NPAC
Particle Physics
Course 2 – Symmetries

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Program

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   3.1 Flavour SU(2) - Isospin
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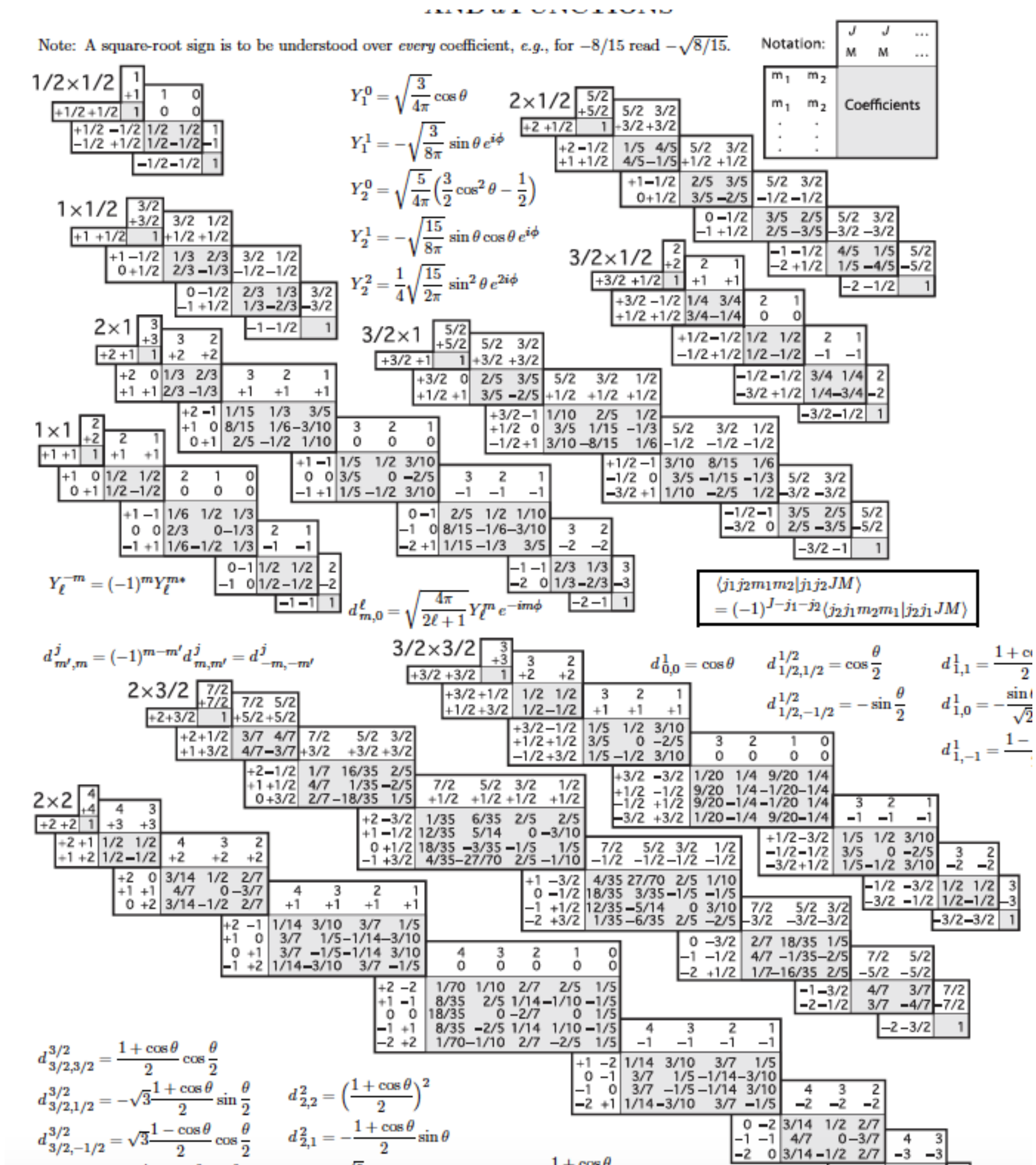
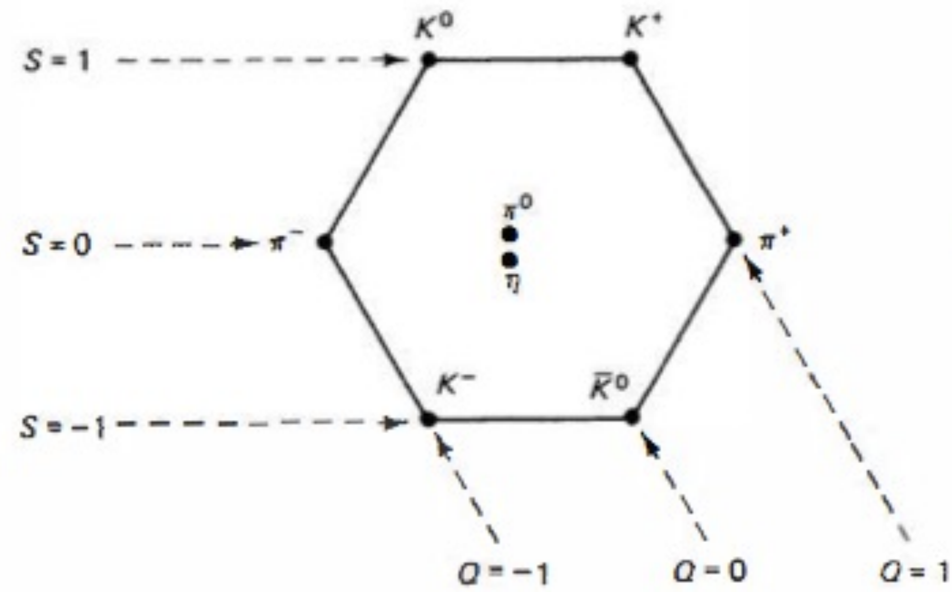
Clebsch-Gordon coefficients



