Collisions and kinematics

Particle Physics

Fabrice Couderc / Eli BenHaim

NPAC 2023

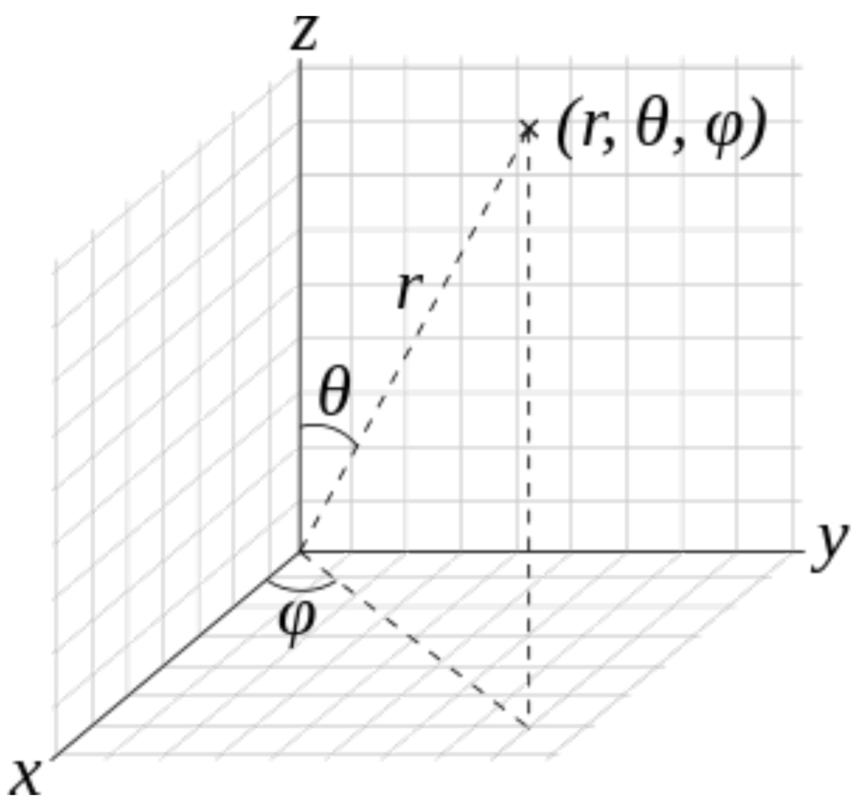
Overview

1. Special relativity for particle physicists
    1. Reminders
    2. Rapidity
    3. Pseudo-rapidity
2. Cross sections
    1. Introduction
    2. First example
    3. Cross section master formula
    4. Two-body phase space
    5. Luminosity
    6. Resonances
3. Particle total width
    1. Total width definition
    2. Branching ratio and partial width
    3. Master formula

Reminder of special relativity

Properties	Classical relativity	Special relativity
Coordinates	time universal (same in all frames)	$x \equiv x^\mu = (t, \vec{x})$ frame dependent
	$\vec{x}$ frame dependent	
Frame change	Galilean group	Poincare group
	space-time translations + rotations	space-time translations
	space rotations	space rotations
	+ Galielan transfo	+ Lorentz boost
Invariant	time is invariant	$ds^2 = \eta_{\mu\nu}\, ds^\mu ds^\nu$
	$d\vec{x}^2 = \sum_i (dx^i)^2$	$ds^2 = c^2\, dt^2 - d\vec{x}^2$
		$ds^2 = c^2\, dt^2\,(1 - \vec{\beta}^2) = \frac{1}{\gamma^2}\, c^2 dt^2$
		$\gamma = \frac{1}{\sqrt{1-\vec{\beta}^2}} > 1$
length	frame independent	frame dependent
time	frame independent	frame dependent

A few definition



Lorentz boost along z with velocity β

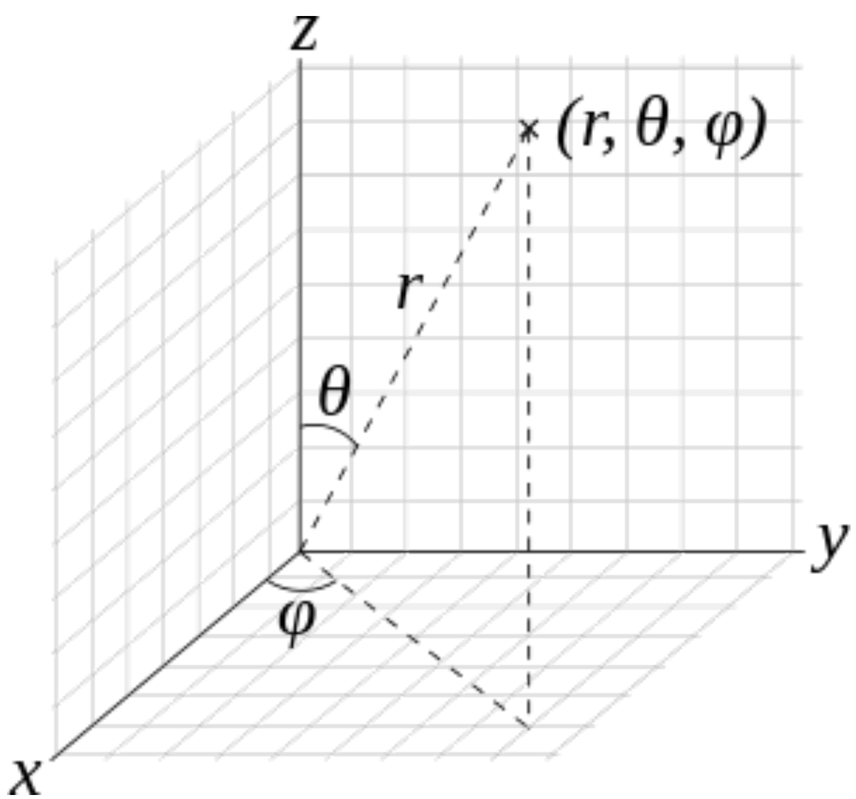
$$Y = \text{arctanh}(\beta)$$

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} \cosh Y & 0 & 0 & -\sinh Y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh Y & 0 & 0 & \cosh Y \end{pmatrix}$$

Hyperbolic trgonometry, $\text{arctanh}(x) = \frac{1}{2} \ln \frac{1+x}{1-x},$

Express the rapidity of a particle of mass m and momentum |p| ?

A few definition



Lorentz boost along z with velocity β

$$Y = \text{arctanh}(\beta)$$

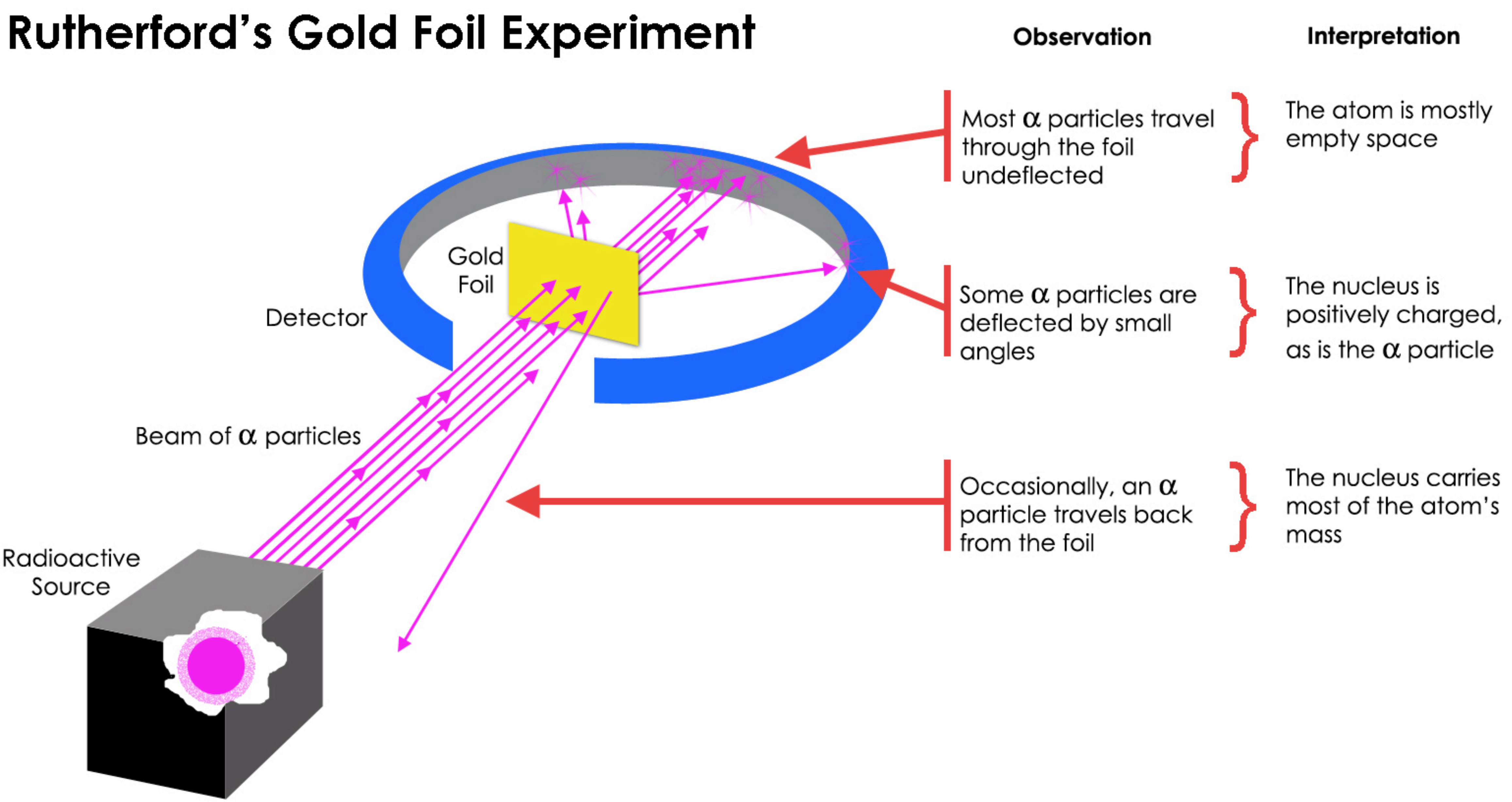
$$\Lambda^\mu_\nu = \begin{pmatrix} \cosh Y & 0 & 0 & -\sinh Y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh Y & 0 & 0 & \cosh Y \end{pmatrix}$$

Hyperbolic trgonometry, $$\text{arctanh}(x) = \frac{1}{2} \ln \frac{1+x}{1-x},$$

Express the rapidity of a particle of mass m and momentum |p| ?

