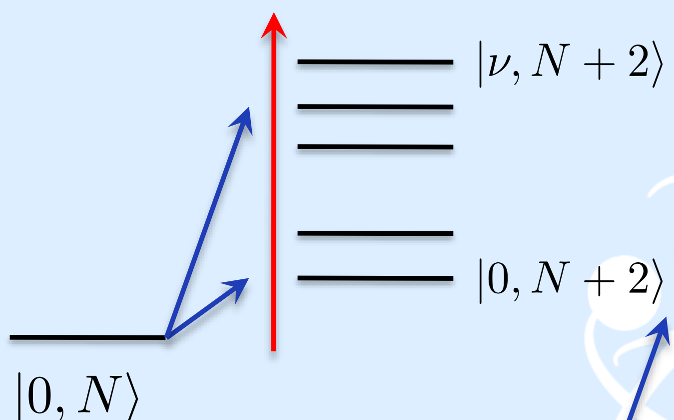
Taking the Time-dependent Schrödinger picture (6D problem)

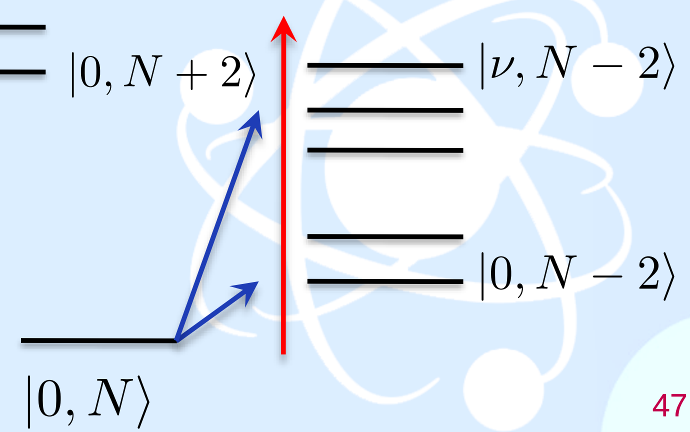
$$i\hbar\frac{d}{dt}\Psi(\mathbf{r}_1,\mathbf{r}_2,t) \simeq \left\{\frac{\mathbf{p}_1^2}{2m}+\frac{\mathbf{p}_2^2}{2m}+V_A(\mathbf{r}_1)+V_A(\mathbf{r}_2)+V_B(\mathbf{r}_1-\mathbf{R}(t))+V_B(\mathbf{r}_2-\mathbf{R}(t))+V_{\rm cor}(\mathbf{r}_1,\mathbf{r}_2)\right\}\Psi(\mathbf{r}_1,\mathbf{r}_2,t)$$

- The problem can eventually be decomposed on a set of correlated 2-body states with coupled angular momentum
- Then the problem can be formally reduced to the previous perturbative approach
- In HFB theory (superfluid systems), the problem can be seen as the nuclear response to an external field inducing 2 nucleon addition and/or removal

2n-transfer reactions



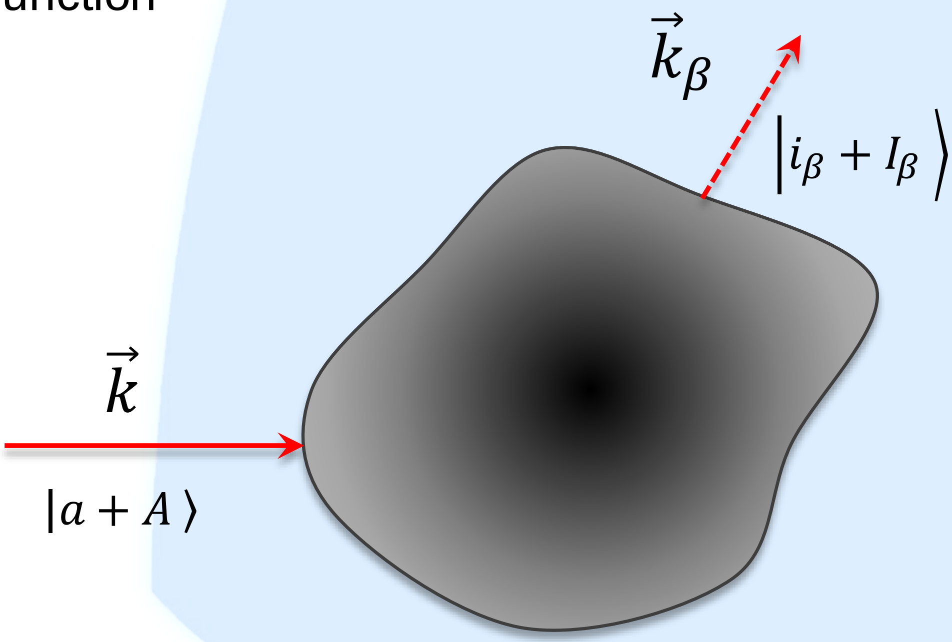




When semi-classical methods fail, it is necessary to use fully quantum approach to scattering (see lecture 2-3)

