Final exam of Particle Physics
Monday February 6$^{\text{th}}$ 2023

Duration: 3 hours
8 printed pages

Allowed material: PDG booklet, simple calculator.
Solve on two separate sheets exercise I-II and exercises III-IV.

Approximate duration per exercise:

Ex. I:	20 min.	Ex. II:	70 min.
Ex. III:	20 min.	Ex. IV:	70 min.

Exercise I

Short questions on the lectures and general understanding

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. Explain the role of the BEH mechanism in the Standard Model: why is it introduced (the issues to solve and the way they are solved)? Do we have any experimental proof that this mathematical conception is actually realised?

2. Using the relations in the appendix, show that the spinor u transforms by the parity operator into $u' = \gamma^0 u$, and that $\bar{u}' = u^\dagger$. Explain.

3. In the context of QCD and hadron collisions, define and explain what is an infrared-safe observable. Give an example and a counter-example of such an observable, along with a short explanation.

4. An observable that drew much attention from particle physicists recently is the ratio

$$R_K = \frac{\Gamma(B^+ \to K^+\mu^+\mu^-)}{\Gamma(B^+ \to K^+e^+e^-)}\,,$$

measured in intervals of the invariant mass of the dilepton ($\mu^+\mu^-$ or e^+e^-) system. What is the approximate expected value for this observable in the standard model? Use qualitative arguments to explain your answer. Why is this observable interesting to measure?

Exercise II

Z production at e^+e^- colliders

In this exercise, we consider the interaction $e^-e^+ \to f\bar{f}$, where f is a fermion, at LEP or SLC. LEP and SLC were e^-e^+ colliders, SLC had in addition the capability of polarizing the electron beam. We consider the energy $\sqrt{s} = 91.2$ GeV and as a consequence **we neglect the EM interaction**.

$$\begin{array}{ll} e^2 = 4\pi\alpha & \alpha = 1/128 \\ s_w^2 \equiv \sin^2\theta_w = 0.23 & m_Z = 91.2\text{ GeV} \end{array} \tag{1}$$

The couplings of the Z boson to fermions are given by:

$$g_X^f = i\frac{e}{\cos\theta_w \sin\theta_w} \times \left(I_{f3}^{X} - Q_f \sin^2\theta_w\right)\,, \tag{2}$$

with $X \in [L, R]$. The Feynman rules are given in the appendix. We will note P_X the projector on the chirality X.

1. *Z boson partial and total widths*

   (a) What are the values of I_{f3}^{X}, Q_f, for the following particles: $\nu_{\tau X}$, μ^-_X, c_X, b_X for $X \in [L, R]$.

   (b) Draw the Feynman diagram for the decay $Z \to f\bar{f}$ and the corresponding matrix element. We note p and p' the 4-momenta of f and $\bar{f}$, respectively.

   (c) In the SM, what are the values of $\Gamma(Z \to f_L\bar{f}_L)$ and $\Gamma(Z \to f_R\bar{f}_R)$? Justify.

   (d) We quote $\Gamma_L^f \equiv \Gamma(Z \to f_L\bar{f}_R)$ and $\Gamma_R^f \equiv \Gamma(Z \to f_R\bar{f}_L)$. We remind you that for a two-body decay:
   $$\frac{d\Gamma}{d\Omega}(Z \to f\bar{f}) = \frac{p_f^*}{32\,\pi^2\,m_Z^2}|\mathcal{M}(Z \to f\bar{f})|^2\,. \tag{3}$$
   We quote $\mathcal{A}_X = \bar{u}(p)\gamma^\mu\, P_X\, v(p')\epsilon_\mu(Z)$ for $X \in [L, R]$ and we remind you that $|\mathcal{A}_L|^2 = |\mathcal{A}_R|^2 = 2m_Z^2$. Compute the value of Γ_L^f and Γ_R^f as a function of m_Z, g_X^f (justify any approximation if need be).

   (e) Compute the numerical value of $\Gamma(Z \to \nu_e\bar{\nu}_e)$. Quid of the other neutrino families? Compare this result to the PDG value: $\Gamma(Z \to \text{invisible}) = 499.0 \pm 1.5$ MeV.

