Corrections final exam of Particle Physics

Exercise I

Short questions on the lectures and general understanding

1. Explain the role of the BEH mechanism in the Standard Model: why is it introduced (the issues to solve and the way they are solved)? Do we have any experimental proof that this mathematical conception is actually realised?

   The BEH mechanism is named after Brout Englert Higgs. It was introduced in the SM by Weinberg to solve two different issues:

   - In order have a gauge theory of the EWK interaction, the lagrangian needs be invariant under the $SU(2)_L$ global symmetry which is impossible for massive fermions (the mass term of a particle mixes the chirality: $m\overline{\psi}\psi = m\overline{\psi_L}\psi_R + m\overline{\psi_R}\psi_L$). This issue is solved by introducing Yukawa couplings between fermions and the Higgs field.
   - Promoting $SU(2)_L$ as a gauge symmetry implies that EWK gauge bosons should be massless in contradiction the experimental evidence that EWK interaction was short distance. This is solved by the BEH mechanism (and the "BEH potential").

   To solve both this issues, the EWK symmetry $SU(2)_L U(1)_Y$ needs to be broken. After the break, we have massive gauge bosons, massive fermions and the remaining gauge symmetry is $U(1)_E M$. This mechanism introduces a new particle which is called the Higgs boson. The Higgs boson was discovered at the LHC in 2012 and is the experimental evidence that this is indeed the BEH mechanism which is at play to break the EWK symmetry.

2. Using the relations in the appendix, show that the spinor u transforms by the parity operator into $u' = \gamma^0 u$, and that $\bar{u}' = u^\dagger$. Explain.

   We can explicitly write the Dirac equation in terms of the energy and the momentum of the particle, multiply it on the left by γ^0, and move it to the left of the spinor u, according to the anti-commutation rule.

$$
\begin{aligned}
&(\gamma^0 E - \gamma^1 p_x - \gamma^2 p_y - \gamma^3 p_z - m)u = 0 \\
&((\gamma^0)^2 E - \gamma^0\gamma^1 p_x - \gamma^0\gamma^2 p_y - \gamma^0\gamma^3 p_z - \gamma^0 m)u = 0 \\
&(\gamma^0 E + \gamma^1 p_x + \gamma^2 p_y + \gamma^3 p_z - m)\gamma^0 u = 0\,.
\end{aligned}
$$

   We obtained the parity-transformed Dirac equation, $(\not{p}' - m)u' = 0$ (still valid as it involves only kinematics), where $\not{p}'$ is obtained from the parity-transformed energy momentum 4-vector $p'^\mu \Leftrightarrow (E, -\vec{p})$. We then conclude

$$u' = \gamma^0 u\,.$$

   Following the definition of $\bar{u}$

$$\bar{u}' = u'^\dagger\gamma^0 = (\gamma^0 u)^\dagger\gamma^0 = u^\dagger\gamma^0\gamma^0 = u^\dagger\,.$$

3. In the context of QCD and hadron collisions, define and explain what is an infrared-safe observable. Give an example and a counter-example of such an observable, along with a short explanation.

In QCD, the cross-section of *soft* and *colinear* gluon is divergent (A soft particle is a particle with small momentum). An infrared-safe observable, depending on several objects (tracks and neutral particles) in a detector, is, by definition, not altered if two colinear objects are grouped, or if a soft particle is absorbed into another particle. In other words, for a physical event, infrared safety means that the actual event gives approximately the same result as when the hadrons in a jet are combined to make a few parton jets. This may also be written, for an observable O_n depending on n objects, where x_j is either soft, or colinear with x_i, as:

$$O_n(x_1, x_2, \cdots, x_i, x_j, \cdots, x_n) = O_{n-1}(x_1, x_2, \cdots, x_i + x_j, \cdots, x_n)\,.$$

Such observables may be computed in a reliable way using perturbative-QCD methods, as infrared infinities cancel. Examples and counter-examples that we mentioned in the course are inclusive and non-inclusive cross-sections (e.g. the cross-section of hadron production in e^+e^- collision is inclusive). We also gave the example of jets. A jet which is naively defined by all the hadrons included in a cone around some thrust axis is not infrared-safe. On the contrary, there exist several iterative infrared-safe jet algorithms that cluster objects following some criteria until a condition is met. We gave examples of the top-down cone algorithm with seed, the inclusive k_T algorithm, and the Anti-k_T algorithm

4. An observable that drew much attention from particle physicists recently is the ratio

$$R_K = \frac{\Gamma(B^+ \to K^+\mu^+\mu^-)}{\Gamma(B^+ \to K^+e^+e^-)}\,,$$

measured in intervals of the invariant mass of the dilepton ($\mu^+\mu^-$ or e^+e^-) system. What is the approximate expected value for this observable in the standard model? Use qualitative arguments to explain your answer. Why is this observable interesting to measure?