Scattering wave-function

$$\Psi_{\rm scatt}(r,\theta,\varphi) \longrightarrow e^{i\mathbf{k}_\alpha \mathbf{r}}\Psi_a\Psi_A + \sum_\beta f_\beta(\theta,\varphi)\frac{e^{ik_\beta r}}{r}\Psi_b\Psi_B$$



$$\frac{d\sigma_\beta}{d\Omega} = \frac{v_\beta}{v_\alpha}|f_\beta(\theta,\varphi)|^2 \quad \Rightarrow \quad \frac{d\sigma_{\beta\alpha}}{d\Omega} = \frac{\mu_\alpha\mu_\beta}{(2\pi\hbar^2)^2}\left(\frac{k_\beta}{k_\alpha}\right)|T_{\beta\alpha}(\mathbf{k}_\alpha,\mathbf{k}_\beta)|^2$$

SCATTERING FORMALISM (REMINDER)

The scattering states are the solution of

$$(E - H_\beta - K_\beta)\Psi_\alpha^+ = V_\alpha \Psi_\alpha^+$$

Using the same method as before

$$\langle \beta | \Psi_\alpha^+ \rangle (\mathbf{k}_\alpha, \mathbf{k}_\beta) = e^{i\mathbf{k}_\alpha \mathbf{r}} \delta_{\alpha\beta} - \left(\frac{\mu_B}{2\pi\hbar^2}\right) \underline{\int \frac{e^{i\mathbf{k}_\beta |\mathbf{r}_\beta - \mathbf{r}_{\beta'}|}}{|\mathbf{r}_\beta - \mathbf{r}_{\beta'}|} \langle \beta | V_\beta | \Psi_\alpha^+ \rangle d\mathbf{r}_{\beta'}}$$

$$= \langle \Phi_\beta(\mathbf{k}_\beta) | V_\beta | \Psi_\alpha^+(\mathbf{k}_\alpha) \rangle = T_{\beta\alpha}$$

Born approximation

$$\Psi^+(\mathbf{k}_\alpha) \to |\Phi_\alpha(\mathbf{k}_\alpha)\rangle$$

Distorted Wave approximation

$$(E - H_\beta - K_\beta - U_\alpha)\Psi_\alpha^+ = W_\beta \Psi_\alpha^+$$

$$(E - H_\beta - K_\beta - U_\alpha)\chi_\beta^- = 0$$

$$\langle \Phi_\beta(\mathbf{k}_\beta) | V_\beta | \Psi^+(\mathbf{k}_\alpha) \rangle \to \langle \chi_\beta^-(\mathbf{k}_\beta) | W_\beta | \Psi^+(\mathbf{k}_\alpha) \rangle$$

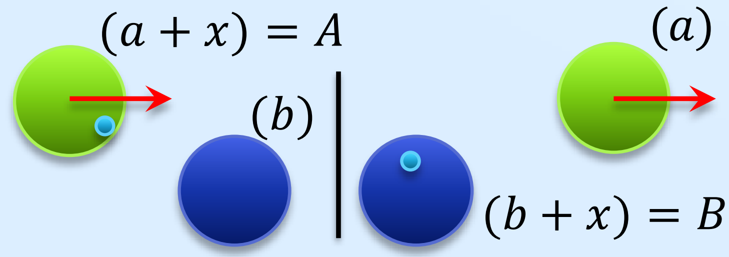
→ **DWBA**

NUCLEON TRANSFER

DWBA APPLIED TO NUCLEAR TRANSFER REACTION

$\langle \chi_\beta^-(\mathbf{k}_\beta)|W_\beta|\Psi^+(\mathbf{k}_\alpha)\rangle$?

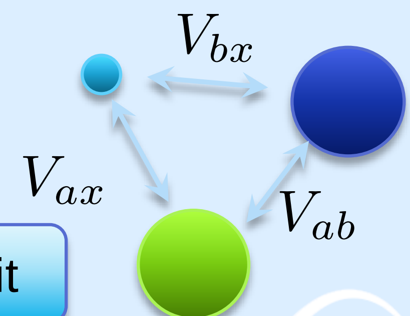
Starting point



The transferred particle (n, p, d, …) should be considered explicitly

$$A + b \longrightarrow a + B \quad \Rightarrow \quad (a + x) + b \longrightarrow a + (b + x)$$

The interaction potentials should be between all Constituents [3-body problem]:



Entrance

Exit

$$\Psi_A = C_{ax}^A[\varphi_a \varphi_x \Phi_{ax}(\mathbf{r})]^{J_i M_i} + \ldots \quad \Psi_B = C_{bx}^B[\varphi_b \varphi_x \Phi_{bx}(\mathbf{r})]^{J_f M_f} + \ldots$$

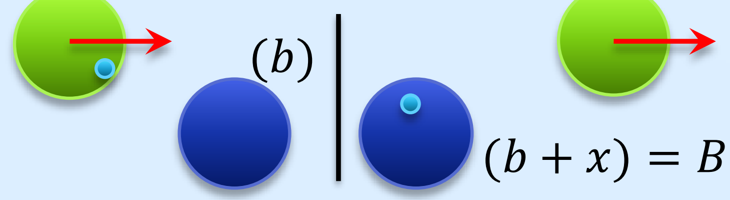
C_{ax}^A Spectroscopic amplitude

$|C_{ax}^A|^2$ ~ probability to find the configuration $(a + x)$ in the nucleus A

Irène Joliot-Curie
Laboratoire de Physique des 2 Infinis

$\langle \chi_\beta^-(\mathbf{k}_\beta)|W_\beta|\Psi^+(\mathbf{k}_\alpha)\rangle$?