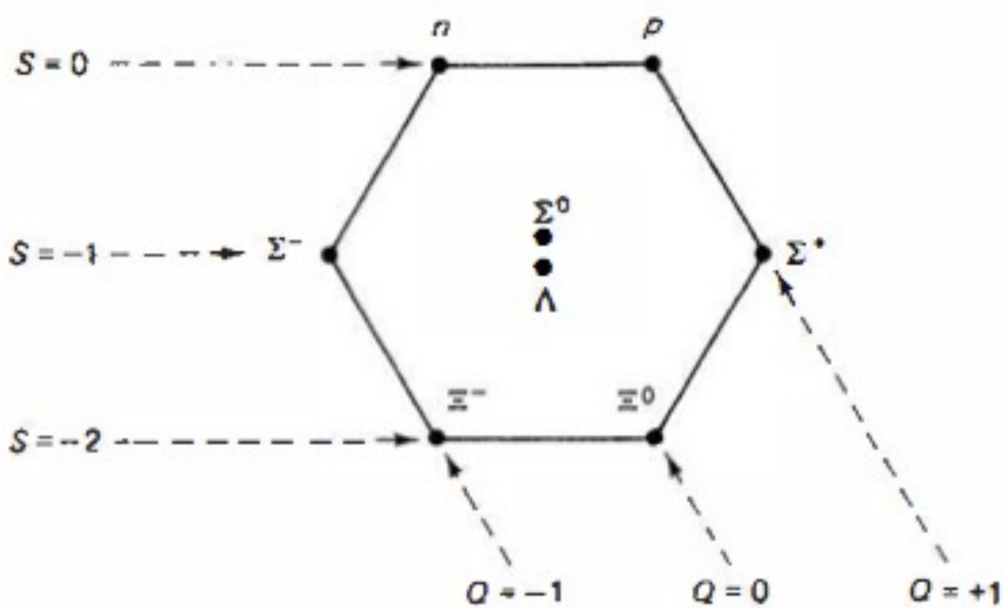
Meson and baryon octets

Another meson octet
($J^P=1^-$)
$\boldsymbol{\pi}\rightarrow\boldsymbol{\rho}$ (same quark content)
$\boldsymbol{\eta}\rightarrow\boldsymbol{\phi}$ (pure $s\bar{s}$ state)
$K\rightarrow K^*$ (same quark content)



