NPAC Accelerator Physics Examination

Longitudinal dynamics and space charge

16 December 2021

We consider the proton linac for the European Spallation source ESS. The project is to accelerate 2 GeV protons up to a target to make spallation reactions and produce neutrons. The neutrons are then used for different experiments like neutronography. A layout of the proton linac for ESS is shown in Figure 1.

One project is to upgrade ESS and to accelerate by the same time H^- ions. The H^- ions are then accelerated up to 2 GeV, stripped (in H^+ ions) and accumulated into an accumulator ring (see Figure 3). The accumulator ring enables to compress the pulses before sending into a target. The target converts the protons into pions which are focused by a magnetic horn. The pions decay into neutrinos which are used for neutrino physics like study of their oscillation.

We remind some constants and useful formulae:

Speed of light	$c = 299\,792\,458\,\mathrm{m\,s^{-1}}$	Proton rest mass	$E_p = 938.27\,\mathrm{MeV}$
$\gamma = \frac{1}{\sqrt{1-\beta^2}}$	$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$	$\sin(x) = \frac{e^{\imath x} - e^{-\imath x}}{2\imath}$	$\cos(x) = \frac{e^{\imath x} + e^{-\imath x}}{2}$
Transition gamma	$\gamma_t = \frac{1}{\sqrt{\alpha}}$		

Part 1: The proton linac

1. The ESS proton linac is made of different accelerating structures like DTL (Drift Tube Linac) or Spokes. Explain why we use different accelerating structures.

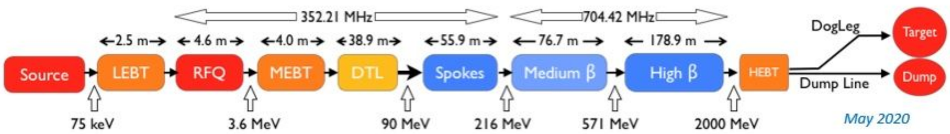

Figure 1: Layout of the ESS proton linac.

2. The ion source uses electrodes to accelerate the protons up to 75 keV. For which reason do we use radiofrequency (RF) acceleration instead of electrostatic for the following?

3. Remind the principle of the RF acceleration (boundary conditions, energy exchange, shunt impedance).

4. For which purposes do we use an RFQ (RadioFrequency Quadrupole)?

Part 2: The low-energy beam transport (LEBT)

We consider now the low-energy beam transport line. For simplification purposes, we assume that the beam has a uniform cylindrical distribution with an average current I. We consider a continuously focusing channel of external force k_0. We assume that the beam is perfectly centred: $\langle x \rangle = \langle x' \rangle = \langle y \rangle = \langle y' \rangle = 0$. Let be η the tune depression of the core.

1. Explain why space-charge forces strongly decrease with energy.

2. Show the spectrum of the particle motion in the beam distribution in the case with no space-charge, with space-charge for a uniform beam and for a Gaussian beam.

3. The envelope equation ($\tilde{x}$ is the RMS beam size) along the longitudinal direction s is given by:

$$\tilde{x}'' + k_{x,0}^2(s) \cdot \tilde{x} - \frac{K/2}{\tilde{x} + \tilde{y}} - \frac{\tilde{\epsilon}_x^2}{\tilde{x}^3} = 0 \qquad (1)$$

   with $K = \dfrac{q \cdot I}{2\pi\epsilon_0 m \left(\gamma\beta c\right)^3}$ the generalized beam perveance.

   We assume that the beam is perfectly matched: the beam size $\tilde{x}$ does not change with s. Express the beam size $\tilde{x}_0$ in this case.

4. We have now a small mismatch. We assume now that $\tilde{x}(s) = \tilde{x}_0(1 + \delta(s))$ with $\delta(s) \ll 1$. Express $\delta(s)$ by solving Eq (1). What is the frequency of the mismatching when space-charge is negligible?

5. In the LEBT, the vacuum is not perfect and you have some residual gas inside. When the pressure increases, the beam may ionize the residual gas. With no calculation, may this effect have some effects on space-charge forces? Explain.