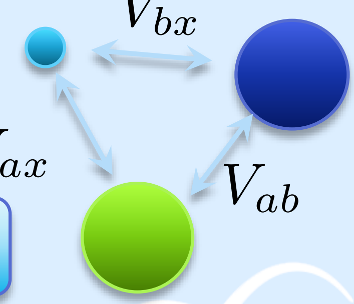
Starting point



The transferred particle (n, p, d, …) should be considered explicitly

$$A + b \longrightarrow a + B \quad \Rightarrow \quad (a+x) + b \longrightarrow a + (b+x)$$

The interaction potentials should be between all Constituents [3-body problem]:



Entrance

Exit

$$\Psi_A = C^A_{ax}[\varphi_a\varphi_x\Phi_{ax}(\mathbf{r})]^{J_iM_i} + \ldots \quad \Psi_B = C^B_{bx}[\varphi_b\varphi_x\Phi_{bx}(\mathbf{r})]^{J_fM_f} + \ldots$$

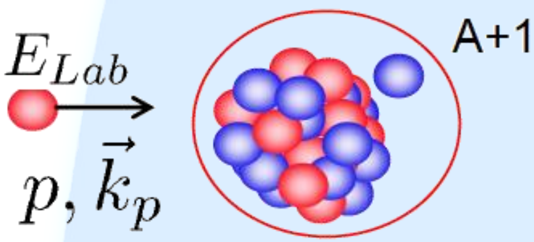
χ^- χ^+

$$\langle \Psi_B\Psi_a\chi_{aB}(\mathbf{R})|\mathcal{V}_{\text{int}}|\Psi_A\Psi_b\chi_{Ab}(\mathbf{R})\rangle$$

Adapted from a lecture of J. Tostevin, Ecole Joliot-Curie 2012

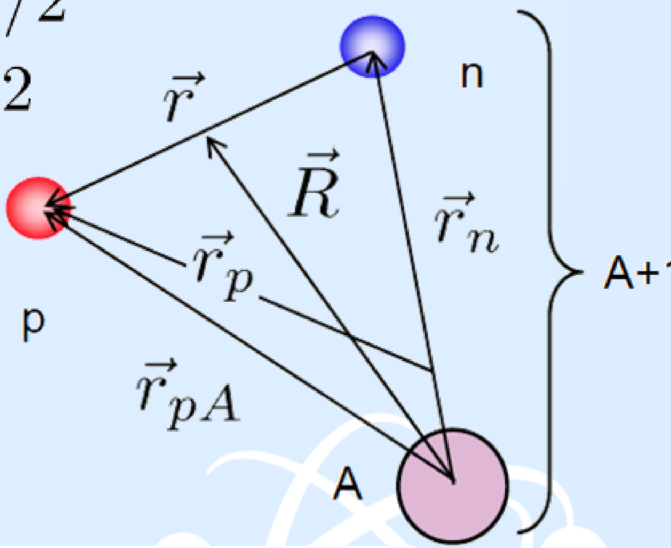
A typical illustration (p,d) transfer experiments

Entrance channel $(p + A)$

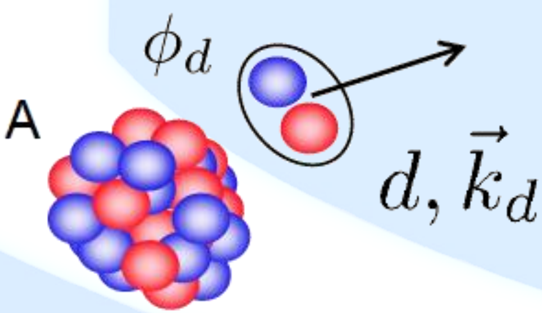


$$\vec{R} = [\vec{r}_n + \vec{r}_p]2$$
$$\vec{r}_{pA} = \vec{R} + \vec{r}/2$$
$$\vec{r}_n = \vec{R} - \vec{r}/2$$