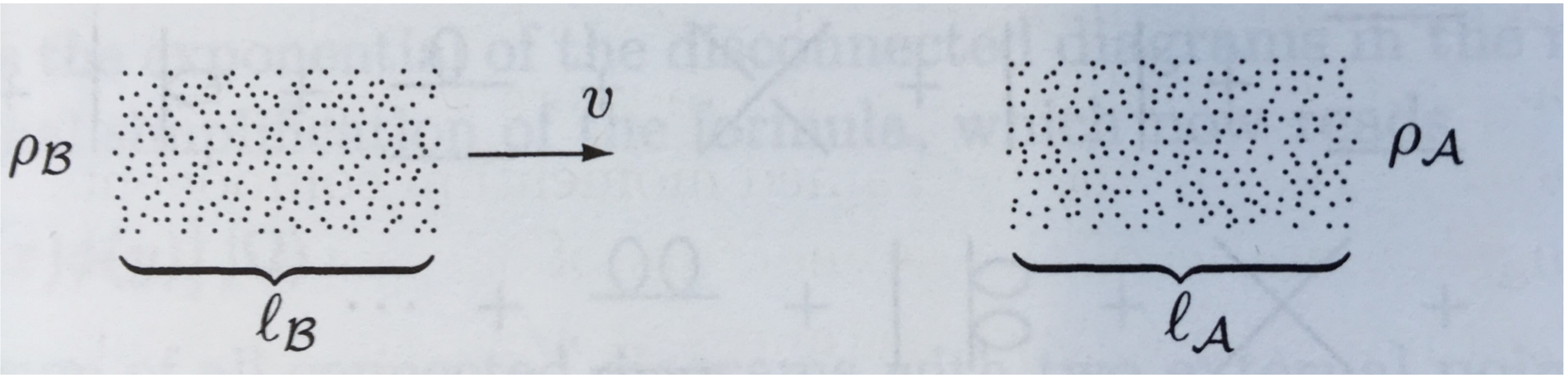
Particle physics $$y \equiv \text{arctanh}\left(\frac{p_z}{E}\right) = \frac{1}{2} \ln \left(\frac{E+p_z}{E-p_z}\right)$$

Cross section definition



The atomic model introduces the concept of cross section

Cross section definition

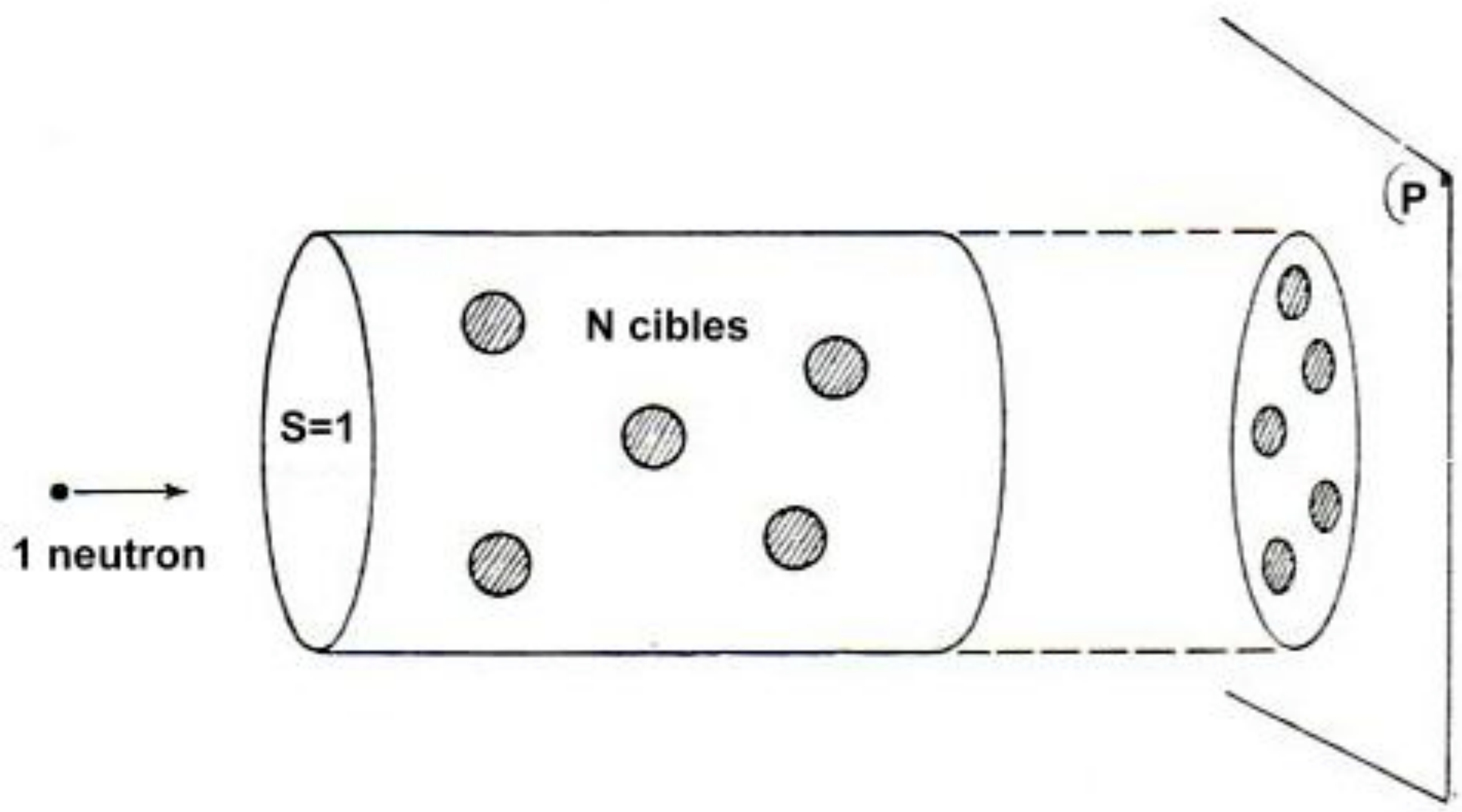


Consider a beam of particles B smashed on a target of particles A

$$\frac{\sigma}{A} = \frac{\text{Number of scattering events}}{N_A\, N_B}$$

With A the area of the beam(s)

Cross section definition

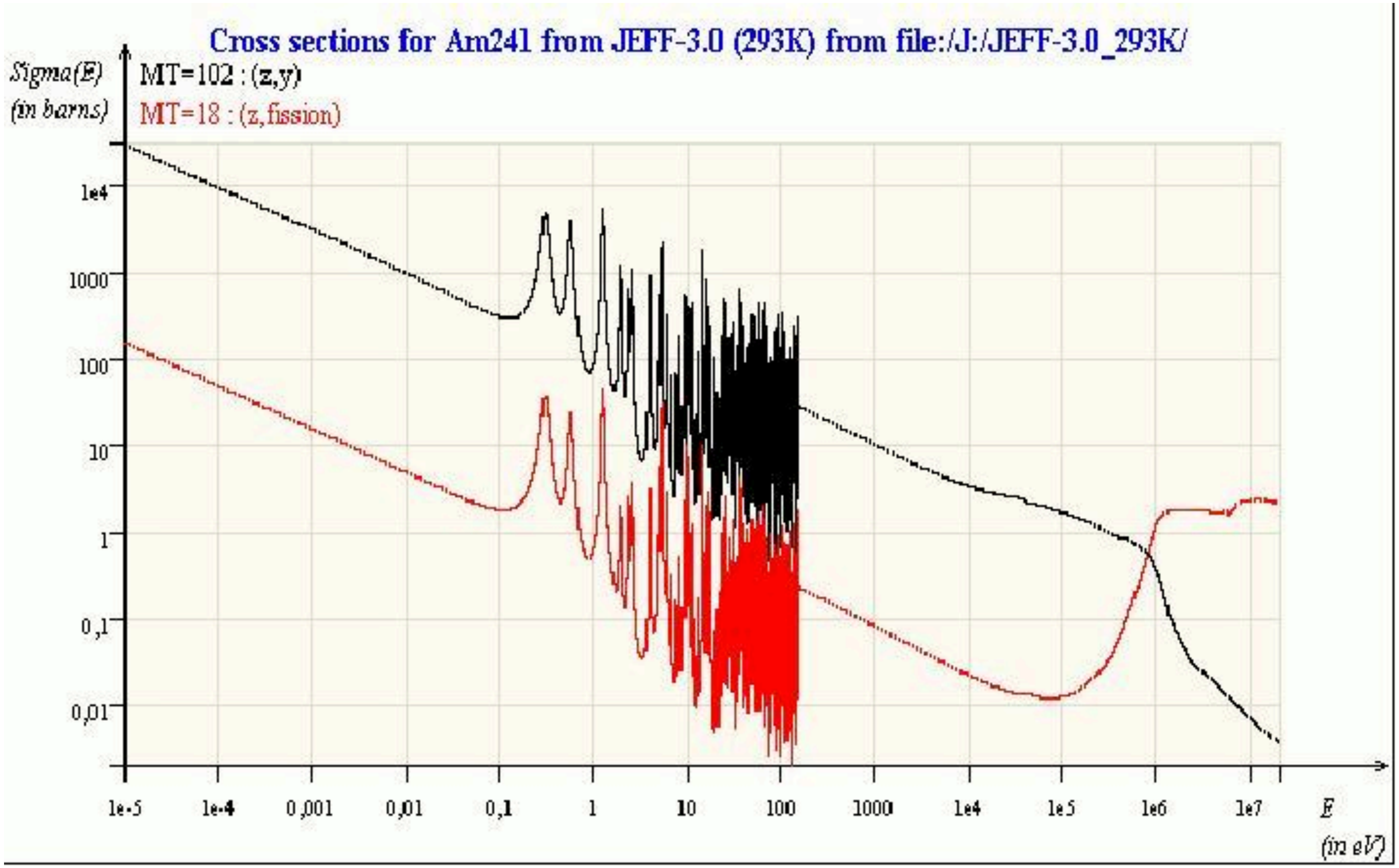


Assuming the projectile surface has a negligible surface, the probability of interaction is

P = (cross section) / (total area) = (sum of gray area) / S

Cross section unit: 1 barn = 10^{-24} cm^2 = 10^{-28} m^2

A first example



Cross section of neutron on Americium 241, showing low energy 1/E dependence

$$\sigma(n-T) \propto \pi \left(R + \lambda(E_n)^2\right)$$

Cross section and matrix element

A lot is given in the booklet

http://pdg.lbl.gov/2018/reviews/rpp2018-rev-kinematics.pdf

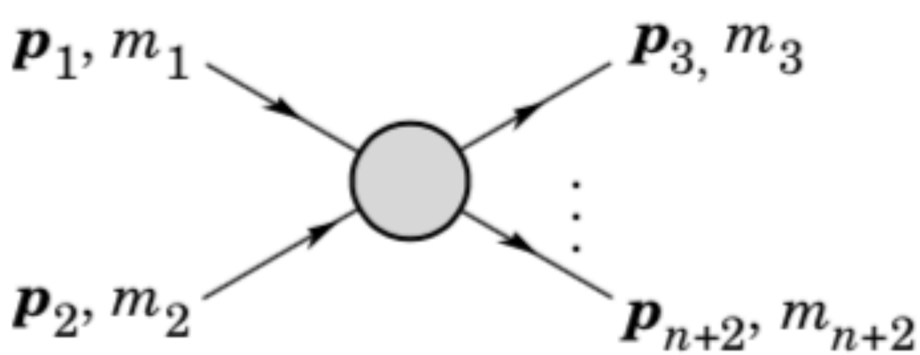


Figure 47.5: Definitions of variables for production of an n-body final state.