The meson octet

$J^P=0^+$



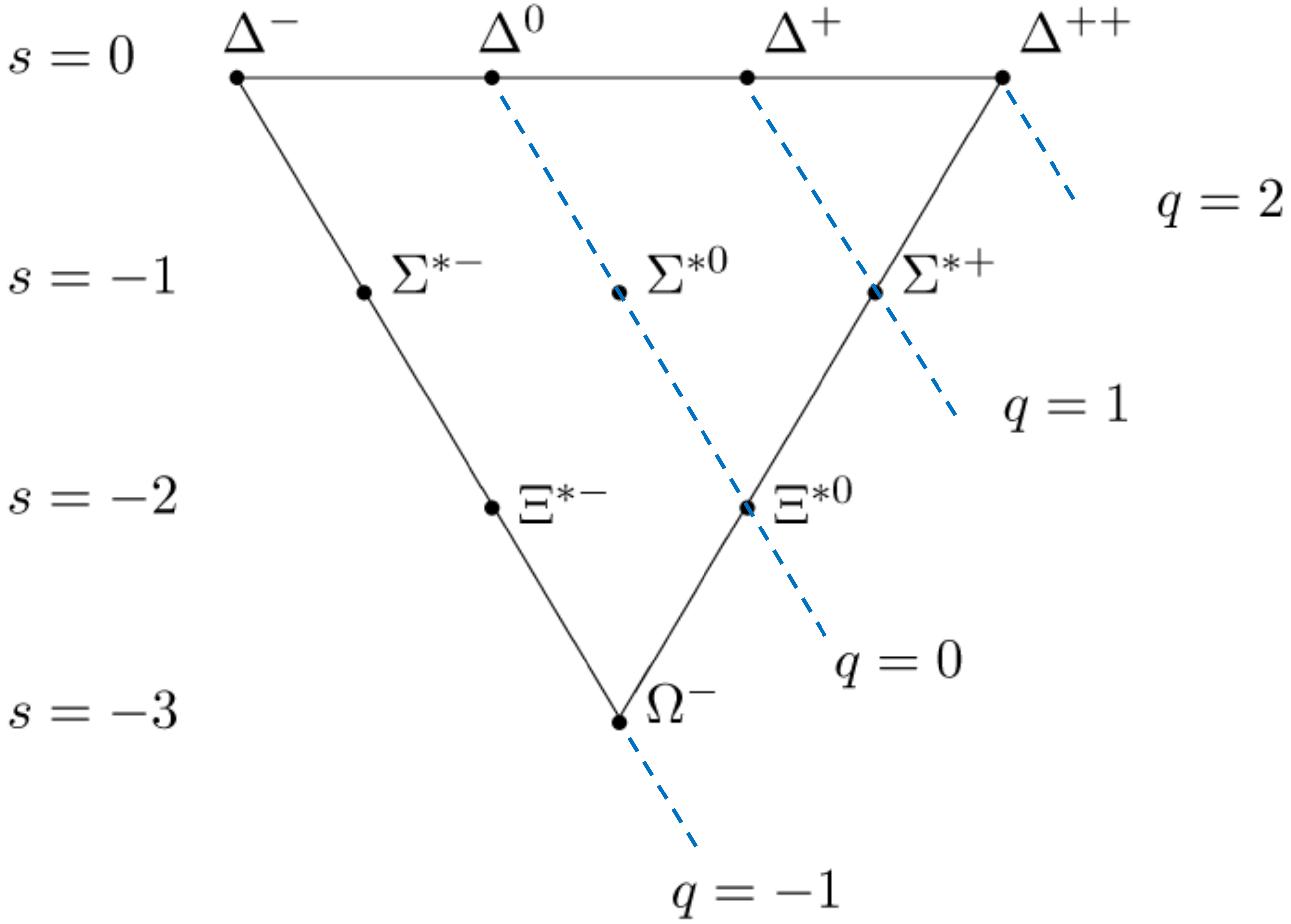
The baryon octet

$J^P=½^+$

Remark: we will discuss the parity (P) in the next paragraph

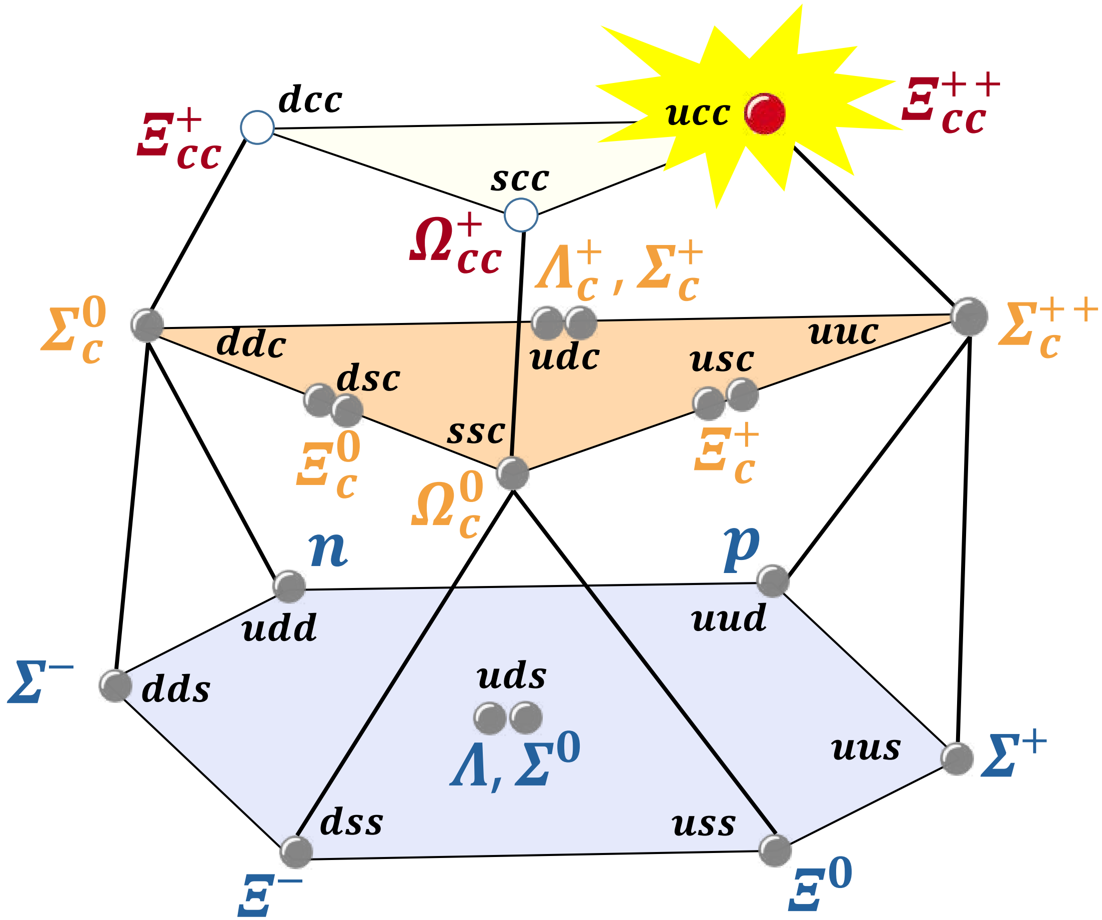
Baryon decuplet

$J^P = 3/2^+$



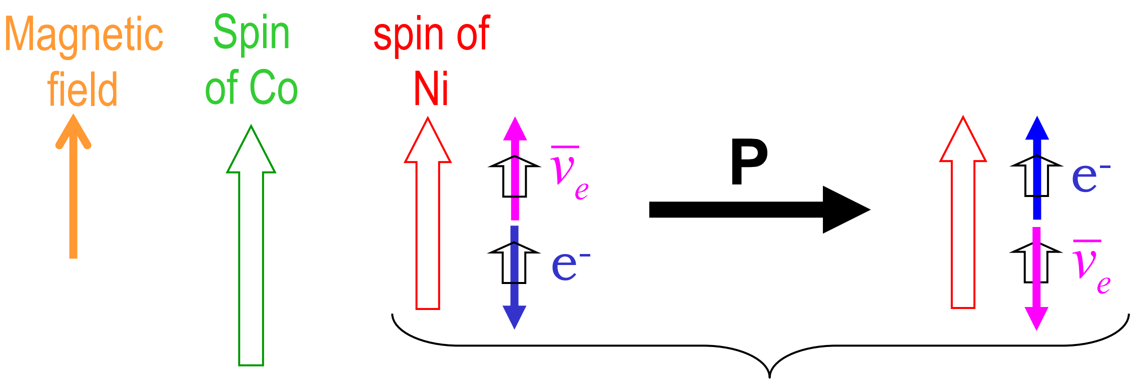