3-body problem



Exit channel $(p + A)$

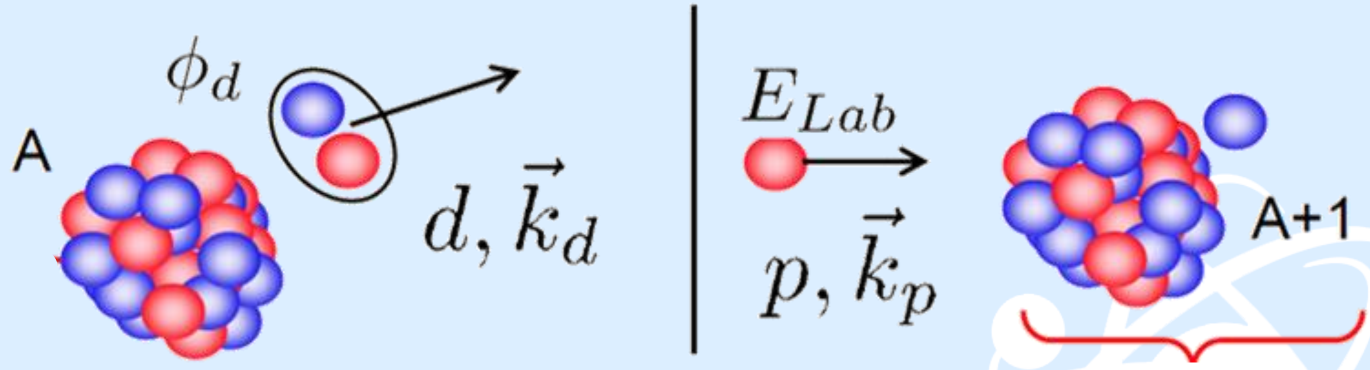


Interaction potential

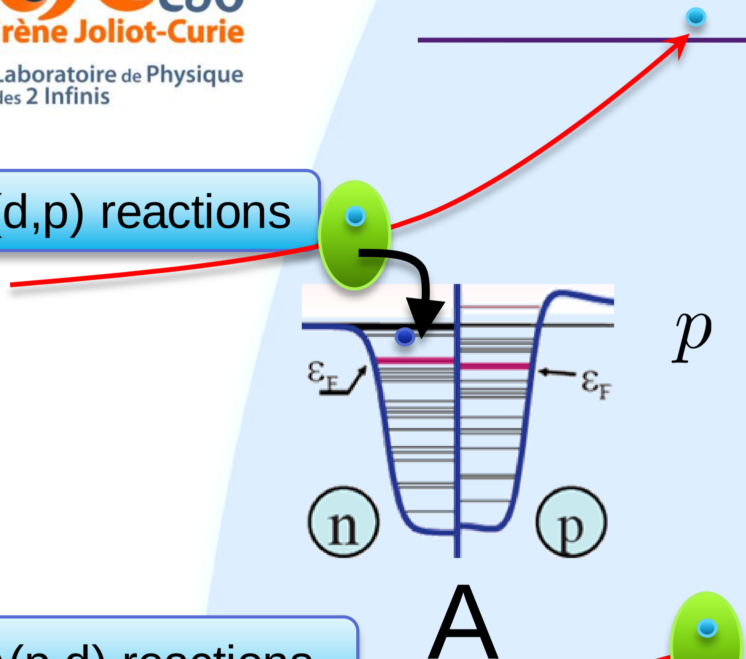
$$V_p(\vec{r}_{pA}) = V_p(\vec{R} + \vec{r}/2)$$
$$V_n(\vec{r}_n) = V_n(\vec{R} - \vec{r}/2)$$
$$V_{np}(\vec{r})$$

Transfer reaction transition amplitudes - DWBA

$$T(p,d) = \langle\, \underbrace{\chi^{(-)}_{d,\vec{k}_d}\,\phi_d\;\Phi(A,J_f)}_{\text{exit channel}}|V_{np}|\underbrace{\chi^{(+)}_{p,\vec{k}_p}\;\Phi(A+1,J_i)}_{\text{entrance channel}}\rangle$$



A(d,p) reactions



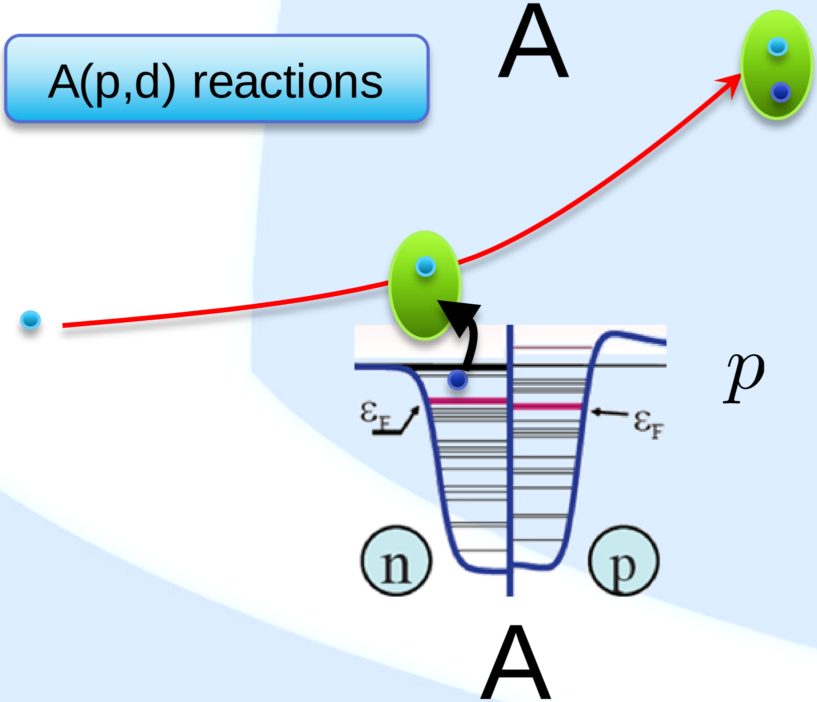
For the nucleus A this might be seen as

$$[A]^{J_iM_i} + [n]^{nljm} \to [A+1]^{J_fM_f}$$

We anticipate that the cross-section is sensitive to the quantity

$$\Rightarrow \ |\langle A+1, J_fM_f|a^\dagger_{nljm}|A, J_iM_i\rangle|^2$$

A(p,d) reactions



$$[A]^{J_iM_i} \to [A-1]^{J_fM_f} + [n]^{nljm}$$

$$\Rightarrow \ |\langle A-1, J_fM_f|a_{nljm}|A, J_iM_i\rangle|^2$$

SPECTROSCOPIC FACTOR

The spectroscopic factors are defined as the average over all spin projection (unpolarized beam):

Removal of a nucleon (p,d)

$$S_{nlj} = \frac{1}{2J_i + 1} \sum_{M_i M_f m} |\langle A-1, J_f M_f | a_{nljm} | A, J_i M_i \rangle|^2$$

Then the cross-section simply writes

$$\sigma_{nlj}^{A \to A-1}(\theta, \varphi) = S_{nlj} F_{nlj}(\theta, \varphi)$$

Contains the reaction part

Spectroscopic factors

Addition of a nucleon (d,p)