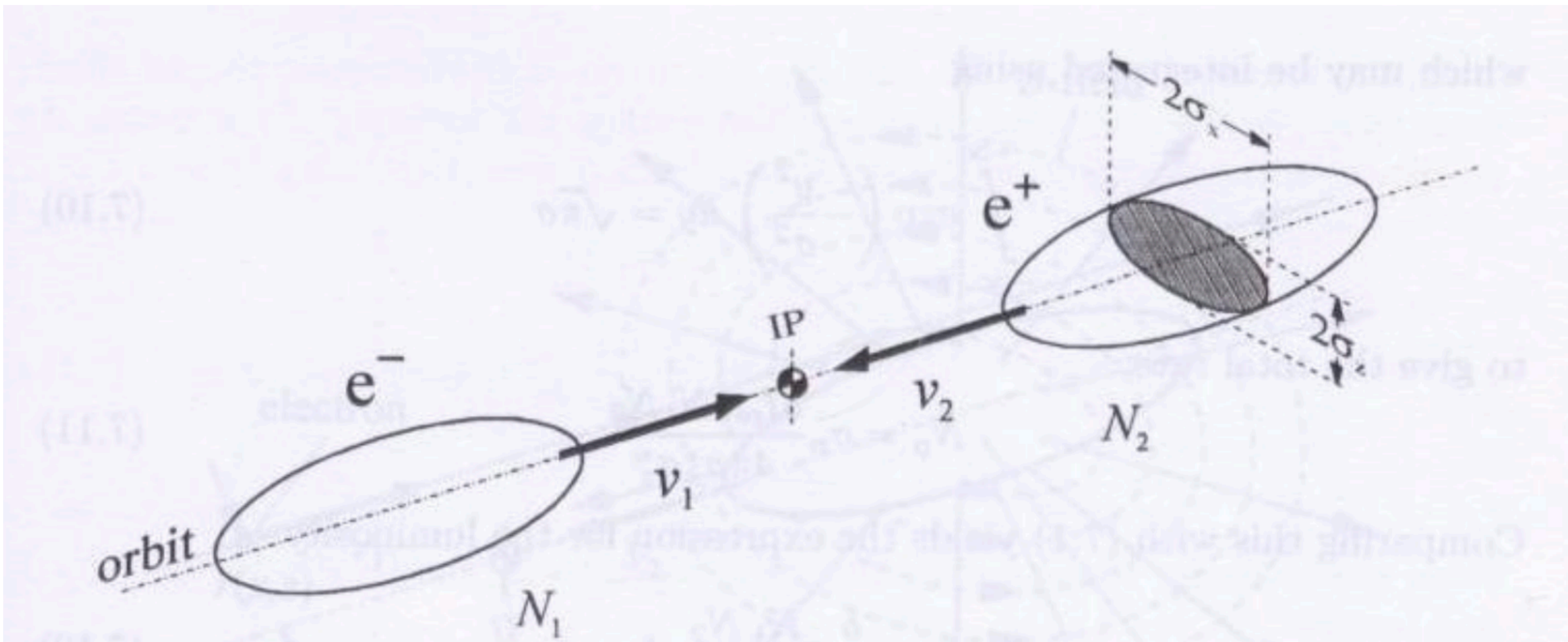
The differential cross section is given by

$$d\sigma = \frac{(2\pi)^4|\mathscr{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}$$

$$\times \; d\Phi_n(p_1 + p_2;\; p_3, \ldots, p_{n+2}) \; . \qquad (47.27)$$

$$d\Phi_n(P;\; p_1, \ldots, p_n) = \delta^4 \left(P - \sum_{i=1}^{n} p_i\right) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

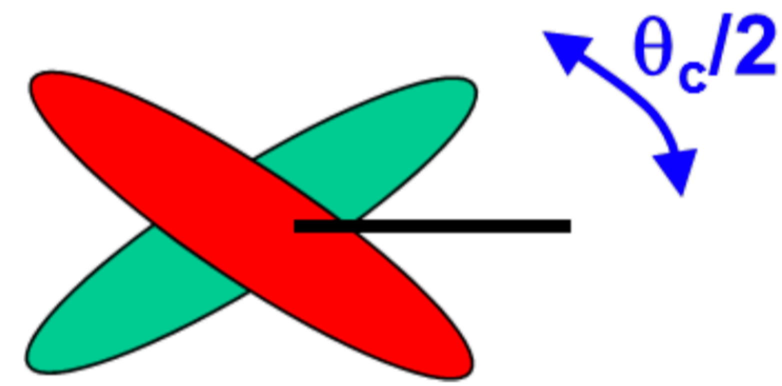
Collider luminosity



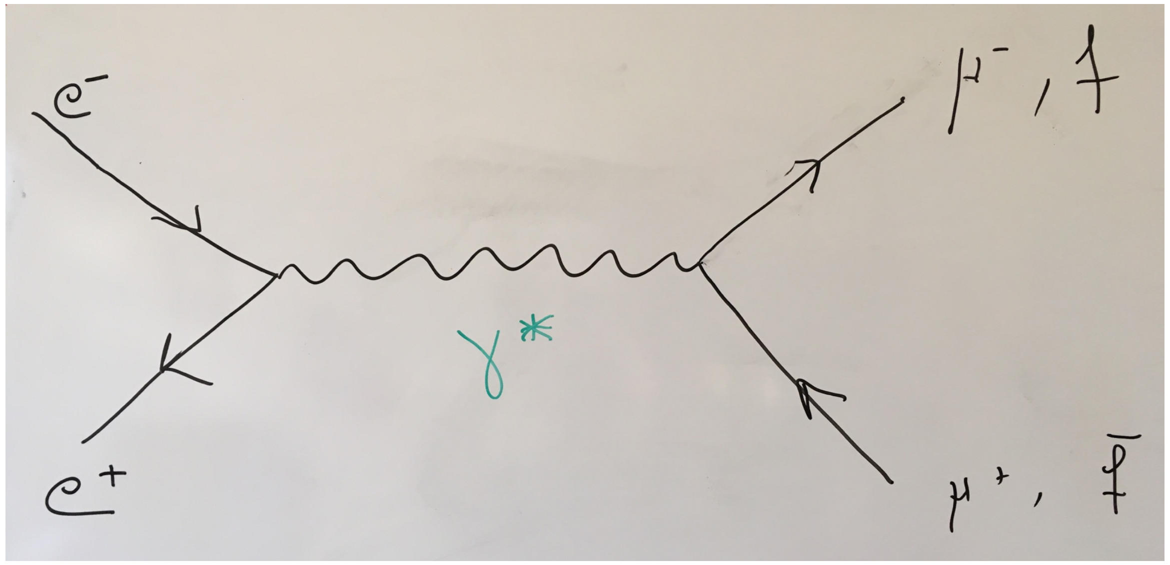
$$\mathcal{L} = R_\phi \times \frac{N_1\ N_2\ f\ n_{bunch}}{4\pi\sigma_x^*\sigma_y^*}$$

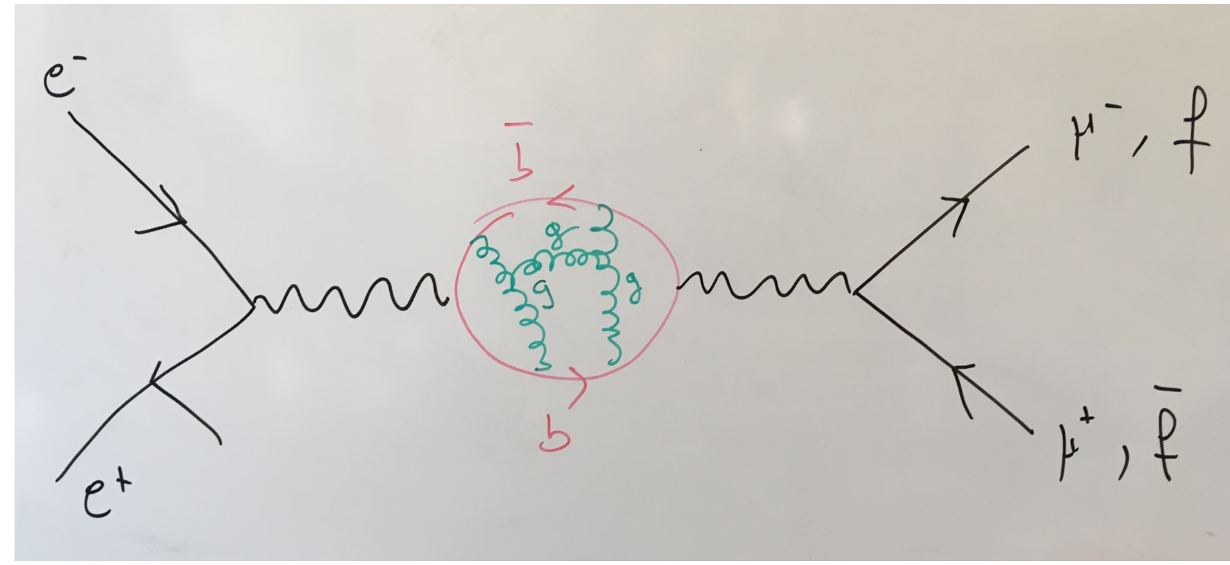
$$\mathcal{L} = R_\phi \times \frac{N_1\ N_2\ f\ n_{bunch}}{4\sqrt{\epsilon_x^*\beta_x^*\epsilon_y^*\beta_y^*}}$$

with $R_\phi = \frac{1}{\sqrt{1+\phi^2}}$ *and* $\phi = \frac{\theta_c\sigma_z}{2\sigma_x}$