SU(4) 20-plet

Ξ_{cc}^{++} discovered in 2017 (LHCb at CERN), Ξ_{cc}^{+} and Ω_{cc}^{+} still missing.



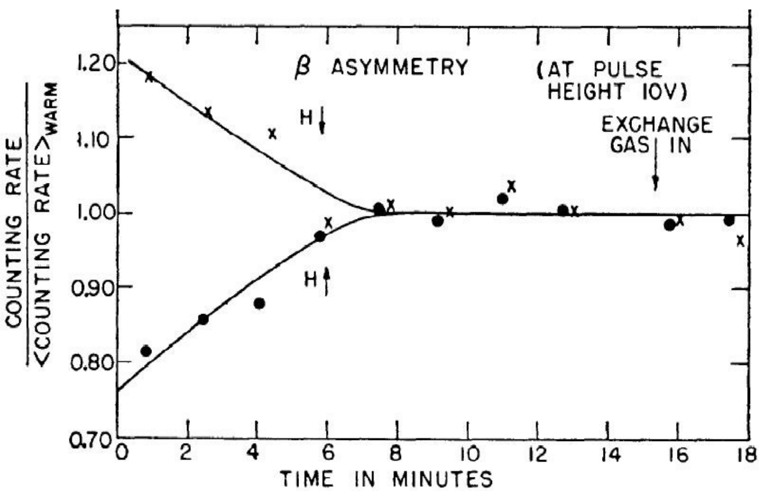
The Wu et al. experiment

- Decay : $\mathrm{Co}^{60}\ (J=5) \rightarrow \mathrm{Ni}^{60*}\ (J=4)\, e^{-}\,\overline{\nu}_e$ $\qquad \mathrm{n} \rightarrow \mathrm{p}\, e^{-}\,\overline{\nu}_e$
- The experiment:
  - The spins of cold Co^{60} atoms are aligned in a magnetic field
  - Detection of the e^- (knowledge of the direction of its momentum)