With the same definition

$$\sigma_{nlj}^{A \to A+1}(\theta, \varphi) = \frac{2J_f + 1}{2J_i + 1} S_{nlj} F_{nlj}(\theta, \varphi)$$

- Spectroscopic factors contains the selection of channels (only specific [nljm] can be probed)

$$\langle A-1, J_f M_f | a_{nljm} | A, J_i M_i \rangle \propto \langle J_f M_f | jm J_i M_i \rangle$$

For instance $\mathbf{J}_f = \mathbf{J}_i + \mathbf{j} = \mathbf{J}_i + \mathbf{l} + \mathbf{s}$

- (d,p) reaction on an even-even nucleus can only populate $J_f = l \pm \frac{1}{2}$ state

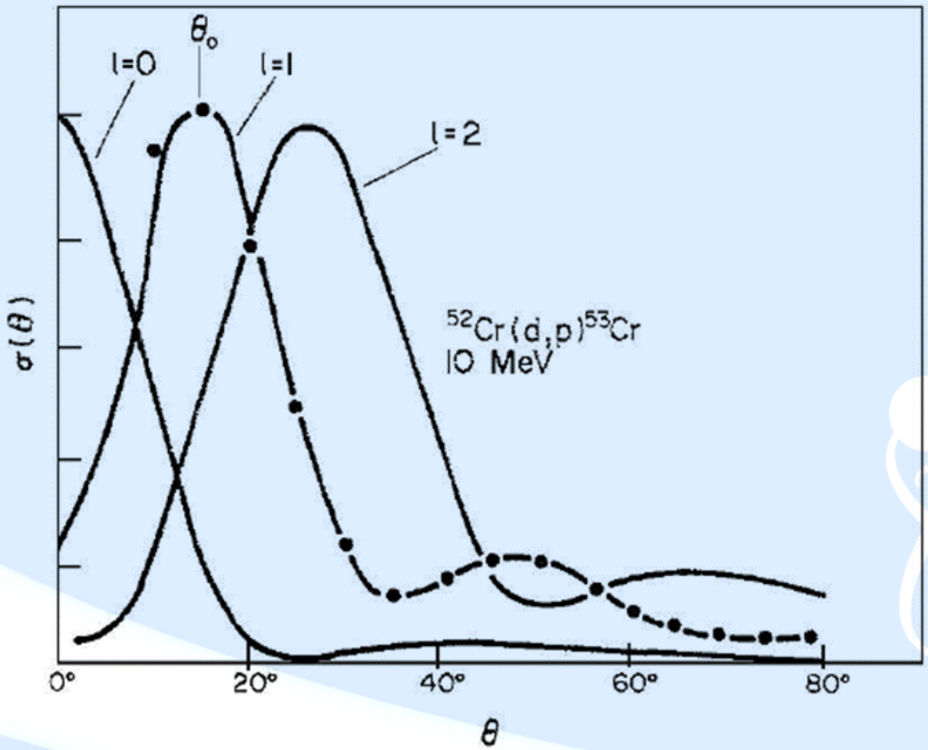
- Spectroscopic factor contains Clebsh coefficients