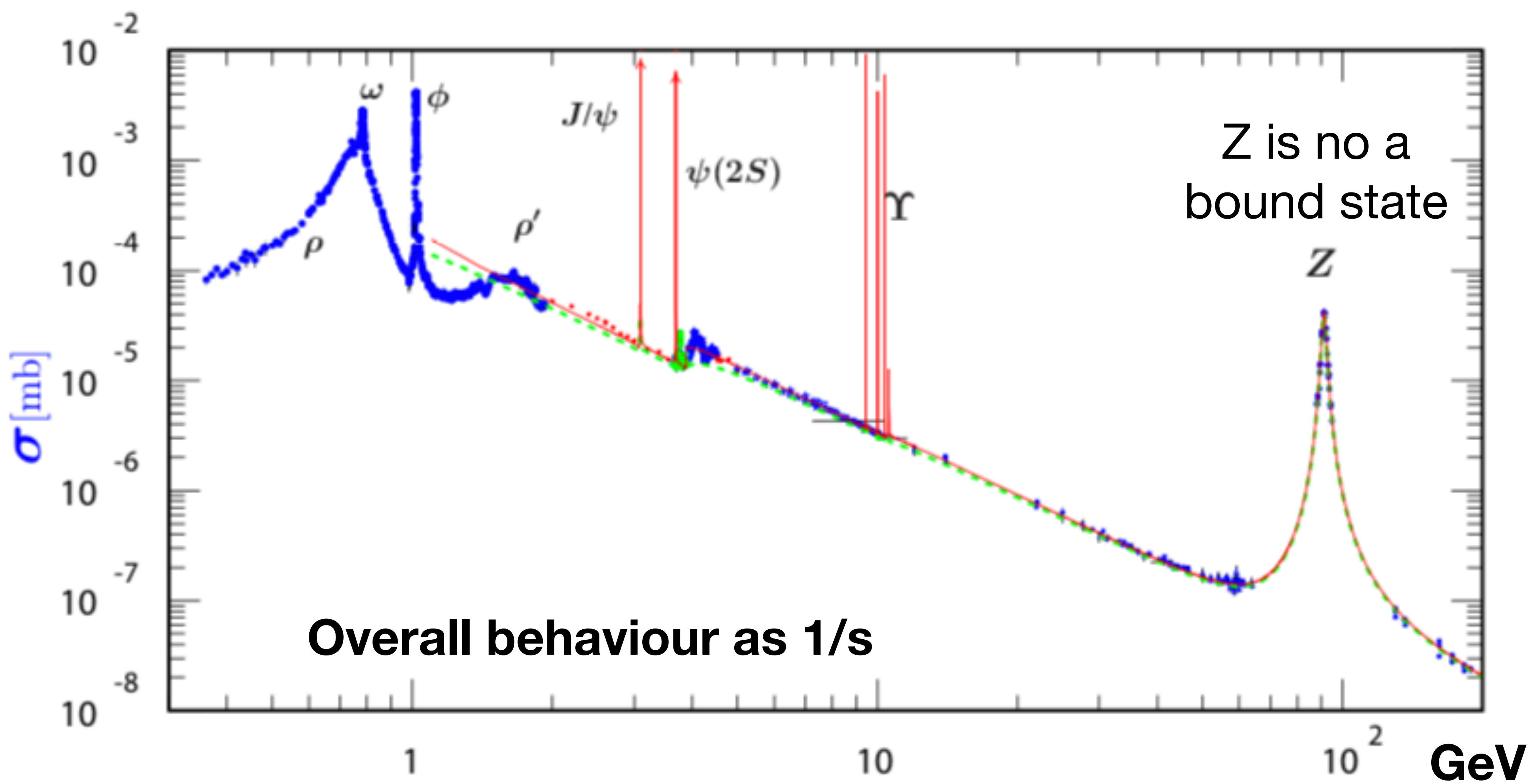
Resonances and e+e- cross section





LO: usually dominates the σ

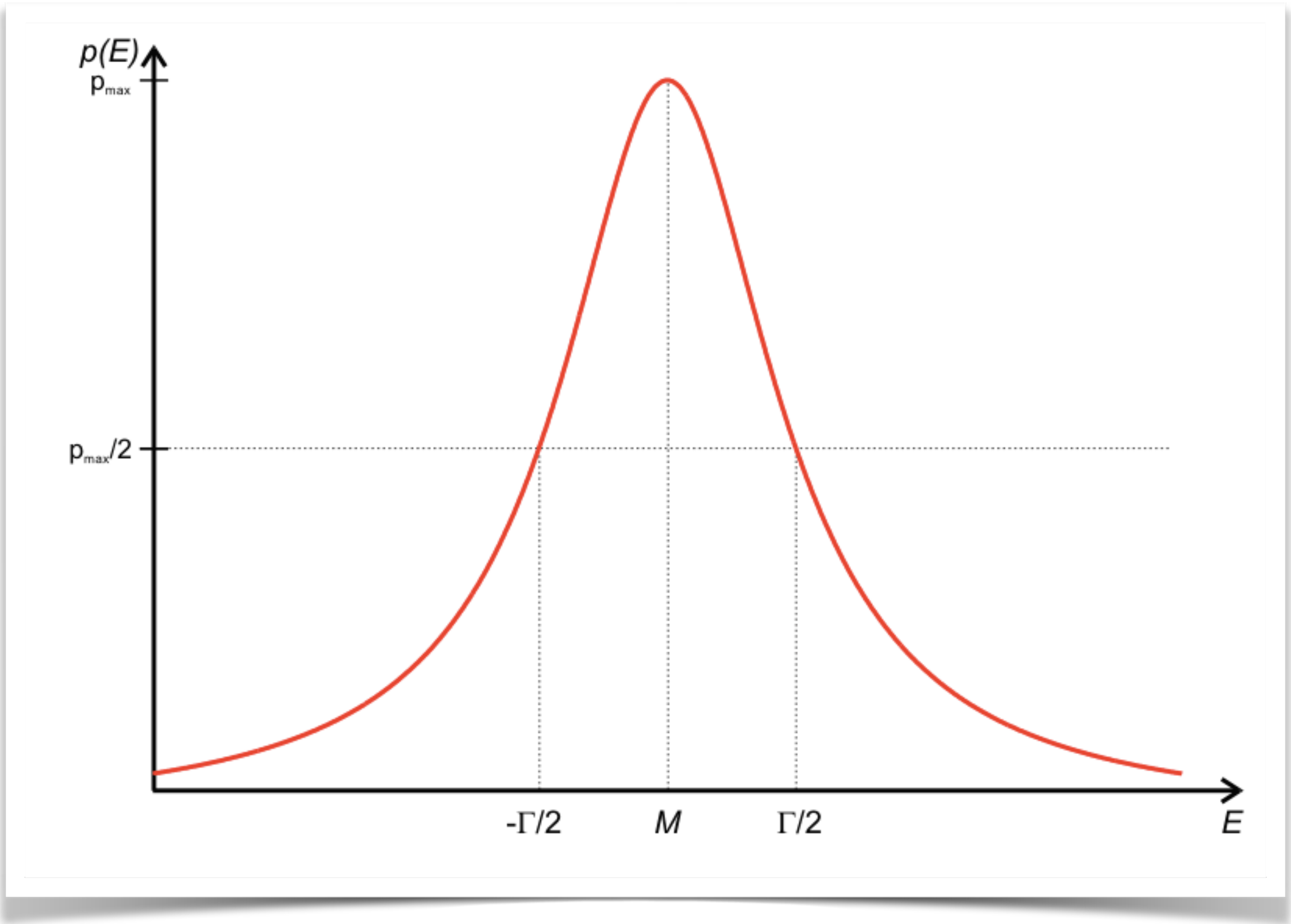
Loop correction: Bound state qq̄



Lifetime and width

- Particles stable at the detector level
  - ✓ γ stable
  - ✓ e stable
  - ✓ p stable
  - ✓ n : $c\tau = 2.6\ 10^8$ km
  - ✓ $\mu^\pm$: $c\tau = 658$ m
  - ✓ $\pi^\pm$: $c\tau = 7.8$ m
  - ✓ $K^\pm$: $c\tau = 3.7$ m
  - ✓ K_L : $c\tau = 15.3$ m
- Short-lived particles
  - ✓ K_S, Λ… : 10^{-10}s , $c\tau = O(1\text{cm})$
  - ✓ D mesons : $c\tau = O(100\ \mu\text{m})$
  - ✓ B mesons : $c\tau = O(500\ \mu\text{m})$

Total width



$$\frac{\mathrm{d}N}{\mathrm{d}m} = \frac{\Gamma/2}{(m-m_0)^2 + \Gamma^2/4}$$

Partial width and branching fraction

$$\Gamma_{tot} = \sum_i \Gamma_i(M \to \{f\}_i) \qquad \mathcal{B}(M \to \{f\}_i) = \frac{\Gamma_i}{\Gamma_{tot}}$$

$D^{\pm}$

$I(J^P) = \frac{1}{2}(0^-)$

$D^{\pm}$ MASS

The fit includes $D^{\pm}$, D^0, $D_s^{\pm}$, $D^{*\pm}$, D^{*0}, $D_s^{*\pm}$, $D_1(2420)^0$, $D_2^*(2460)^0$, and $D_{s1}(2536)^{\pm}$ mass and mass difference measurements.

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
1869.65± 0.05 OUR FIT					
1869.5 ± 0.4 OUR AVERAGE					
1869.53± 0.49±0.20	110 ± 15	ANASHIN	10A	KEDR	e^+e^- at $\psi(3770)$
1870.0 ± 0.5 ±1.0	317	BARLAG	90C	ACCM	π^- Cu 230 GeV
1869.4 ± 0.6		[1] TRILLING	81	RVUE	e^+e^- 3.77 GeV

$D^{\pm}$ MEAN LIFE

Measurements with an error $> 100 \times 10^{-15}$ s have been omitted f
Listings.