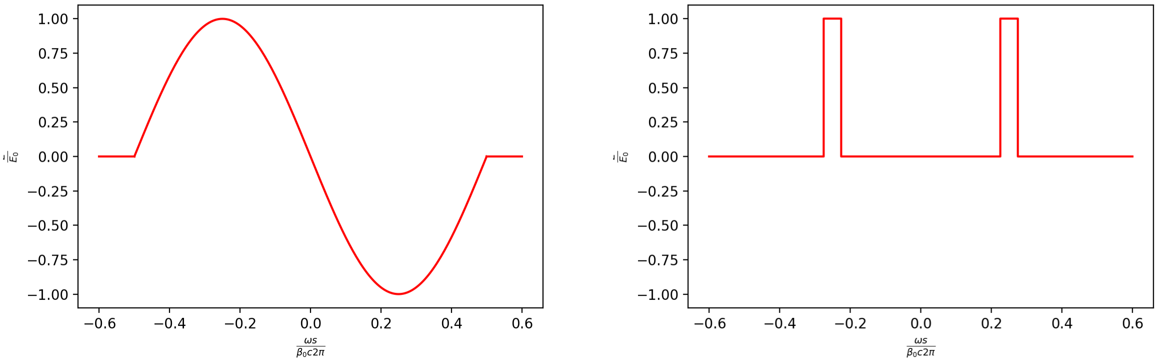

Figure 2: Possible structures for acceleration

Part 3: The low β linac

We consider two structures to accelerate the protons (see Figure 2). The RF pulsation of the cavity is ω. Let be β_0 the reduced velocity of the beam to be accelerated. We will assume that the speed does not change while accelerating in the RF structure.

The expression of the first structure is

$$E_1(s) = \begin{cases} E_0 \sin \dfrac{\omega s}{\beta_0 c} & \text{if } |s| < \pi \dfrac{\beta_0 c}{\omega} \\ 0 & \text{if } |s| \geq \pi \dfrac{\beta_0 c}{\omega} \end{cases} \tag{2}$$

The expression of the second structure is

$$E_2(s) = \begin{cases} E_0 & \text{if } \left| s \pm \pi \dfrac{\beta_0 c}{2\omega} \right| < d \\ 0 & \text{if } \left| s \pm \pi \dfrac{\beta_0 c}{2\omega} \right| \geq d \end{cases} \tag{3}$$

$$d = \pi \frac{\beta_0 c}{10\omega} \tag{4}$$

1. Intuitively, what is the structure we should choose for the most efficient acceleration?
2. Give the expression of the transit time factor T for the cavity you have selected as a function of $\beta = v/c$, and ω.

$$\text{Transit time factor : } T(\beta) = \frac{\left| \int E(s) \exp \imath \dfrac{\omega s}{\beta c} ds \right|}{V_0} \tag{5}$$

$$\text{Total voltage : } V_0 = \int |E(s)| \, ds \tag{6}$$

3. Check the value of $T(\beta_0)$ and conclude if you have chosen the right structure.

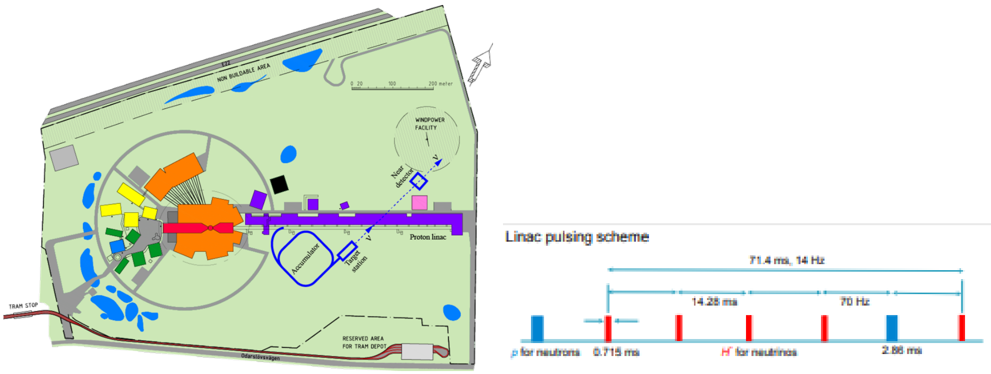

Figure 3: Layout of the complex (left) and pulsing scheme (right) in the linac

Part 4: The accumulator

To accelerate H^- ions, the source, LEBT, RFQ, and MEBT are duplicated. The H^+ and H^- ions are then accelerated in the same linac with the pulse structure shown in Figure 3. When the H^- ions are deflected into the accumulator, they are stripped into H^+ ions by crossing a thin foil.

The circumference of the accumulator ring is 375.88 m. The injection kinetic energy in the accumulator is 2 GeV. Its momentum compaction α is 10^{-2}. The RF frequency of the cavity is 704.42 MHz.

1. Calculate the revolution time and the harmonic number.

2. Compare the length of the pulse (714.46 µs) and the revolution time. Explain the interest to strip the ions when injecting into the accumulator.

3. The accumulator does not accelerate the beam. Do we need an RF cavity? Explain why.

4. We assume we use that $E_z = E_0 \sin(\phi + \varphi_s)$. What synchronous phase should we choose?

5. Show in the longitudinal phase plane $(\phi, \delta E)$ the separatrix, the bucket and the beam (we assume it matched). Show by an arrow in which direction the particles are moving.