$$\sigma_{nlj}^{A\to A-1}(\theta,\varphi) = S_{nlj} F_{nlj}(\theta,\varphi)$$

Occupation probability

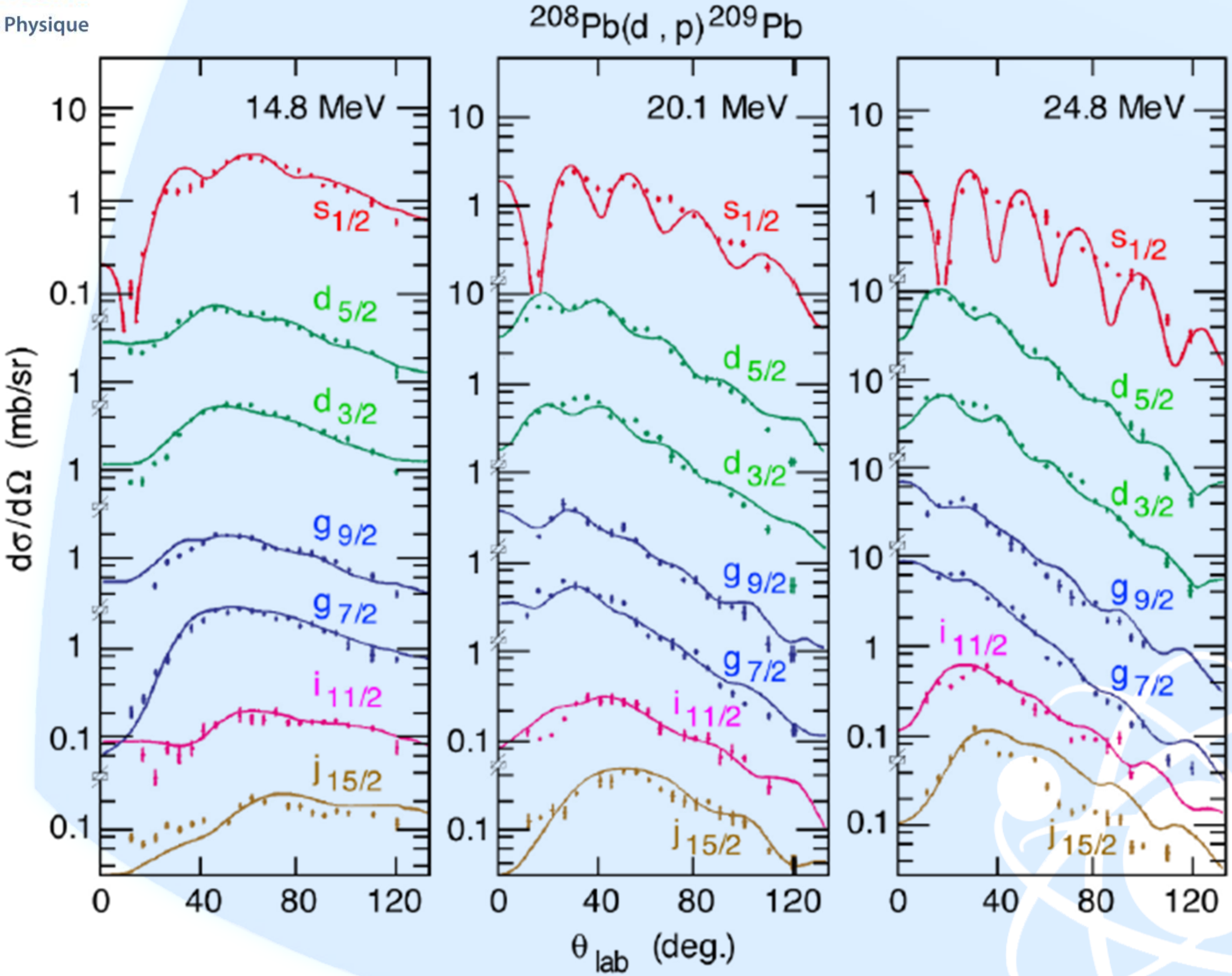
Reaction part

The reaction part [model dependent] depends strongly of the initial (final) single-particle state properties (quantum numbers, extension, …)



NUCLEON TRANSFER AS A SPECTROSCOPIC TOOL

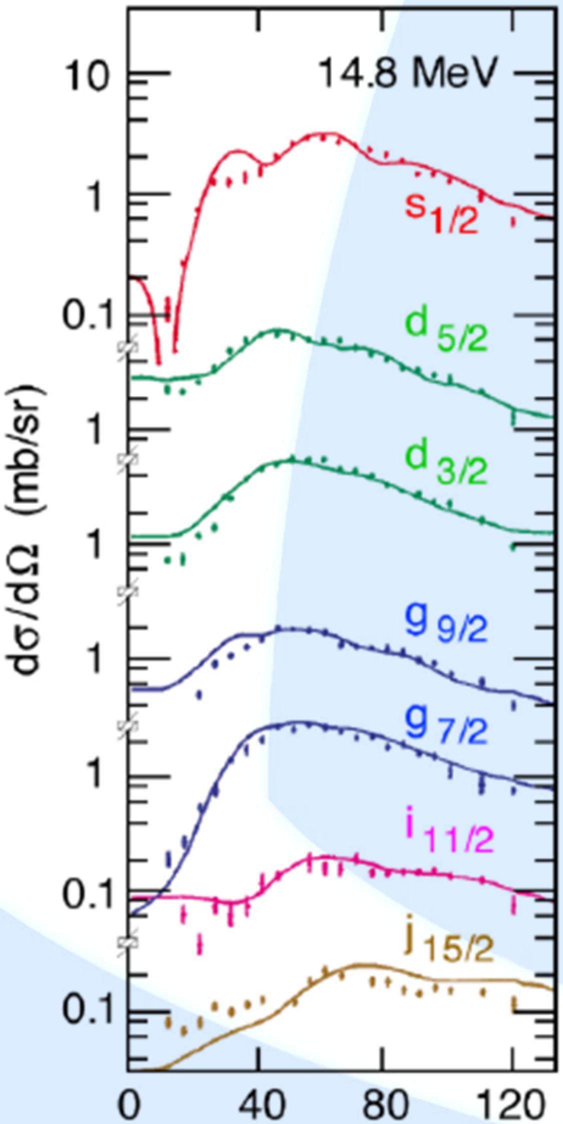
FROM EXPERIMENT TO SF



Muehllener et al.
Phys. Rev. 159, 1043 (1967)

NUCLEON TRANSFER AS A SPECTROSCOPIC TOOL

FROM EXPERIMENT TO SF

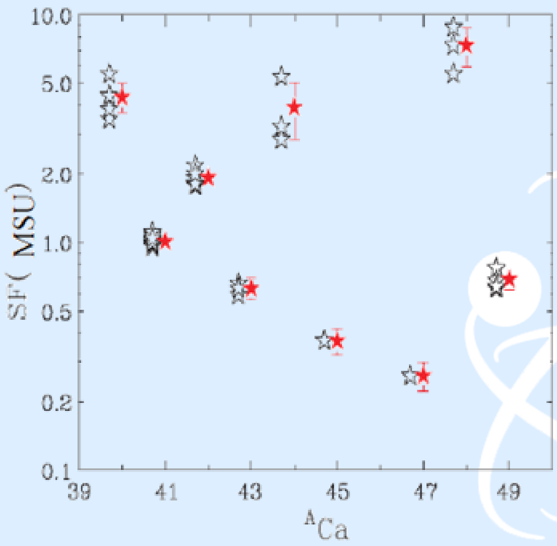


➡ From the shape of the cross-section one attributes the (nlm)