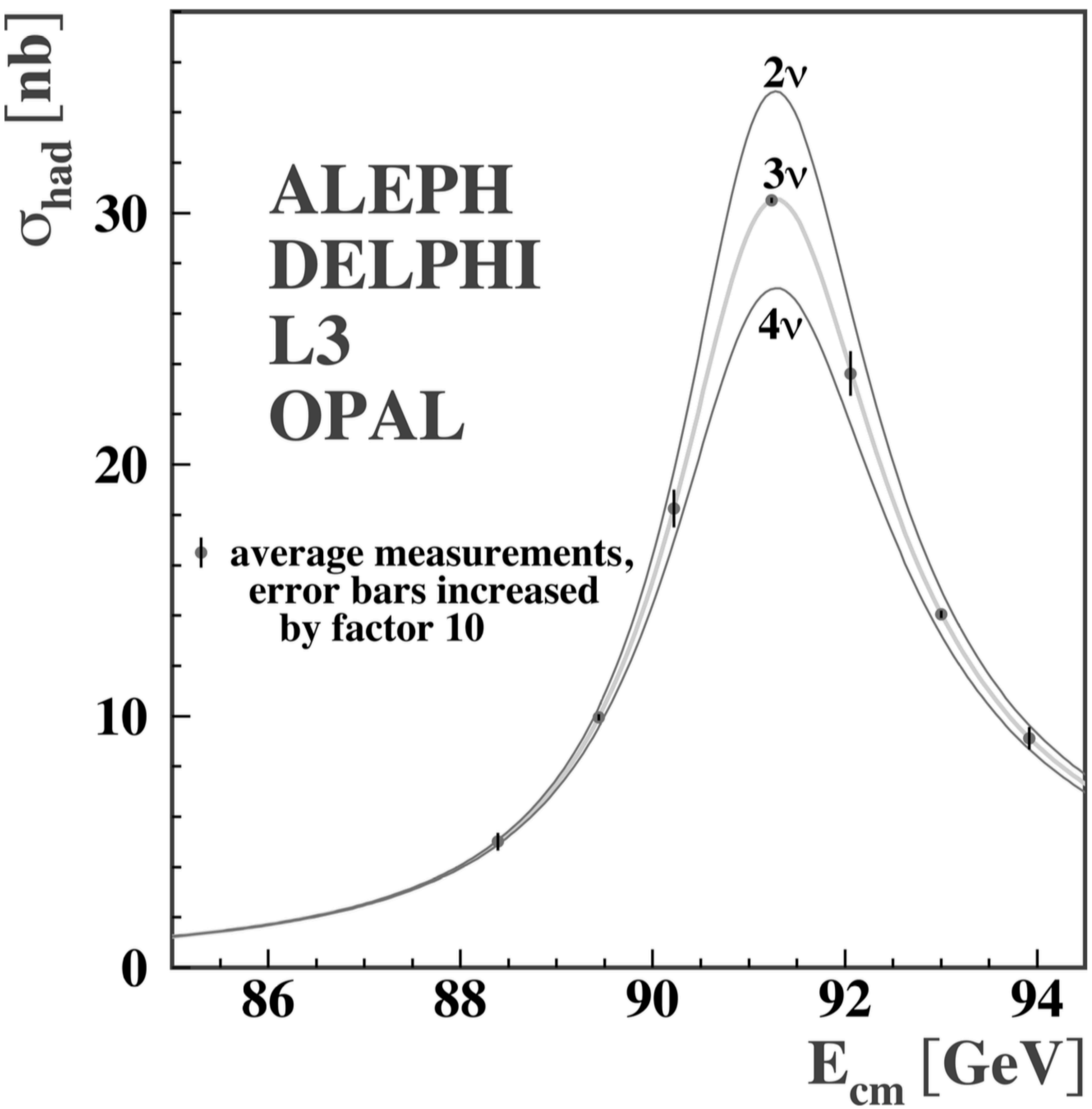
VALUE (10^{-15} s)	EVTS	DOCUMENT ID		TECN	COMME
1040 ± 7 OUR AVERAGE					
1039.4± 4.3± 7.0	110k	LINK	02F	FOCS	γ nucle
1033.6±22.1 $^{+\ 9.9}_{-12.7}$	3.7k	BONVICINI	99	CLEO	e^+e^-
1048 ±15 ±11	9k	FRABETTI	94D	E687	$D^+ \to$

Hadronic modes with a $\overline{K}$ or $\overline{K}K\overline{K}$

Γ_{41}	$K_S^0\pi^+$		(1.47±0.08) %
Γ_{42}	$K_L^0\pi^+$		(1.46±0.05) %
Γ_{43}	$K^-2\pi^+$	[a]	(8.98±0.28) %
Γ_{44}	$(K^-\pi^+)_{S-\text{wave}}\pi^+$		(7.20±0.25) %
Γ_{45}	$\overline{K}_0^*(700)^0\pi^+$, $\overline{K}_0^*(700) \to K^-\pi^+$		
Γ_{46}	$\overline{K}_0^*(1430)^0\pi^+$,	[b]	(1.19±0.07) %

Partial width cont'd

$$\Gamma_Z = \Gamma(Z \to \text{had}) + 3 \times \Gamma(Z \to \ell^+\ell^-) + N_\nu \times \Gamma(Z \to \nu\bar{\nu})$$

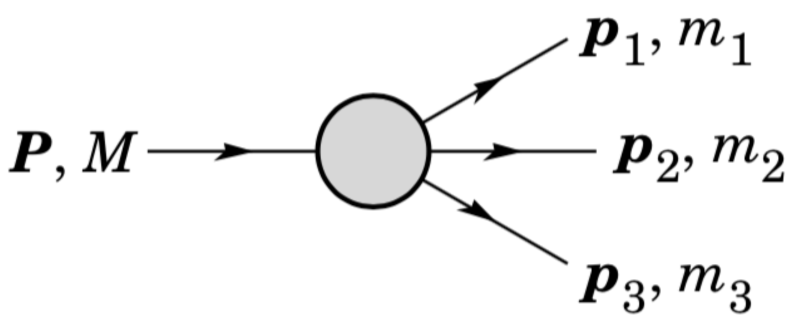


Measurement of the number of neutrino flavours at LEP

NB:
this is measured with the hadronic decays of the Z only, i.e. the width in hadronic decay is the total Γ_Z and not only Γ_{had}.

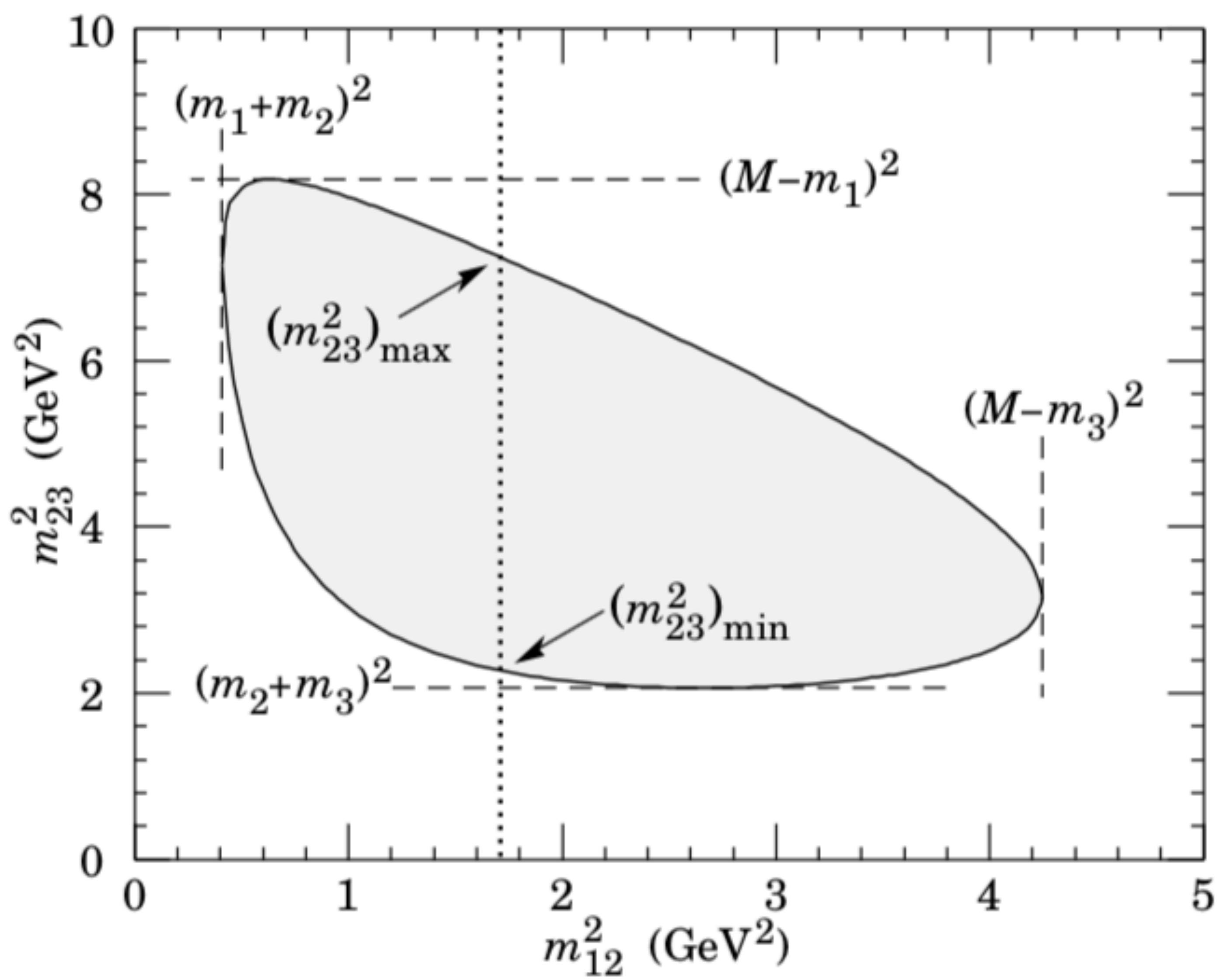
Partial widths are not a measurable quantity only the total width is.