   (f) We define the fermion left-right asymmetry as:
   $$A_f = \frac{\Gamma_L^f - \Gamma_R^f}{\Gamma_L^f + \Gamma_R^f}\,. \tag{4}$$
   Define A_f as a function for g_x^f. Explicit your result as a function of e, θ_W, $I_3^f \equiv I_{f3}^{L}$ and Q_f.

   (g) Find the numerical value of A_e.

2. *Z boson production*

The SLC collider had a polarized electron beam. The polarization is defined as:

$$\mathcal{P}_e = \frac{N^+_{e^-} - N^-_{e^-}}{N^+_{e^-} + N^-_{e^-}} \ , \tag{5}$$

with $N^+_{e^-}$ ($N^-_{e^-}$) the number of electrons with a positive (negative) helicity.

(a) Draw the Feynman diagram for the process $e^- e^+ \to Z^* \to f\bar{f}$. Write the corresponding matrix element as a function g^e_X and g^f_Y.

(b) Define a potential connection between helicity and the chirality for fermions at play in this reaction.

(c) We note

- $d\sigma^f{}_{LL} \equiv d\sigma(e^-_L e^+_R \to f_L \bar{f}_R)$, $\quad d\sigma^f{}_{LR} \equiv d\sigma(e^-_L e^+_R \to f_R \bar{f}_L)$,
- $d\sigma^f{}_{RL} \equiv d\sigma(e^-_R e^+_L \to f_L \bar{f}_R)$, $\quad d\sigma^f{}_{RR} \equiv d\sigma(e^-_R e^+_L \to f_R \bar{f}_L)$.

What is the value of $d\sigma^f{}_L \equiv d\sigma(e^-_L e^+ \to f\bar{f})$ as a function for the aforementioned $d\sigma^f{}_{XY}$? Same question for $d\sigma^f{}_R \equiv d\sigma(e^-_R e^+ \to f\bar{f})$.

(d) With a polarized beam, the cross section is given by:

$$\frac{d\sigma_f}{d\cos\theta} \equiv \frac{d\sigma(e^-e^+ \to f\bar{f})}{d\cos\theta} = \frac{3}{8}\,\sigma^0_f\ \left[(1+\cos^2\theta)(1-\mathcal{P}_e A_e)\ + 2\,\cos\theta(A_e - \mathcal{P}_e)A_f\right], \tag{6}$$

with A_e, A_f defined in Eq. (4), and σ^0_f the total cross section for fermion f in the case of an unpolarized beam, $\mathcal{P}_e = 0$.
Which value of $\mathcal{P}_e$ would you need to measure σ^f_R (justify without any computation)? Same question for σ^f_L.

(e) Demonstrate that the total cross section σ_f from Eq. 6 is not sensitive to A_f.

(f) Propose a method that would allow to measure A_f (even if $\mathcal{P}_e = 0$).

(g) Compute the total cross sections $\sigma^{\pm}_{\text{tot}}$ for $\mathcal{P}_e = \pm|\mathcal{P}_e|$.

(h) Propose an asymmetry to determine A_e and compute this asymmetry as a function of A_e and $|\mathcal{P}_e|$.

(i) The number of Z bosons collected with $\mathcal{P}_e = +|\mathcal{P}_e|$ (resp. $\mathcal{P}_e = -|\mathcal{P}_e|$) was 234748 (resp. 291775), while the average polarisation was $|\mathcal{P}_e| = 71.5$ %. What is the numerical value of A_e?

(j) The LEP collider produced about 18 million Z bosons. Despite having a sample 36 times larger than SLC, the precision on A_e is identical for both colliders. Find the main reason (hint: you can think about the size of the sample used in the method you proposed in question 2.f).