➡ From the absolute value, one deduce the SF

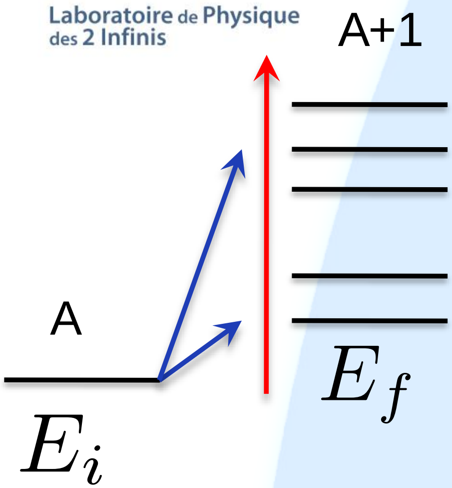
$$SF = \frac{\sigma_{\exp}}{\sigma_{\rm th}^{\rm sp}}$$

Illustration



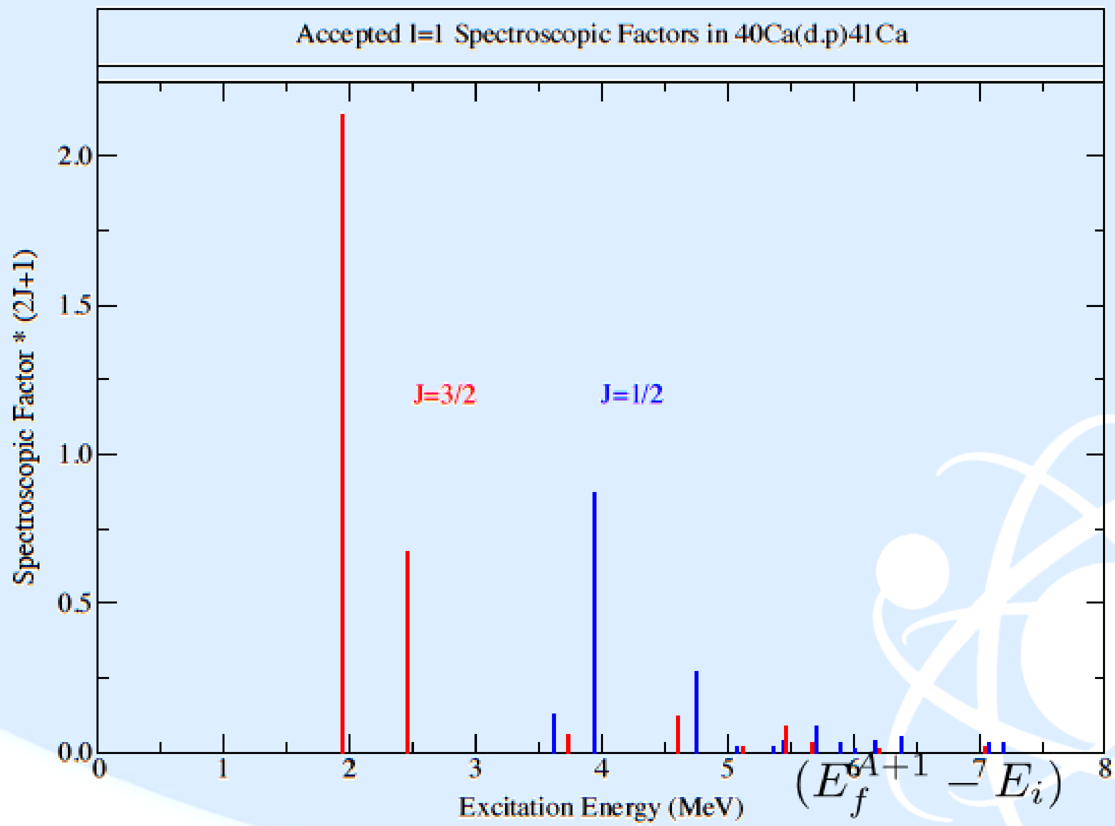
J. Lee et al, Phys. Rev. C75 (2007) 064320

SINGLE-PARTICLE ENERGIES
FROM EXPERIMENTS

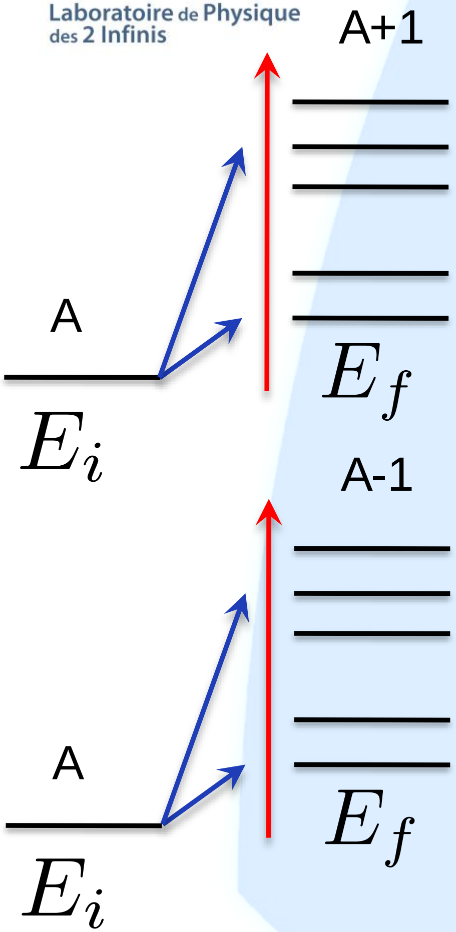


$$a^\dagger_{jlm}|A\rangle = \sum_f c_f |A+1, f\rangle$$

➡ Single-particle states are "fragmented"