Dalitz plot



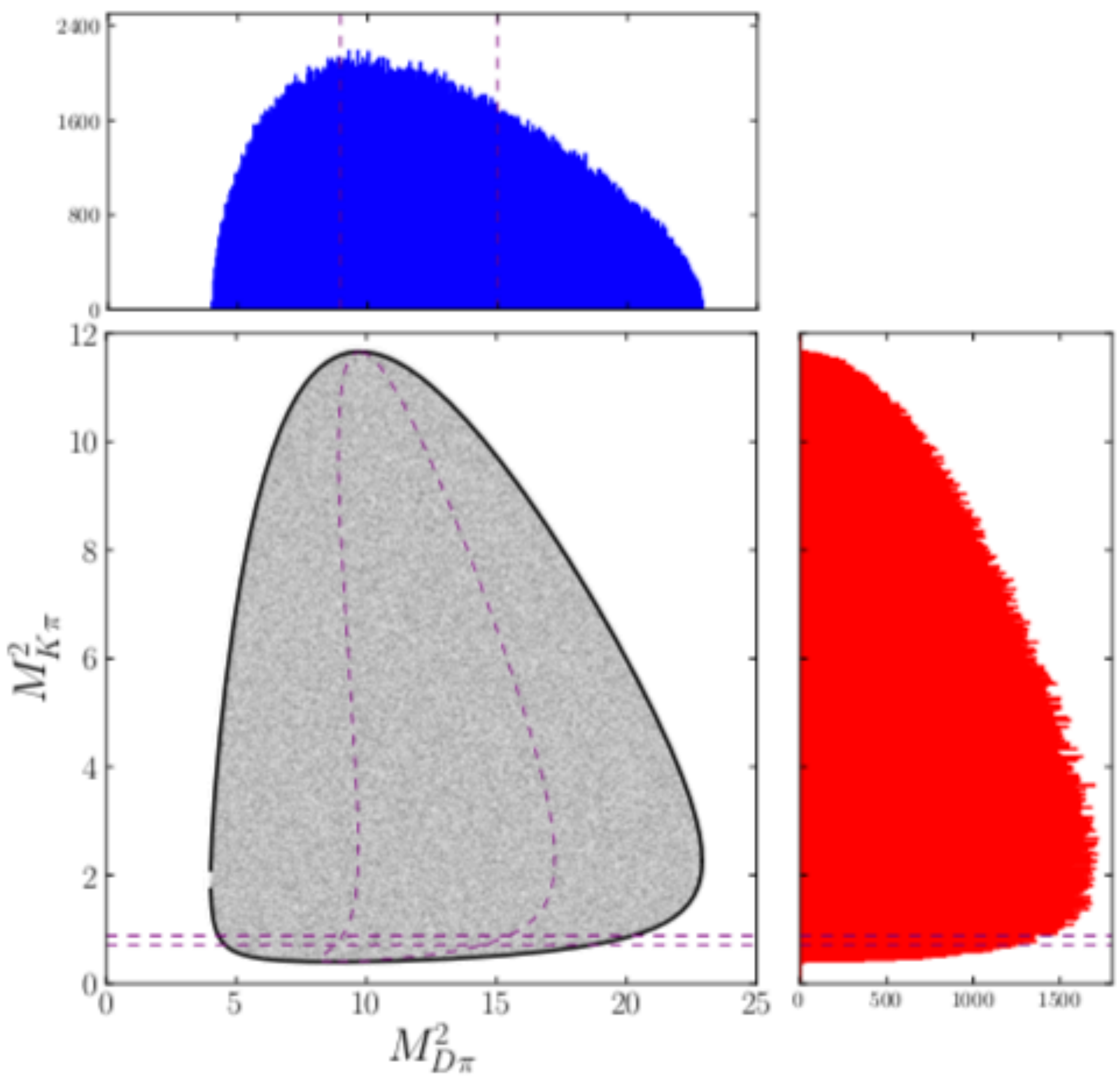
$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{8M}\overline{|\mathscr{M}|^2}\,dE_1\,dE_3$$

$$= \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|\mathscr{M}|^2}\,dm_{12}^2\,dm_{23}^2$$