The standard-model Feynman diagrams for these two modes (electroweak penguin or box topologies) are identical apart from the lepton flavour. The CKM matrix elements at play are the same as in both modes, as the two transitions are $b \to s$, with a dominant virtual top quark. The couplings of the Z and W bosons to the three lepton families are the same (lepton universality). The matrix elements of the two modes are thus very similar, with only a small difference due to the lepton masses. Phase space is also affected by these masses. In both modes, they are expected to have a small influence on the amplitude, because of the large mass of the B^+ meson and available phase space. The standard-model prediction for the ratio R_K is thus very close to 1, up to small and well-understood corrections. The ratio is interesting to measure because if the result significantly differs from unity, this means that new physics (new particles and interactions), which unlike the standard model does not have the feature of lepton universality, could be at play. It is, in general, better to compare the theoretical predictions and experimental measurements of ratios rather than those of absolute rates. From the theoretical point of view, in the ratio, some ill-known ingredients (e.g. QCD corrections) may simplify, and from the experimental point of view, some systematic effects affect in the same way the numerator and the denominator and therefore have less influence.

Exercise II

Z production at e^+e^- colliders

1. *Z boson partial width*

   (a) ${I_f}_3^R = 0$ for all type of fermions. Q_f is the electric charge and I_{3f}^L the isospni: $+\frac{1}{2}$ for ν, c; $-\frac{1}{2}$ for μ, b

   (b) Cf course. Note that the Z boson in this case is an external particle and it is associated to ϵ_μ. The matrix element is therefore:

   $$i\mathcal{M}(Z \to f\bar{f})_X = g_X^f \epsilon_\mu \bar{u}(p)\gamma^m u P_X v(p') = g_X^f \mathcal{A}_X \tag{1}$$

   (c) In the SM; $\Gamma(Z \to f_R \bar{f}_R) = \Gamma(Z \to f_L \bar{f}_L) = 0$ because the Z boson couples only L-fermion and R-antifermion (as well as R-fermion and L-antifermion) due to its vectorial nature.

   (d) In the case of a Z-boson decay, all the fermions can be considered massless. Therefore the momentum of f in the center of mass of the Z boson is given by $p_f^* = m_Z/2$. In addition we need to average over the helicities of the Z boson, hence there is a factor 1/3 in front of the matrix element. For each chirality, the partial width is given by:

   $$\begin{aligned}\frac{d\Gamma_X}{d\Omega} &= N_c^f \frac{p_f^*}{32\pi^2 m_Z^2}|g_X^f|^2 \times \frac{1}{3} \times |\mathcal{A}_X|^2 \\ &= N_c^f \frac{1}{3}\frac{1}{64\pi^2 m_Z}|g_X^f|^2 \times 2\, m_Z^2 \\ &= N_c^f \frac{1}{3}\frac{m_Z}{32\pi^2}\frac{e^2}{c_w^2 s_w^2} \times ({I_f}_3^X - Q_f s_w^2)^2 \\ &= N_c^f \frac{1}{3}\frac{m_Z}{8\pi}\frac{\alpha}{c_w^2 s_w^2} \times ({I_f}_3^X - Q_f s_w^2)^2\end{aligned} \tag{2}$$

   Integrating over Ω:

   $$\Gamma_X^f = N_c^f \frac{m_z}{6}\frac{\alpha}{c_w^2 s_w^2} \times ({I_f}_3^X - Q_f s_w^2)^2, \tag{3}$$

   with N_c^f the number of colors for fermion f (3 for quarks and zero for leptons).

   (e) For neutrinos, we therefore obtain:

   $$\Gamma(Z \to \nu_e \bar{\nu}_e) = \Gamma_L^{\nu_e} = \frac{m_z}{6}\frac{\alpha}{c_w^2 s_w^2} \times \left(\frac{1}{2}\right)^2$$

   $\Gamma(Z \to \nu_e \bar{\nu}_e) = 167.6$ MeV. This is the same value for the other neutrino family. $\Gamma(Z \to \text{invisible})$ represents to total width to *not detectable* particles, in the SM this only neutrinos, therefore in the SM: $\Gamma(Z \to \text{invisible}) = 3\Gamma(Z \to \nu_\mathrm{e}\bar{\nu}_2) = 502.3$ MeV in good agreement with the experimental value given in the PDG.