if P is conserved, these 2 configurations are equally probable

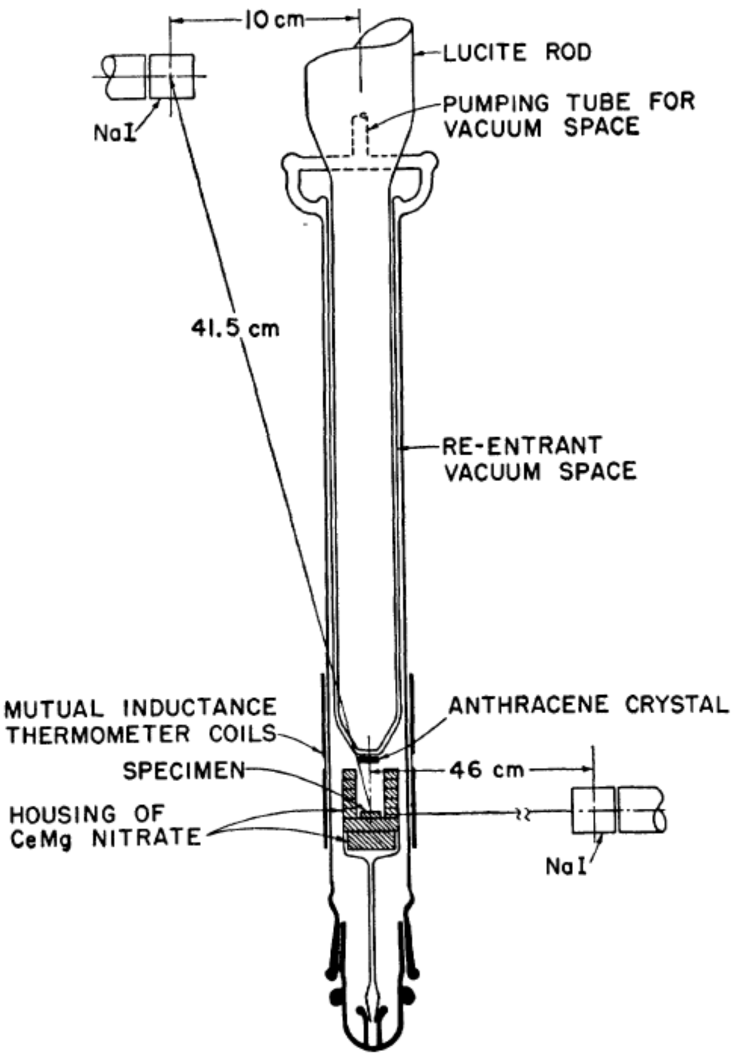
Result: e^- preferentially emitted in the opposite direction to that of spin of the Co (asymmetry!)



Wu et al. also swap the direction of the magnetic and repeat the experiment. The result is still the same.

Parity is violated by weak int.

The Wu et al. experiment: scheme of apparatus



4.3 G-Parity

- It is a generalization of charge conjugation, applicable to isospin multiplets and systems of particles.
- The operator G is defined as:

$$G = C\exp(i\pi I_2)$$

charge conjugation op. 2nd isospin component

(180° rotation about the 2nd isospin axis followed by a charge conjugation).

- The G-parity (eigenvalue) of particles belonging to isospin singlets and triplets is:

$$\eta_G = \eta_C(-1)^I$$

Eigenvalue of the charge conjugation of the (single!) C-eigenstate of the multiplet

Isospin of the multiplet (0 or 1)

Example: $\eta_G(\pi^+) = \eta_C(\pi^0)(-1)^1 = (+1)(-1) = -1$

(confirm with the PDG)

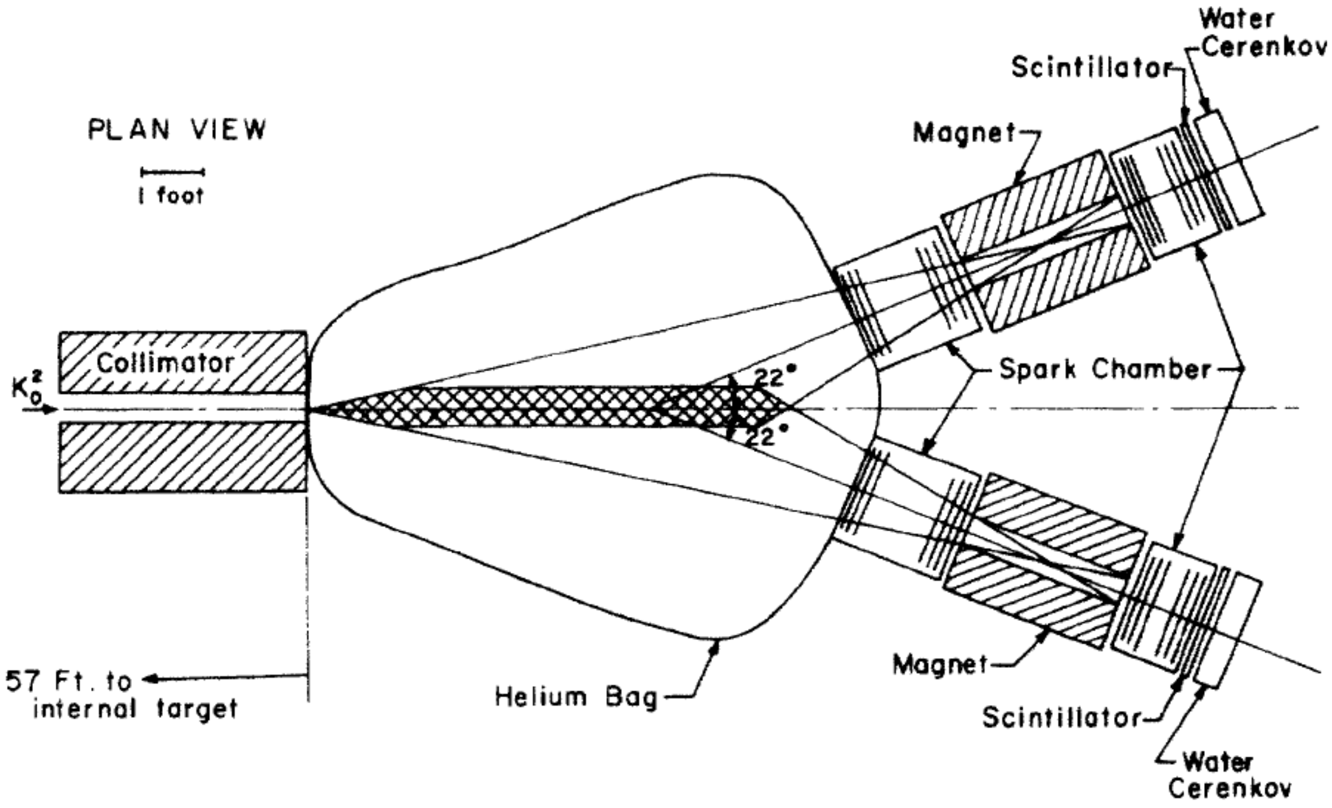
- G-parity is a multiplicative quantum number.