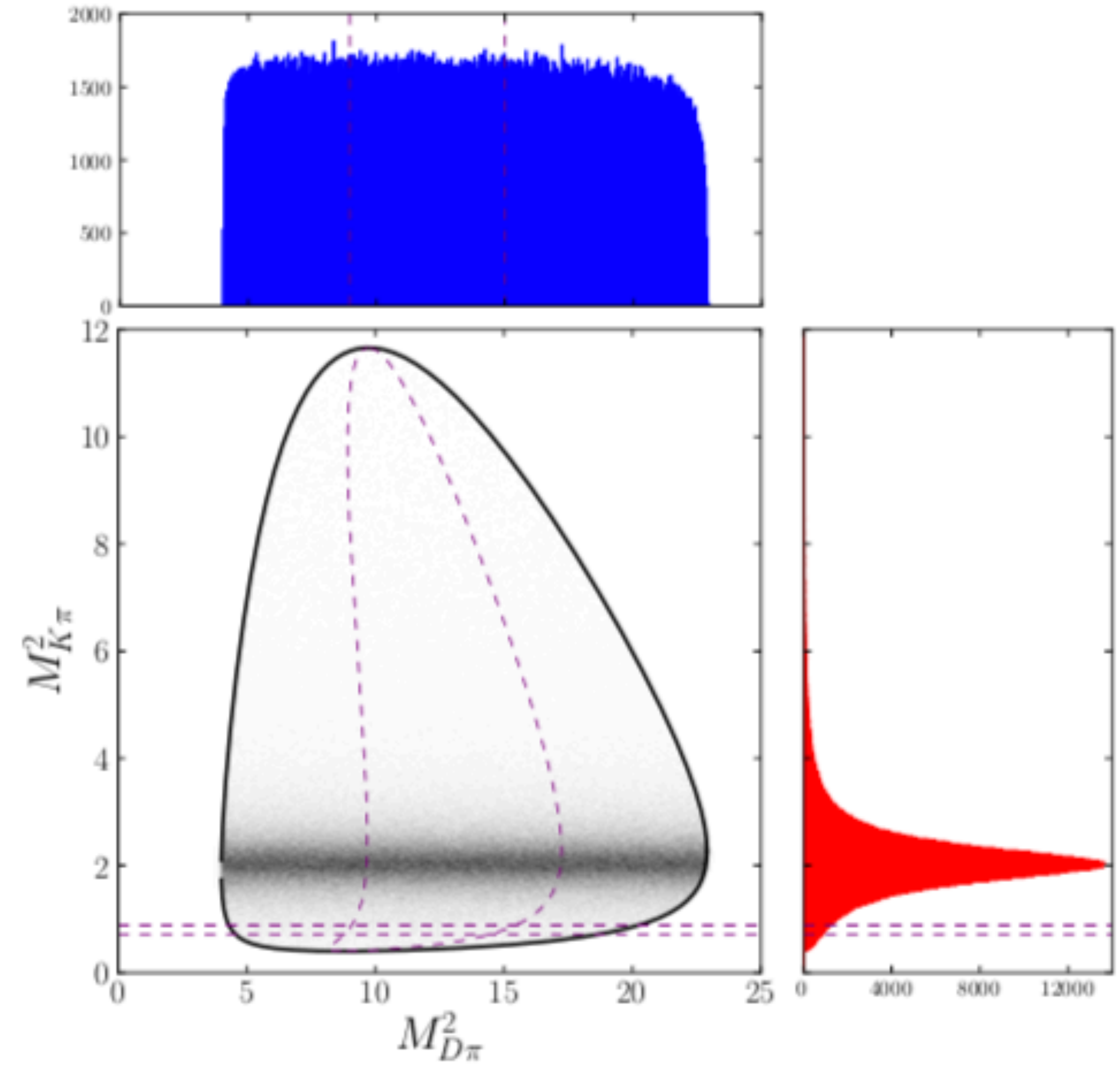


$B \to D K \pi$

Pure phase space



Actual plot

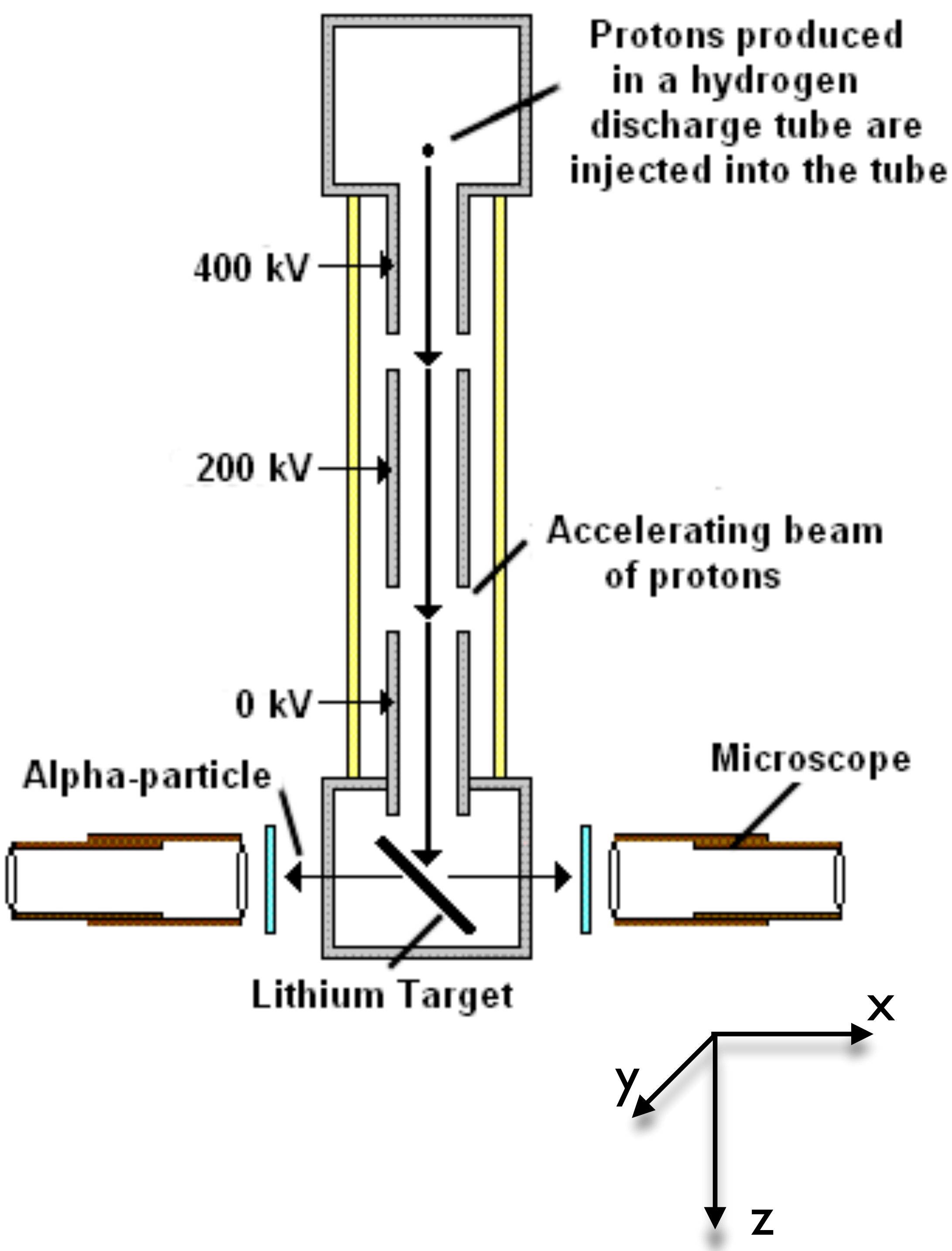


Most of the decay pass through the D* resonance

Complements

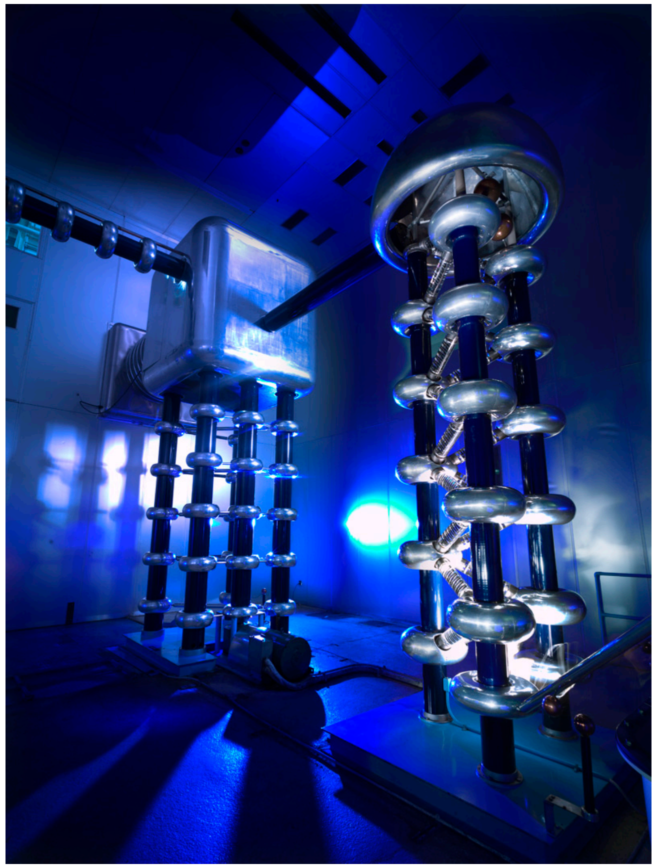
Cockcroft Walton experiment

- 1932, first transmutation of a nuclei
- Smash $E_{kin}[p] \approx 800$ keV on a Lithium target
- $Li^7 + p \rightarrow \alpha + \alpha$
  - ✓ $M[Li^7] = 6535.4$ MeV
  - ✓ $M[p] = 938.3$ MeV
  - ✓ $M[\alpha] = 3728.4$MeV



Cockcroft Walton experiment

- Compute the total mass of the initial system ?
- Kinetic energy of p in the center of mass
- Justify that the center-of-mass frame is approx. the lab frame
- General 2-body decay
  - ✓ M → p1 + p2
  - ✓ Momentum of |p| of p1 and p2 in M rest-frame
  - ✓ Apply that to M → 2 alphas
    - ✓ Energy of alpha in the center of mass
    - ✓ Kinetic energy of each alpha in the lab ?
  - ✓ Solve in 2 lines with the conservation of energy in the center-of-mass frame



Cockcroft-Walton generator, Fermilab

Cockcroft Walton experiment

Disintegration of Lithium by Swift Protons

In a previous letter to this journal [1] we have described a method of producing a steady stream of swift protons of energies up to 600 kilovolts by the application of high potentials, and have described experiments to measure the range of travel of these protons outside the tube. We have employed the same method to examine the effect of the bombardment of a layer of lithium by a stream of these ions, the lithium being placed inside the tube at 45° to the beam. A mica window of stopping power of 2 cm. of air was sealed on to the side of the tube, and the existence of radiation from the lithium was investigated by the scintillation method outside the tube. The thickness of the mica window was much more than sufficient to prevent any scattered protons from escaping into the air even at the highest voltages used.

On applying an accelerating potential of the order of 125 kilovolts, a number of bright scintillations were at once observed, the numbers increasing rapidly with voltage up to the highest voltages used, namely, 400 kilovolts. At this point many hundreds of scintillations per minute were observed using a proton current of a few microamperes. No scintillations were observed when the proton stream was cut off or when the lithium was shielded from it by a metal screen. The range of the particles was measured by introducing mica screens in the path of the rays, and found to be about eight centimetres in air and not to vary appreciably with voltage.

To throw light on the nature of these particles, experiments were made with a Shimizu expansion chamber, when a number of tracks resembling those of α-particles were observed and of range agreeing closely with that determined by the scintillations. It is estimated that at 250 kilovolts, one particle is produced for approximately 10^9 protons.

The brightness of the scintillations and the density of the tracks observed in the expansion chamber suggest that the particles are normal α-particles. If this point of view turns out to be correct, it seems not unlikely that the lithium isotope of mass 7 occasionally captures a proton and the resulting nucleus of mass 8 breaks into two α-particles, each of mass four and each with an energy of about eight million electron volts. The evolution of energy on this view is about sixteen million electron volts per disintegration, agreeing approximately with that to be expected from the decrease of atomic mass involved in such a disintegration.

Experiments are in progress to determine the effect on other elements when bombarded by a stream of swift protons and other particles.

J. D. Cockcroft.
E. T. S. Walton.

Cavendish Laboratory,
Cambridge,
April 16.

[1] Nature, **129**, 242, Feb. 13, 1932.

No. 3261, Vol. 129]

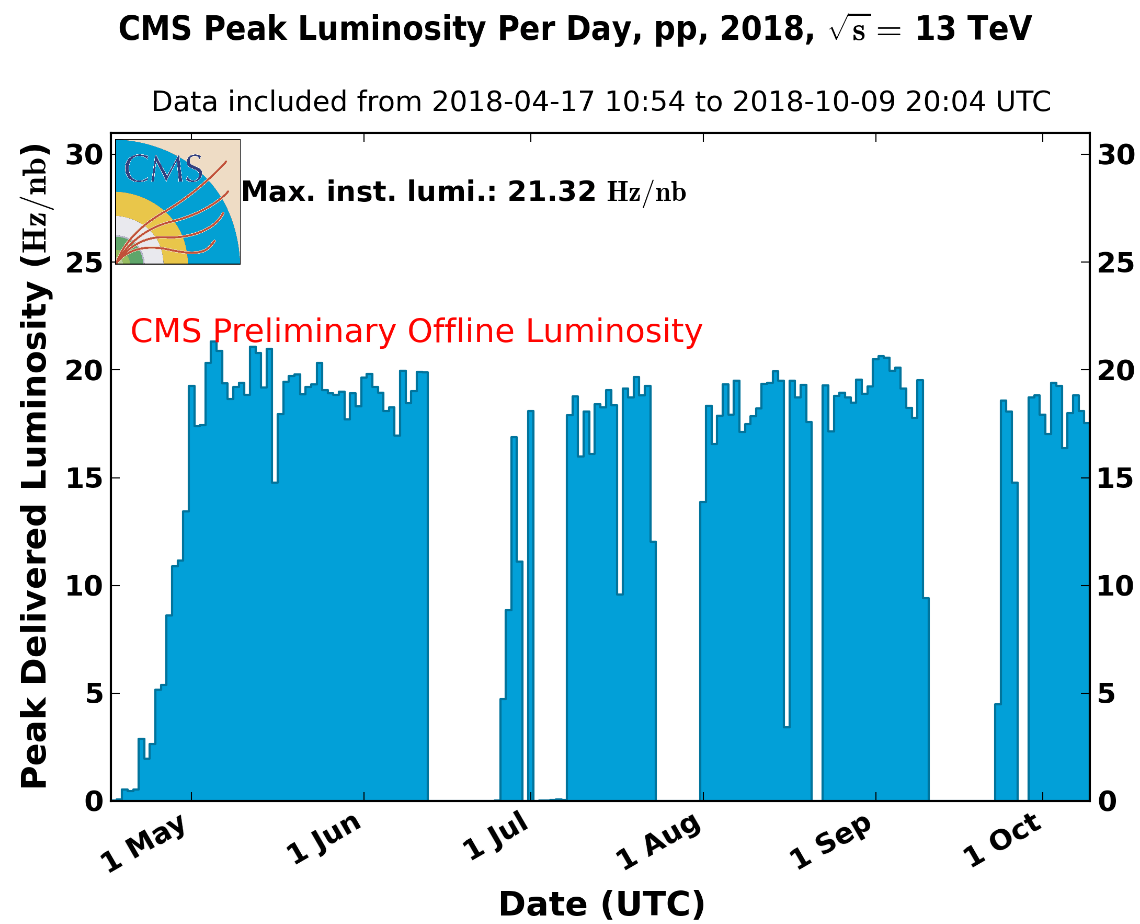
Nobel 1951

Known as the first experimental proof that mass can be converted into kinetic energy

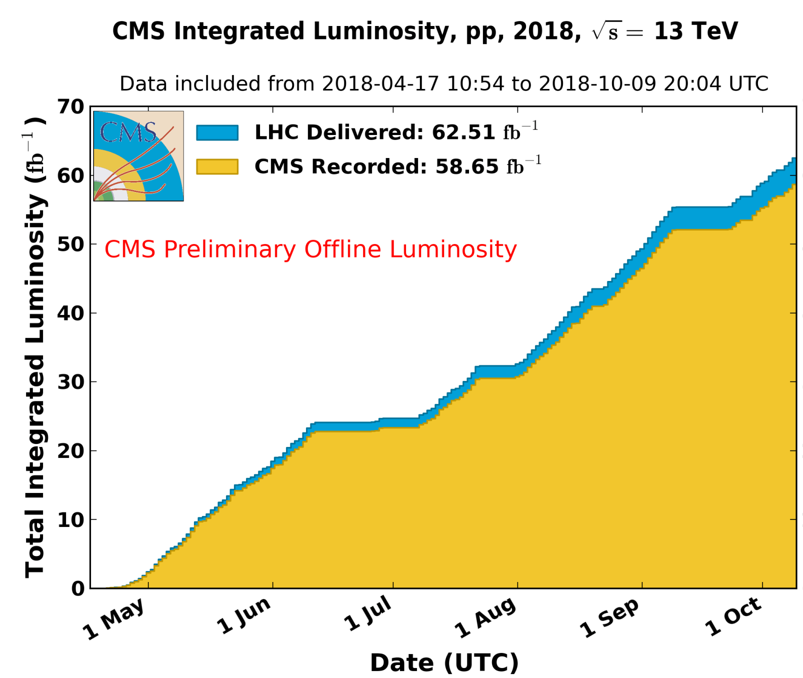
Also the first actual alchemists !