   (f) The LR asymmetry is defined by:

   $$\begin{aligned}A_f &= \frac{\Gamma_L^f - \Gamma_R^f}{\Gamma_L^f + \Gamma_R^f} \\ &= \frac{(g_L^f)^2 - (g_R)^2}{(g_L^f)^2 + (g_R)^2} \\ &= \frac{\left(I_3^f - Q_f s_w^2\right)^2 - Q_f^2 s_w^4}{\left(I_3^f - Q_f s_w^2\right)^2 + Q_f^2 s_w^4}\end{aligned} \tag{4}$$

(g) For the electron $I_3^f = -1/2$ and $Q_e = -1$,

$$A_e = 0.159$$

2. *Z boson production*

(a) The matrix element corresponding to the Z production is given by:

$$i\mathcal{M}_{XY} = g_X^e \, g_Y^f \quad \bar{v}(k_{e^+})\gamma^\mu P_X u(k_e^-) \; \frac{i\eta_{\mu\nu}}{q^2 - m_Z^2 + i\Gamma_Z m_Z} \; \bar{u}(p_f)\gamma^\nu P_Y v(p_{\bar{f}}) \tag{5}$$

(b) The electron and positron masses, as well as any fermion f from the Z decay can be considered massless w/r to the Z-boson mass. Therefore we can consider that the helicity is identical to the chirality. This means that left-particle have helicity $-1/2$. Similarly right particles have helicity=$+1/2$.

(c) In this case, the positron beam has a random polarisation so we need to average over the 2 e^+ helicities. Therefore

$$\begin{aligned} d\sigma_L^f &= \frac{1}{2}\left(d\sigma_{LL}^f + d\sigma_{LR}^f\right) \\ d\sigma_R^f &= \frac{1}{2}\left(d\sigma_{RR}^f + d\sigma_{RL}^f\right) \end{aligned} \tag{6}$$

(d) In order to measure σ_R^f, we need all electrons to be right, so they all have an helicity $+1/2$, this can be obtained with $\mathcal{P}_e = +1$. Similarly σ_L^f can be measured with $\mathcal{P}_e = -1$.

(e) When integrating Eq. 6 over $d\cos\theta$, we obtain:

$$\begin{aligned} \sigma_f &= \frac{3}{8}\sigma_f^0 \left\{ \int_{-1}^{1} (1+\cos^2\theta) d\cos\theta (1+\mathcal{P}_e A_e) \int_{-1}^{1} (\cos\theta) d\cos\theta (A_e - \mathcal{P}_e) A_f \right\} \\ &= \sigma_f^0 \, (1+\mathcal{P}_e A_e) \end{aligned} \tag{7}$$

which is independent of A_f. Therefore A_f can not be accessed via the total cross-section.

(f) In order to assess A_f we need the term in $\cos\theta$ to no vanish. So it can be measured by integrating for $\cos\theta > 0$ (Forward cross section) and for $\cos\theta < 0$ (Backward cross section) and then comparing the two. This methodology was used at LEP to measure, it is called the Forward-Backward asymmetry. One can also think of measuring the cross section as a function $\cos\theta$ and fitting the observed distribution.

(g) $\sigma_{\text{tot}}^+ = \sum_f \sigma_f^+$, with σ_f the cross section for fermion f measured with a polarization $\mathcal{P}_e = +\mathcal{P}_e$. Therefore:

$$\sigma_{\text{tot}}^{\pm} = \left(\sum_f \sigma_f^0\right) \times (1 + \pm \mathcal{P}_e Ae) \tag{8}$$

(h) Therefore we can compute the asymmetry $A_{+-} \equiv \frac{\sigma_{\text{tot}}^+ - \sigma_{\text{tot}}^-}{\sigma_{\text{tot}}^+ + \sigma_{\text{tot}}^-}$.

$$A_{+-} \equiv \frac{\sigma_{\text{tot}}^+ - \sigma_{\text{tot}}^-}{\sigma_{\text{tot}}^+ + \sigma_{\text{tot}}^-} = |\mathcal{P}_e| \; A_e \tag{9}$$

Note that this measurement requires a non null polarization (the larger the polarization, the better).

(i) $\sigma_{\text{tot}}^{\pm}$ are directly proportional to the number of Z boson collected in each polarisation. Thus, we obtain:

$$A_e = \frac{1}{0.715} \; \frac{291775 - 234748}{291775 + 234748} = 0.151$$

(j) The LEP collider was had a 36 times larger sample, yet to measure A_e, it had to use the process $e^+e^- \to Z^* \to e^+e^-$ while SLC was able to use the total cross section (without looking for a distinct final state). Therefore the LEP measurement total sample for this measurement was only $36 \times \mathcal{B}(Z \to e^+e^-)$ larger. Since $\mathcal{B}(Z \to e^+e^-) = 3.3$ %, thus the number of Z bosons available for this measurement was about the same for the 2 colliders.