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Exercice III

Allowed and forbidden processes, Feynman diagrams

For each of the processes below, determine whether it is allowed or forbidden in the standard model. For the forbidden processes, explain why they are forbidden, giving *all the possible reasons* (here we do not require to take into account multiplicative quantum numbers and angular momentum). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Give all the relevant arguments you find to justify the interaction, and, when applicable, name the topology of the Feynman diagram. Note on the diagram the names of all real and virtual particles. When relevant, indicate near the vertex the CKM matrix elements that contribute and give their orders of magnitude in terms of $\lambda = \sin\theta_c$ (θ_c is the Cabibbo angle). Then, give the total order of magnitude of the diagram in terms of λ. In the case of Penguin or box diagrams, do this for the dominant intermediate quarks. In general, tree processes are favoured compared to penguin or box processes. We will thus try to privilege tree diagrams when several topologies are possible. Also, if possible, we will try to privilege colour allowed to colour suppressed diagrams.

1. $\Omega^- \to \Xi^-\pi^0$
2. $\Omega^- \to \Lambda\pi^-$
3. $e^+e^- \to p\overline{p}n$
4. $B^- \to \tau^-\tau^+\tau^-\nu_\tau$
5. $\pi^+ n \to \Lambda_b^0 B^+$
6. $B_s^0 \to \overline{K}^0\tau^+\tau^-$

Exercice IV

B meson decays into $K_S^0K^+K^-$

This exercise will study a few aspects of B meson decays into three kaons. We will look in particular into the mode $B^0 \to K_S^0K^+K^-$, with special attention to the resonant mode $B^0 \to \phi K_S^0$ ($\phi \to K^+K^-$), where the K^+K^- pair originates from the decay of a $\phi(1020)$ meson, denoted ϕ.

Parts 1-4 below are independent.

1. Kinematics

   We consider the decay of a particle of mass M into three particles of masses m_1, m_2 and m_3, with momentum-energy 4-vectors P_1, P_2 and P_3, respectively. The spins of all these particles are 0. The dynamics of such processes is often represented in a Dalitz plot: the plane of square invariant masses m_{12}^2, m_{23}^2, where $m_{ij}^2 = (P_i + P_j)^2$. A generic Dalitz plot is shown in Figure 1-left, and the distribution of $B^0 \to K_S^0K^+K^-$ over the Dalitz plot, as observed by the BABAR experiment, is shown in Figure 1-right.

   (a) Show that, in the considered three-body decay, two degrees of freedom are needed to describe the final state.

   (b) Show that the global maximum of m_{23} is $M - m_1$. What is the corresponding kinematic configuration of the three final-state particles?

   (c) Show that the global minimum of m_{23} is $m_2 + m_3$. What is the corresponding kinematic configuration of the three final-state particles?

   (d) In Figure 1-right, there is a cluster of events on the left of the Dalitz-plot. Suggest an explanation for this.