LHC parameters

	design	June 2012
Beam energy	7 TeV	4 TeV
transv. norm. emittance	3.75 μm	2.6 μm
beta*	0.55 m	0.6 m
IP beam size	16.7 μm	19 μm
bunch intensity	1.15×10^{11}	1.48×10^{11}
luminosity / bunch	3.6×10^{30} cm^{-2}s^{-1}	1.1×10^{30} cm^{-2}s^{-1}
# bunches	2808	1380
bunch spacing	25 ns	50 ns
beam current	0.582 A	0.369 A
rms bunch length	7.55 cm	≥9 cm
crossing angle	285 μrad	290 μrad
"Piwinski angle"	0.64	≥0.69
luminosity	10^{34} cm^{-2}s^{-1}	6.8×10^{33} cm^{-2}s^{-1}



Find the instantaneous luminosity and convert it in nb^{-1} s^{-1}



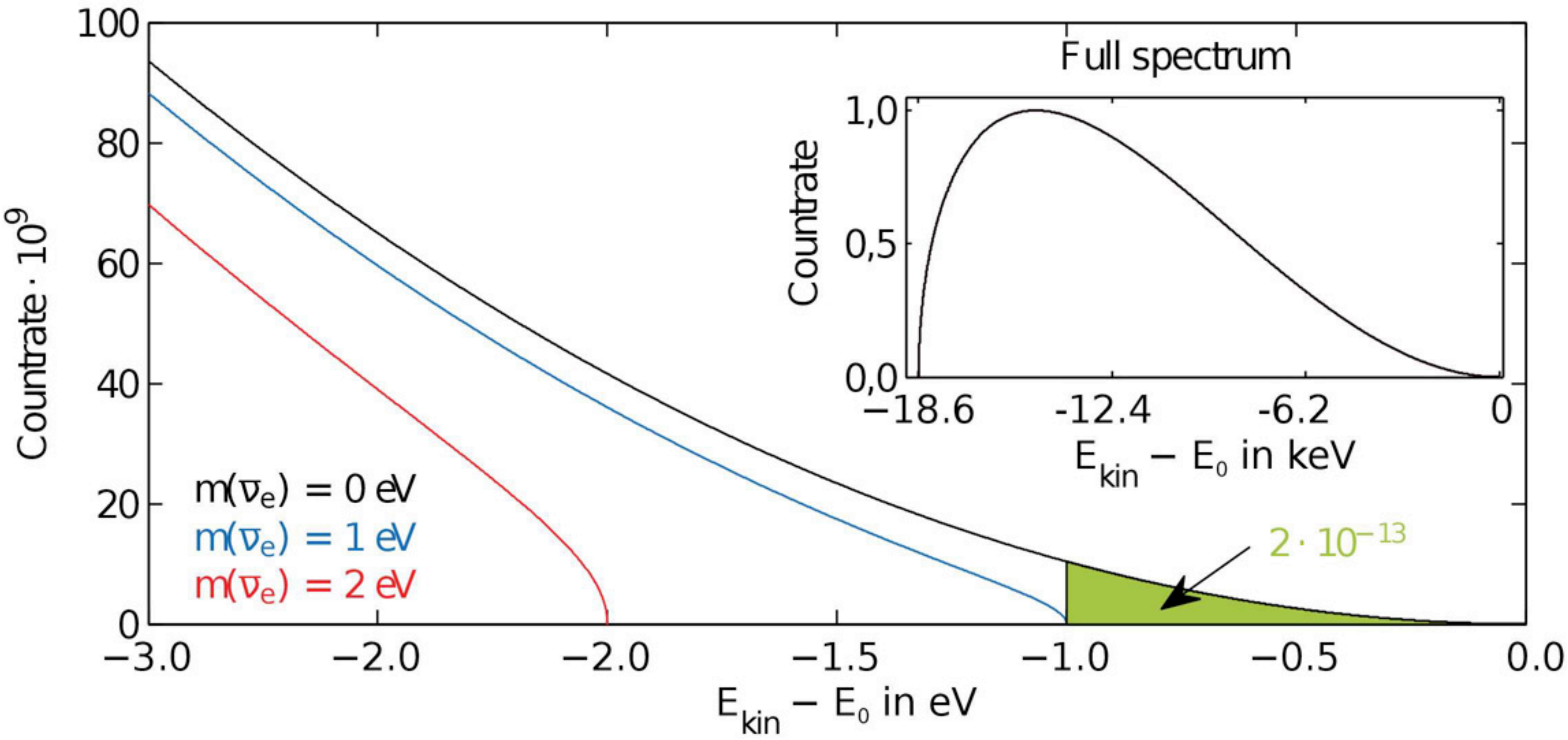
Kurie plot

- Consider the reaction: $n \to p\, e^- \nu_e$
- From the maximum energy released in the reaction justify
  - ✓ $E_{p/e} = m_{p/e} + K_{p/e}$ and that $K_p << K_e$
- Integrate the phase-space over d^3p_p
- Then over $dp_{\nu e}\, \delta(E_f - m_n)$
- Find

$$\frac{1}{G_F^2 p_e E_e} \frac{d\Gamma(n \to\to p\beta^- \bar{\nu}_e)}{dK_e} \propto (Q - K_e)^2 \times \sqrt{1 - \frac{m_\nu^2}{(Q - K_e)^2}}$$

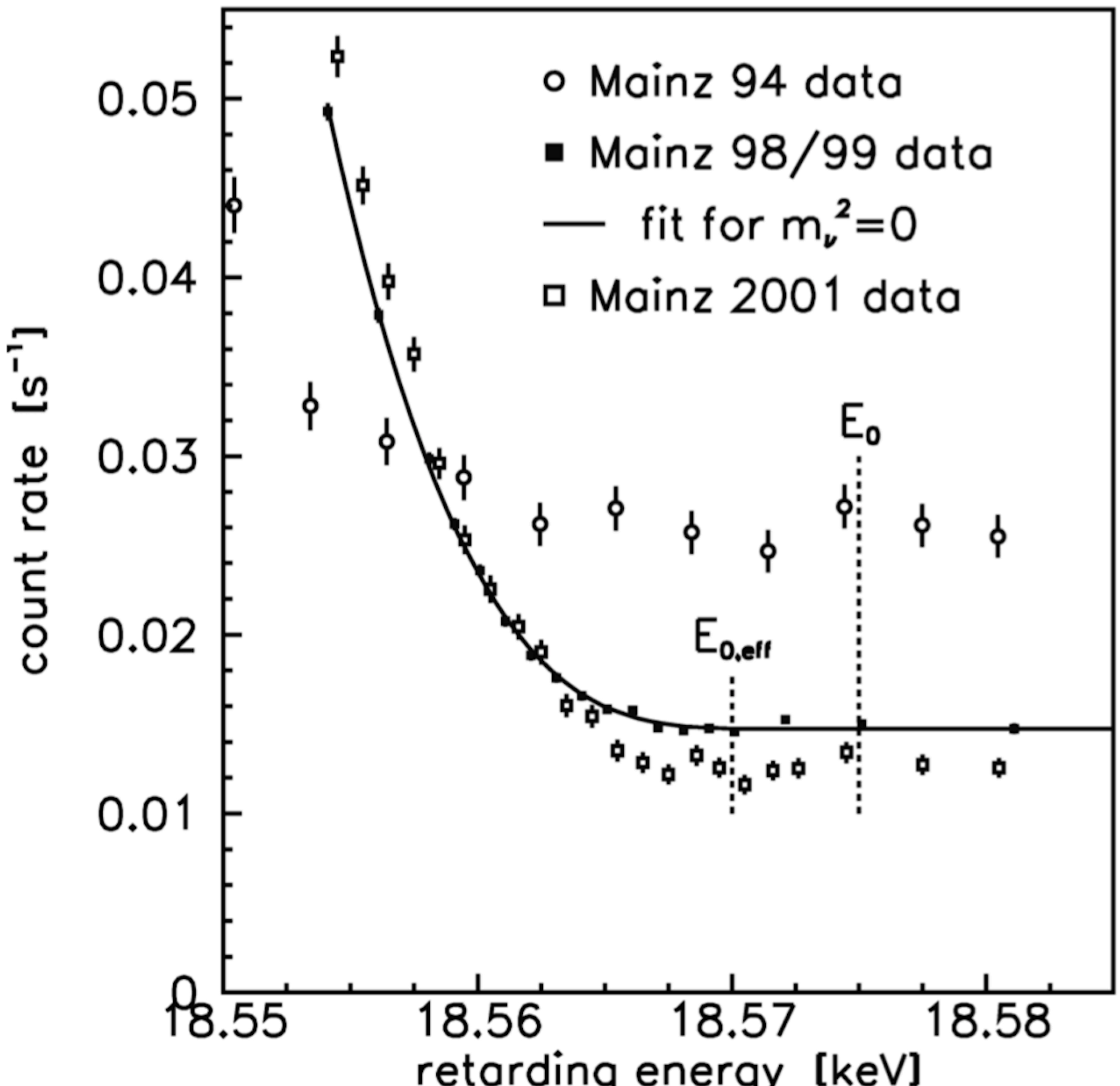
$$Q = m_n - m_p - m_e$$

Kurie plot



Full spectrum
Countrate
1,0
0,5
0,0
−18.6
-12.4
-6.2
0
$E_{kin} - E_0$ in keV
Countrate · 10^9
100
80
60
40
20
0
−3.0
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
$E_{kin} - E_0$ in eV
$m(\bar{\nu}_e) = 0$ eV
$m(\bar{\nu}_e) = 1$ eV
$m(\bar{\nu}_e) = 2$ eV
$2 \cdot 10^{-13}$

Constraint on neutrino mass



Mν< 2eV