- Strong interaction conserves both C and the isospin.
  ➔ If a process is forbidden by charge conjugation, other processes obtained by isospin rotation are also forbidden.
  Note that this is valid even though the charge conjugation is not defined for certain particles in the “isospin-rotated” process.
  $\Rightarrow$ **G-Parity is conserved by the strong interaction**

- This is an illustration of the fact that the SU(2) symmetry does not consist only in an exchange between two states. It is a symmetry with respect to rotation in the space of the two states.
- Some “non-observations” of processes are easily explainable by G-parity violation (and not in other ways).

  Example: non observation of $\rho \to \pi\pi\pi$ by strong interaction.

Cronin and Fitch experiment: scheme of apparatus



Cronin and Fitch experiment: results

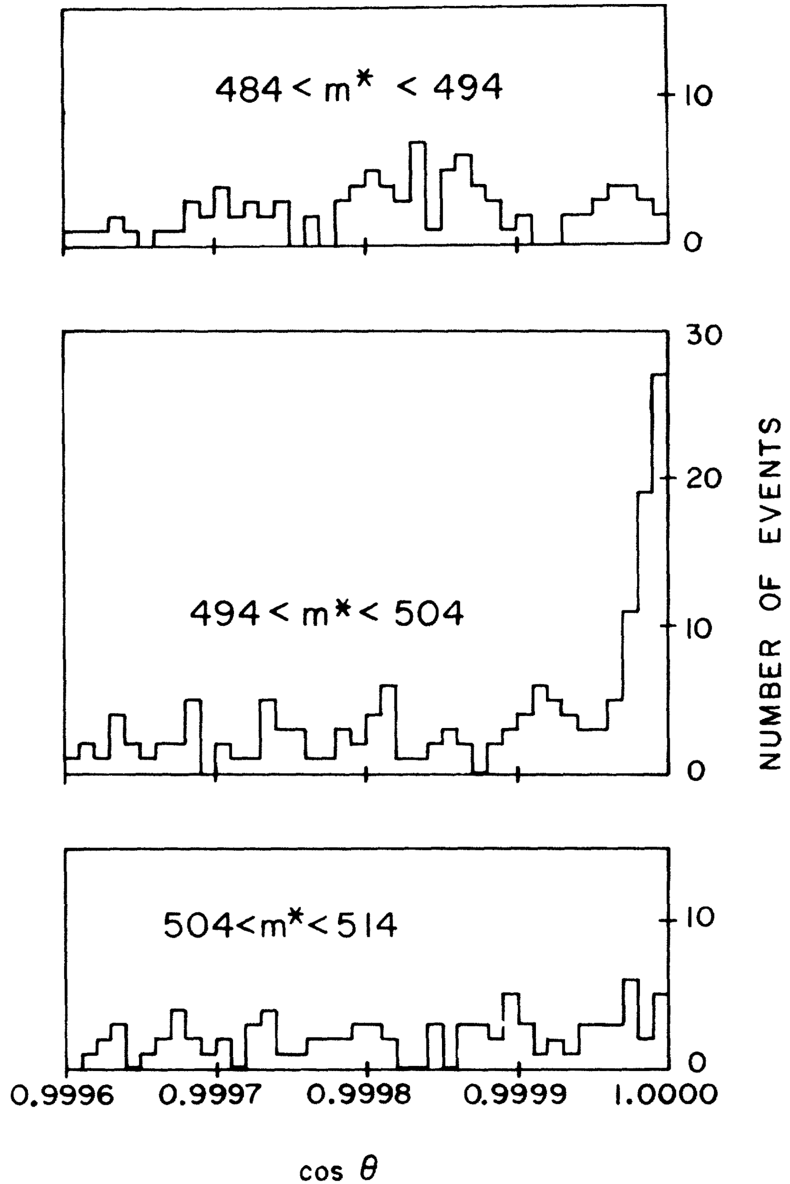


FIG. 3. Angular distribution in three mass ranges for events with $\cos\theta > 0.9995$.

Flavor oscillations in the neutral kaons system

(case with no CP violation) – we will see this in the CPV chapter

- Amplitude of an instable particle (e.g. K_S): $$a_s(t) = a_s(0) e^{-\left(\frac{im_s}{\hbar}\right)t} e^{-\left(\frac{\Gamma_s}{2\hbar}\right)t}$$

describes "mass" — "lifetime" (exp. law) with $\Gamma = \hbar/\tau$

- Probability:

$$\Gamma(t) = a_s(t) a_s^*(t) = a_s(0) a_s^*(0) e^{-\left(\frac{\Gamma_s}{\hbar}\right)t} = \Gamma(0) e^{-\left(\frac{\Gamma_s}{\hbar}\right)t}$$

- For the K^0- $\overline{K}^0$ system:

$$K_S: \quad a_S(t) = a_S(0) e^{-\left(\frac{\Gamma_S}{2\hbar} + \frac{im_S}{\hbar}\right)t}$$

$$K_L: \quad a_L(t) = a_L(0) e^{-\left(\frac{\Gamma_L}{2\hbar} + \frac{im_L}{\hbar}\right)t}$$

t=0: pure beam of K^0. Given that: $\left|K^0\right\rangle = \sqrt{\frac{1}{2}}\left(\left|K_S^0\right\rangle + \left|K_L^0\right\rangle\right) \Rightarrow a_L(0) = a_S(0) = \frac{1}{\sqrt{2}}$ ← CP conservation

At time t (in natural units):

$$\Gamma\left(\left|K^0\right\rangle(t)\right) = \frac{\left(a_S(t) + a_L(t)\right)}{\sqrt{2}} \cdot \frac{\left(a_S^*(t) + a_L^*(t)\right)}{\sqrt{2}} = \frac{1}{4}\left\{e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t} \cos \Delta m t\right\}$$

$-$ for $\overline{K}^0$ $\quad (\Delta m = |m_L - m_S|)$

$$\Gamma\left(\left|K^0\right\rangle\right) - \Gamma\left(\left|\overline{K}^0\right\rangle\right) = e^{-[(\Gamma_S+\Gamma_L)/2]t} \cos \Delta m t \qquad \Gamma\left(\left|K^0\right\rangle\right) + \Gamma\left(\left|\overline{K}^0\right\rangle\right) = \frac{1}{2}\left(e^{-\Gamma_S t} + e^{-\Gamma_L t}\right)$$

The K^0 - $\overline{K}^0$ oscillation frequency is Δm