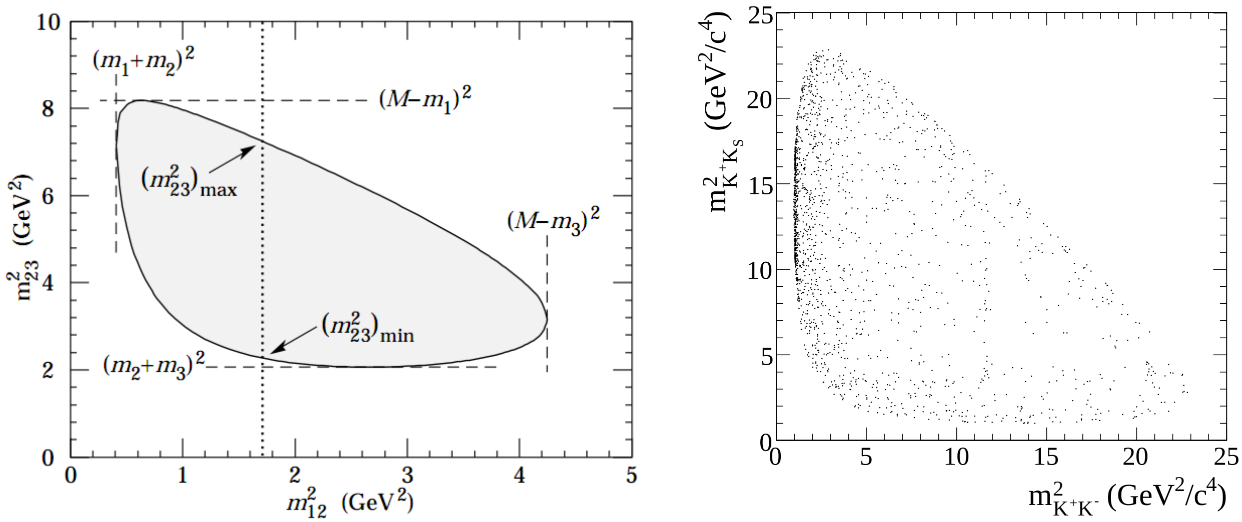


Figure 1: (Left) A Dalitz plot describing a decay of a particle of mass M into three particles of masses m_1, m_2 and m_3. The shaded area shows the kinematically-allowed region. (Right) Dalitz-plot distribution of $B^0 \to K_S^0 K^+ K^-$ decays, obtained by the BABAR experiment [PRD 85, 112010 (2012)].

2. Study of the ϕ-meson decays

   The ϕ meson is a pure $s\bar{s}$ state, decaying predominantly into either K^+K^- or $K^0\overline{K}^0$ via strong interaction.

   (a) Briefly explain why the quark content of the ϕ meson is not compatible with the hypothesis that SU(3)-flavour is an exact symmetry.

   (b) Using the PDG, give the two branching fractions of $\phi \to K^+K^-$ and $\phi \to K^0\overline{K}^0$ (given as $\phi \to K_S^0 K_L^0$). Compare them to the branching fraction of the decay $\phi \to \pi^+\pi^-$. What is the explanation for the difference? What is the role of isospin in this difference?

3. Angular distribution of the decay products in the process $\phi \to K^+K^-$

   We define the z axis as the flight axis of the ϕ meson in the laboratory frame, and the z' axis as the flight direction of the K^+ in the centre of mass of the ϕ meson. The angle between the two axes is denoted θ.

   (a) Obtain the angular distribution of the decay products as a function of θ, when the spin of the ϕ meson is aligned with the z axis.

   (b) Using a simple argument, justify the values obtained at $\theta = 0$, $\theta = 180°$.

4. CP violation in $B^0 \to \phi K_S^0$ (with $\phi \to K^+K^-$) decays

   The time-dependent CP asymmetry is written

$$A_{CP}(\Delta t) = \frac{\Gamma(\overline{B}^0 \to \phi K_S^0) - \Gamma(B^0 \to \phi K_S^0)}{\Gamma(\overline{B}^0 \to \phi K_S^0) + \Gamma(B^0 \to \phi K_S^0)} = S \sin(\Delta m_d \Delta t) - C \cos(\Delta m_d \Delta t)\,,$$

   with

$$S = \frac{2\mathfrak{Im}\lambda^{CP}}{1+|\lambda^{CP}|^2} \quad , \quad C = \frac{1-|\lambda^{CP}|^2}{1+|\lambda^{CP}|^2} \quad , \quad \lambda^{CP} = \frac{q}{p}\frac{\bar{A}_{\phi K_S^0}}{A_{\phi K_S^0}}\,.$$

(a) Remind what is the physical meaning of the parameters Δm_d, q and p. What are $A_{\phi K^0_S}$ and $\bar{A}_{\phi K^0_S}$?

(b) Which types of CP violation can be studied using the decay $B^0 \to \phi K^0_S$? Explain.

We will now, step by step, study the expression of λ^{CP}. The Wolfenstein parameterisations of the CKM matrix at $\mathcal{O}(\lambda^3)$ and $\mathcal{O}(\lambda^5)$ (where λ is the sine of the Cabibbo angle) are given in the appendix, as well as the expression of the matrix in term of angles of unitarity triangles (CKM angles) at $\mathcal{O}(\lambda^4)$.

(c) briefly justify the fact that $\frac{q}{p} = \frac{V^*_{tb}V_{td}}{V_{tb}V^*_{td}}$, and obtain its expression at $\mathcal{O}(\lambda^4)$ as a function of the CKM angle β.