SINGLE-PARTICLE ENERGIES
FROM EXPERIMENTS



$$a^\dagger_{jlm}|A\rangle = \sum_f c_f |A+1, f\rangle$$

➡ Single-particle states are "fragmented"

Nevertheless, one can define effective single-particle energies

$$\varepsilon_i \simeq \sum_f (SF)_i^{A\to A+1}(E_f^{A+1} - E_i)$$

A more precise definition should include both removal and addition SF

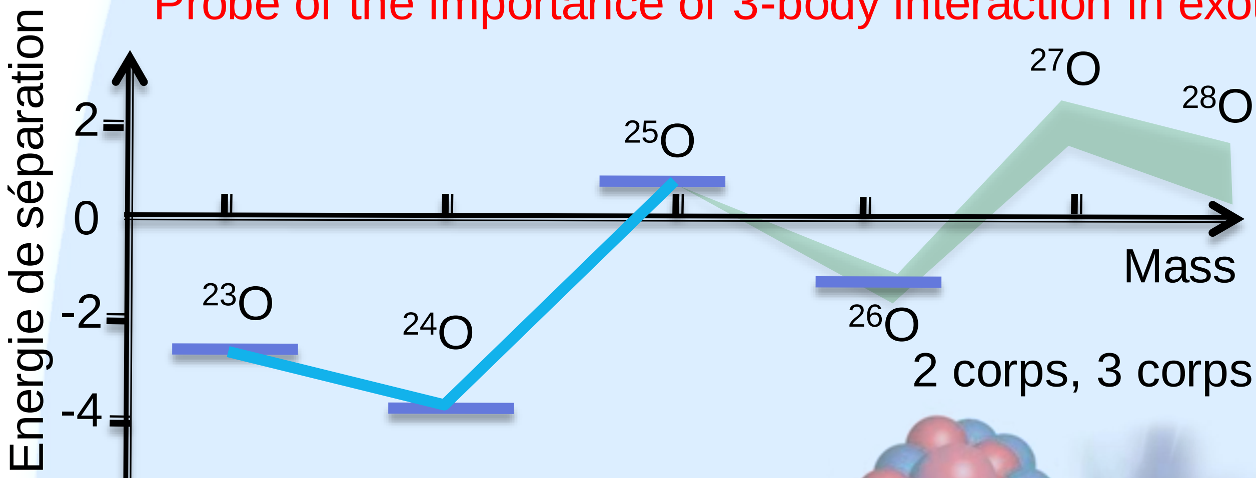
$$\varepsilon_i \simeq \sum_f (SF)_i^{A\to A+1}(E_f^{A+1} - E_i) + \sum (SF)_i^{A\to A-1}(E_i - E_f^{A-1})$$

See formal definition (Baranger Nucl. Phys. A149 (1970))

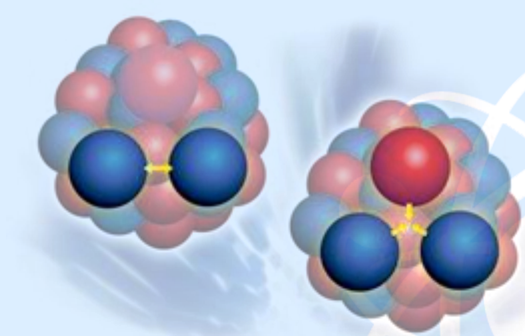
SINGLE-PARTICLE ENERGIES
FROM EXPERIMENTS

Illustration of recent use

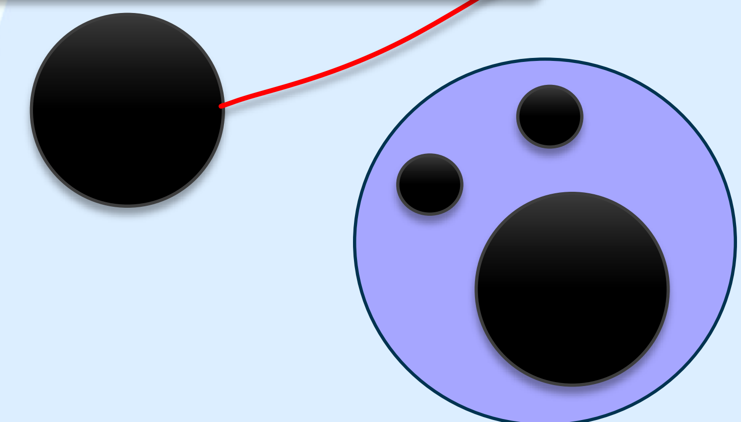
Probe of the importance of 3-body interaction in exotic oxygen



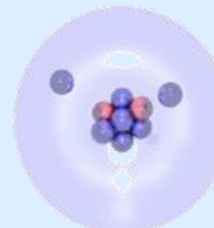
2 corps, 3 corps ?



Two nucleon transfer [4 body problem]



Example:
Halo nucleus ^{6}He, ^{11}Li…



Break-up and continuum effect

- Nucleons are sometimes very weakly bound [500 keV]
- Break-up (emission of particle) is a strongly competing channel.
- Requires to introduce a continuum of exit channels

(see next lecture)