Accelerator Physics Tutorials

Master NPAC (Nuclei, Particles, Astroparticles, Cosmology)

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Exercise 1: Cyclotron

C_{12}^{+} ions are accelerated in a cyclotron with a diameter of 2 m and a magnetic field of 1 T.

1. Give the exit kinetic energy T of ions.
2. The injection kinetic energy is 10 keV and the final energy is obtained after 30 turns. Give the average energy gain per accelerating gaps.
3. RF frequency is set to be synchronized with ion rotation at 50 cm radius, calculate the RF frequency.
4. Gap voltage amplitudes are 100 kV, calculate (with Excel) the injection phase and the energy gain in each gap.

Exercise 2: 3 gap cavity

Let's consider a 3-gap cavity with respective lengths g_2, g_1, and g_2, consecutive gap centres being separated by a distance $d = 10\,\text{cm}$. The accelerating electric field is:

$$\begin{aligned} E &= -E_0 \sin \omega t && \text{in outer gaps,} \\ E &= E_0 \sin \omega t && \text{in the inner gap} \end{aligned}$$

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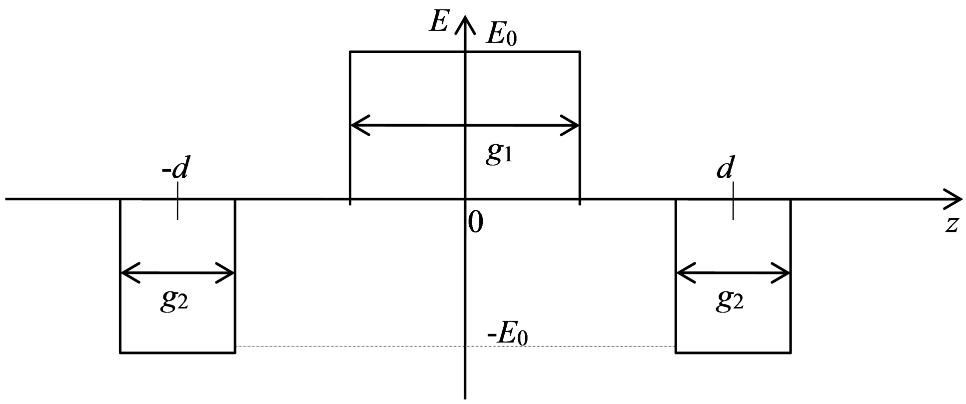

1. Intuitively, what is the particle velocity v_{opt} having an optimal acceleration. Do numerical application with 100 MHz RF frequency.

2. Give the expression of the transit time factor T as a function of $\beta = v/c$, ω, d, g_1 and g_2.

3. For $d = g_1 = 2 \cdot g_2 = 10\,\text{cm}$,

   a) Plot $T(\beta)$.

   b) Give the numerical values of β_{opt} (precision 1×10^{-3}) and associated T_{opt},

   c) Conclude on your intuition.

Exercise 3: Storage ring

One considers an electron storage ring with 2.15 GeV kinetic energy. Its circumference is $C = 336\,\text{m}$. The 32 dipoles produce 1.555 T magnetic field. The effective voltage over one turn is $V = 2\,\text{MV}$ and the RF frequency is $f = 500\,\text{MHz}$. The momentum compaction is $\alpha = 5.5 \times 10^{-4}$.

1. Calculate the curvature radius in dipoles.

2. Calculate the electron energy loss per turn ΔE ($\Delta E(\text{keV}) = 88.4\frac{E^4(\text{GeV})}{\rho(\text{m})}$).

3. Calculate the harmonic number h (ratio between RF frequency and turn frequency) and the synchronous phase φ_s to compensate this energy loss.

4. What is the maximum energy acceptance ($\Delta E/E$)?

5. Calculate the cavity effective voltage V to set $\Delta E/E = \pm 4\%$.

6. Calculate the rms energy dispersion of a matched beam with longitudinal rms emittance: $430\pi\,^\circ\,\text{MeV}$.

7. Calculate the synchrotron oscillation pulsation Ω_s (for low amplitude).
8. Betatron wave numbers being respectively $\nu_x = 18.30$ and $\nu_y = 8.30$, what is the ratio between them and the longitudinal one?

Exercise 4: Space-charge

Let's consider a continuous axisymmetric beam of current I and of velocity βc with two kinds of charge distribution ρ:

- Uniform distribution:

$$\rho_u(r) = \begin{cases} \rho_u(0) \text{ if } r < R \\ 0 \text{ if } r \geq R \end{cases}$$

- Gaussian distribution:

$$\rho_g(r) = \rho_g(0) \exp\left(-\frac{r^2}{r_0^2}\right)$$

$\rho_u(0)$ and $\rho_g(0)$ are the charge densities on the axis.
R is the beam size for a uniform beam.

1. By using Gauss theorem, calculate the components of the electric fields $E_u(r)$ and $E_g(r)$ for each charge distribution. Write down them as a function of I, βc, ϵ_0 and:
   - R for the uniform distribution,
   - r_0 for the Gaussian distribution.
2. Calculate the RMS beam size (or standard deviation) for each charge distribution.
3. Write down the ratio between electric field $E_g(r)$ for the Gaussian distribution and the electric field $E_u(r)$ for a uniform distribution when the RMS beam size and the current are the same in both distributions.
4. By using the definition of the space charge depression, write down the ratio between the square of the radius depression $\eta_g^2(r)$ for a Gaussian distribution (RMS beam size equivalent) and the square of the radius depression $\eta_u^2(r)$ for a uniform distribution.
5. What is this value on the axis?
6. In the case of a Gaussian distribution (RMS beam size equivalent), for which value of $\eta_u(0)$ are the space charge forces greater than the external focusing strengths?