The ratio of amplitudes may be written as

$$\frac{\bar{A}_{\phi K^0_S}}{A_{\phi K^0_S}} = \eta^{CP}_{\phi K^0_S} \frac{\bar{A}_{\phi \bar{K}^0}}{A_{\phi K^0}} \frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}} \tag{7}$$

(d) Obtain $\eta^{CP}_{\phi K^0_S}$, the CP eigenvalue of the final state ϕK^0_S in the decay under scrutiny, using the approximation that the K^0_S is a CP eigenstate with $CP = 1$.

(e) Besides $\eta^{CP}_{\phi K^0_S}$, explain the factorisation of the rest of the expression of Eq (7) in two parts, and the source of the term $\frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}}$. Drawing a Feynman diagram may be useful.

(f) Obtain this term at $\mathcal{O}(\lambda^4)$.

(g) Draw the Feynman diagrams (gluonic penguins) of $B^0 \to \phi K^0$ and $\overline{B}^0 \to \phi \overline{K}^0$.

(h) Use these diagrams to obtain the term $\frac{\bar{A}_{\phi \bar{K}^0}}{A_{\phi K^0}}$ of Eq. (7), at $\mathcal{O}(\lambda^4)$ as a function of one of the CKM angles. Justify.

(i) Finally, obtain the expressions of λ^{CP}, S and C.

(j) Which CKM angles are measured by studying the time-dependent CP asymmetry in $B^0 \to \phi K^0_S$ decays? Explain.

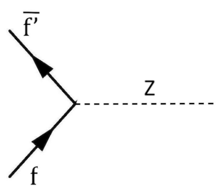
(k) This mode is known to provide a clean measurement of the CKM angle β. Comment on this statement, also in light of your answer to the last question.

(l) The time-dependent CP asymmetry may be used to probe the existence of particles and interactions beyond the standard model (new physics). Explain qualitatively why and how.

Appendix

QED Feynman rules and couplings of the Z and Higgs bosons. s denotes the (anti-)fermion spin and λ the photon helicities. P_X are the chirality projectors ($P_L = \frac{1-\gamma^5}{2}$, $P_R = \frac{1+\gamma^5}{2}$).

Incoming fermion:		$\rightarrow$	$u\ (\vec{p}, s)$
Incoming antifermion:		$\rightarrow$	$\overline{v}\ (\vec{p}, s)$
Outgoing fermion:		$\rightarrow$	$\overline{u}\ (\vec{p}, s)$
Outgoing antifermion:		$\rightarrow$	$v\ (p, s)$
Incoming/Outgoing scalar :		$\rightarrow$	**1**
Incoming vector boson :	μ	$\rightarrow$	$\epsilon_\mu(\vec{k}, \lambda)$
Outgoing vector boson :	μ	$\rightarrow$	$\epsilon_\mu(\vec{k}, \lambda)^*$



$\rightarrow \quad i\frac{e}{\cos\theta_w \sin\theta_w} \times \left(I_3^{f_X} - Q_f \sin^2\theta_w\right) \times P_X\gamma^\mu$

Useful relations regarding the γ matrices, spinors and the Dirac equation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$
$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$
$$\{\gamma^\mu, \gamma^5\} = 0$$
$$\not{p} = \gamma^\mu p_\mu$$
$$\bar{u} = u^\dagger \gamma^0$$
$$(\not{p} - m)u = 0$$
$$\bar{u}(\not{p} - m) = 0$$

The Wolfenstein parameterisation of the CKM matrix at orders λ^3 and λ^5

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$V_{\text{CKM}} = V_{\text{CKM}}^{\text{W3}} + \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5\left(1-2\left(\rho+i\eta\right)\right) & -\frac{1}{8}\lambda^4\left(1+4A^2\right) & 0 \\ \frac{1}{2}A\lambda^5(\rho+i\eta) & \frac{1}{2}A\lambda^4\left(1-2\left(\rho+i\eta\right)\right) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$